Mechanical Unit Operations
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Lecture No. 7
Methods of Size Reduction.

Welcome to the MOOCs course Mechanical Unit Operations. In this particular lecture we will be discussing methods of size reduction. We have seen the requirement of the size reduction and then clean analysis, differential analysis, cumulative analysis, those kind of problems we have seen, how to find out the details of the surface area of the individual screen increment or the surface area of the entire sample or specific surface those kind of details we have seen till now.

But in order to do size reduction what are the methods available and then how much power should we provide for a given size reduction requirement. Let us say you have a kind of initial 50 mm particle size. If you wanted to reduce the size of those particle sizes to the average of 1 mm particle size so then what are the forces available and then how much energy should be provided for those kind of reduction. Those things we are going to see. That is basically methods of size reduction we are going to see in this lecture and then in the coming lecture we will be discussing the equipments available for these kind of size reduction operations.

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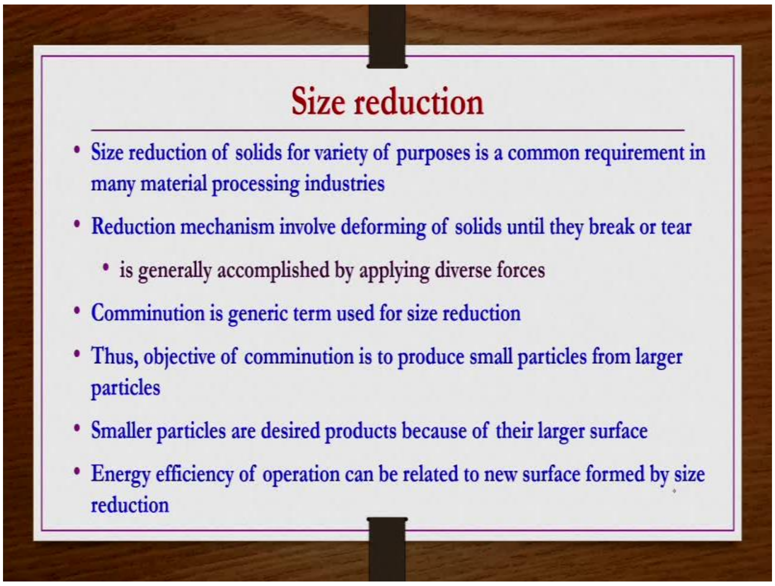

Size reduction

- Size reduction of solids for variety of purposes is a common requirement in many material processing industries
- Reduction mechanism involve deforming of solids until they break or tear
  - is generally accomplished by applying diverse forces
- Comminution is generic term used for size reduction
- Thus, objective of comminution is to produce small particles from larger particles
- Smaller particles are desired products because of their larger surface
- Energy efficiency of operation can be related to new surface formed by size reduction

So, as already we know the basic purpose of the size reduction is you know to reduce the size of the bigger size particle to the smaller size particles because smaller size particles are very much convenient to use in any of the given unit operations or unit processes or you know even for the transport or conveying as well. So that is size reduction of solids for variety of purposes is a common requirement in many material processing industries. We will see some examples also. The reduction mechanism involves deforming of solids until they break or tear.

And then how that can be done that can be done by applying diverse type of forces, different types of forces you can do, you can apply like compaction, nut cracking kind of thing. You can apply hammering kind of forces, you can apply the filing, rubbing kind of forces even you can apply cutting kind of forces if you have kind of specific size. So, different types of forces are available so you can apply as per your requirement of the application. As per the process requirement you can choose a kind of force which can provide you the required size of reduction.

And then in general one particular force may not be sufficient to get the required size reduction you maybe applying more than one kind of forces in general for this kind of size reduction. Those details also we are going to see sooner. For the size reduction is another generic term that is comminution. Right? Comminution is also a kind of generic term used for size reduction, so objective of the comminution is to reduce the bigger particle into smaller particles or to reproduce smaller particles from larger particles.

Why because smaller particles having a kind of larger surface in this particular size reduction operations we call, we designate the smaller particles as a kind of desired product because it is a size reduction operation, so whatever the reduce size particles are there they are the products. So, in this particular model whatever the size reduction operation have been carried out.

So finally size reduced smaller particles are taken as a kind of a products whereas the initial feed whatever is there bigger size particles we will be considering it as a feed. So we may be having to terminologies in power calculation is also, power requirement for a given size reduction operation that we are going to see. So, in those cases we are going to have two terminologies the feed size and the product size.

Feed size is the given material average size whatever the bigger size particles their average size is a kind of feed size. And the product size is a kind after size reduction whatever the final particle smaller particles are there their average size is a kind of product size. So these two sizes are important in general in order to find out the power requirement for even efficiency of a operation. So energy efficiency of operation can be related to new surfaces formed by size reduction when you are doing the size reduction.

So new particles, smaller particles are forming that is the new surfaces are forming, so this kind of indication of energy efficiency. Energy efficiency of operation now can be related to the new surfaces have been formed by size reduction. So what are the common applications of size reduction? Though we have seen several common and individual application specific applications given specified to a given problem type of problem so however we have a kind of a recapitulation again.

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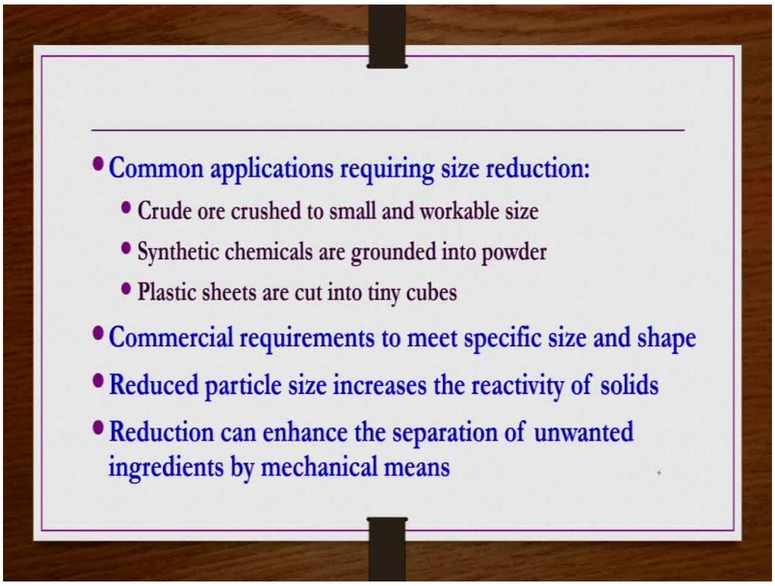

- Common applications requiring size reduction:
    - Crude ore crushed to small and workable size
    - Synthetic chemicals are grounded into powder
    - Plastic sheets are cut into tiny cubes
- Commercial requirements to meet specific size and shape
- Reduced particle size increases the reactivity of solids
- Reduction can enhance the separation of unwanted ingredients by mechanical means

So, common applications requiring size reduction are Crust Ore or Crude Ore that we crush into smaller and workable sizes because from the natural resources we get big-big rocks kind of things as a kind of raw material. We crush them into smaller sizes because big-big rocks you cannot handle in any kind of unit operation or unit processes properly even if you handle them the overall productivity or overall efficiency of the product will not be good enough.

So what you do you crush them into smaller and workable size so that you can conveniently take them into the concerned unit operations or the concerned unit processes if at all reaction is there you can do the required process comfortably or you can comfortably transport them in a kind of smaller size particles rather in a kind of bigger size particles. Then synthetic particles that are formed after a given reaction so they need to be grounded into a specified powder sizes.

So there also after reaction some kind of synthetic and chemicals formed in a solid form they were not been required size. So there are also you need to do a kind of size reduction so that you can have a final product and I have specified powder size as per the requirement of the consumer or as per the requirement of the subsequent process if it is not the final product. In general plastic sheets are cut into tiny cubes sometimes you know what happens you need to make a kind of a particles size reductions into very specific definite size and shape.

