Mechanical Unit Operations
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Lecture 5
Size Analysis by Screening

Welcome to the MOOCS course Mechanical Unit Operations, the title of this lecture is Size Analysis by Screening. We have seen in the previous lecture how to do the screening, what are the other basic concepts about the screening. What we have screen is a kind of you know metal pan with forming bottom as screens, whatever the screens are there, they form a bottom of a metal pan and then those metal pans having screens at the bottom can be arranged one onto the other.

So we arrange them in such a sequence that you know at the bottom most finest screen would be there or the screen with the smallest opening would be at the bottom and the next smallest would be on the top of that one like that gradually we can arrange the screens one on the other and then at the top will be having a screen which is having a opening largest compared to the all other screens.

So then whatever the crushed material is there that will be keeping on the top screen and then we will be mechanically agitating it or vibrating it for the 20 minutes or 30 minutes. Then based on the size of the material the particles may be passing through some of these screens and then retaining on some of the screen, so based on the size each screen may be having some amount of the material being collected on individual screen, right?

And then these screens are arranged in such a way that in this particular way we name them as a screen increment. Screen increment numbering we do, at the bottom one is will be giving i is equal to 1 then i is equals to 2 is the next above it and like that the topmost one would be given the i the largest number of the whatever the total number of screens we have, let us say we have a 20 screens in the state then i would be 20 for the top one like that, right?

Then whatever the material collected individual screens will be weighed or taken out weighed and then the mass is reported are the mass is converted into the mass fraction by dividing the mass of the total entire sample that that has been taken as a feed. Then, the average size we have assigned a kind of average size for the material that is being retained on each individual screen, so let us say on screen 2 whatever the material is retained, the average size of the particles of that particular sample retain and the screen increment of second screen, so that would be the size opening, the average size would be the arithmetic average of the screen aperture of screen 2 plus screen aperture of screen 3/2 like this, increments we give from bottom to top so then accordingly we have to do.

So now in this process that whatever the average diameter state that I can study mass fraction of the material or mass of the material retained on that particular screen we tabulate them. This is how we do the screen analysis and then tabulate the experimental observation. So the important thing here what we see this screen opening, right? Screen opening is a kind of a screen aperture is a kind of very much important factor. So we should have a kind of some kind of standard screens which can be used with a kind of proper reference.

So are there any standard screens or not? That first we see and then from there we go to the analysis, size analysis, finding out the different equivalent diameters for the samples and then surface area for the individual fraction and then specific surface for the entire sample, etc. Those kind of things we do, so in order to do this one, we need to know this screen opening, so for that we have to see what are the types of standard screens are available.

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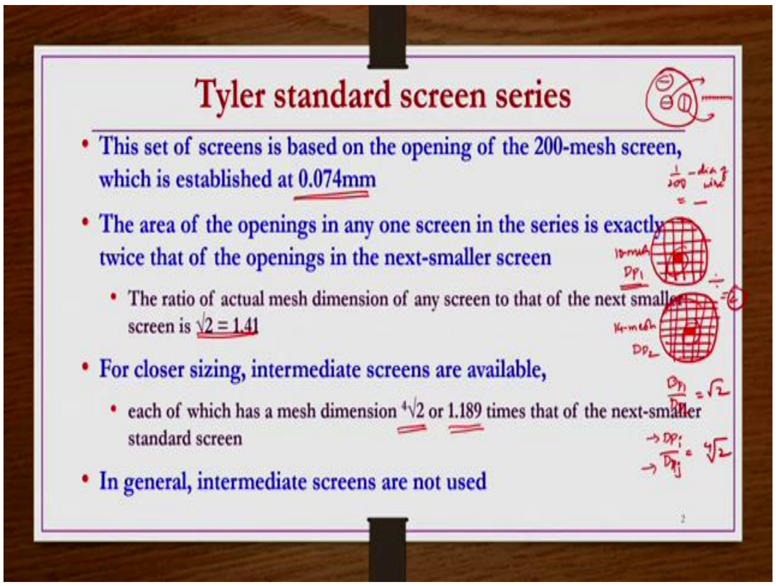

So Tyler screen series is the kind of one of the standard one, right? It has been developed or design based on the opening of the 200 mesh screen which is established at 0.074 mm. So 200 mesh screen in the sense, let us say this is the screen that you have, so out of which let us say this particular dimension is you know having one linear inch dimension, so with one linear inch dimension will be having 200 openings like this.

So then that particular screen is known as a kind of a 200 mesh. Like that you can take anywhere linear dimension one inch that also if you see that will be same, right? In any one direction. Like that any location if you see. So per linear inch dimension of that particular screen there will be 200 opening, so the diameter of that particular opening screen aperture is 1/200 minus dia of the wire that has been used for this construction. So whatever that number is there that will be coming in some kind of inches, that number if you convert into the mm that is nothing but 0.074 mm. That has been taken as a kind of standard one and then based on that one other screens have been designed.

How they have other screens have been designed? So let us say next the mesh number is larger that means the aperture is the smaller. So see largest mesh number will have the finest opening. So next opening whatever is next to smaller opening whatever you have like that you have to construct, so there should be a kind of proper ratio should be maintained. So in this particular screen what has been done, let us say this is one screen, so we have seen the D screens are a kind of you know, they constructed apertures, they are in this square size, right?

So like that you can take the next smaller one, so here also now you have this screen opening, let us say like this, right? Let us say this is 10 mesh and then let us say this is 14 mesh. So then whatever this the area one particular square here, in this 10 mesh that you take this area and then you take a area of another I mean anyone square aperture of 14 mesh, right? If you take this area ratio, the area of this particular aperture divided by this area of this particular aperture, if you see that will always be equal to 2. The area ratio will always be equal to 2 for this, this is how this measures have been constructed, that is the reason after 10 mesh you will not be having 11 mesh or 13 mesh, you will be having the 14 mesh so that we have a kind of standard series can be designed.

So this is how these screens are designed, right? So this is the area ratio if you take the dimension ratio, that is the deep this whatever the let us say Dp_1 here for 10 mesh the screen aperture opening is Dp_1 and then 14 mesh, this screen aperture is opening is Dp_2. So then Dp_1 by Dp_2 should be what? It should be $\sqrt{2}$, because the area ratio is 2 so then the dimension ratio should be $\sqrt{2}$

$$\frac{D_{p_1}}{D_{p_2}} = \sqrt{2}$$

because they are in the square shape. So this is how the screens are constructed.

Sometimes you may also need to have a kind of intermediate screens between these two. So there are some kind of intermediate screens are also designed so that you know whatever the ratio is there, let us say intermediate screen between Dp_i and then Dp_j is let us say you have the intermediate screen so then that linear dimension would be having $\sqrt[4]{2}$ and the ratio between those two linear dimension, the regular one divided by the intermediate one that value will always be $\sqrt[4]{2}$.

$$\frac{D_{p_i}}{D_{p_j}} = \sqrt[4]{2}$$

So this is how it in this style a standard screen screens have been designed.

So the area of the openings in any one screen in the series is exactly twice that of the openings in the next smaller screen that is the area ratio is 2 or otherwise the ratio of actual mesh dimension of any screen to that of the next smallest screen would be square root of 2 or 1.41, okay. For closure sizing intermediate screens are also available, right? So for which the each of which has a kind of mesh dimension of $\sqrt[4]{2}$ that is 1.18 times that of the next smaller standard

screen. So this is the basis that Tyler standard screens have been designed, right? So but in general intermediate screens are not used they are used only for specified applications wherever the requirement is there but in general they are not used. So this is what the Tyler standards screen series.

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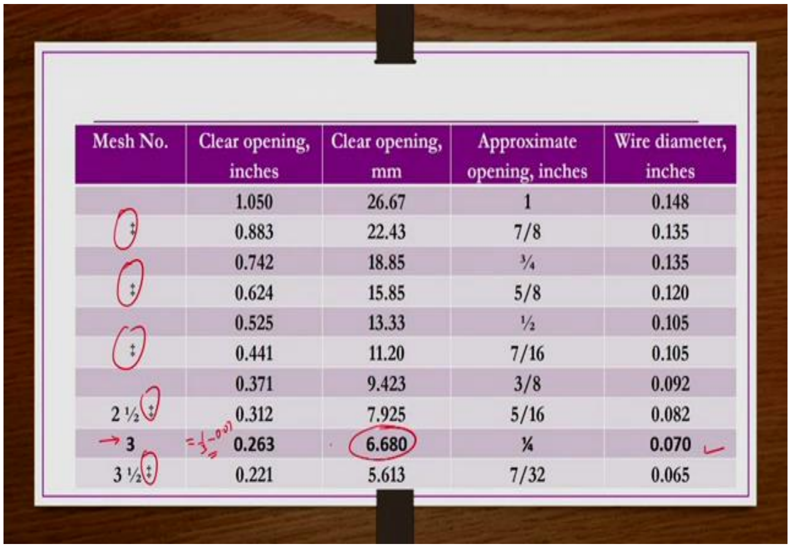

Mesh No.	Clear opening, inches	Clear opening, mm	Approximate opening, inches	Wire diameter, inches
	1.050	26.67	1	0.148
‡	0.883	22.43	7/8	0.135
	0.742	18.85	¾	0.135
‡	0.624	15.85	5/8	0.120
	0.525	13.33	½	0.105
‡	0.441	11.20	7/16	0.105
	0.371	9.423	3/8	0.092
2 ½‡	0.312	7.925	5/16	0.082
3	0.263	6.680	¼	0.070
3 ½‡	0.221	5.613	7/32	0.065

Now we see different mesh number, what is the mesh opening, what is the diameter of the wire that has been used in a kind of tabular column. So whatever this numbers I mean like these symbols are there they indicate a kind of intermediate screen. So let us not go for them immediately. Let us say you have mesh 3, so this opening should be 1/3 because per linear inch 3 aperture so will be there, 3 openings would be there, right. So that is written 1/3 minus the diameter used wire diameter that has been used for constructing these three mesh is 0.07. So that value is nothing but (1.333 - 0.07) if you do you will get 0.263, the same thing if you convert in mm you will get 6.68, right?

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Mesh No.	Clear opening, inches	Clear opening, mm	Approximate opening, inches	Wire diameter, inches
4	0.185	4.699	3/16	0.065
5‡	0.156	3.962	5/32	0.044
6	0.131	3.327	1/8	0.036
7‡	0.110	2.794	7/64	0.0328
8	0.093	2.362	3/32	0.032
9‡	0.078	1.981	5/64	0.033
10	0.065	1.651	1/16	0.035
12‡	0.055	1.397		0.028

Let us say the mesh 4 is there, so it is opening should be 1/4 minus diameter of the wire that has been used 0.065. So that is 1/4 is, how much? 0.25, (0.25 - 0.65) would be 0.185. So the clear opening of that inches is having the Dp_i for that 4 mesh would be 0.185, if the same thing if you convert in like you know, mm it is 4.699 mm. So similar is mesh 5 that is the regular one. Now if you take the ratio of this 4.699÷3.327 you get 1.41 that is in the ratio of $\sqrt{2}$.

Similarly, 4.699÷3.962 intermediate one if you do you will get 1.189, so that is how the standard screens are developed up like this. So all any two successive mesh numbers you take and then you check the ratio you will get the same thing these numbers only.

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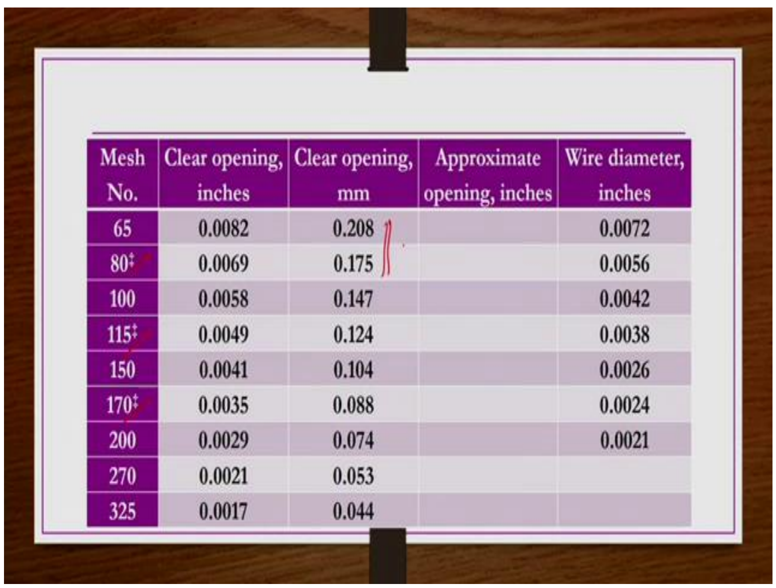

Mesh No.	Clear opening, inches	Clear opening, mm	Approximate opening, inches	Wire diameter, inches
65	0.0082	0.208		0.0072
80‡	0.0069	0.175		0.0056
100	0.0058	0.147		0.0042
115‡	0.0049	0.124		0.0038
150	0.0041	0.104		0.0026
170‡	0.0035	0.088		0.0024
200	0.0029	0.074		0.0021
270	0.0021	0.053		
325	0.0017	0.044		

Likewise all other mesh numbers like 65, 100, 150, 200 like that 270, 325 are there, these are the kind of intermediates one with the symbol given here. They indicate for the intermediate screen so that the ratio between these two let us say here that will be 1.189. Now we do the size analysis, this the information whatever the Tyler series is there that you know screen opening information that would be used as a Dp_i for each screen increment and then $\overline{D}_{pi}$ would be taken like Dp_i + Dp_i + 1/2 like that, we take the average ones, okay? So that information we need $\overline{D}_{pi}$ information is required for the size analysis that is the reason the information about this Tyler standard screens are important. So that is how we have we got them here and, right?

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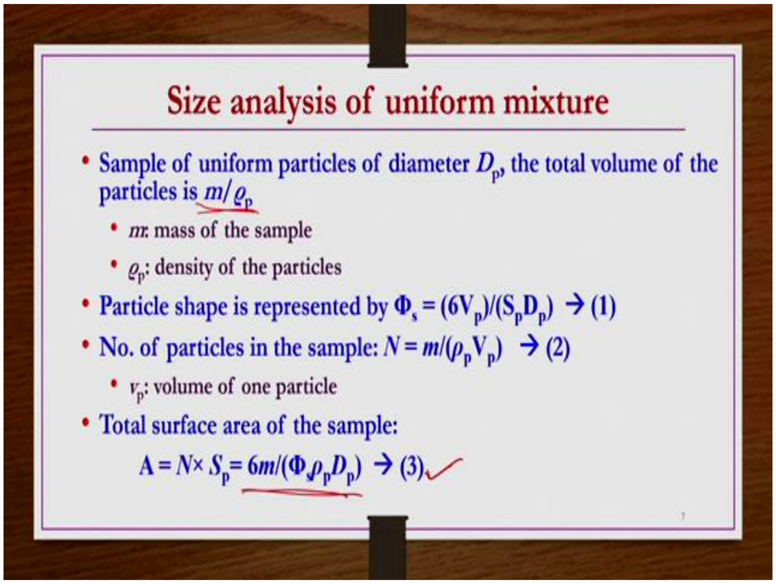

So now we see the size analysis of uniform mixtures. Uniform mixtures in the sense they may be any shape, irregular shape but their size is the same, size and density of those particle is same, right? So let us say you have a sample of uniform particles of diameter D_P then

$$total\ volume\ of\ particles = \frac{m}{\rho_P}$$

uniform mixture here, they have the uniform size D_P is same for all the particle. Let us say you have glass beads of spherical shape so the diameter of each and every glass bit would be almost same, if not exactly the same they will be almost enclose to each other.

So we can say that particular mixture is a kind of a uniform mixture having the constant or the same particle diameter. Let us say the glass beads all of them are having 1 mm, so they will definitely be 1 mm or 0.995 mm or 1.005 mm like that, they are in the 1 mm range within plus or minus 1 or 2 percent error. So the sample of uniform particles have diameter Dp so then the total volume of the particle should be $\frac{m}{\rho_P}$, m is the mass of the sample and then ρ_P is the density of the particles. Then if they let us say particle is not spherical shape, it is having some other shape then you need to know the sphericity of that particular particle shape also.