You cannot have a kind of different size and shape so you want to have a very specified size, so accordingly you need to cut them. or sometimes these kind of cutting into the specified or definite size happens in many food processing applications like you know slicing of vegetables, mincing of meat extracts those kind of applications. So like that if you keep on listing the end there are n number of applications where size reduction is kind of very much essential as discussed now we have seen previously several applications.

Sometimes this commercial requirement is also is the reason for kind of size reduction. Sometimes what happens commercial requirements to meet specific size and shape also makes a kind of point that you need to go for a kind of size reduction. And then the reduced size increase the reactivity of solids in general if you have a kind of heterogeneous reaction where solid particles are reacting with given fluid then if your particle solid particle size small then in general the reactivity of that particle or the heterogeneous reaction increases like if you take the smaller particle size.

If you take bigger long kind size particle size then the reactivity of the particle solids may be very less in general so for that reason also you need to do a kind of size reduction. Then reduction can enhance the separation of unwanted ingredients by mechanical means. Sometimes what happens you to separation based on the size not only for this operation sometimes you know it is required to do the separation based on the size also.

So when you do the size reduction so it can be convenient to separate out unwanted ingredients from the main reactance or the main product or you can separate out the bi-products from the main product sometimes by this size reductions. In general it can be good valid kind of thing for the reactance where the natural over that may be having several ingredients, so when you do the size reduction it may be possible that sum of the unwanted ingredients can be easily taken away by the mechanical separation. So because of that reason also we need to go for the size reduction.

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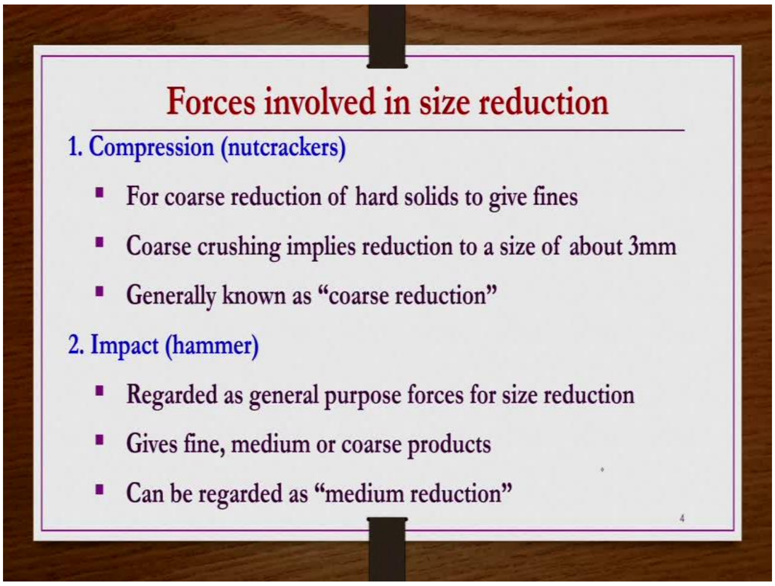

Forces involved in size reduction

1. Compression (nutcrackers)
   - For coarse reduction of hard solids to give fines
   - Coarse crushing implies reduction to a size of about 3mm
   - Generally known as “coarse reduction”
2. Impact (hammer)
   - Regarded as general purpose forces for size reduction
   - Gives fine, medium or coarse products
   - Can be regarded as “medium reduction”

Then what are the forces involved in the size reduction. We know that the importance we realised the importance of the size reduction especially in solids, solid operations or solid fluid operations have seen several kind of applications examples. But we need to know what are the forces that are required or water forces involved in size reduction. Right so let us say you have a big rock a very big rock and you wanted to get a very finer particle. So if you are doing something of rubbing or attrition kind of thing you may not get the required product even if you are getting that you may be giving lot of energy and then a lot of time to get that final particle from that the your particle.

So what you do in general you try to break the bigger particle into smaller ones or the medium ones and then you further hammer those medium kind of particles so that you can get a kind of a finer particles. So like that you know you have a kind of compressive forces, you have the hammering forces you can have the attrition forces. Even you have the specified kind of cutting kind of forces in size reduction. So those forces we need to understand first before going into you know equipment and then power required for the given size reduction operation.

So we list out the forces involved in size reduction now. First one is the compression where the nut cracker kind of action takes place. For coarse reductions of hard solid to give fines one can apply these compressive forces. And then coarse crushing implies reduction of reduction of sample to be a size of about 3 mm. So in general whatever the compressive forces that you apply from there whatever the material that you get out of which coarse material in general have a kind of 3 mm about to have a 3mm size.

So that is the reason the reduction by this compressive forces is also known as a kind of a coarse reduction. Please be noted that whatever the force that we apply whether the compression or the hammering and then attrition or the cutting kind of other subsequent forces that we are going to see. One particular force will not give only one type of particle that is coarse particle or medium particle or fine particles. It is possible that when you apply one particular kind of forces all three types of coarse medium and fine particles may be possible to get as a kind of product.

But depending on that how much fraction of a given material given product is having coarser, how much fraction of given product is having medium, how much fraction given product is having finer one. So these forces or these reduction processes are also given as a terminology as a coarse reduction, medium reduction and then fine reduction. So, whatever the compressive nut crackers are there they are generally given a terminology or known as the coarse reduction because when you apply these can forces in the product you have the majority of the fraction you know coarse material.

Next one is the impact on hammering can forces. So here they are regarded as general-purpose forces for size reduction. Here they produce, again as coarse, medium and then fine all of them they produce in general. But however in this one majority of the fraction of the product by hammering whatever the products of the solids you get are size reduced solids you get out of which you know you have the majority of the fractions as a kind of medium fraction. So that is the reason this impact or hammering is also known as the medium reduction. Though it will also produce coarse and fines but the majority of the fractions would be in the kind of medium size.

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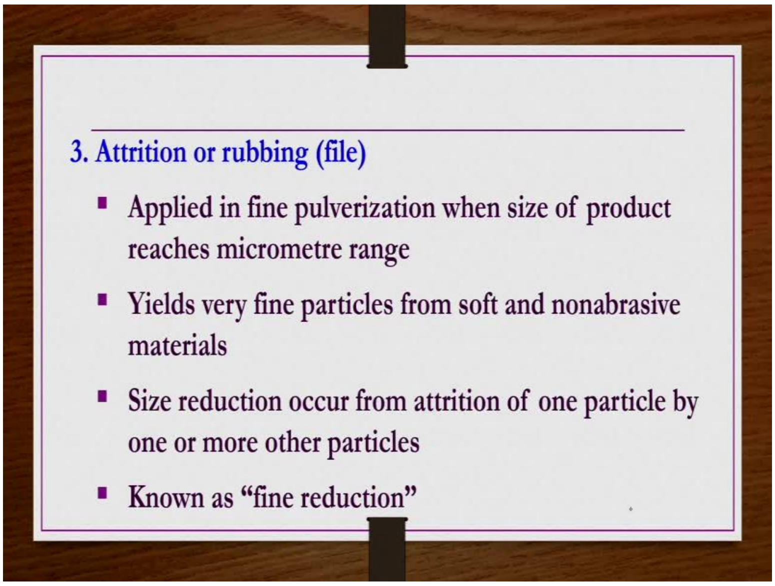

3. Attrition or rubbing (file)

- Applied in fine pulverization when size of product reaches micrometre range
- Yields very fine particles from soft and nonabrasive materials
- Size reduction occur from attrition of one particle by one or more other particles
- Known as "fine reduction"

The third type of force that is involved in size reduction is attrition or rubbing. Here is this rubbing her attrition is in general applied in fine pulverization when size of the product reaches micro-meter range. That is when you need to have a kind of fine particle so then you apply attrition or rubbing right. But how will direct you cannot apply rubbing for a break the rock size in general as I mentioned first you need to apply the compressive force then reduce the coarse material that coarse material probably if you do hammering and then after the hammering whatever the material is there if you applied attrition then it is possible that you get the majority of the fraction and the product will be having fine particles.

So, size reduction occur from attrition of one particle by one or more other particles when they are the tumbling out to each other they may be going through a kind of attrition, rubbing kind of amongst themselves and then a kind of size reduction may also take place. And then because of this region that they produce many of majority fraction, fine material, this attrition of the size reduction by attrition or rubbing is also known as fine reduction. These are the three processes in general. Right and then despite of whatever the uniformity of the feed the product that you get after the size reduction you may be having the very wide distribution you may not have the very uniform size distribution.

Let us say you take the glass bits, you take glass bits of 1 centimetre size, you take several numbers of such glass bits of 1 centimetre, you take them in a kind of a crushing equipment and then when you apply the size reduction operation, when you crashed them would you finally get the product. So, initially feed you have taken uniform feed you have taken uniform feed, all the particles are having 1 centimetre size but after crushing whatever the product that you will get will get the same product are you expecting to get the product of the uniform size?

So it is not possible after crushing you may be having again coarse, medium and then finer fractions and then you have the wider distribution of the particles. Right, so that is also possible. That is the reason based on the dominating fraction in the product the size reduction forces size reduction by different forces are given different terminology as coarse reduction. Compressive forces when you apply you get the coarse reduction and then if you apply the hammering forces you get medium reduction, when you apply the attrition forces you get the fine reduction.

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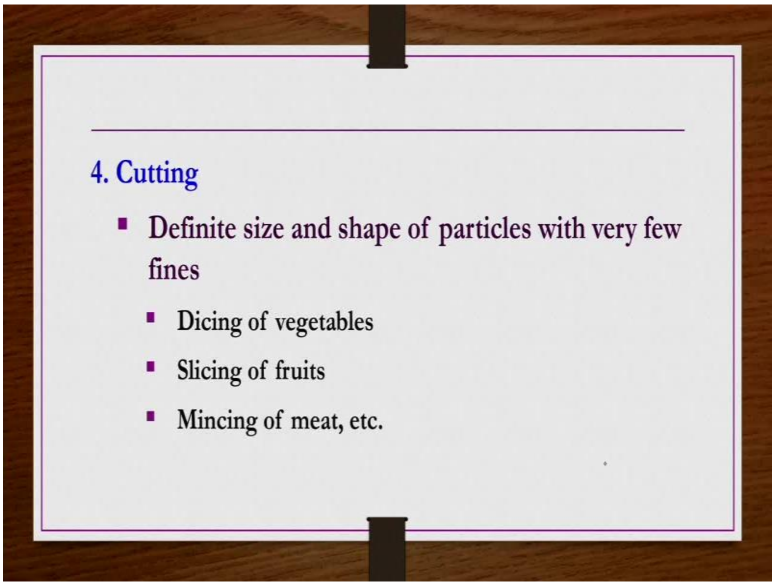

4. Cutting

- Definite size and shape of particles with very few fines
  - Dicing of vegetables
  - Slicing of fruits
  - Mincing of meat, etc.

Fourth one is the cutting, cutting is the one when it's applied even you need a kind of very specified size of the particles or the very specified shape then you have to go for this kind of cutting and this is in general product oriented. Product you know after the reactions for whatever the process has been taken if you pack the product in a specified size and shape then cutting is very much essential. So it provides definite size and shape of the particles with very few fines or almost no fines. Applications we find mostly it would in many industries such plastics and other things but in general food processing industries also they have plenty of applications like dicing up with vegetables, slicing off roots, mincing of meat etc.

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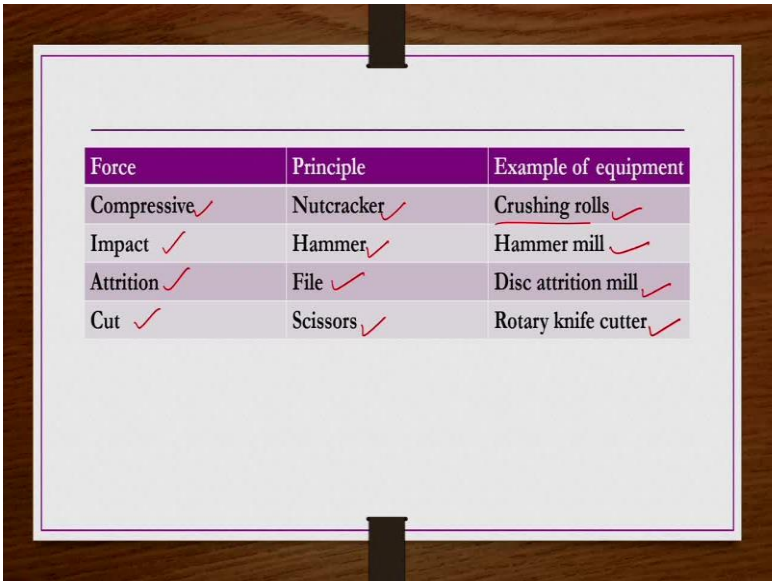

Force	Principle	Example of equipment
Compressive ✓	Nutcracker ✓	Crushing rolls ✓
Impact ✓	Hammer ✓	Hammer mill ✓
Attrition ✓	File ✓	Disc attrition mill ✓
Cut ✓	Scissors ✓	Rotary knife cutter ✓

So by summary what we understand if you have a kind of compressive force then principal action that is taking place is nut cracker a kind of action. And then example of equipments were that this kind of compressive forces or dominating for the crushing roles there are different types of size reduction equipments available that we are going to see in next lecture probably. So crushing role is one of the type of size reduction equipment in that primarily compressive force is applied.

And then there is a kind of hammer mill very applied impact kind of force where the hammering kind of action is taking place. Then we have disc attrition mill where the attrition forces are applied in and then principal of a action that is causing this reduction is the filing action. Then rotary knife cutters are there where the scissors are provided which produce the particles of different size and shape by cutting force.

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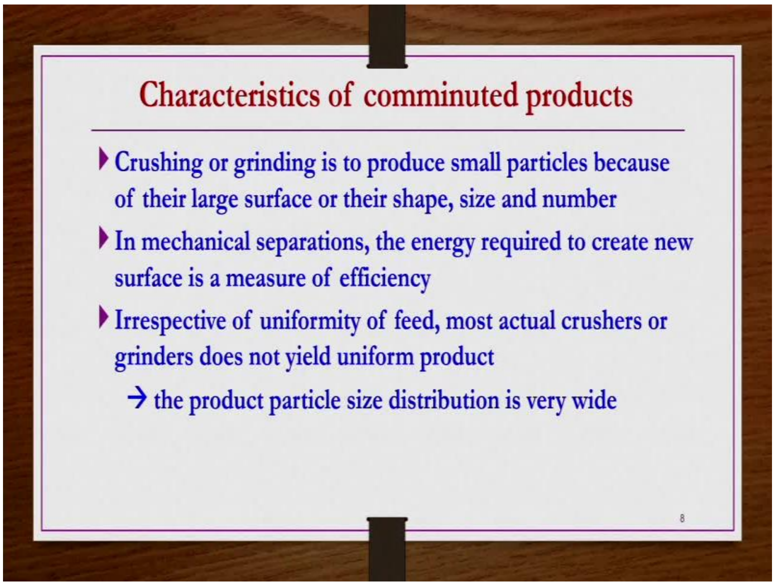

Characteristics of comminuted products

- Crushing or grinding is to produce small particles because of their large surface or their shape, size and number
- In mechanical separations, the energy required to create new surface is a measure of efficiency
- Irrespective of uniformity of feed, most actual crushers or grinders does not yield uniform product
  - → the product particle size distribution is very wide

Now characteristics of comminuted products. Crushing and grinding is to produce small particles because of their large surface or their shape, size and number as well. Sometimes, that is the reason we do crushing or grinding crushing and grinding are some kind of a size reduction operation. Okay, so we are doing the size reduction operations to produce small particles.

In mechanical separations of the energy required to create new surfaces is a measure of efficiency so what happens how much power you supplied for a given size reduction operation but how much is consumed by the particles you know for the creation of the new surfaces by size reduction is a kind of measure of efficiency of the given process. Those things we are going to see in calculations anyways.