So for that reason we have this. $\emptyset_s = \frac{6V_p}{S_pD_p} \quad \longrightarrow (1)$

Now then if you wanted to know the total number of particles present in that particular uniform mixture, what you can do? You take the total volume of the sample $\frac{m}{\rho_P}$ that you already know divided by the volume of single particle, so then you will get the total number of particles. So that is number of particles in the sample $N = \frac{m}{\rho_P V_p} \quad \rightarrow (2)$, Vp is nothing but the volume of single particle, one single particle. So the total surface area of the sample would be n number of particles multiplied by the surface area of one single particle, since this mixture is a uniform mixture, you do not need to worry about the changes in the surface area from one particle to other particle because all the particles are having the same size same diameter. Then if you know the surface area of one particle that particular value you multiply by the total number of particles, you will get the $A = N \times S_p$.

So N is number of particles that is $\frac{m}{\rho_P V_p}$ and then Sp from this equation 1, what we get Sp is nothing but $\frac{6\,Vp}{\phi_s\ Dp}$ if you simplify you will get $\frac{6m}{\phi_s D_p \rho_p}$ as a kind of total surface area of the uniform mixture, whatever the uniform mixture that you have taken, so that means if you know the mass of the sample and then density of the particles and then size of the one single particle, then you can know the total surface area using this one without worrying the other kind of information like what is the volume, what is the surface area and then what is the difference from one particle to the other particle? You do not need to worry because of the uniformity of the mixture.

$$A = N \times S_p = \frac{6m}{\phi_s D_p \rho_p} \quad \rightarrow (3)$$

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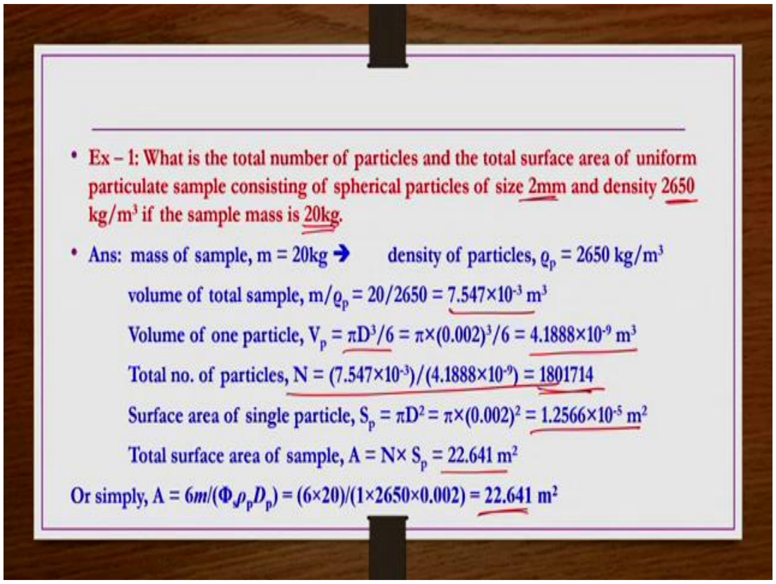

Then, if you have a kind of non-uniform mixture, how to do it? But before going to the non-uniform mixtures, we take a problem, example problem uniform mixture, right? Size analysis of a uniform mixture. So what we do? We calculate the total number of particles and then total surface area of uniform particulate sample consisting of spherical particles size 2 mm and density 2650 kg/m^3 and then mass of the sample is 20 kg. So what is the total number of particles? And then what is the total surface area of the sample?

Because it is a uniform sample so you do not need much information other than these 3, so all information is given sphericity is required but that for that is also equal to 1 for spherical particle. So simply we can get this information by using the equations. So what would be the volume of the total sample? So that would be $\frac{m}{\rho_p}$, m is given $\rho_p$is given so total volume would be this one.

$$m = 20kg, density\ of\ particle = 2650\ {}^{Kg}/_{m^3}$$

$$volume\ of\ total\ sample, \frac{m}{\rho_P} = \frac{20}{2650} = 7.547 * 10^{-3} m^3$$

Likewise, volume of one particle because it is a spherical particle the volume of one single spherical particle is $\frac{\pi\, D3}{6}$, D is the size of the particle.

$$volume\ of\ one\ particle, V_p = \frac{\pi D^3}{6} = {\pi * (0.002)^3}/{6} = 4.1888 * 10^{-9} m^3$$

So then you get volume of single particle like this.

If you divide the volume of total sample by volume of one single particle, you will get the total number of particles 1801714 you get here.

$$Total\ no\ of\ particles, N = {7.547 * 10^{-3}}/{4.1888 * 10^{-9}} = 1801714$$

Likewise, the surface area of single particle is π D^2 for spherical particles, hat is given this one number this number you get. Now the total surface area of the sample that you can get by the multiplying these two numbers that is total number of particles multiplied by the surface area of one single particle. When you do this, you get 22.641 m^2.

$$Surafce\ area\ of\ single\ particle, S_p = \pi D^2 = \pi * (0.002)^2 = 1.2566 * 10^{-5} m^2$$

$$total\ surface\ area, A = N * S_p = 22.641 m^2$$

Let us say, you are not asked to calculate the number of particles but directly only surface area of the sample if you need to calculate you can use this equation directly A is equals to $\frac{6M}{\phi_s D_p \rho_p}$ and then substitute these numbers, you will get the same number 22.641 m^2 in one single step, this we are doing because you also need to obtain the total number of particles.

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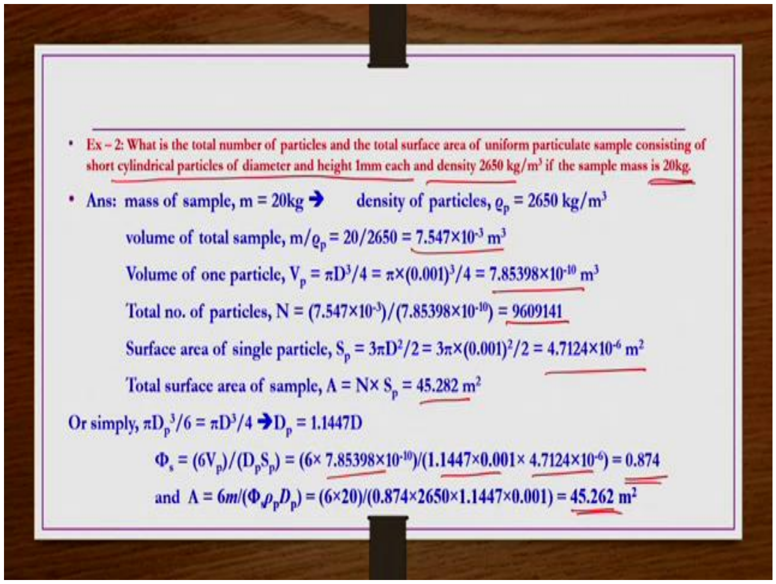

Now what we do, we take another example here we take a short cylindrical particles of diameter 1 mm each rather than spherical particles, okay and then density is say 2650 as the previous problem, but the mass is also is 20 kg as the previous problem. So only thing that we are changing the sample like now using a kind of short cylinders of $D = 1mm$ and $H = 1mm$ if $L = D$ then we can call those cylinders as a kind of short cylinders. Then mass of the sample, density of the sample is given so total volume of the sample would be $\frac{m}{\rho_p}$. So then you get this number, then volume of one particle would be $volume = \frac{\pi D^3}{4}$ for short cylinder so that you get this number.

So the total number of particles is the total volume or the volume of total sample divided by volume of one particle. So you get 9609141 particles, right? So surface area of single particle for short cylinder with L=D we have $\frac{3\pi D^2}{2}$ that comes out to be this one. Then total surface area of the particle will be the total number of particles multiplied by the surface area of single particle when you do this one. So similarly if you do not want to calculate the total number of particles but you want to calculate directly the total surface area, so then you can use the $A = \frac{m}{\phi_s D_p \rho_p}$ equation directly to get that information.

But ϕ_s is required that for short cylinders we have already calculated it as a kind of 0.874 how we get Dp we need to know. So $D_p = \frac{\pi D^3}{4}$ of a short cylinder equated to the volume of a spherical particle size Dp which is the same volume as the particle volume$\frac{\pi D^3}{4} = \frac{\pi {D_p}^3}{6}$ So from there $D_p = 1.1447D$

At the time of calculating sphericity we have also seen that the sphericity does not depend on the size of the particle. So without substituting their true volume or the true surface area, etc. We can directly substitute their formulas and get the results but however this will be a kind of demonstration that even if you substitute the true volume and surface area of the particles, you will get the $\emptyset_s = 0.874$ this shows that it is independent of the size anyway, right? Then you $A = \frac{6m}{\phi_s D_p \rho_p} = 45.262m^2$ anyway. So this is how the size analysis of uniform mixtures one has to do very simply without worrying too much.

$$m = 20kg, density\ of\ particle = 2650\ {}^{Kg}/_{m^3}$$

$$volume\ of\ total\ sample, \frac{m}{\rho_P} = \frac{20}{2650} = 7.547 * 10^{-3}m^3$$

$$volume\ of\ one\ particle, V_p = \frac{\pi D^3}{4} = {}^{\pi * (0.001)^3}/_{4} = 7.85398 * 10^{-10}m^3$$

$$Total\ no\ of\ particles, N = {}^{7.547 * 10^{-3}}/_{7.85398 * 10^{-10}} = 9609141$$

$$Surafce\ area\ of\ single\ particle, S_p = {}^{3\pi D^2}/_{2} = {}^{3\pi * (0.002)^2}/_{2} = 4.7124 * 10^{-6}m^2$$

$$total\ surface\ area, A = N * S_p = 45.282m^2$$

$${}^{\pi D_p^3}/_{6} = 4 = 1.1447D$$

$$\emptyset_s = \frac{6V_p}{S_p D_p} = \frac{\left(6 * 7.85398 * 10^{-10}\right)}{\left(1.1447 * 0.001 * 4.7124 * 10^{-6}\right)} = 0.874$$

$$A = \frac{6m}{\phi_s D_p \rho_p} = \frac{6*20}{0.0874*2650*1.1447*0.001} = 45.262m^2$$

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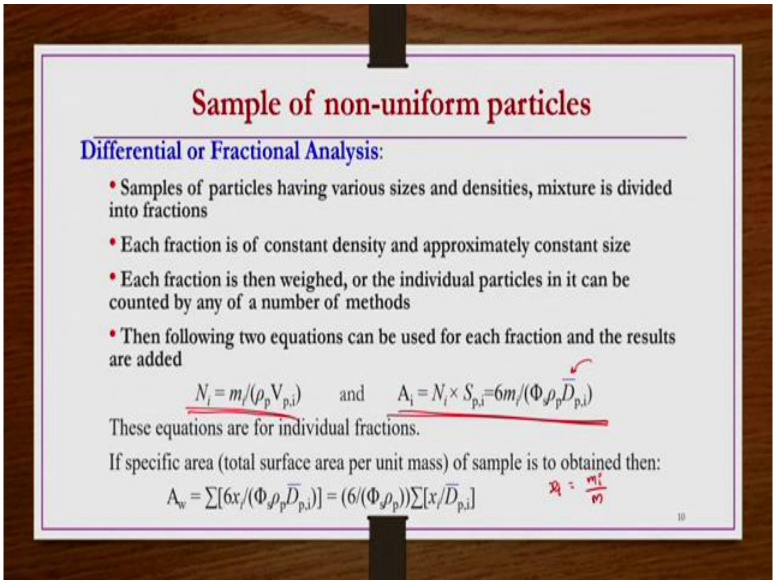

But coming to the sample of non-uniform particles, so the analysis could be slightly different or bit lengthier process than what we have done, okay. Uniform mixtures you know particles are same size so then you can do as a one single fraction, but here non-uniform mixture the particle size distribution may be very wide. So what you do you do the screening and then divide it into the in different-different fractions, each fraction maybe having the more uniform particle size distribution so that you can assign one average diameter.

So one average diameter would be assigned to one fraction, another average diameter would be assigned to the another fraction like that. You can assign different fractions, different average diameter and then for individual fractions you can use the principles of same uniform mixture size analysis of uniform mixture principles because individual fractions now they are treated as a kind of uniform, they are not uniform but the size range variation in size range is very narrow that you can assign one average diameter.

So then for individual fraction you can apply the principles of the size analysis of uniform mixtures like that you have to obtain the surface area of individual fraction of all fractions indeed and then add them together so then you get the surface area of the entire sample okay. If you divide by the mass of that entire sample then you will get this specific surface of the entire sample, that is how we have to do but here this non-uniform particles we have two approaches for reporting the results are obtaining the some kind of size analysis results.

One is the differential analysis, which is also known as the fractional analysis another one is the cumulative analysis. In the differential or fractional analysis, whatever the samples of particles having various sizes and densities, mixture is divided into fractions by doing this screening operations as we have discussed before. Then each fraction is of constant density and approximately constant size, not the constant exactly same size each particle will not have same size within each fraction, but approximately constant size, so $\overline{D_{pi}}$ can be assigned, right?

Each fraction is then weighed or the individual particles in it can be counted by a number of methods available, okay. Then following two equations can be used for each fraction and the results are added. What are this equation?

$$N = \frac{m}{\rho_P V_{pi}}$$

the same as exactly the uniform mixtures. There we do not how this subscript i, i indicates the screen increments, right? i can be 1 to 20 if you have the 20 screens, i can be 1 to 10 if you have 10 screen increments within this take where you have done the fractionation of the sample into individual fractions, right?

For ith screen increment number of particles would be the mass of the sample that has been retained on that particular screen divided by the density of one single particle that is retained and this particular sample. So in this $V_{p,i}$, we do not use like you know individual particle diameters of that one we take the average diameter that is $\overline{D_{pi}}$ for that increment that we take in order to get the volume of single particle. So similarly the surface area of the sample retained on particular screen increment i would be $A = N * S_p = \frac{6m_i}{\phi_s \overline{D_{pi}} \rho_p D}$. So now this here it is not D_p but $\overline{D_{pi}}$, the average diameter of you know sample that whatever that ith increment is having is I mean retained on that ith increment. Then these equations are a kind of purely for individual fractions, they are not for the entire non-uniform mixtures.

$$N_i = \frac{m_i}{\rho_P V_{p,i}} \text{ and } A_i = N_i * S_{p,i} = \frac{6m}{\phi_s \overline{D_{p.i}} \rho_p}$$

So basically non-uniform mixtures what we are doing? Non-uniform mixtures we are subdividing into several fractions but each fraction is almost a kind of uniform mixture that is it. And then for each fraction, we are applying the principles of uniform mixture and then we are adding them together in order to get the you know numbers required number for the entire non-uniform mixture.

So if specific area of sample is to be obtained for that non uniform mixture, then that you know the A_i should be divided by the whatever the total mass of the non-uniform mixture. So then that you do so $\frac{m_i}{M}$ let us say m is the mass of the total sample that should be kind of mass fraction and then for a given fraction individual fraction in general the sphericity may remain same and then particle density may also be same. So then those things may be separated out. If they are not same then they should be kept inside the summation, okay.

So individual fractions whatever are there A_i whatever they are all the A_i are added together and divided by the total mass of the sample, so that we get the specific area of the non-uniform mixture.

$$A_w = \Sigma\left[{}^{6x_i}\!\big/_{(\emptyset_s \rho_s \overline{D_{p.\iota}})}\right] = \left({}^{6}\!\big/_{(\emptyset_s \rho_s)}\right)\Sigma\left[{}^{x_i}\!\big/_{(\overline{D_{p.\iota}})}\right]$$

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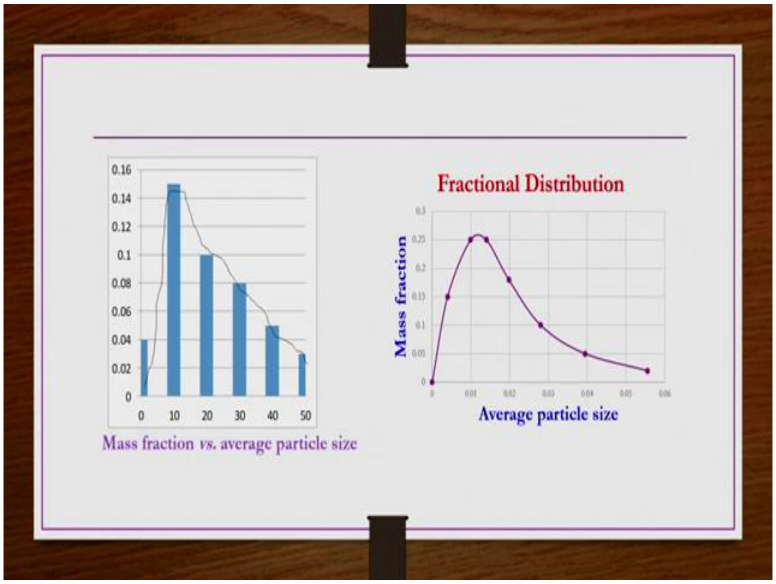

Then whatever the mass fraction versus average particle size the information you there you get that can be plotted like a histogram like this shown here or they can be plotted like a kind of graph as shown here. So you may get the distribution like this non-uniform mixtures.