And then as I already mentioned irrespective of uniformity of feed whatever the feed you have taken it may be very uniform in size however after crushing or grinding kind of size reduction operations many apply to that feed you may not get the uniform product though the feed is a very uniform so that means a product particle size distribution is very wide in general.

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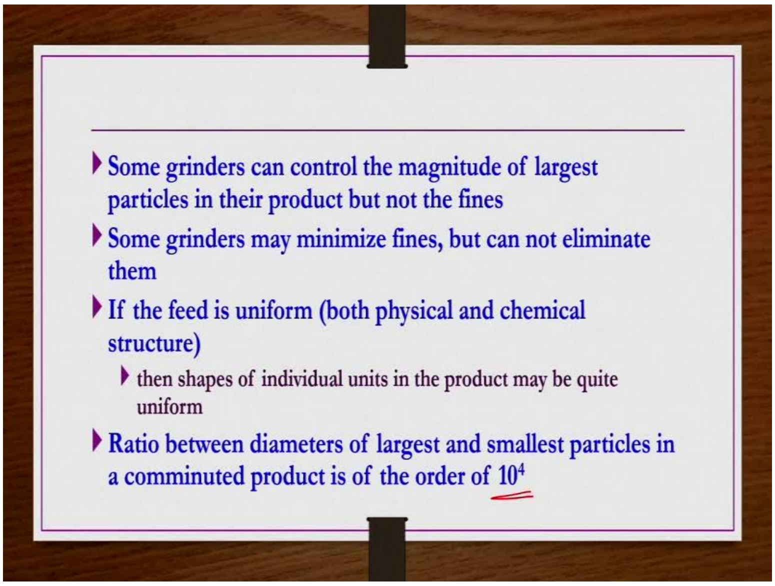

- Some grinders can control the magnitude of largest particles in their product but not the fines
- Some grinders may minimize fines, but can not eliminate them
- If the feed is uniform (both physical and chemical structure)
  - then shapes of individual units in the product may be quite uniform
- Ratio between diameters of largest and smallest particles in a comminuted product is of the order of 10^4

Sometimes grinders can control the magnitude of the largest particles in the product but not the fines. Whatever the size reduction forces you apply you know you get all kind of these things coarse, medium, fine kind of thing. But you cannot avoid a kind of fines by whatever the forces that you apply in general. Right so whether it is grinder or pressing definitely there will be some kind of fines. You can control the larger size of particles but you cannot control the smaller finer particles. Some grinders may minimise these fines you can reduce these fines but you cannot eliminate them.

If the feed is uniform both physical and chemical structure then shapes of individual units in the product may be quite uniform. Let us say the micro structure of the particles in general having a role in size reduction. When you take a material and do the size reduction so whatever the particles that you are going to get, they shape primarily will have a kind of some resemblance with their micro structure. Let us say galano always produces if you take a big lumps of galano and then do the size reduction, you primarily get a particles of cubicle shape. Not exactly cubicles but resemblance to the cubicle shape you can have.

So, that is because of the micro structure of the galano. As I already mentioned whatever the size reduction you do compression, hammering or attrition you know the particle size distribution is going to be very wide irrespective of the uniformity of the feed the product the size reduced material is going to we have a very wide distribution of the size. So that is the ratio between the largest size particle and then smallest particle of the product is in general of the order of 10 power plus 4 such wide is the product size distribution for the product obtained after the size reduction.

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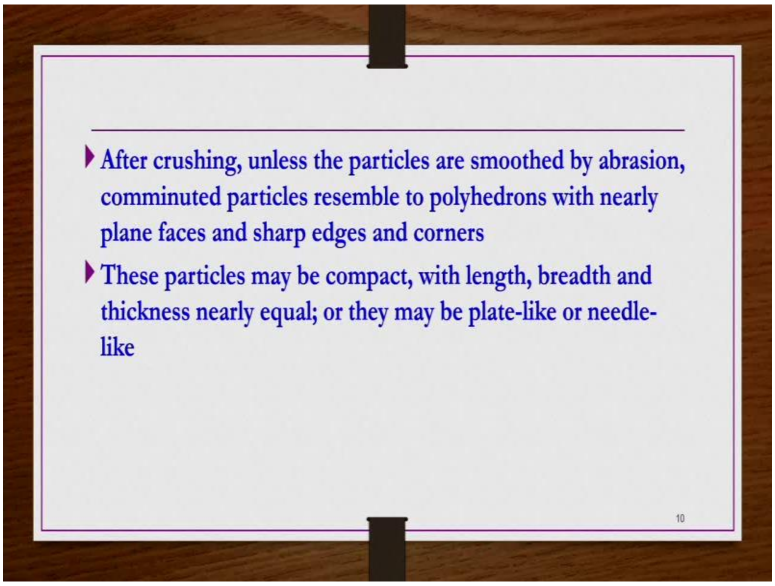

- After crushing, unless the particles are smoothed by abrasion, comminuted particles resemble to polyhedrons with nearly plane faces and sharp edges and corners
- These particles may be compact, with length, breadth and thickness nearly equal; or they may be plate-like or needle-like

Then after crushing unless the particles are smoothed by abrasion comminuted particles resembled to polyhedrons with nearly plain faces and sharp edges and corners. This polyhedrons in general may have four to seven faces depending upon the type of material that has been crushed. So, if you have more number of faces then sometimes they can also by abrasion while process they go through the abrasion and they become spherical shape otherwise in general the polyhedrons will have four to seven faces, they may be compact with length, breadth and width almost nearly equal. However, they also the kind of plate-like needle-like shape also.

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Energy and power requirements in comminution

- Major expense in crushing and grinding is the power cost
  - Factors that control power cost are very important
- During size reduction, the particles of feed material are first distorted and strained
  - Work necessary to strain the particles is stored temporarily in the solids as mechanical energy of stress (as in coiled spring)
- Then additional force is applied to the stressed particles so that they distorted beyond their ultimate strength and suddenly ruptured into fragments and new surfaces are generated

So, having seen the size reduction requirements and then their general characteristics, now we see the energy requirement for a given size reduction operation so energy and power requirements in comminution. Major expense in crushing and grinding is the power cost. We have variety of equipments, let us say we have a ball mill, ball mill is a size reduction equipments details we will be starting later again. In this ball mill usually you have a cylindrical container, in that cylindrical container you have a very basic size iron ball, metal balls like you know 10 centimetres 5 centimetres there may be some balls of 1 centimetre.

Those balls are kept inside the container, in that cylindrical container having these balls, metal or heavy balls we pour the solid material which we need to do the size reduction. And then we rotate the cylinder when we rotate the cylinders whatever the balls are there metal balls are there inside they will lifted up and then when they go to the top surface of the cylinder because of the gravity the fall down, when they fall down they show some kind of a force on the solid particles and those solid particles break depending upon the ultimate strength.

So this is how the size reduction takes place. Now, you need to provide the power for rotating the cylinders and then power for lifting these particles are things and then power for bearing, fitting etc. to avoid the friction between bearings and fittings etc. These many I mean this in the equipment you have to provide power for those many kind of things. But only a fraction of power is utilised by the particles were size reduction. Whatever the power required for lift the ball up, metal balls up and then they are falling down because of the gravity.

Only that much fraction in general is a kind of utilised by the particle for size reduction. All other power that is given for operating, for operation only, not for the size reduction of the given operations, like that you can see many other equipments also there are heavy kind of parts which need more power for operating than required for the original size reduction. Okay so that is the reason major expense in crushing and grinding is the power cost and that is the reason factors that control power cost are very much important.

So one has to find out what are the forces that are controlling the power cost and accordingly one has to tune the parameters so that the power expenses can be reduced. Then during size direction the particles of freedom material are first distorted and strained. Work necessary to strain the particles is stored temporarily in the solids as mechanical energy of stress. Then additional forces applied to the stressed particles so that they distorted beyond they are ultimate strength and suddenly ruptured into fragments and new surfaces are generated.

This is the mechanism in general how the size reduction taking place. The particles of the materials are first distorted and strained whatever the work necessary to strain the particles that is stored as temporarily in the solids as mechanical energy of stress. Then additional force when you apply to the stressed particle they are distorted beyond their ultimate strength and suddenly they are ruptured into fragments into the smaller particles and new surfaces are generated. This is the mechanism in general size reduction takes place.