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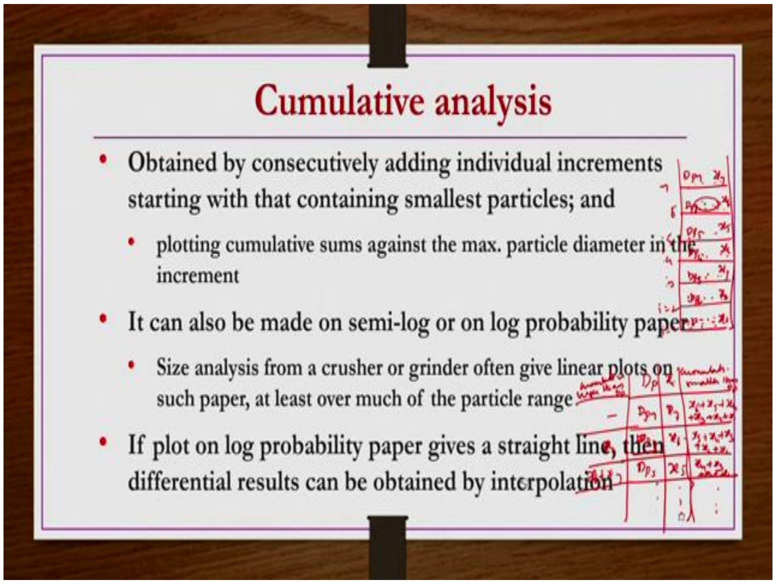

Then coming to the cumulative analysis here it is obtained by consecutively adding individual's increments starting with that containing smallest particles and plotting cumulative sums against the maximum particle diameter in the increment. What we have let us say, we have you know these many increments let us say this is i is equal to 1 for the pan, i is equal to 2 like this we have i is equal to 3, 4, 5, 6, 7. Now there would be some material retained on each of these things, screens including the top one also based on the screen analysis that we do and then based on the size of the you know nature of the sample and then you know other properties of the sample, okay?

On screen opening etc. So whatever the material that has been retained on this one you assign some fraction x_{ir} x_6 this is x_7 this is let us say x_5, x_4, x_3, x_2 and x_1 let us say, the screen opening here is Dp_7 for the seventh one, likewise Dp_6, Dp_5, Dp_4, Dp_3, Dp_2 and this is actually pan so it should be 0 opening. So now cumulative analysis what we do, for Dp_7 row the cumulative fraction x_7 what is the fraction of material that is you know, having the opening or that is having this size smaller than Dp_7, right? So this is cumulative analysis actually in two types, cumulative oversized, undersized, right?

Undersized will say let Dp_7 first we take so in the column here, so how do you find for against Dp_7? Cumulative fraction should be what so again x_2, x_7 the cumulative fraction smaller than Dp let us say, this is Dp so first one is Dp_7. So like this Dp_8 and then here you have this x_i, x_7, x_6, Dp_5, x_5 and so-and-so like this out. So now here what should be the number here? The cumulative fraction smaller than Dp_7. So this is Dp_7 having fraction x_7 and then Dp_6 and all of them Dp_5 are having the smaller opening because the coarsest one is at the top we arrange, we arrange screen such a way.

So now the whatever the fraction other than this column this entire thing whatever the things are there they are all you know having size smaller than Dp_7 Dp_6 Dp_5 Dp_4 and all that are having size smaller than Dp_7, Dp_7 is the largest one. So this fraction the value should be here x_5+x_5, x_6, x_4 + x_3 so that is $x_6+x_5+x_4+x_3+x_2+x_1$. Likewise here when you come to here we come to this point. What is the fraction of material which is having and the size smaller than Dp_6 Dp_5 Dp_4 Dp_3 Dp_2 Dp_1 all of them are having the size smaller than Dp 6. So then corresponding fraction should be added here cumulative that is $x_5+x_4+x_3+x_2+x_1$.

Likewise here it should be $x_4+x_3+x_2+x_1$ like this we have to add and then this cumulative fraction versus Dp_1 has to plot that will give the cumulative fraction or the cumulative analysis smaller than Dp, right? Like that we can do the exactly the other reverse one the cumulative fraction larger than D_p. If you do the larger than D_p here so this case it should be 0, this case would be x_7, this case would be $x_6 + x_7$ like this we have to do. Let us say cumulative fraction larger than Dp, we take example problem also so I am just explaining here so it will come like this, okay. It will be opposite to the other one.

So this is how we have to do the cumulative analysis. So plotting cumulative sums against in against the maximum particle diameter in the increment is a kind of gives the cumulative analysis. This cumulative analysis one can plot on a log-log or semi-log plot as well and then size analysis from a crusher or a grinder often give linear plots on a log-log or similar papers on log paper if you get a straight line by this cumulative analysis, then whatever the differential analysis results are there that you can obtain by just linear interpolation, you do not need to worry about the other details, right? So this is how we have to do the cumulative analysis, right?

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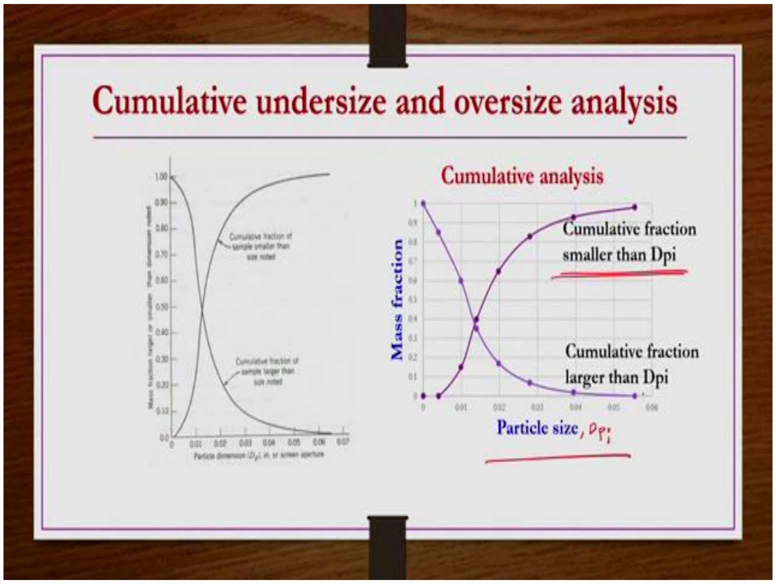

So cumulative undersized and oversized analysis if you see so this is whatever the way that mass fraction x_i versus particle size Dp if you plot or $D_{P,i}$ if you plot, you know the ones that have drawn here, this is what we have done cumulative fraction smaller than Dp A would be like this and then cumulative fraction larger than Dp A would be like this. So they are opposite to each other. This is you know, undersized cumulative undersized analysis, this is cumulative oversized analysis.

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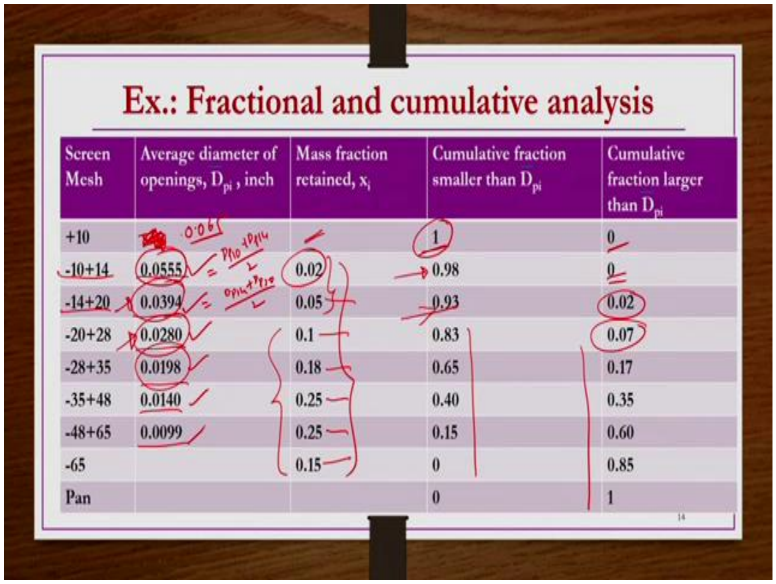

Now we take an example problem, so let us say fraction and cumulative analysis we take this example problem. We have the mesh screen +10, -10, +14,-14, -20 like this you know screen openings I mean mesh numbers are there that is what they arrange that is means at the bottom they have the pan above it with they have 65 mesh above it they have 48 mesh, about it they have a 35 mesh above it 28 mesh like that at the top 10 mesh ester, right? Whatever the material retained on 14 mesh that should be -10 +14.

So what do you mean by -14+20? The material that is passed through 14 mesh but retained on the 20 mesh. Like that their corresponding mass fractions are given, so 14 mesh, 10 mesh their Dp the screen openings from the Tyler screens we have already seen, right? If you take the average of those Dp's let us say what is the Dp of 10 mesh? Plus what is the Dp of 14 mesh divided by 2? Then you get this number 0.055 in inches, they are in inches. Like that other averages also like this.

So our for simplicity I write Dp 10 + Dp 14 / 2 is this number, this number is Dp 14+Dp 20 / 2 that is now screen opening of 14 mesh plus screen opening of 20 mesh divided by 2 arithmetic average. So all this average diameter$\overline{D_{pi}}$ are given, then mass fractions are given, right? So now cumulative fraction and smaller than Dp_i bar here see the average diameters are given, so we do in a kind of Dp_i bars but they should be in general done with respect to Dp, okay? So now first one, what is the cumulative fraction smaller than Dp_i? What is this actually this size would be more than 0.055.

So smaller fractions if you see, what is the fraction of material that is having size greater than 0.055 or smaller than 0.055? Not greater and smaller than 0.055 is a is like this, all these fractions so this all this is 0.0555 this thing, this thing all these things are smaller sizes. Their corresponding fractions or you know they should be added and then written here. Likewise now for this let us say we do the second one. What is the fraction of material which is having size less than 0.0555? This is you know, let us say I repeat once again.

Let us say the size opening, the average diameter not the average diameter for mesh 10 the screen opening is 0.065 inch. So what is the fraction of material whose size is smaller than 0.065? So all these material 0.0555, 0.0394, etc. All of them their sizes are smaller. So then corresponding fractions whatever are they should be added and then reported here, so 0.02 + 0.05 + 0.1 + 0.18 and so and so you will get 1 here. Similarly, in the second row here, what is the fraction of material whose size is smaller than 0.0555?

So this 0.0394, 0.028 and all of them are you know smaller than the 0.0555 so their corresponding fractions whatever are there so they should be added together and then written here, so that is 0.98. Likewise, the third row what is the fraction of material whose size is smaller than 0.0394? So all these 0.02, 0.018 all of them are having you know smaller size compared to the 0.0394. So these fractions whatever are they should be added together and written here, so that is 0.093.

Like that we have to construct all of them and then same in the similar way we have to do cumulative fraction larger than Dp_i here. So now 0.065 first row, what is the fraction of material whose size is less than 0.065? There is nothing so 0, see the fraction of material whose size is more than 0.065 is 0, so that is 0. What is the fraction of material whose size is more than 0.0555? So there is no fraction here so again this one has also 0. What is the fraction of material whose size is more than 0.0394? Is you know only 0.0555 is more than 0.034 so only this 0.02 should come here.

Then next one is, what is the fraction of material whose size is larger than 0.0280? So only two fractions are there 0.0394, 0.0555 are the larger one. Their corresponding fractions 0.02 plus 0.05 that should come here 0.07. Likewise we have to construct all of them, then you can plot them.

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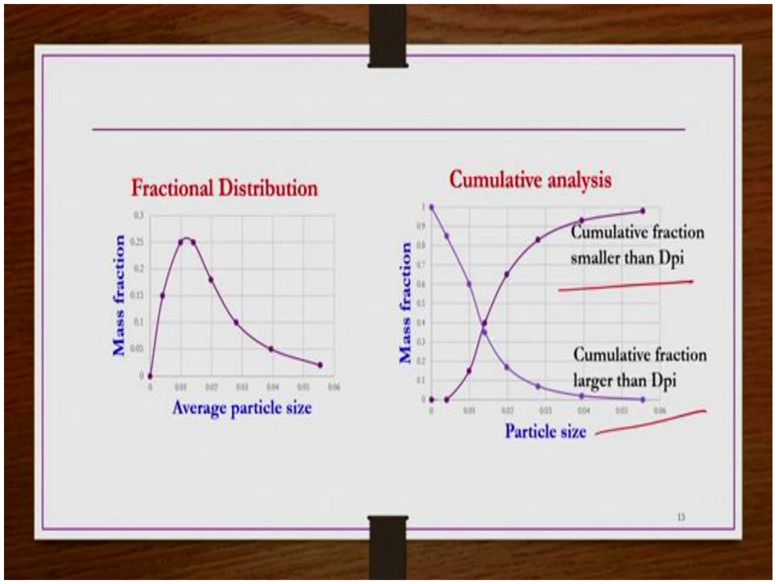
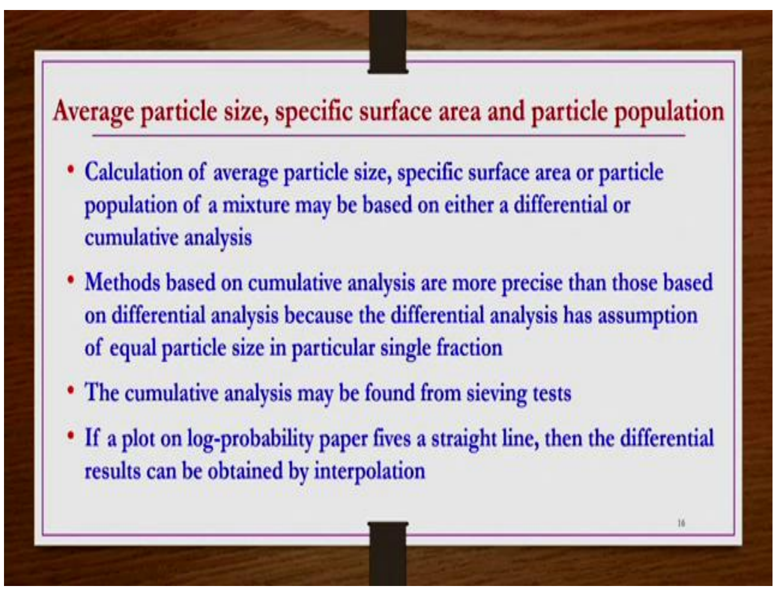


So when you plot the cumulative fraction smaller than Dp_i versus x_i, you will get plots like this. This is you know, cumulative fraction smaller than Dpi this is cumulative fraction larger than Dp_i. This fractional distribution this anyway we have already done. (Refer Slide Time: 38:51)



Now if you wanted to know the average particle size specific surface area and particle population, then we need to do some kind of calculations. So the calculation of average particle size, specific surface area or particle population that is number of particles may be based on error fractional or differential analysis or by the cumulative analysis. However, the methods based on the cumulative analysis are more precise than based on the differential analysis because the differential analysis has assumption of equal particle size in particular single fraction, within the single fraction whatever the material is there that we are assuming that you know, it is having a kind of uniform diameter.

But those kind of assumptions are not there in the cumulative analysis, that is the reason if you get the numbers using the cumulative analysis, it would be more precise. The cumulative analysis may be found from this sieving test as well and they can be plotted on the straight lines, okay.

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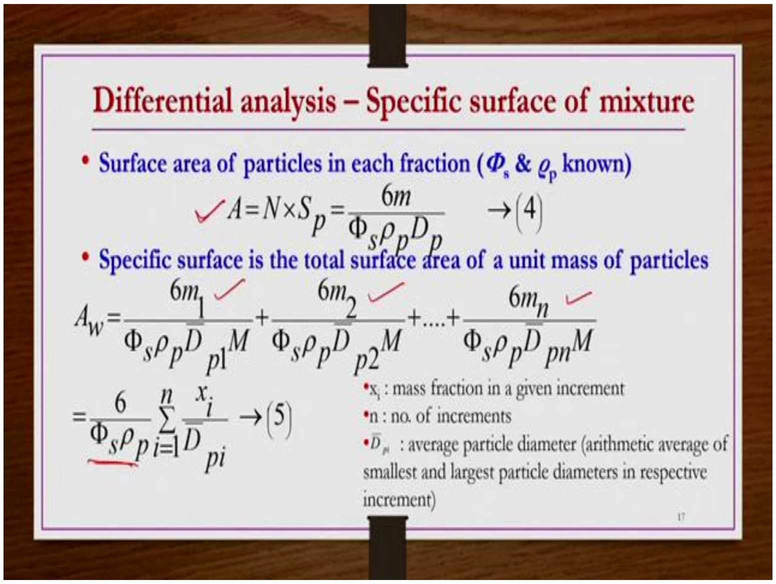

By differential analysis we do specific surface of non-uniform mixture calculations, how to do that one? So surface area of particles in each fraction, let us say $\varphi_s \rho_p$ are known, okay. Then the surface area of particles in that particular given fraction would be number of particles multiplied by the Sp that is the surface area of those particles. So then that comes out to be $A = N \times S_p =$

$\frac{6m}{\phi_s D_p \rho_p}$, it is same as a kind of uniform mixture but it is valid only for individual fraction. Now specific surface is the total surface area of a uniform of a unit mass of particle.

So specific surface is the total surface area of a sample per unit mass of the particles or whatever the sample that you have taken. So this A if you divide by m and then add them together for all the increments this is for one increment, like that you have a all increments, screen increment, let us say this is for screen increment 1, this is for screening increment 2, and so on so this is nth the screen increment. So if you add them together and then $\emptyset_s \rho_p$assume the same for the entire mixture, so $\frac{6m}{\phi_s \rho_p} \Sigma x_i / \overline{D_{pi}}$ because $\frac{m_1}{M} = x_1$ $\frac{m_2}{M} = x_2$ like that, you know all x_i's are there.

So $\frac{x_i}{\overline{D_{pi}}}$so here $\frac{x_1}{\overline{D_{p1}}} + \frac{x_2}{\overline{D_{p2}}+}$..., so we can write it as $\frac{\Sigma x_i}{\overline{D_{pi}}}$ .Sometimes this phi s rho p may not be same for a given fraction individual fractions so they should also be brought into their kind of summation. Let us say you have a kind of a non-uniform mixture, but you know different types of particles like spherical particle, short cylinder particle, cubicle particle like that then would be $\emptyset_s$ different for each type of particle, so then that should be taken inside the summation.

So this is how you can get the specific surface of a non-uniform mixture. That is the total surface area of the mixture divided by the mass of the sample. Here x_i is the mass fraction in a given increment, n is the number of increments how many increments that you have, how many screens that you arranged in the experiment 1, 2, 3, 4 up to 10 or 20 that is that number is indicated by the N, $\overline{D_{pi}}$ is the average particle diameter that is the arithmetic average of smallest and larger particle diameters in respect to increments.

$$A = N * S_p = \frac{6m}{\phi_s D_p \rho_p} \qquad \rightarrow (4)$$

Specific surface area is the total surface area of unit mass of particles

$$A_w = \frac{6m_1}{\phi_s \overline{D_{p_1}} \rho_p M} + \frac{6m_2}{\phi_s \overline{D_{p_2}} \rho_p M} + \cdots + \frac{6m_n}{\phi_s \overline{D_{pn}} \rho_p M} = \frac{6m}{\phi_s \rho_p} \Sigma \frac{x_i}{D_{p_i}} \qquad \rightarrow (5)$$

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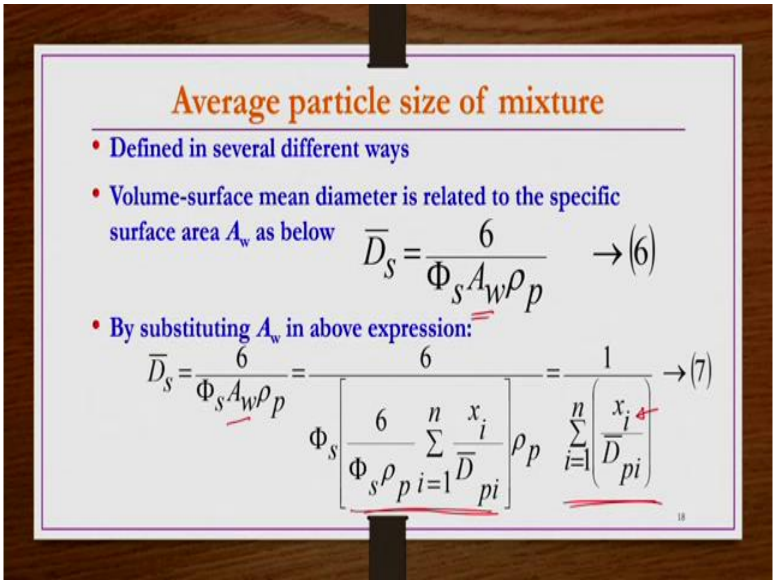

Then average particle size of mixture defined in several different ways actually, it is not possible to define them in one single way there are several types of ways available, okay and then all of them will give a different values, so selection of that which average value should be taken that again comes with the experience and then from process to process where the samples are going to be used.

The first one is volume surface mean diameter, it is related to the surface area Aw as given by $\overline{D_s} = \frac{6}{\phi_s A_w \rho_p}$, right? So here, you know Aw we have already calculated we have seen for a non-uniform mixture that you substitute here then this $A_w = \frac{6m}{\phi_s \rho_p} \Sigma \frac{x_i}{D_{p_i}}$ that just now we have seen you substitute here and then simplify. So you will get$\overline{D_s} == \frac{1}{\sum_{i=1}^{n}\left[\frac{x_i}{D_{p_i}}\right]}$ So this is the volume surface mean diameter. It is also known as the Sauter mean diameter. Sauter mean diameter is T one of the reliable one compared to the other one for most of the applications, however, not for all the applications. And then this average particle size of mixtures you calculate either if you know the mass fraction of the material retained on that particular increment or if you know the total number of particles in that particular increment. Let us say for each increment rather mass fraction, if you know the number of particles how to get this volume surface mean diameter or Sauter mean diameter that we will see.

So this is if you know what is the mass fraction of the material on each increment? In place of x_i if you know the N_i that is the total number of particles in that fraction then you have to use this equation.

By substituting A_w in above expression:

$$\overline{D_s} = \frac{6}{\phi_s A_w \rho_p} = \frac{6}{\phi_s \rho_p \left(\frac{6m}{\phi_s \rho_p} \Sigma \frac{x_i}{\overline{D}_{p_i}}\right)} = \frac{1}{\sum_{i=1}^{n} \left[\frac{x_i}{\overline{D}_{p_i}}\right]} \qquad \longrightarrow (7)$$

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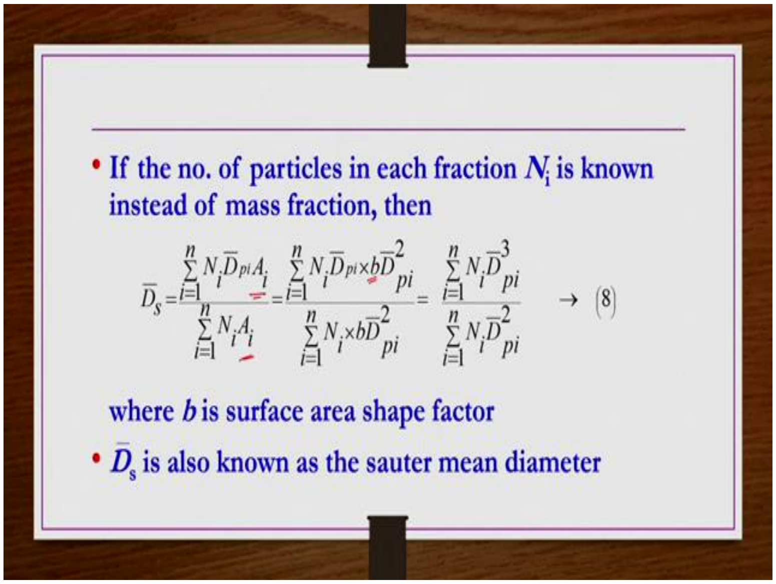

$\overline{D_s} = \frac{\sum_{i=1}^{n} N_i \overline{D}_{pi} A_i}{\sum_{i=1}^{n} N_i A_i}$, so here this Ai is nothing but the surface area is there so that particular thing is there. So usually it is you know $\overline{D}_{pi}^2$ multiplied by some surface area shape factor would be there. Let us say particles are spherical then this b is b the surface area shape factor, whatever is there that would be π.

Let us say if you have a short cylinders L is equals to then $L = D$ then this surface area shape factor $b = \frac{3\pi}{2}$. So that is the that particular multiplication factor of the Dp so that may be different and then for non-uniform mixtures you cannot have one single value. So but that is the reason we are assuming that let us say that constant is b and then it is same for all the particles within that particular mixture, so that is this one. So when you substitute here also $\overline{D_s} = \frac{\sum_{i=1}^{n} N_i \bar{D}_{pi} A_i}{\sum_{i=1}^{n} N_i A_i} = \frac{\sum_{i=1}^{n} N_i \bar{D}_{pi} * b\bar{D}_{pi}^2}{\sum_{i=1}^{n} N_i b\bar{D}_{pi}^2} = \frac{\sum_{i=1}^{n} N_i \bar{D}_{pi}^3}{\sum_{i=1}^{n} N_i \bar{D}_{pi}^2} \rightarrow (8)$ is nothing but the volume surface mean diameter $\overline{Ds}$ and this $\overline{Ds}$ is also known as the Sauter mean diameter.

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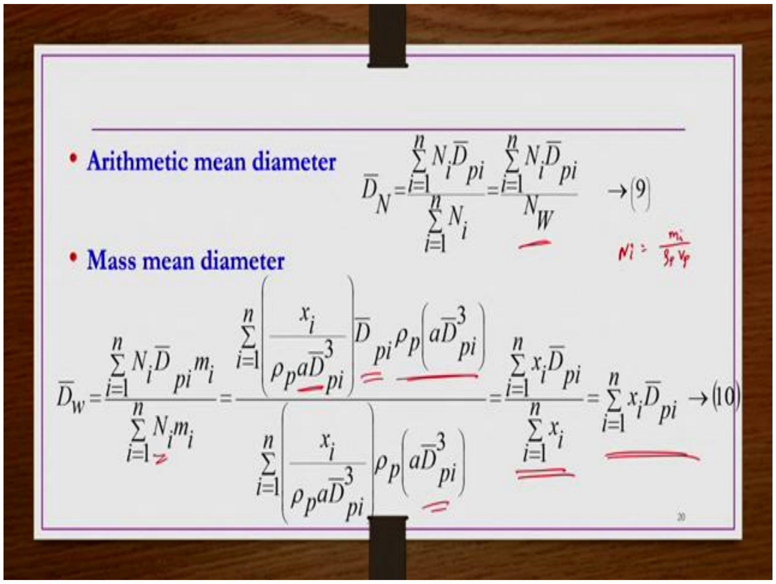

Arithmetic mean diameter$\bar{D}_N = \frac{\sum_{i=1}^{n} N_i \bar{D}_{pi}}{\sum_{i=1}^{n} N_i}$, so $\sum N_i = N_W$ that is the total number of particles in the sample $\bar{D}_N = \frac{\sum_{i=1}^{n} N_i \bar{D}_{pi}}{N_W} \rightarrow (9)$. Likewise mass mean diameter, you have$\bar{D}_w = \frac{\sum_{i=1}^{n} N_i \bar{D}_{pi} m_i}{\sum_{i=1}^{n} N_i m_i}$. So $N_i = \frac{x_i}{\rho_p V_p}$, actually this $N_i = \frac{m_i}{\rho_p V_p}$, is the $volume\ of\ the\ all\ particle = \frac{m_i}{\rho_p}$ present in that particular increment ith increment and then Vp is volume of one single particle of that particular I mean sample for that taken from that particular increment.

So here but we are doing in a kind of per unit mass of the sample, so if you divide by M so you get $\frac{x_i}{\rho_p V_p}$ and then $V_p = a\ \bar{D}_{pi}{}^3$. The average diameter of the sample that particular sample for that the given increment is $\bar{D}_{pi}$. So it should volume should be some constant multiplied by $\bar{D}_{pi}{}^3$that constant is a or the shape factor, volume shape factor a like b is surface area shape factor a is volume shape factor, okay. And then this $\bar{D}_{pi}$ as it is, mi is nothing but rho p into that volume of the particles whatever the single particle is there so that mi is rho p multiplied by the volume of the all the particles in the in that particular mixture is $a\,\overline{D}_{pi}^{\,3}$, okay.

Like that here also Ni substitute and then mi you substitute then you simplify you get $\sum_{i=1}^{n}\left[\frac{x_i\overline{D}_{p_i}}{x_i}\right]$and then$\sum x_i = 1$, for a given sample. So mass mean diameter $\overline{D}_w$ is nothing but

$$\overline{D}_w = \frac{\sum_{i=1}^{n} N_i \overline{D}_{pi} m_i}{\sum_{i=1}^{n} N_i m_i} = \frac{\sum_{i=1}^{n}\left(\frac{x_i}{\rho_p a\,\overline{D}_{pi}^{\,3}}\right)\overline{D}_{pi}\rho_p a\,\overline{D}_{pi}^{\,3}}{\sum_{i=1}^{n}\left(\frac{x_i}{\rho_p a\,\overline{D}_{pi}^{\,3}}\right)\rho_p a\,\overline{D}_{pi}^{\,3}} = \sum_{i=1}^{n}\left[\frac{x_i\overline{D}_{p_i}}{x_i}\right] = \sum_{i=1}^{n}\left[x_i\overline{D}_{pi}\right] \qquad \rightarrow (10).$$

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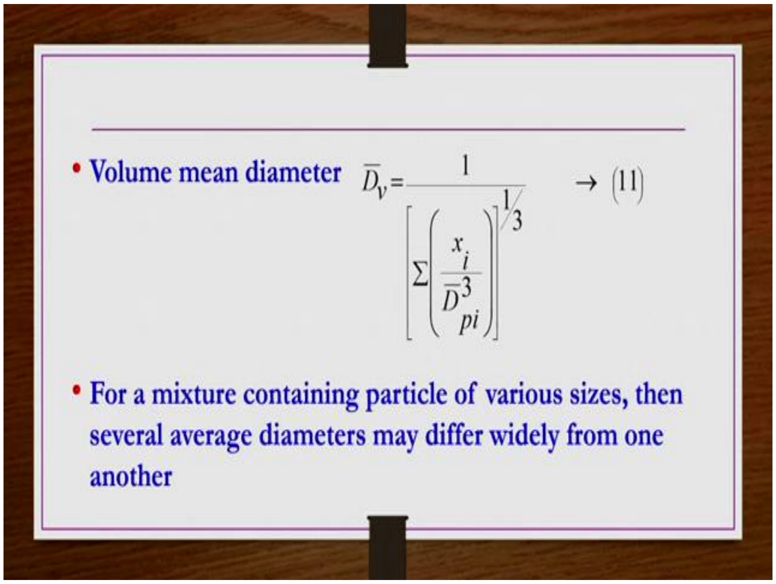

Like that we can have the value mean diameter $\overline{D}_v = \frac{1}{\left[\sum\left(\frac{x_i}{\overline{D}_{pi}^{\,3}}\right)\right]^{1/3}}$ So as I already mentioned for a given mixture containing particles of various sizes then there will be several average diameters and they may be different widely from one another. Let us say for a given sample you do the analysis and then you obtain the$\overline{D}_s$ you obtain the $\overline{D}_v$ and then you obtain the $\overline{D}_n$ and then you obtain the $\overline{D}_w$. So all four types of average diameter you calculate all four of them may be giving different number. So one should be careful based on the application which number should be selected which could be the most appropriate.