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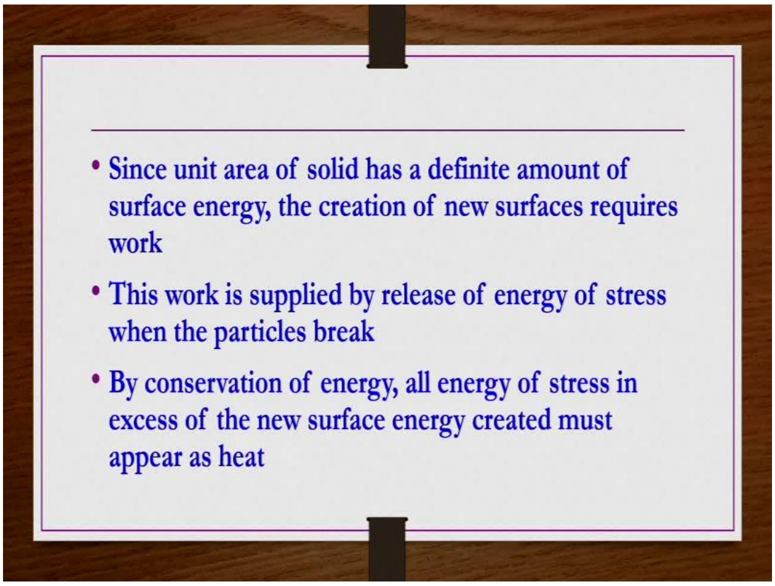

- Since unit area of solid has a definite amount of surface energy, the creation of new surfaces requires work
- This work is supplied by release of energy of stress when the particles break
- By conservation of energy, all energy of stress in excess of the new surface energy created must appear as heat

However, since unit area of solid has a definite amount of surface energy, the creation of new services requires work and then that is a kind of miser of efficiency of the process. This work is supplied by release of energy of stress when the particles break and then by conservation of energy all energy or stress in access of the new surface energy created must appear as feed. So whatever the energy that we or the power that we supply for the operating of the equipment and all that.

The all that energy is not being consumed by the particles so that they break, only fraction of the energy is being utilised for the solid reduction or the reduction of the solid particles but rest everything is not utilised it is only for the operating so that has to be dissipated and that usually dissipated as heat so that is the reason many of the size reduction operations during the operations during the operation or at the end of the operation when touches the surface of the equipment one can clearly feel the temperature is very much higher. The equipment is becoming very-very hot.

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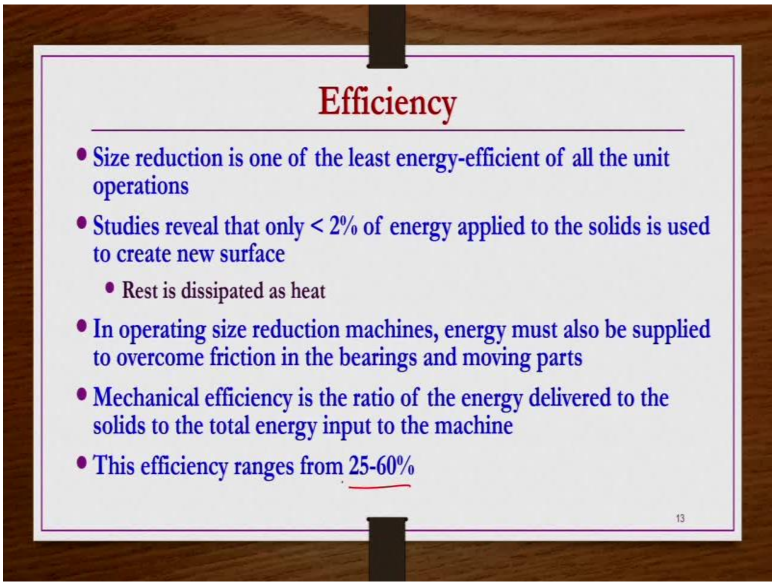

Efficiency

- Size reduction is one of the least energy-efficient of all the unit operations
- Studies reveal that only < 2% of energy applied to the solids is used to create new surface
  - Rest is dissipated as heat
- In operating size reduction machines, energy must also be supplied to overcome friction in the bearings and moving parts
- Mechanical efficiency is the ratio of the energy delivered to the solids to the total energy input to the machine
- This efficiency ranges from 25-60%

Efficiency, size reduction is one of the least energy-efficient of all unit operations. Why because as I mentioned you know for a given example of ball mill I have taken that example if you see only only small fraction of power that is supplied is utilised by the particles for the size reduction. Rest all you know for operating of the equipment and all that. So that is the reason you know the size reduction operation is one of the least energy efficient operations among the all unit operation of chemical engineering.

And then you will be surprised to know studies reveal that only less than 2% of the energy applied to the solid is used to create the new surface. Whatever the let us say you giving 100 kilo watt power for the required size reduction so only 2 or less than 2 kilowatts is being utilised for the size reduction rest everything is for the operating of the equipment and then overcoming the friction and bearing and all those things.

So all that energy rest energy is going into that one, so that is the reason you know this is a kind of very least energy efficient process, whatever the remaining energy is there that is dissipated as heat. In operating size reduction machines energy must also be supplied to overcome friction in the bearing and moving parts as I already mentioned. And then mechanical efficiency is ratio of energy delivered to the solids to the total energy input to the machine.

So since we have already seen that only fraction of energy is been solids for a given size reduction operation so that utilise energy divided by the total supply to the size reduction equipment that will give a kind of measure of mechanical efficiency. So the mechanical efficiency is the ratio of the energy delivered to the solids which are been reduced to form a new surface divided by the total energy input to the machines. The ratio of the energy delivered to the solids to the total energy into the machines that is the mechanical efficiency. And then this efficiency in general is very small in between 25 to 60%.

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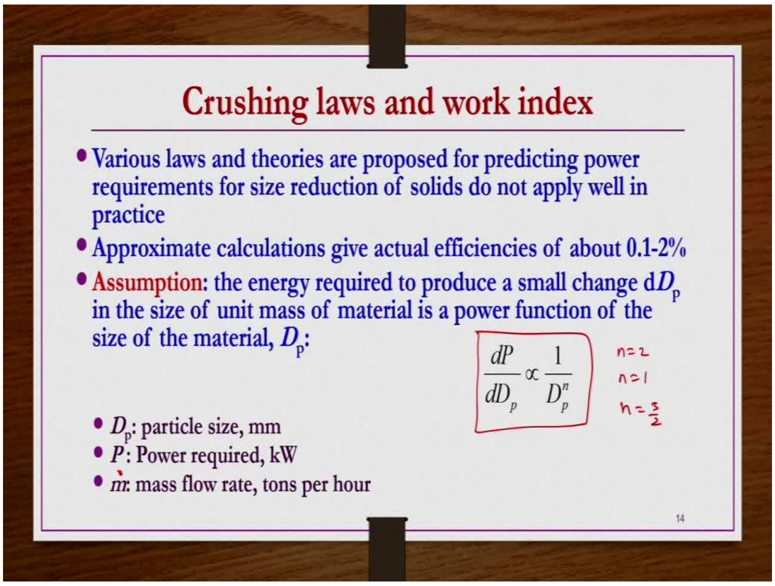

Crushing laws and work index

- Various laws and theories are proposed for predicting power requirements for size reduction of solids do not apply well in practice
- Approximate calculations give actual efficiencies of about 0.1-2%
- **Assumption**: the energy required to produce a small change dD_p in the size of unit mass of material is a power function of the size of the material, D_p:

$$\boxed{\frac{dP}{dD_p} \propto \frac{1}{D_p^n}}$$

$n=2$
$n=1$
$n=\frac{3}{2}$

- D_p: particle size, mm
- P: Power required, kW
- $\dot{m}$: mass flow rate, tons per hour

So, having seen the energy requirements what we see now we try to see the available crushing loss and walk index. So, like crushing laws in the sense they will be give a kind of a prior how much power one should supply for size reduction per unit KG of the feed. So let us say initially field you are having 50 mm particle sizes and then in the newer reducing them by size reduction so that you get 1 mm particle sizes in general. So that you know from 50 mm particle sizes to 1 mm average particle size if you wanted to convert. For a given equipment how much power is required per kg of the feed.