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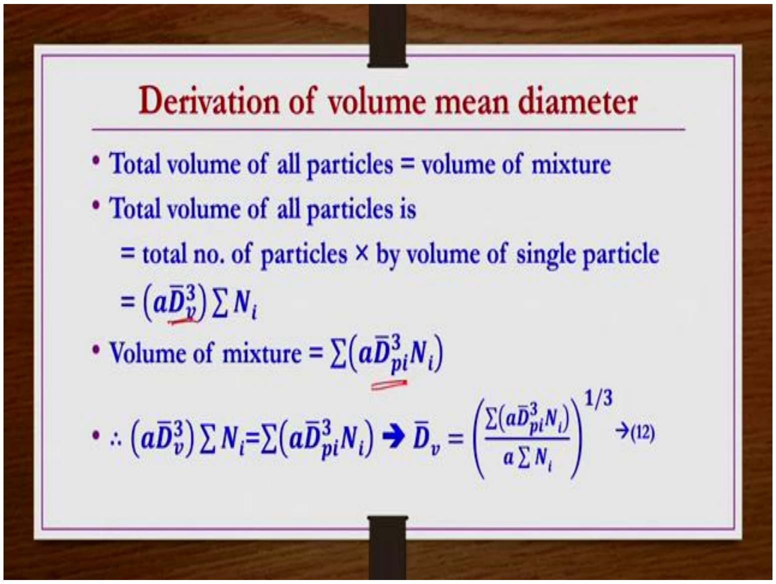

So derivation of volume mean diameter we can see because the $\bar{D}_s$, $\bar{D}_n$, $\bar{D}_w$ we have seen in a simple way, so the similar way it is difficult to do volume in diameter but the other way we can do. The total volume of all particles should be the volume of mixture and then total volume of all particles should be the total number of particles multiplied by volume of single particle but that volume calculated using this volume mean diameter$\bar{D}_v$. So that is $a\bar{D}_v{}^3\sum(N_i)$, $\sum(N_i)$is the number of total number of particles in the sample and then volume of single particle having the average diameter volume mean diameter $volume\ of\ mixture =$.

So $a\bar{D}_v{}^3\sum(N_i)$, volume of mixture so now should be $\sum\left(a\bar{D}_{pi}{}^3 N_i\right)$ so this is for each individual increment, you know the $\bar{D}_{pi}$, so for each increment what is the volume that you know, for each increment what is the number of particles Ni you know, you multiply them and then you do the summation for all increments, so that is what this one, this is$\bar{D}_{pi}$ is that you know average diameter for that increment, for that screen increment whereas $\bar{D}_v$is that the volume mean diameter for the entire sample, right?

So that means all the particles we are assuming that they are having a mean diameter, value mean diameter ofD_v, okay. Whereas coming to the mixture, we are assuming that the all the particles are having the you know average diameter of $\bar{D}_{pi}$for a given ith increment, okay. So when you equate these two now, you will get $\bar{D}_v$like this.

$$a\bar{D}_v{}^3 \sum (N_i) = \sum \left(a\bar{D}_{pi}{}^3 N_i\right) \to \bar{D}_v = \left(\frac{\sum \left(a\bar{D}_{pi}{}^3 N_i\right)}{a\sum N_i}\right)^{1/3} \quad \to (12)$$

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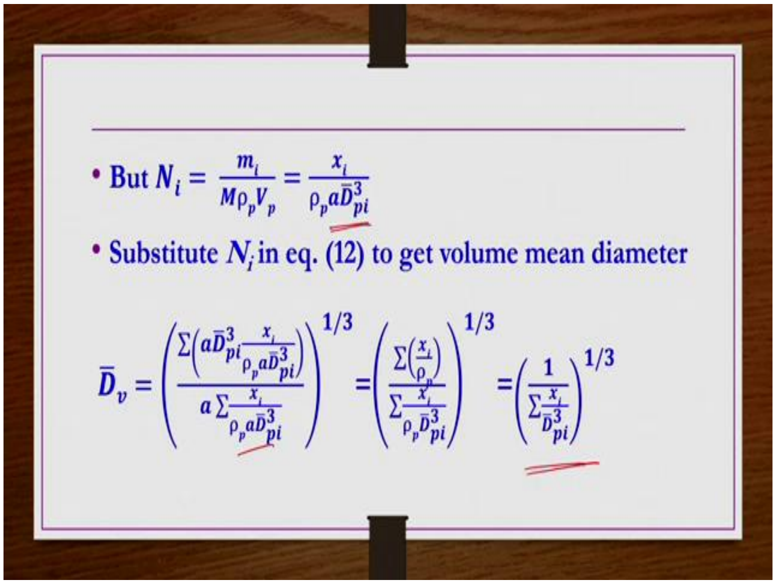

And then here you substitute $N_i = \frac{x_i}{\rho_P V_p}$. Then $\rho_P$is as it is V_p is nothing but for given increment ith screen increment $a\bar{D}_{pi}{}^3$ right? So that you substitute in the above equation here N_i wherever this N_i and then this is $\sum N_i$ and then do the simplification. You will get

$$\bar{D}_v = \left(\frac{\Sigma\left(a\bar{D}_{pi}{}^3 \frac{x_i}{\rho_P a\,\bar{D}_{pi}{}^3}\right)}{a\,\Sigma\frac{x_i}{\rho_P a\,\bar{D}_{pi}{}^3}}\right)^{1/3} = \left(\frac{\Sigma\left(\frac{x_i}{\rho_P}\right)}{\Sigma\frac{x_i}{\rho_P\,\bar{D}_{pi}{}^3}}\right)^{1/3} = \left(\frac{1}{\Sigma\frac{x_i}{\bar{D}_{pi}{}^3}}\right)^{1/3}.$$

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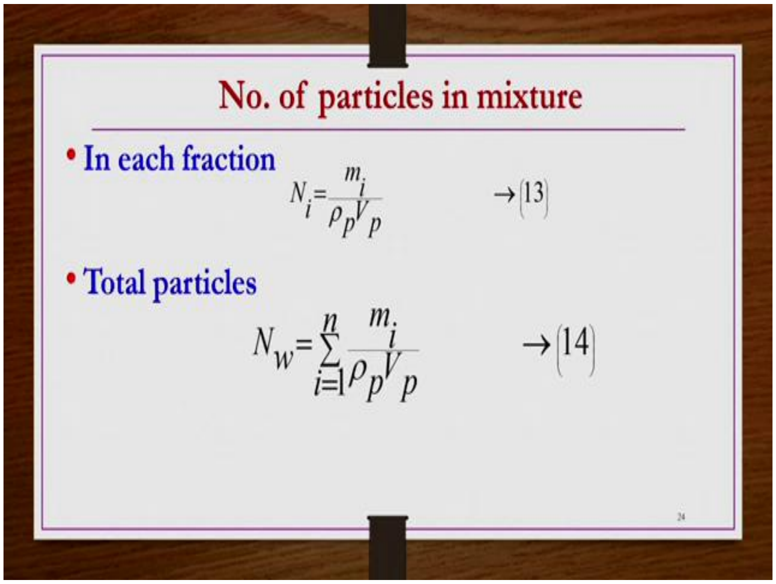

Then number of particles in mixture already we have discussed that is $N_i = \frac{m_i}{\rho_p V_p} \rightarrow (13)$, for one particular fraction. Total number of fraction would be $N_w = \sum_{i=1}^{n} \frac{m_i}{\rho_p V_p} \rightarrow (14)$ the total number of particles for a given mixture.

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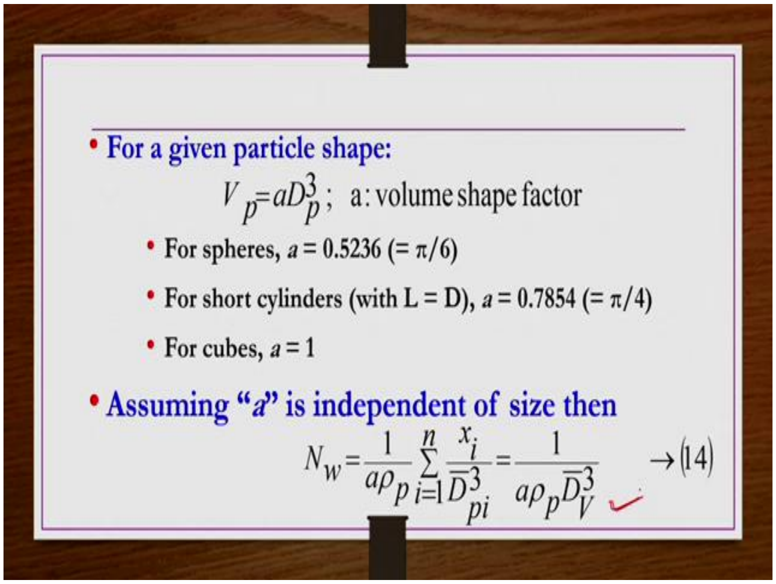

And then for a given particle shape factors let us say $V_p = aD_p^3$. So a is the volume shape factor as I mentioned for sphere it is $a = \frac{\pi}{6} = 0.5236$. If the particles are kind of short cylinders $a =$

$\frac{\pi}{4} = 0.7854$. If it is cubicle particle then $a = 1$ and then assuming a is independent of size then N_w you can calculate using this expression if you know the volume mean diameter $\bar{D}_v$.

$$N_w = \frac{1}{a\rho_p}\sum_{i=1}^{n}\frac{x_i}{\bar{D}_{pi}^{\,3}} = \frac{1}{a\rho_p\,\bar{D}_v^{\,3}}$$

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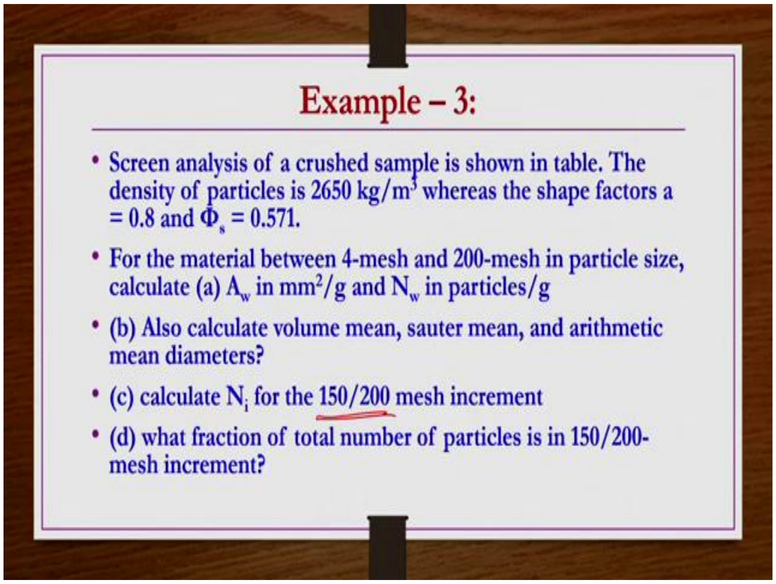

So now we take an example, screen analysis of a crust sample is shown in table. The density of particles is 2650 kg per meter cube whereas the shape factors are 0.8 and then that is whatever the shape factors are the 0.8 and then sphericity is 0.571. For the material between 4 mesh and 200 mesh in particle size calculate Aw, that is a specific area of the sample in mm square per gram and then Nw in particles per gram that is total number of particles per gram of the sample.

Then also calculate the volume means, Sauter mean and arithmetic mean diameters also calculate Ni only for the 150/20 mesh screen increment not for the entire sample only for that particular increment. Then what is the fraction of total number of particles? Is in this 150/20 mesh increment.

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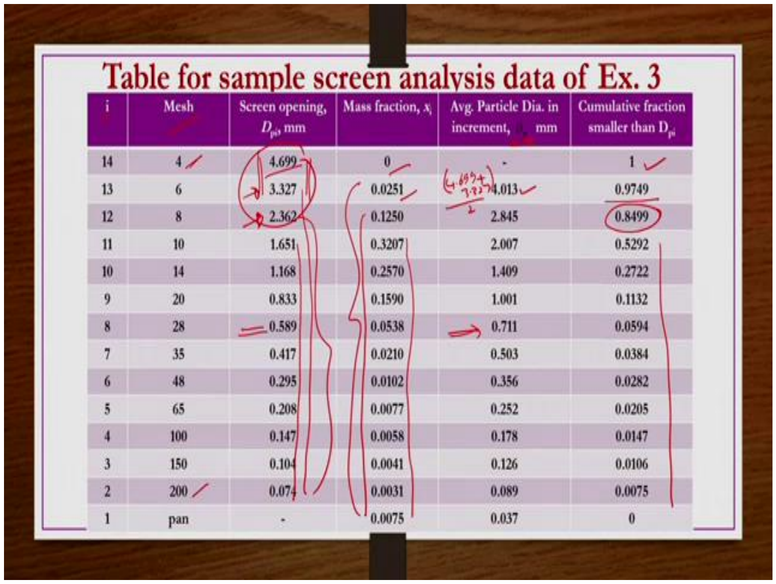

Table for sample screen analysis data of Ex. 3

i	Mesh	Screen opening, D_{pi}, mm	Mass fraction, x_i	Avg. Particle Dia. in increment, mm	Cumulative fraction smaller than D_{pi}
14	4	4.699	0	-	1
13	6	3.327	0.0251	4.013	0.9749
12	8	2.362	0.1250	2.845	0.8499
11	10	1.651	0.3207	2.007	0.5292
10	14	1.168	0.2570	1.409	0.2722
9	20	0.833	0.1590	1.001	0.1132
8	28	0.589	0.0538	0.711	0.0594
7	35	0.417	0.0210	0.503	0.0384
6	48	0.295	0.0102	0.356	0.0282
5	65	0.208	0.0077	0.252	0.0205
4	100	0.147	0.0058	0.178	0.0147
3	150	0.104	0.0041	0.126	0.0106
2	200	0.074	0.0031	0.089	0.0075
1	pan	-	0.0075	0.037	0

That is actually we see the table here, this is the information 150/20 mesh is this one so that is excluding the pan whatever I mean, what are the finest particles are there that we are going to calculate anyway. Screen increment I and then mesh number are given in screen increment we in general give the largest number at the top and then smallest first number at the pan for the pan so pan is 1, 2, 3, 4 like that so 14 increments are there.

The finest mesh that is used 200 mesh that is at the bottom and then closest to mesh that is used is the 4 mesh and that is at the top their corresponding opening D_{pi}'s are given here, so they are not D_{pi}, $\overline{D}_{pi}$ they are D_{pi} only. So mesh 4 is having the screen opening of 4.699 mm and then their corresponding mass fractions, let us say what is the mass fraction that is retained on 4 mesh, is 0, what is the mass fraction that is retained on screen or mesh 6, is 0.0251 like that all these things are given.

So now we need average diameter $\overline{D}_{pi}$, so average diameter is you know first one is nothing as it is or 4.6991 can write because it is only one. So the second one is the average of these 2 that is $(4.699 + 3.327)/2 = 4.013$. So likewise all the other diameters average diameter is also, so cumulative fraction smaller than D_{pi} here so now D_{pi} here is 4.699. So we are doing only smaller one, so what is the fraction of the material whose size is smaller than 4.699 mm? So all these

3.321 all of them are having smaller than 4.699 mm. So their corresponding mass fractions all of them should be added and then retained so that is 1.

Next one is what is the fraction of material whose size is less than 3.327? D_{pi} is now here in for this row 3.327. So except these two all other things are you know having the size smaller than 3.327. So all the corresponding fractions this one from here to here we have to add together. So that comes out to be 0.9749. Likewise, what is the fraction of material smaller than D_{pi} 2.362, right? Except these three all other these are smaller than 3.262 so their corresponding mass fractions are this one 0.3207 + 0.2570 and so and so 0.0075. If you add them together you get 0.8499. Likewise, we have to construct all of them this will give you the cumulative fraction smaller than Dpi.

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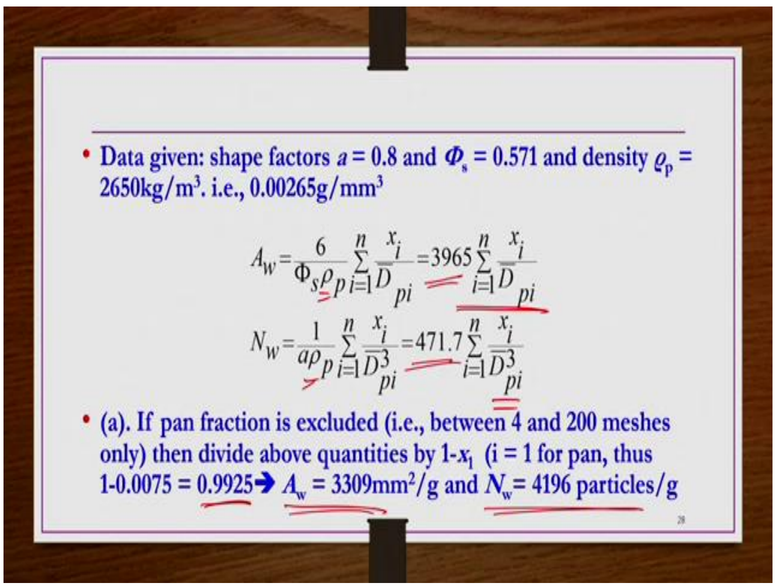

So the first part of the question is you know obtaining the, $A_w = \frac{6m}{\phi_s \rho_p} \Sigma \frac{x_i}{\bar{D}_{p_i}}$. So here in this table we have x_i for each you have to have the another column $x_i/\bar{D}_{pi}$here. So whatever the x_i , this x_i and then corresponding $\bar{D}_{pi}$here, so that should be 0 so here it should be $0.0251/3.327$, like that all the values we have to have here and then we have to do the summation.

When you do that one you get this particular $\Sigma \frac{x_i}{\bar{D}_{p_i}}$. So remaining fractions are $3965 \sum_{i=1}^{n} \frac{x_i}{\bar{D}_{p_i}}$. . Similarly,

$$N_w = \frac{1}{a\rho_p\,\bar{D}_v^{\,3}}$$

, a is also given as 0.8 so $\frac{1}{\rho_p a} = 471.7$ and then here in this one we have to have another column $\frac{x_i}{\bar{D}_{pi}^{\,3}}$. So this one also you do the similar one and then do the summation whatever the value is there that you should put here that will be coming giving us the numbers.

But the question is that you know, total number of particles between 4 mesh to 200 mesh. So whatever the particles they are present in the pan, they should not be considered they should be excluded. When you exclude them so you get the two number of picture otherwise you get different numbers. So whatever the pan fraction is there 0.0075 so 1-x_1, one for the pan $so\ 1 - 0.0075 = 0.9925$. So whatever this numbers are there, if you divided by 0.9925, you will get the true numbers that is Aw is 3309 mm square per gram and then Nw is 4196 number of particles by per gram, right?

Here this densities, etc. They have taken in the gram per mm cube because we want to report these values mm square per gram. Also in the question it is mentioned to report them in mm square per gram.

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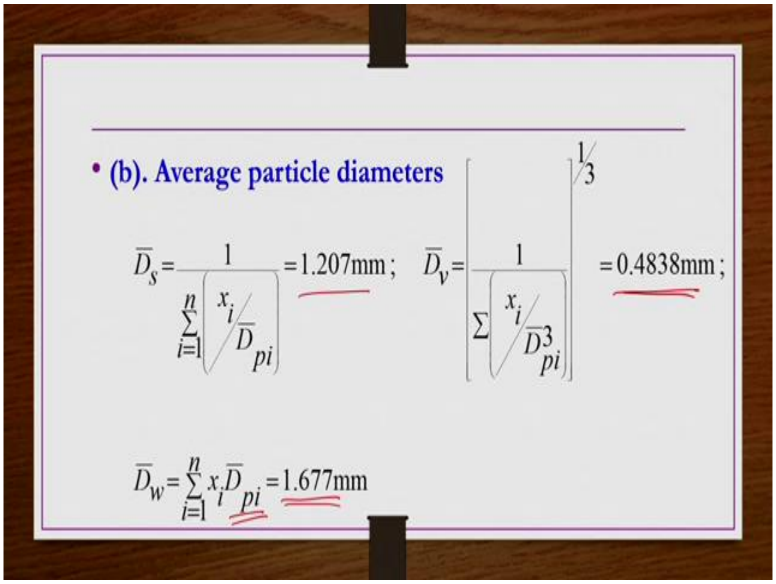

So similarly average particle diameters so we have $\overline{D}_s = \frac{1}{\sum x_i \overline{D}_{pi}}$. So for each row you obtain what is $x_i \overline{D}_{pi}$ and then add them together and then you take the reciprocal of that one, you get 1.207 mm. Likewise for each row in the table you calculate what is $x_i \overline{D}_{pi}^3$ and then you add them together for all the rows then you take the reciprocal of that and then you take the cube root of that one, you get 0.4838 mm and then mass mean diameter for each row you get $x_i \overline{D}_{pi}$ and then for all the rows you do like that and calculate them and then add them together you get, you know, 1.677 mm. Now you can see all three numbers are very much different from each other, not even close to each other.

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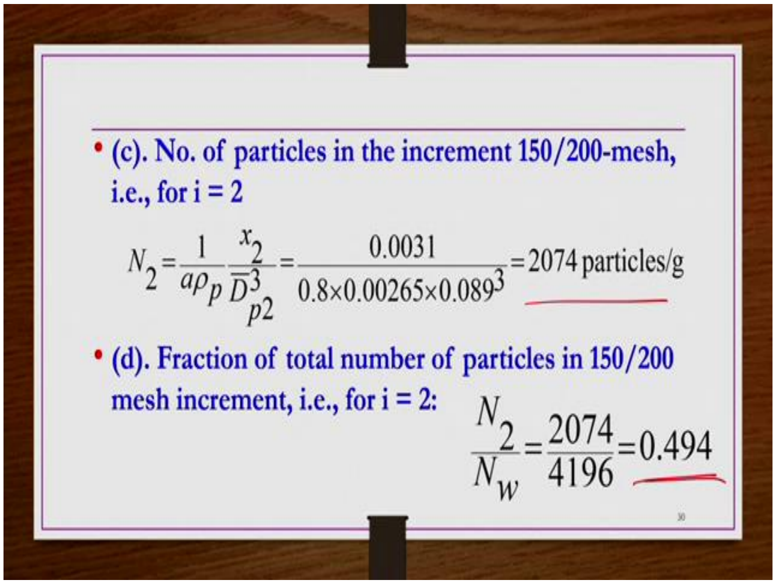

The number of particles in increments 150 by 20 mesh is $N_2 = \frac{1}{a\rho_p} \frac{x_2}{\overline{D}_{p2}^3}$ because that is 150 by 20 increment is nothing but i is equals to 2 so. So when you substitute these things you get 2074 particles per gram. So in the pan we have these many particles. So the fraction of total number of particles in 150/20 mesh is nothing but this 2074/4196=0.494 that is 50%, 49.4 almost 50 percent of the particles are you know in 150/200 mesh increments. So that indicates that the sample is having so many finer particles almost 50 percent of the sample is having finer material, okay. This is how one has to do the problems.

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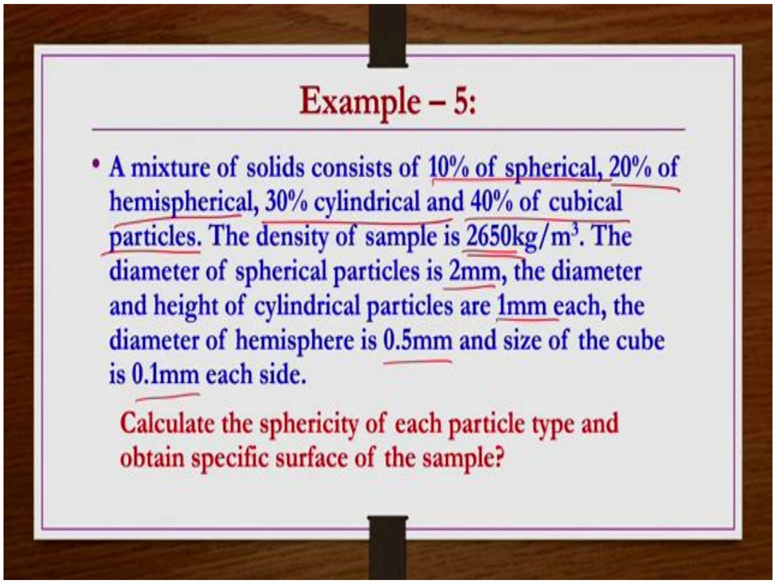
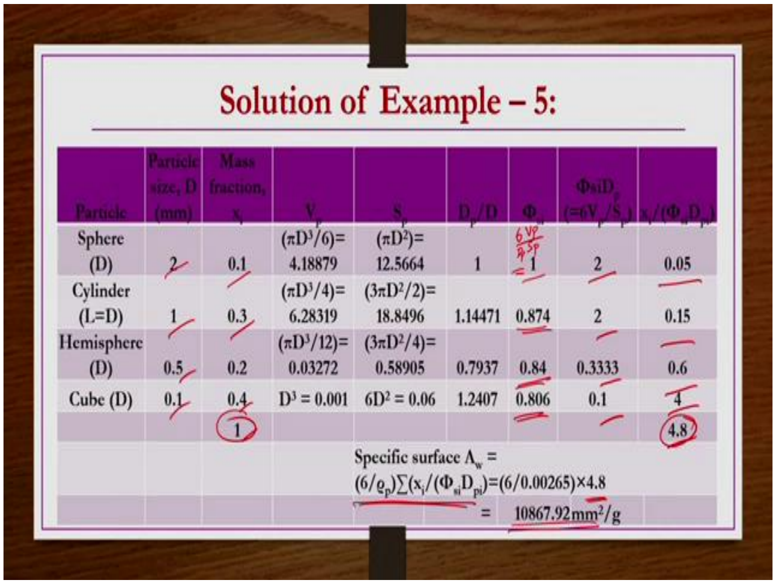


So we take one example where we do not take the screen analysis results, but we have a mixture of different types of particles. We take let us say 10% of spherical particles, 20% of the hemispherical particle, 30 cylindrical particles and 40% cubicle particles that is a kind of non-uniform mixture we have. But the density is same for all of them and then particle shapes are also given, diameter for spherical particle is 2 mm and then short cylinders 1 mm and then hemisphere 0.5 mm and then cube is 0.1 mm each. So for this what you need to do, we have to find out the sphericity for each particle type and then obtain specific surface of the sample. (Refer Slide Time: 61:58)



So particle size is given mm and then corresponding mass fractions are given here in this second and third column, okay, this is the summation when you add together this fraction is 1. So Vp the area of particle each particle and then Sp the surface area of each particle is calculated here. So Dp/D if you do that you will get here and then ϕ_{si} we already know that it is spherical particle 1 and then cylindrical particle, this you know this particle diameter if you equate to the particle volume, if you equate to the volume of a spherical particle whose diameter is D_p so then that D_p you can use in the sphericity definition.

So you get 0.874 this anyway we have already calculated so this phi s is nothing but $\emptyset_s = \frac{6V_p}{S_p D_p}$, when you do for spherical particle 1 and then for this short cylinders 0.874, this hemisphere 0.84 and then cube 0.806, these things we have already calculated in one of the previous lecture.

The specific surface area here is $S_p = \frac{6}{\rho_p \sum x_i \phi_{si} D_{pi}}$. Now the sphericity is different for one particle increment one fraction to the other fraction. So, you know that ϕ_{si} is coming into the summation. So you have here $\phi_{si} D_{pi}$ you already D_p you calculated here. So now ϕ_{si} you calculated you can get in get $\phi_{si} D_{pi} = \frac{6}{V_p S_p}$. So you get these numbers.

Now $x_i \phi_{si} D_{pi}$ you get these numbers, you add them together so you get 4.8 so $\frac{6}{\rho_p} = 0.00265 \frac{g}{mm^3}$ multiplied by sigma $\sum x_i \phi_{si} D_{pi} = 4.8$. So that gives us you know, $10867.92 \frac{mm^2}{g}$. This is how we have to solve the problems related to the non-uniform mixtures.

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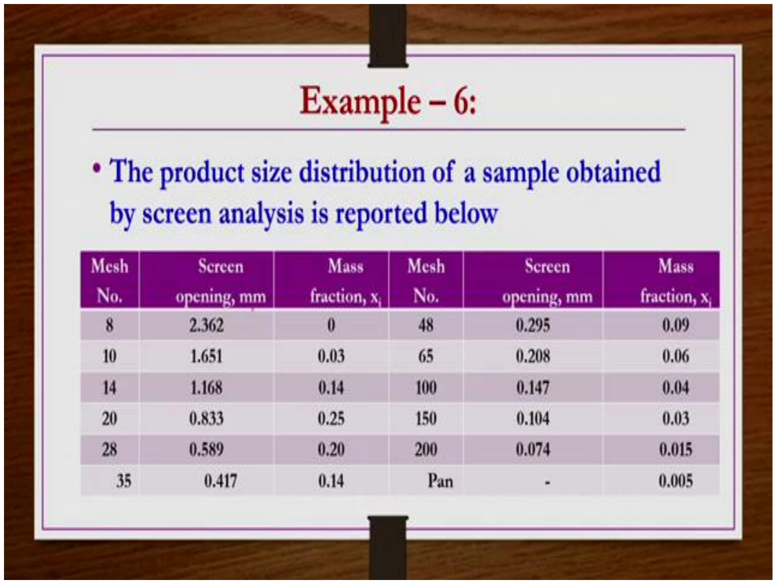
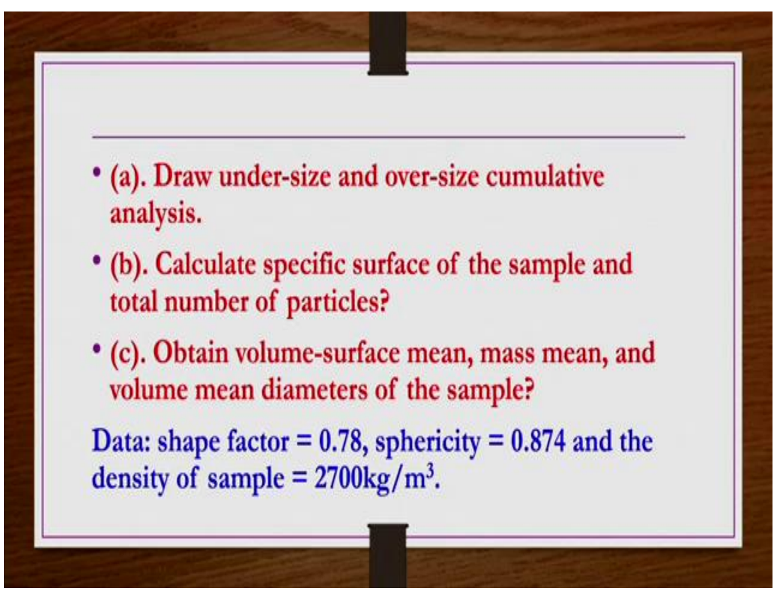


So now we take another example problem, example 6, here what we do we see the product size distribution of sample obtained by screen analysis given in a table like mesh number 8, 10, 14, 20 and so on, so up to 200 pan it is given and their corresponding screen opening in mm also given along with their mass fractions. So screen analysis has been done using the screens between 8 to 200 and then whatever the mass fractions or mass retained on each of the screen have been converted into the mass fractions and then they are reported against individual screen increments. So we have a screen increments from pan to 8 mesh in increasing order, right? (Refer Slide Time: 65:03)



So the question is draw the undersized and oversized cumulative analysis and then calculate specific surface of the sample and total number of particles, also obtain volume surface mean, mass mean and then volume mean diameters of the sample. It is quite similar as a one of the previous problem that we have already seen but in addition to that problem we have to draw the undersized and then oversized cumulative analysis as well. So for this problem the shape factor and then sphericity required which are given here 0.78 and 0.874 respectively and density of the sample is 2700 kg/m^3.