Per kg of the sample that you have taken that a priori information one can get by this crushing laws and then this crushing laws the different types of crushing loss we take only few of them which are found to be reliable. Various laws and theories are proposed for producing power requirements for size reduction of solids and unfortunately they do not apply well in practice. You apply these crushing loss and then you have a prior information how much power requirement for a given size reduction operations.

But according to those calculations if you go and try to implement in a kind of real life situations you may end up that you know you may not be getting the same thing. So that means this crushing laws are a kind of a indication how much power is required but they cannot given a kind of accurate number the relatively accurate number of the power that is required for size reduction.

So that is not possible because of the some limitation that we are going to see anyway. So, approximate calculations give actual efficiency of about 0.1 to 2% only. By using these things whatever the crushing laws that you use and then you do the approximate calculations you get actual efficiency of about 0.1 to 2% only. So, let us say actual efficiency something but when you apply these things you may get only 0.1 to 2% of that efficiency you may not get the actual efficiency. So, this is the other drawback however it solves some kind of kind of basis for selection of power how much power should be given kind of thing.

In deriving these crushing laws there are kind of assumption but there is a common one common assumption what is that assumption is the energy required to produce small a change dD_p in the size of unit mass of material is a power function of the size of the material Dp. Let us say initially you have a I know Dp size. Particle size of the size of the particle is Dp you wanted to reduce its size by small change dD_p. To bring that small change in the size of the initial particle size Dp how much power is required per unit mass?

That power requirement per unit mass is proportional to is a kind of power functions of this particle size. That is

$$\frac{dP}{dD_p} \propto \frac{1}{D_p^n}$$

. This is a kind of generalise assumption, if you take n equals to 2 you may get one law if you take n equal to 1 you may get another law. If you can take n equal to 3 by 2 you may get another crushing law so different crushing laws or you know generalised by this particular assumption. So, here dp's particle size which is in mm P is power requirement in kilo watts and then m. Is a kind of mass flow rate which is in ton/h. So, now we take one by one each of this crushing laws.

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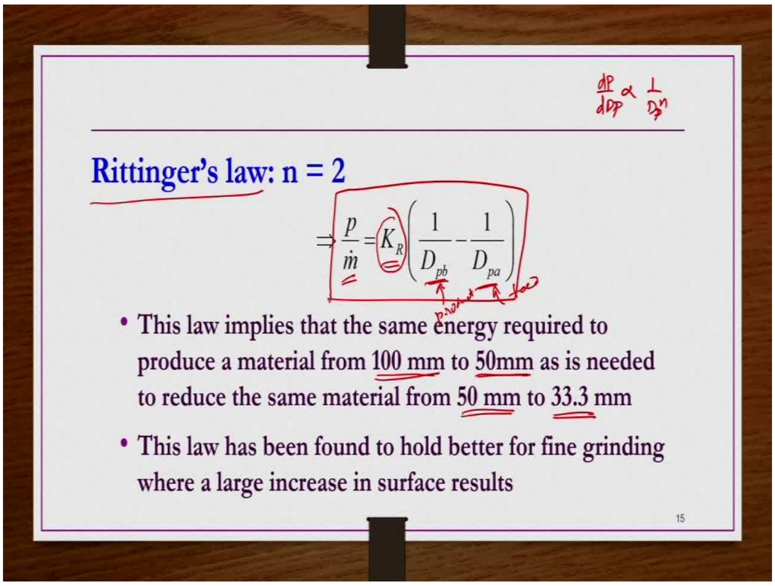

$\frac{dP}{dD_p} \propto \frac{1}{D_p^n}$

Rittinger's law: n = 2

$$\Rightarrow \boxed{\frac{p}{\dot{m}} = K_R\left(\frac{1}{D_{pb}} - \frac{1}{D_{pa}}\right)}$$

product feed

- This law implies that the same energy required to produce a material from 100 mm to 50mm as is needed to reduce the same material from 50 mm to 33.3 mm
- This law has been found to hold better for fine grinding where a large increase in surface results

First we take n=2 if you take n=2 in this equation here if you take n=2 $\frac{dP}{dD_p} \propto \frac{1}{D_p^n}$ and then do the integration and substitute the limits of feed size to the product size then you will get $\frac{P}{\dot{m}} = K_R\left(\frac{1}{D_{pb}} - \frac{1}{D_{pa}}\right)$ that constant is known as the Rittinger's law constant. If n is equal to 2 if you substitute n equal to 2 in this particular equation whatever the crushing that you get that is known as the Rittinger's law

. And then after integration substitute the limits so you will get that constant K_R that K_R is known as the Rittinger's constant and it is dimensional quantity it is not dimensional is it is having some dimensions.

And then here D_{pb} is the average size of the particle in the product. That is after the size reduction whatever the that you take to hope that you get the average size of the product is D_{pb} and then D_{pa} is the average size of the feed before crushing whatever the feed that you have taken the size of the of that particular feed is D_{pa}. Okay and then m. Is the mass rate of the feed and then p is the power requirement for this size reduction to reduce the size of this particle from D_{pa} to D_{pb}.

So this is for the feed size, this is for the product size. this equation as long as the $\frac{1}{D_{pb}} - \frac{1}{D_{pa}}$ remains same the power requirement is going to be same that is what you can understand from this equation because for a given material and for a given equipment K_R is a constant. Right so as long as this $\frac{D_{p1}}{D_{pb}} - \frac{1}{D_{pa}}$remains same you will get the same power requirement.

But that is not true that is not true, if you have a kind of average size of the feed material is 100 mm you are doing some size reduction then you get the product having the average size of the particle is 50 mm. So (1/50 – 1/100) you will get 1/100 right. So whatever the power of you required for that case you know you let us say some x kilowatts. Then you take a feed 50 mm and then product that you get let us say if you are expecting to have 33.3 mm product size average size of the product you want 33.3 mm.

Then again (1/33.3 – 1/50) is again one by hundreds close to 1/100. So that means in this case also the power requirement would be same. But in practical that is not going to be true. That is the reason I mention this crushing law are you not do not hold well with the practical situation, but they gave a kind of a priority information.

So that is not going to be true, indirectly you are saying that you know 100 mm particle size you do you take and then crush them to get a 50 mm size particle size whatever the power requirement is there, the same power will be required said he wanted to reduce that 50 mm size particles to further smaller 33.3 mm particle so that is not going to be true in general. Okay so that is one of the limitations. This limitation is therefore other laws also.

And this law has been found to hold better for fine grinding will wear a large increase in surface results. Okay so wherever you have the fine grinding it is better to use Rittinger’s law in general. So that you have a better reliability these things are you know one find from the experience from the experience of different equipments and different types of material. So in general if you are doing fine grinding then power requirement calculation for that fine grinding if you need to do so it is better to use the Rittinger’s law.

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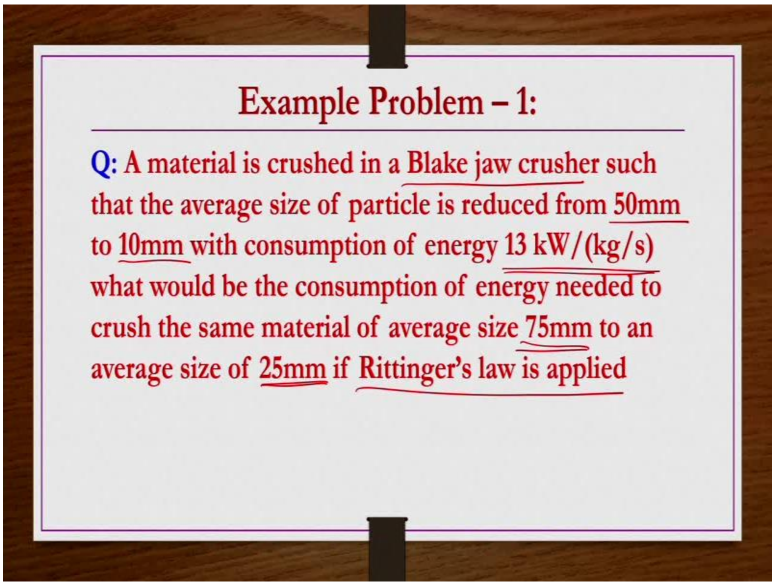

Example Problem – 1:

Q: A material is crushed in a Blake jaw crusher such that the average size of particle is reduced from 50mm to 10mm with consumption of energy 13 kW/(kg/s) what would be the consumption of energy needed to crush the same material of average size 75mm to an average size of 25mm if Rittinger's law is applied

Now take one example problem before going to the next crushing. So a material is crashed in a kind of a crusher the feed size is 50 mm and then you are expected to have a product average size 10 mm so for this case the power energy consumption is 13 kW/kg/s. So kilowatt is power, kg/s is a kind of mass rate. So that means $P/\dot{m}$ Is given. Feed size is given, product size is given and then $P/\dot{m}$ is given. So from this information one can find out the what is constant. What is the proportionality K_R constant Rittinger's constant that you can find out?

Then what would be the consumption of energy needed to crush the same material of average size, now if the feed becomes bigger if you are next time handling bigger size feed 75 mm and then you are trying to get 25 mm product average size, then how much power is required if you apply the Rittingers' law.

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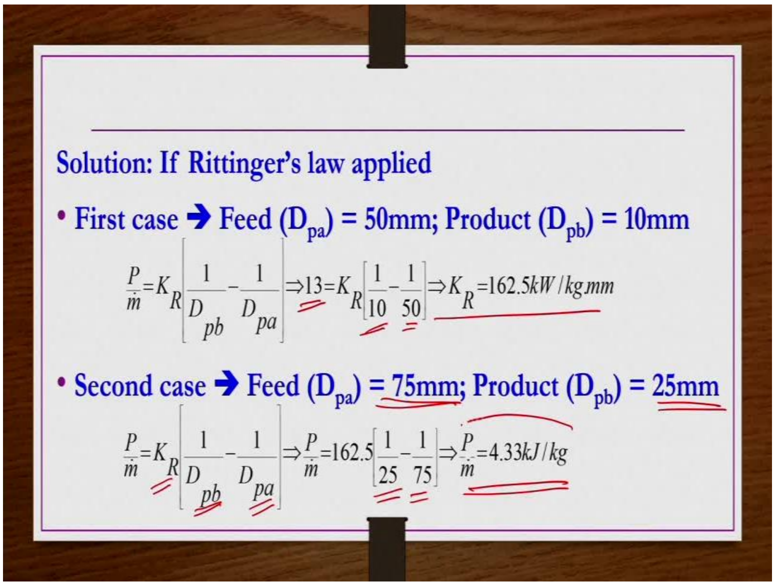

Solution: If Rittinger's law applied

- First case ➔ Feed (D_{pa}) = 50mm; Product (D_{pb}) = 10mm

$$\frac{P}{\dot{m}} = K_R\left[\frac{1}{D_{pb}} - \frac{1}{D_{pa}}\right] \Rightarrow 13 = K_R\left[\frac{1}{10} - \frac{1}{50}\right] \Rightarrow K_R = 162.5 kW/kg\,mm$$

- Second case ➔ Feed (D_{pa}) = 75mm; Product (D_{pb}) = 25mm

$$\frac{P}{\dot{m}} = K_R\left[\frac{1}{D_{pb}} - \frac{1}{D_{pa}}\right] \Rightarrow \frac{P}{\dot{m}} = 162.5\left[\frac{1}{25} - \frac{1}{75}\right] \Rightarrow \frac{P}{\dot{m}} = 4.33 kJ/kg$$

So it is very simple, first case the feed $D_{pa} = 50mm$ the product $D_{pb} = 10mm$ the average size of the feed is 50mm the average size of the product is 10mm now you apply the Rittinger's law

$$\frac{P}{\dot{m}} = K_R\left(\frac{1}{D_{pb}} - \frac{1}{D_{pa}}\right)$$

$$\Rightarrow 13 = K_R\left(\frac{1}{10} - \frac{1}{50}\right) \Rightarrow K_R = 162.5\ {}^{KW}/_{kgmm}$$

Now, in the second case if the feed average size becomes 75 mm and then you are expected to get the product average size 25 mm, how much power is required, that is how much $\frac{P}{\dot{m}}$ required that you need to calculate because $\dot{m}$ is not given.

So, now use the same equation K_R just now you calculated as 162.5, in this case D_{pb} is 25 second case, and then feed in the second case is 75mm, so that you substitute here then you get $\frac{P}{\dot{m}} = 4.33kJ/kg$. This is the power requirement if you apply the Rittinger's law. So, let us remember this value because the same problem using the same values we are going to do for by applying the other law and the we can compare the power requirement, are they giving the same power requirements or not. Right so by Rittinger's law for this problem we are getting power requirement 4.33 KJ/kg okay.

$$\frac{P}{\dot{m}} = K_R\left(\frac{1}{D_{pb}} - \frac{1}{D_{pa}}\right)$$

$$\Rightarrow \frac{P}{\dot{m}} = 162.5\left(\frac{1}{25} - \frac{1}{75}\right) \Rightarrow \frac{P}{\dot{m}} = 4.33\, {}^{kJ}\!/_{kg}$$

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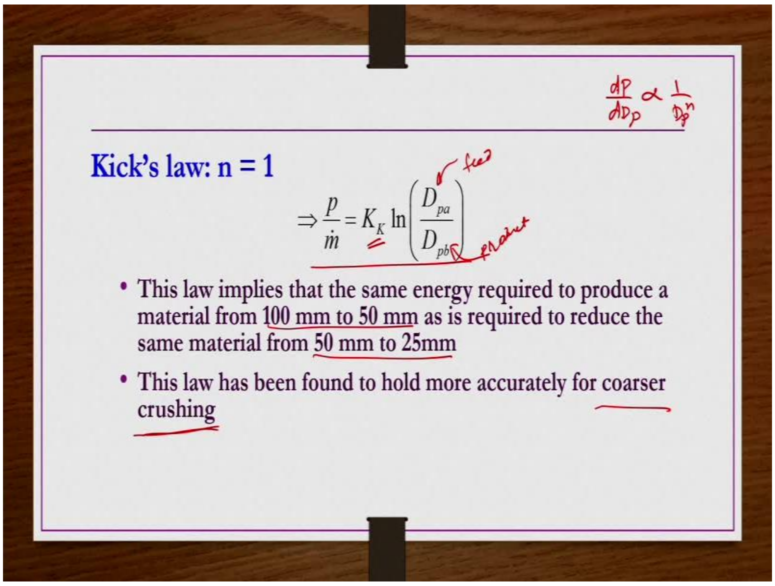

$\frac{dP}{dD_p} \propto \frac{1}{D_p^n}$

Kick's law: n = 1

$$\Rightarrow \frac{p}{\dot{m}} = \underset{=}{K_K} \ln\left(\frac{\overset{\text{feed}}{D_{pa}}}{\underset{\text{product}}{D_{pb}}}\right)$$

- This law implies that the same energy required to produce a material from 100 mm to 50 mm as is required to reduce the same material from 50 mm to 25mm
- This law has been found to hold more accurately for coarser crushing

Next Kick's law, if you substitute in this $\frac{dP}{dD_p} \propto \frac{1}{D_p^n}$, you substitute n is equal to one then integrate and then substitute the limits of D_{pa} and D_{pb} that is the feed average size to average particle product size then you will get this Kick's law by substituting n equal to 1 so that comes out to be $\frac{P}{\dot{m}}$ is equal to some proportionality constant $K_K \ln\left(\frac{D_{pa}}{D_{pb}}\right)$.

$$\frac{P}{\dot{m}} = K_K \ln\left(\frac{D_{pa}}{D_{pb}}\right)$$

This K_k is known as the Kick's law constant. It is again dimensional quantity, it is not a dimensionless quantity it is also having dimensions and then same as the previous case $D_{pa}$is the feed average size and then D_{pb} is product average size. Now here also what we understand as long as D_{pa} by D_{pb} ratio remains same so the power requirement is going to be same.