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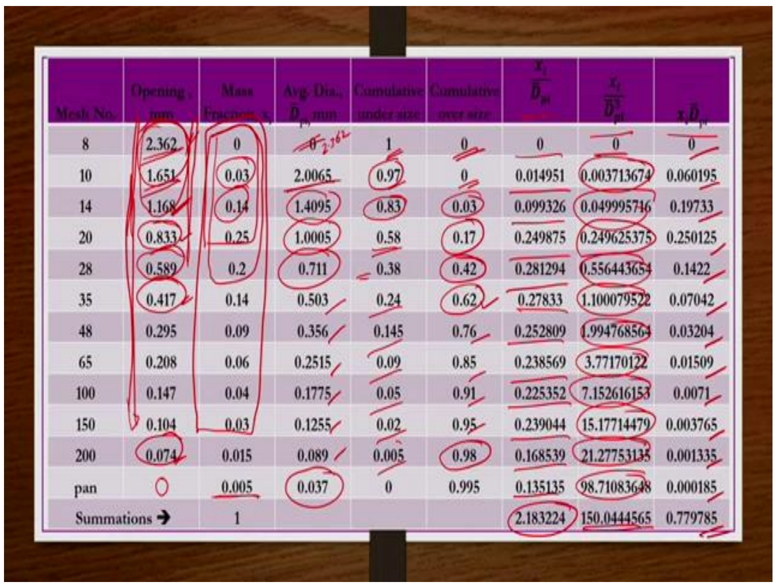

Mesh No.	Opening, mm	Mass Fraction, x_i	Avg. Dia., $\bar{D}_{pi}$, mm	Cumulative under size	Cumulative over size	$\frac{x_i}{\bar{D}_{pi}}$	$\frac{x_i}{\bar{D}_{pi}^3}$	$x_i\bar{D}_{pi}$
8	2.362	0	0	1	0	0	0	0
10	1.651	0.03	2.0065	0.97	0	0.014951	0.003713674	0.060195
14	1.168	0.14	1.4095	0.83	0.03	0.099326	0.049995716	0.19733
20	0.833	0.25	1.0005	0.58	0.17	0.249875	0.249625375	0.250125
28	0.589	0.2	0.711	0.38	0.42	0.281294	0.556443654	0.1422
35	0.417	0.14	0.503	0.24	0.62	0.27833	1.100079522	0.07042
48	0.295	0.09	0.356	0.145	0.76	0.252809	1.994768564	0.03204
65	0.208	0.06	0.2515	0.09	0.85	0.238569	3.77170122	0.01509
100	0.147	0.04	0.1775	0.05	0.91	0.225352	7.152616153	0.0071
150	0.104	0.03	0.1255	0.02	0.95	0.239044	15.17714479	0.003765
200	0.074	0.015	0.089	0.005	0.98	0.168539	21.27753135	0.001335
pan	0	0.005	0.037	0	0.995	0.135135	98.71083648	0.000185
Summations →		1				2.183224	150.0444565	0.779785

So let us tabulate this information in a kind of sequential order, incremental order you know mesh number 8, 10, 14, 20, 28 35, 48, 65, 100, 150, 200 pan and then corresponding openings in mm that is given in the problem or if it is not given we can take them from the Tyler standard screen openings, okay. Then corresponding mass fractions x_i's are given 0, 0.03, 0.14, 0.25 and so on so 0.005 if you add them together or you will get summation 1. So the last row provides the summation wherever it is required, so we need to have a $\overline{D_{pi}}$ that is the average diameter for each increment. So for this case first one there is no kind of you know average diameter because it is the first one so whatever the material that is retained that has been taken as the mass fraction.

But however there is no material retained also as for the mass fraction given so we need not to worry about this one. Actually it is not 0 it should be taken 2.362 if required so but it is not required as of now anyway because there is no mass fraction retained on this particular 8 mesh.

Next one is $\overline{D_{pi}}$ for the second row is the average of these two value. That is $2.362 + 1.651/2 = 2.0065$. Likewise this $\overline{D_{pi}}$would be the average of these two, this$\overline{D_{pi}}$ would be the average of these two and then this $\overline{D_{pi}}$is the average of these two numbers.

Likewise you construct all of them. So let us say this $\overline{D_{pi}}$0.037 you get by taking the half of this one because here now pan it is not having for the pan the opening is 0. So whatever $0 + 0.074/2 = 0.037$. So like that all the $\overline{D_{pi}}$we can include here. Now the first part of the question is the cumulative undersized and then cumulative oversized plots we have to give. So undersized we see first, so undersized in the sense the fraction of material smaller than D_{pi}. So for this case D_{pi} is 2.362, right? What is the fraction of material that is having size smaller than 2.362?

So all these values like you know 1.6512 all this 0.074 all these screen openings are smaller than screen opening 2.362. So what is the combination of the corresponding mass fraction? So all this mass fractions whatever are there so they should be taken. So when you add them together, you will get 1, so that is what this one. Next one is what is the fraction of the material whose size is smaller than 1.651? So except this 1.651 itself and then 2.362 all other fractions all other openings are the, you know smaller than 1.651. So the corresponding fractions whatever are there if you add them together, you will get the value that should be entered here that is 0.97.

Likewise, what is the fraction of material whose size is smaller than D_{pi}? Here D_{pi} is 1.168 so except these three top ones all other screen openings given from mesh 22 pan or mesh 200, all of them are having screen opening smaller than 1.168. So the corresponding mass fractions 0.25 onwards to 0.005 you add them together whatever the number is there 0.83 that you get here.

Likewise, the fraction of material smaller than D_{pi} 0.833 would be 0.58 if you add them together these numbers and then fraction of material smaller than D_{pi} 0.589 would be the summation of all these numbers 0.142, 0.005 all of them add together you get this number, like that you can get all these numbers here, right? Finding out their cumulative fractional smaller than D_{pi} of that particular row that is that particular mesh, okay.

Now we take cumulative oversized. So now for the first row what is D_{pi}? 2.362, what is the fraction of material whose size is more than D_{pi} 2.362? Anything is a there is nothing because the maximum of coarsest opening is 2.362 and there is nothing more than that particular opening. So then all the fractions are 0, next row is D_{pi} is 1.651. What is the fraction of material whose size is more than D_{pi} is equal to 1.651? So only this more than 1.651 only 2.262 opening is there and then corresponding mass fraction is 0.

So then the 0 is coming here. Likewise what is the fraction of material whose size is more than D_{pi} 1.168? So this whatever the fraction corresponding to these two openings because these are the two openings which are having more than D_{pi} is equal to 1.168. So the corresponding mass fraction is 0.003 so that is coming here. Similarly for the next row, what is the fraction of the material whose size is larger than 0.833? Because now here D_{pi} for this row is 0.833 so only these three fractions are having the three meshes they are having opening larger than 0.833. So the corresponding mass fractions, if you add together $0.03 + 0.14 = 0.17$ so that you get here.

Likewise, for the mesh 28, what is the fraction of material whose size is more than D_{pi} 0.589? So all these things 0.833, 1.168, 1.651, 2.362 all the size openings are larger than 0.589 so their corresponding mass fractions are if you add together, so you will get a 0.42 that should be written here. Likewise you know for the next row what is the fraction of material whose D_{pi} is larger than 0.417? So all these 0.589, 0.833, 1.168, 1.651, 2.362 all of these openings are larger than 0.417. So their corresponding mass fractions, if you add together you will get it is as 0.62 that is written here.

Likewise you can get the all other fractions here. So now let us say a mesh 200 if you see the D_{pi} is the 0.074. So what is the fraction of material whose size is larger than D_{pi} 0.074? So all these openings above it 0.104, 0.147 and so on so up to 2.362 they are having openings larger than 0.074. So their corresponding mass fractions if you add together, whatever the number that you get that should be added here. So that should be added here, that should be 0.98, okay. So this is how you how to construct the cumulative undersized and then oversized analysis.

So next in the subsequent calculations that you know average mean diameter, volume mean diameter and Sauter mean diameter, etc. You may be needing this $\frac{x_i}{\overline{D}_{pi}}$ and then $\frac{x_i}{\overline{D}_{pi}^{\,3}}$ and $x_i\overline{D}_{pi}$. So now you have the $\overline{D}_{pi}$ and then corresponding x_i so for each row you do these operations like this x let us say x i by $\overline{D}_{pi}$ for each row you calculate, you insert them here and then add them together like this. Same is $\frac{x_i}{\bar{D}_{pi}{}^3}$ you add them together, before adding you for each row you get these values, right? And then what you do for each row, whatever the $\frac{x_i}{\bar{D}_{pi}{}^3}$ you are getting those values you add together. So you get 150.044. Similarly, $\frac{x_i}{\bar{D}_{pi}}$ if you calculate for each rows and then you add them together what you get? You get 0.779785 this would be a required for the average diameter calculations.

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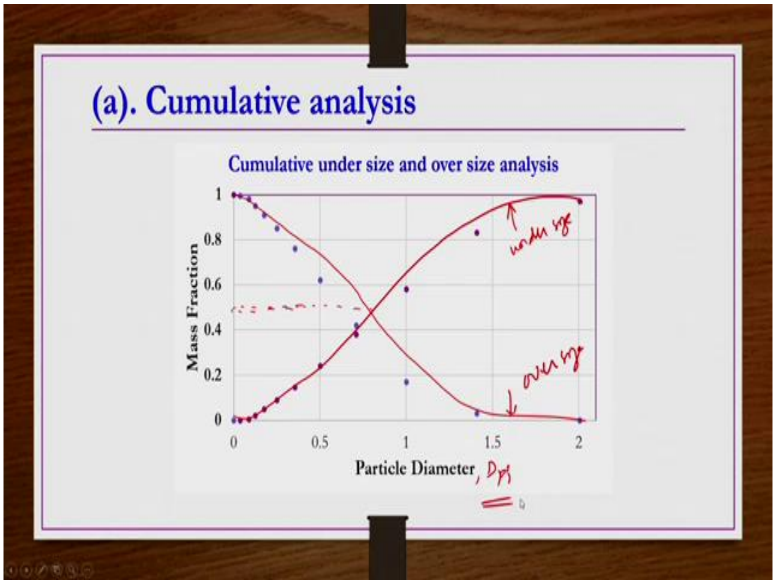

So next what we do, first part cumulative analysis. If you see here we can see cumulative undersized fraction it is coming something like this, okay. Likewise cumulative oversized is coming something like this. So they are actually there may be mismatching exactly at you know the 50 line at mass fraction 0.5. Now proper curve fitting if you, do you get like this. So this is cumulative undersized analysis and then this is cumulative oversized analysis with respect to the particle diameter D_{pi}, this is not $\bar{D}_{pi}$ but this is the should be D_{pi}, okay. So this is the first part of the question.

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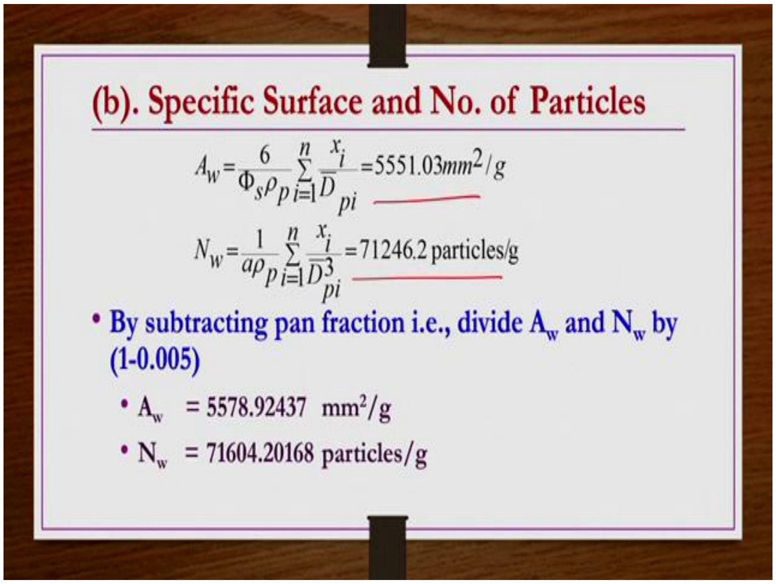

The next part is now a specific surface that is Aw and number of particles that is Nw we have to calculate. $A_w = \frac{6m}{\phi_s \rho_p} \Sigma \frac{x_i}{\bar{D}_{p_i}}$, so when you do this thing you get 5551.03 $\frac{mm^2}{g}$ and then$N_w = \frac{1}{a\rho_p} \sum_{i=1}^{n} \frac{x_i}{\bar{D}_{pi}{}^3}$ =71246.2 particles/g. But subtracting the pan fraction will give the true number, so whatever the pan fraction is there so mass fraction that is present in the pan is 0.005 so 1-0.005 whatever number that you get that you use to divide this A_w and N_w, A_w by that number, N_w by that number that will give the number of material as well as the specific surface of the material without considering the pan. So they are given here, okay.

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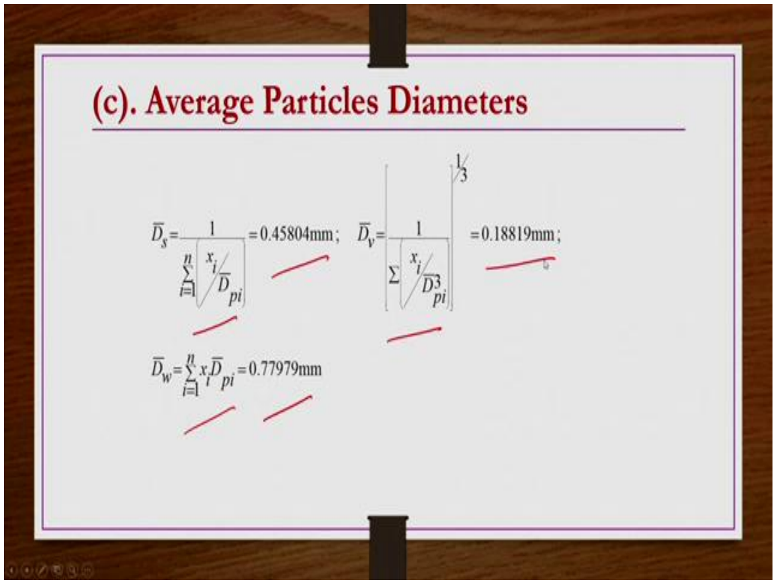

Then average particles diameters, so $\bar{D}_s$ Sauter, mean diameter $\bar{D}_v$, volume mean diameter and $\bar{D}_w$ that is mass mean diameter, so this expressions we have already derived and then we have calculated the summation of $\Sigma \frac{x_i}{\bar{D}_{p_i}}$ and then $\sum \frac{x_i}{\bar{D}_{pi}{}^3}$ and then say $\sum x_i D_{pi}$ the summations we have calculated in the table. So this you substitute you get these numbers. So here also we can see all these three numbers are giving three different values.

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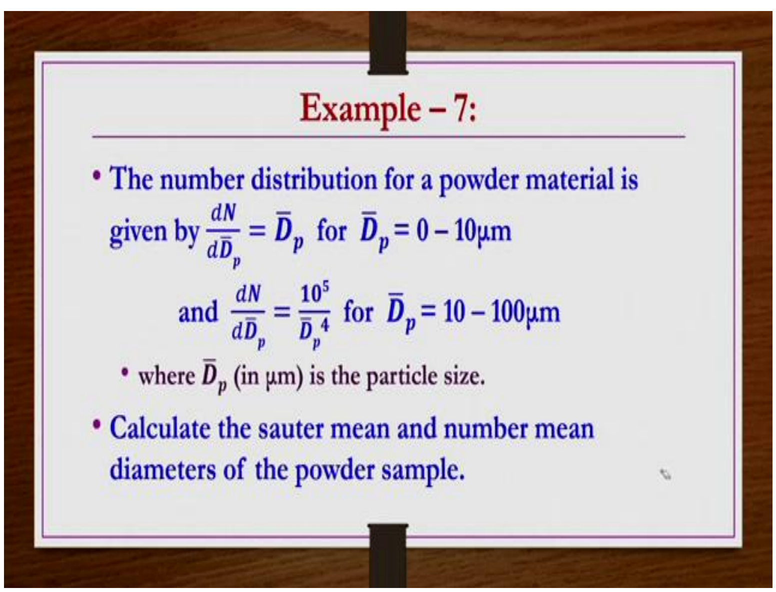

Then we take another example, here in this example rather giving the mesh increment versus you know mass fractions, whatever the Dp bar versus number of particles information is given in a equation form. The number distribution for the powder material is given by $\frac{dN}{d\bar{D}_p} = \bar{D}_p \; for \; \bar{D}_p = 0 - 10\mu m$ and then $\frac{dN}{d\bar{D}_p} = \frac{10^5}{\bar{D}_p^4} \; for \; \bar{D}_p = 10 - 100\mu m$ for two different size distributions, so range of $\bar{D}_p$ two different expressions are given. So where here $\bar{D}_p$is nothing but particle size it is given in microns, calculate the sauter mean and number mean diameters for the powder sample.

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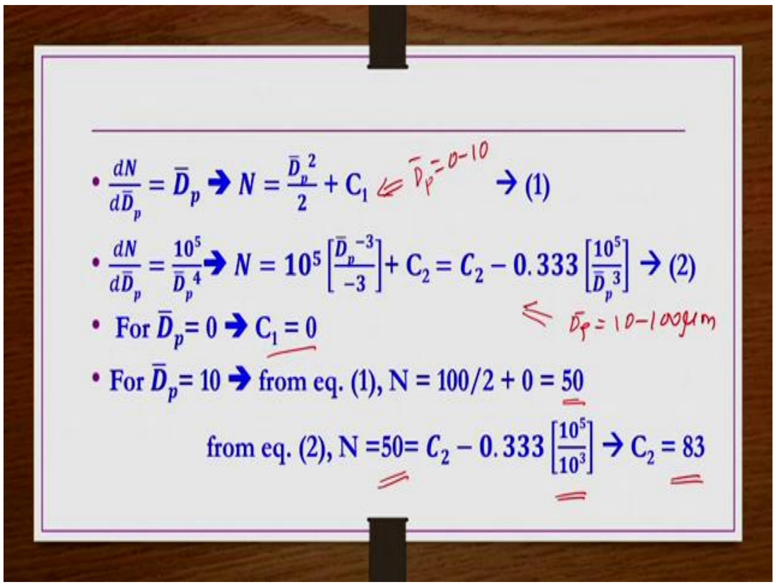

So first we have to find out the N, N distribution because in this by using the number of particles in each increment you know, getting the you know sauter mean diameter and mass mean diameter we need to know N, N_i, N_i for each increment so for that reason we need to find out N from these two expressions given in the problem. So we have $\frac{dN}{d\bar{D}_p} = \bar{D}_p \; for \; \bar{D}_p = 0 - 10\mu m.$ So when you integrate it you get $N = \frac{\bar{D}_p^2}{2} + C_1$and then second equation similarly if integrate you get $N = 10^5\left(\frac{\bar{D}_p^{-3}}{-3}\right) + C_2 = C_2 - 0.333\frac{10^5}{\bar{D}_p^{-3}}$ and then if you substitute boundary condition for $\bar{D}_p \rightarrow 0 \; then \; C_1 \rightarrow 0.$

So because in the first equation this equation is valid for you know for Dp bar 0 to 10 microns. So if you substitute Dp bar is equal to 0 you will get C1 is equals to 0 and then this second equation is valid for both $\bar{D}_p$, for $\bar{D}_p is$ 10 to 100 microns. So here for $\bar{D}_p 10$ from equation 1 what you get N is equals to 100 by 2 plus 0 is equals to 50 and then at $\bar{D}_p = 10 \rightarrow N = \frac{100}{2} + 0 = 50$ from equation number 1 if you substitute 50 for $N = 50 = C_2 - 0.333 \frac{10^5}{\bar{D}_p^{-3}} \Rightarrow C_2 = 83$

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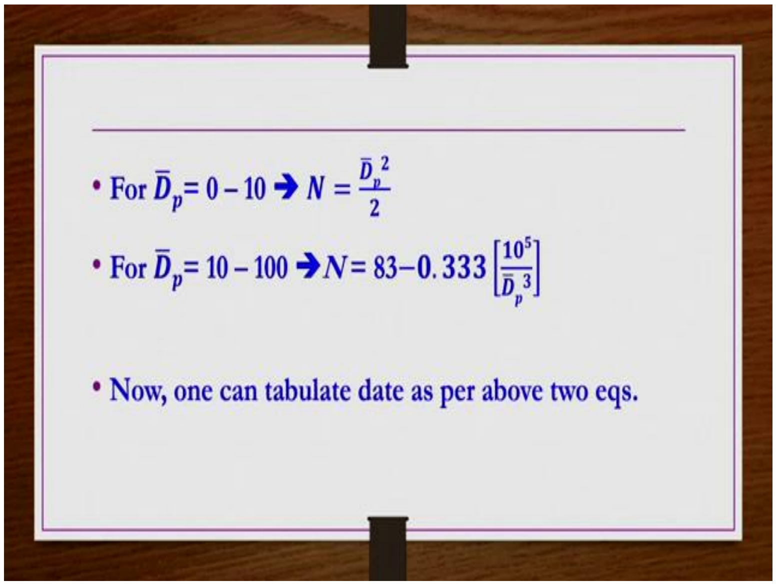
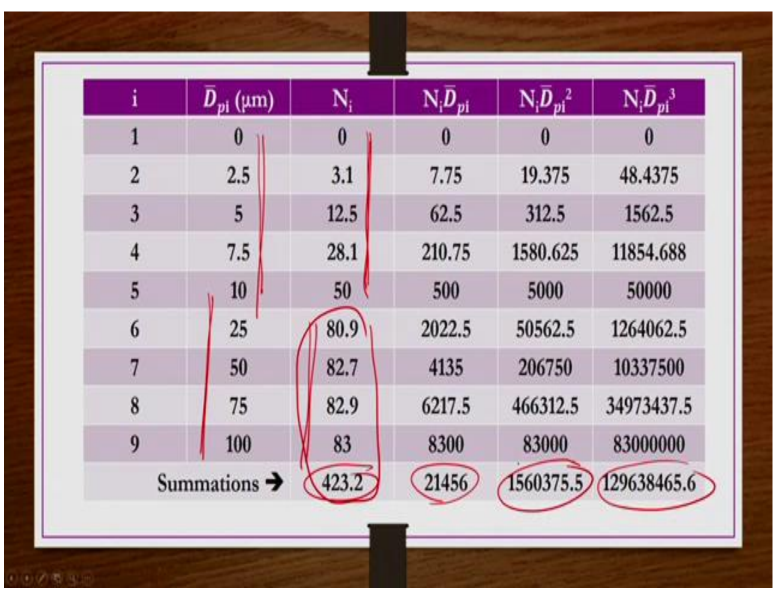


So the particle distribution you get here for 0 to 10 microns $N = \frac{\bar{D}_p^2}{2}$ and then for $\bar{D}_p = 10 - 100\mu m$ $N = 83 - 0.333 \frac{10^5}{\bar{D}_p^{-3}}$, this is what you get, right? (Refer Slide Time: 79:53)



i	$\bar{D}_{pi}$ (μm)	N_i	$N_i\bar{D}_{pi}$	$N_i\bar{D}_{pi}^2$	$N_i\bar{D}_{pi}^3$
1	0	0	0	0	0
2	2.5	3.1	7.75	19.375	48.4375
3	5	12.5	62.5	312.5	1562.5
4	7.5	28.1	210.75	1580.625	11854.688
5	10	50	500	5000	50000
6	25	80.9	2022.5	50562.5	1264062.5
7	50	82.7	4135	206750	10337500
8	75	82.9	6217.5	466312.5	34973437.5
9	100	83	8300	83000	83000000
Summations ➔		423.2	21456	1560375.5	129638465.6

Now one can tabulate data as per the about two equations. So you take i different i's corresponding to you know $\overline{D}_p$. Let us say 0 to 10 microns $\overline{D}_p$ you know you take a few numbers let us say 2.55, 7.5 and then between 10 to 100 also you take a few numbers let us say 25, 50, 75 microns like that. So you use the equation just we derived for N, so you use this for this range, what you do? You apply $N = \overline{D}_p^2$ by two expressions so that you can get this number and then for this beyond 10 you can use the other equation for N which is valid from this 10 to 100 microns that is this equation, so then you get remaining N numbers here like this.

So total number of particles also you can get like this. So number mean, sauter mean diameters as when you know the number of particles you need to know $N_i \overline{D}_p$, $N_i \overline{D}_{pi}^2$, $N_i \overline{D}_p^3$ values so that you can calculate because now you know $N_i \overline{D}_p$ for different increments you can get those numbers and then you can do the summation, so you have the summations so which can be used in a kind of calculations now.

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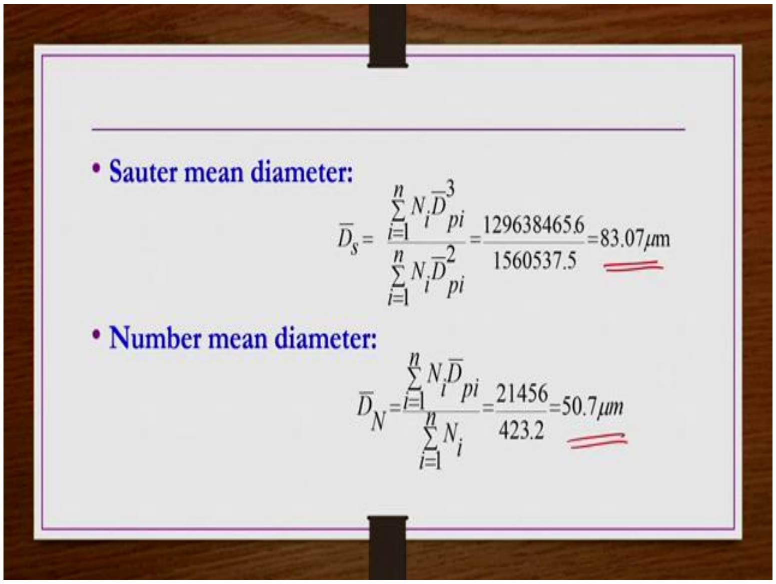

So sauter mean diameter, we know $\overline{D}_s = \frac{\sum_{i=1}^{n} N_i \overline{D}_{pi}^3}{\sum_{i=1}^{n} N_i \overline{D}_{pi}^2}$. So when you substitute this number you get 83.07 microns. Similarly number mean diameter, $\frac{\sum_{i=1}^{n} N_i \overline{D}_p}{\sum_{i=1}^{n} N_i}$ you will get this number as 50.7 microns. So here also we see two different mean diameters are giving different values.

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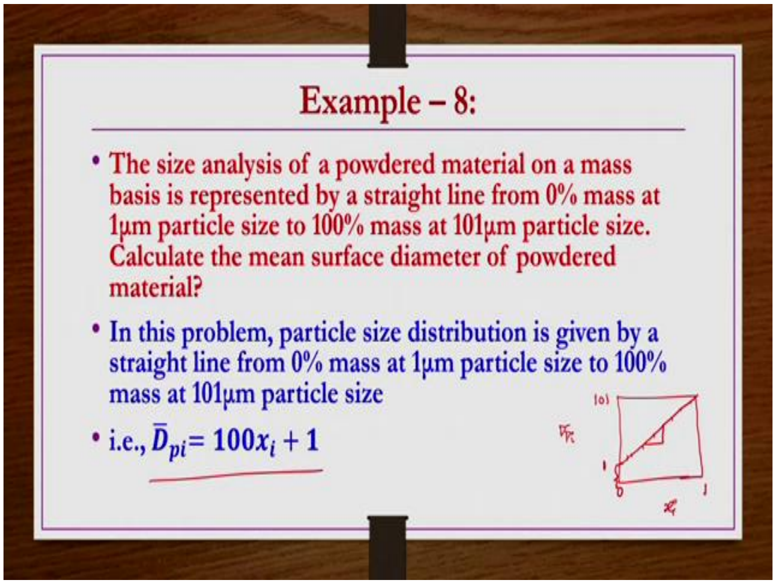

We take last example here the size analysis of a powdered material on a mass basis is represented by a straight line from 0 percent mass at 1 micron particle size to 100 percent mass at 101 micron particle size. So the mass distribution in this problem is given like you know kind of a straight line information. So Y is equal to MX plus format is given so we need to calculate the mean surface diameter of the powdered material.

So first we need to write from the statement of the problem, we have to write the relation between x_i and $\bar{D}_{pi}$. So what we get here, so in this problem particle size distribution is given by a straight line from 0 percent mass at 1 micron particle size to 100 percent mass at 101 micron particle size. So we get $\bar{D}_{pi}$.is equals to let us say this is x_i, it is 0 and then this is 1, so this is let us say $\bar{D}_{pi}$..

So when mass percent or fraction is 0, so $\bar{D}_{pi}$. is 1, let us say this is 1 and then when mass fraction is 100 so $\bar{D}_{pi}$. is 101, let us say 101. So the distribution should be like this and then slope what you get slope from this here? You will get slope as a 100, so you get $\bar{D}_{pi}$. is equal to $100x_i + 1$ this intercept is 1 so $Y = MX + C$ form so you get $\bar{D}_{pi} = 100x_i + 1$.. So the information the D_{pi}.. versus mass fraction information is available, so we can do the similar way as we have done previously.

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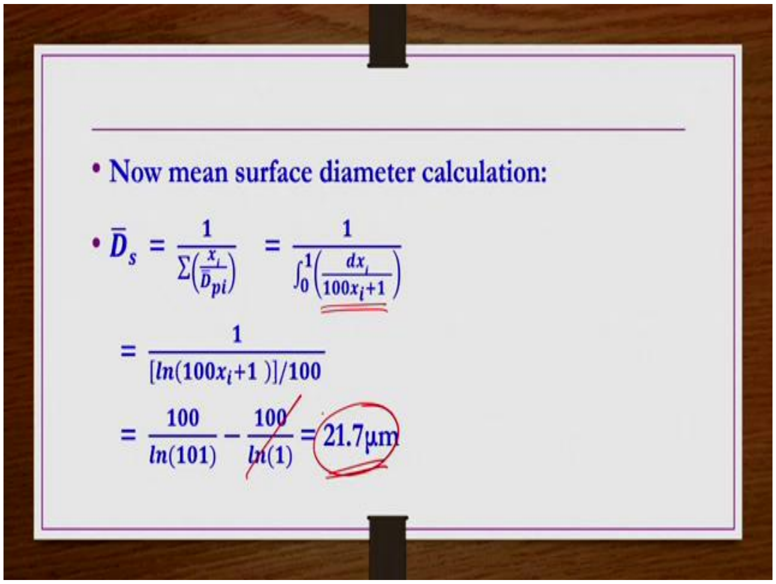

Surface mean diameter calculations first we do $\bar{D}_s$.is nothing but

$$\bar{D}_s = \frac{1}{\sum x_i \bar{D}_{pi}}$$

. So summation so that equation one way that whatever the $\bar{D}_{pi} = 100x_i + 1$ is there so you can take different i and then what is $\bar{D}_{pi}$ corresponding exercise you can take so you need to know both of them are unknown. So you need to from that graph whatever you know that Y is equal to MX plus C graph that we have. So from this this $\bar{D}_{pi}$ versus x_i graph what you can do, different intervals so you can take like x_i like something like this, you know, you get you make a tabulation of a $\bar{D}_{pi}$ versus x_i and then you can do the standard form of calculation as we have been doing for other problems.

But other way we know that this is a kind of sauter mean diameter is $\frac{1}{\sum\frac{x_i}{D_{pi}}}$so it is a kind of summation, integral also we can do because now equation is known so $\frac{1}{\int_0^1 \frac{x_i}{100x_i+1}}$this way also you can do or if you feel comfortable you can make a tabulation on from the you plot a graph and then from graph different xi you go get the what is$\bar{D}_{pi}$ and then you get $\frac{x_i}{D_{pi}}$ for each increment each interval that you have taken and then you add them together.

You take the reciprocal of that one you get the sauter mean diameter, so but this is the simplest one because we are doing summation so the same thing we can get by the integration form like this here or whatever given here. So if you integrate it $= \frac{1}{[\ln 100x_i+1]/_{100}}$. So limits 0 to 1 for x i if you substitute so this is not possible so then we get the valid number is 21.7 microns you get this sauter mean diameter so quickly easily you get it as 21.7 microns, okay.

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Now next what we do? We see the references, the references for this lecture I have taken McCabe, Smith and Harriot, Unit Operations of Chemical Engineering then Ortega-Rivas, Unit Operations of Particulate Solids Theory and Practice then Richardson Harker, Caulson and Richardson's Chemical Engineering series Volume 2 then Geankapolis, Transport Processes and Unit Operations, Brown et al, Unit Operations, Badger and Banchero, Introduction to Chemical Engineering. The problems discussed in this particular lecture are primarily taken from these two books McCabe and Smith and Richardson and Harker and then some problems are also taken from this book by Ortega-Rivas, thank you.