Let us say D_{pa} 100 D_{pb} is 50 so 100/50 is 2 so whatever the power required for reducing the size of you know feed 100mm to get product 50mm then whatever the power requirement is there that is that is that going to be same for the feed 50 mm and then getting product 25 mm because 50/25 is again 2. But that is in general not true that is the reason this kind of restrictions is there here also, here also.

That is this law applies that same energy require to produce a material from 100 mm to 50 mm as is required to reduce from 50 mm to 20 mm because 100/50 is 2 and 50/25 is again 2. So, according to this law as long as D_{pa} by D_{pb} ratio remains same for a given material being crushed in a given equipment that power requirement is going to be same but in reality that is not true in general.

This law has been found to hold more accurately for coarser crushing. That is been used compressive forces etc. or coarse reduction when you do so Kick's law founds to be reliable whereas the Rittinger's law is found to be very much reliable if you are doing fine grinding fine reduction.

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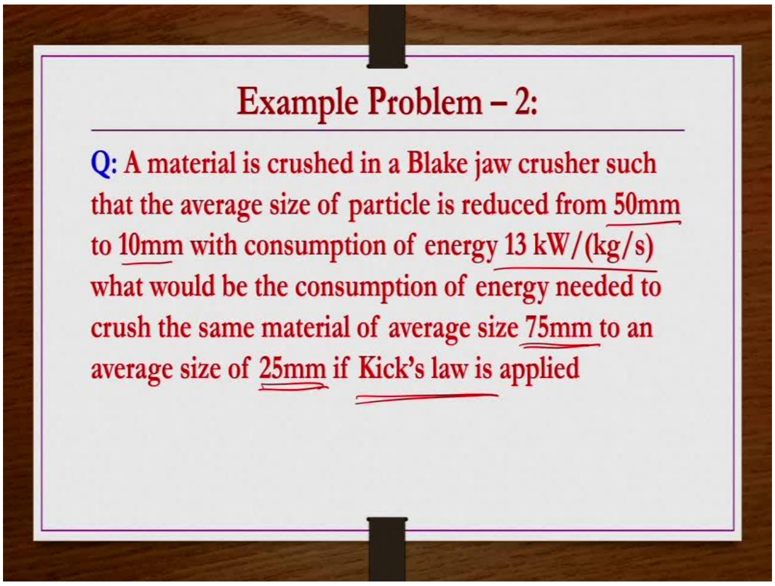

Example Problem – 2:

Q: A material is crushed in a Blake jaw crusher such that the average size of particle is reduced from 50mm to 10mm with consumption of energy 13 kW/(kg/s) what would be the consumption of energy needed to crush the same material of average size 75mm to an average size of 25mm if Kick's law is applied

Now, the same example problem whatever you taken in the Rittinger's law case, now we applied Kick's law that is initial feed size is 50mm and then 10mm is the product average size 30 kilowatt per kg per second power requirement is there and then in the second case if the feed becomes slightly bigger 75mm and then the product becomes 25mm how much power is required. These numbers feed and product sizes and then power requirement in the first case is exactly the same. So second case whatever the power requirement is it going to be the same as in the Rittinger's law or no? That is what we are going to see.

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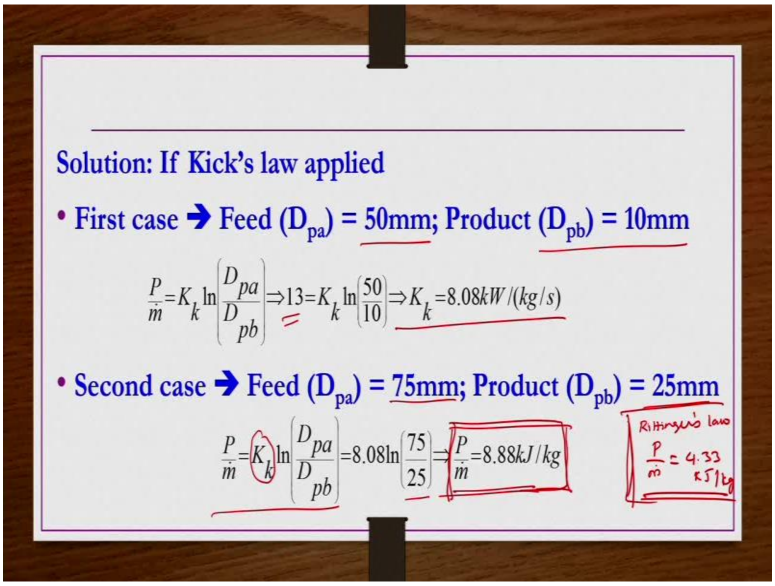

Solution: If Kick's law applied

- First case ➔ Feed (D_{pa}) = 50mm; Product (D_{pb}) = 10mm

$$\frac{P}{\dot{m}} = K_k \ln\left(\frac{D_{pa}}{D_{pb}}\right) \Rightarrow 13 = K_k \ln\left(\frac{50}{10}\right) \Rightarrow K_k = 8.08 kW/(kg/s)$$

- Second case ➔ Feed (D_{pa}) = 75mm; Product (D_{pb}) = 25mm

$$\frac{P}{\dot{m}} = \boxed{K_k} \ln\left(\frac{D_{pa}}{D_{pb}}\right) = 8.08 \ln\left(\frac{75}{25}\right) \Rightarrow \boxed{\frac{P}{\dot{m}} = 8.88 kJ/kg}$$

Rittinger's law

$$\boxed{\frac{P}{\dot{m}} = 4.33 \text{ kJ/kg}}$$

So Kick's law if you apply first case D_{pa}=50mm the product the feed size D_{pa}=50mm, the products size D_{pb}=10mm when you apply the Kick's law $\frac{P}{\dot{m}} = K_K \ln\left(\frac{D_{pa}}{D_{pb}}\right)$

. So, $\frac{P}{\dot{m}} = 13$. D_{pa} is 50 D_{pb} is 10 so K_k you get 8.08KW/kg.s

$$\Rightarrow 13 = K_K \ln\left(\frac{50}{10}\right) \Rightarrow K_K = 8.08 \, {}^{kW}\!/\!{}_{kg}\,/_{s}$$

Now, second case, feed is D_{pa} 75mm average size of feed is 75mm and then average size of the product is 25mm, so then how power is required we apply the same equation $\frac{P}{\dot{m}} = K_K \ln\left(\frac{D_{pa}}{D_{pb}}\right)$, K_K we just now found it as 8.08 that you can substitute here but now D_{pa} is 75 and D_{pb} is 25 so then you get 8.88 kilo joules per kg.

$$\frac{P}{\dot{m}} = K_K \ln\left(\frac{D_{pa}}{D_{pb}}\right)$$

$$\frac{P}{\dot{m}} = 8.08 \ln\left(\frac{75}{25}\right) = 8.88 \, {}^{kJ}\!/\!{}_{kg}$$

But we have seen Rittinger's law just now in the previous case same problem what we have we obtained $\frac{P}{\dot{m}} = 4.33 kJ/kg$. So now you can see by using different types of laws you get the different power requirements. So you need to kind of have an experience in what case what kind of crushing law should you use. So what we have seen if we are doing the fine reduction so Rittinger's law is a better option. If we are doing coarse reduction then Kick's law is the better one or the numbers are requirement numbers power requirement numbers given by Kick's law under coarse reduction is more reliable.

And the power requirements given by Rittinger's law for size reduction is more reliable. In this case both product and feed are you know bigger size in 50mm 10mm like that so they come under coarse size reduction. So for this problem whatever the numbers that gets by Kick's law that is going to be more reliable. So that comes with the experience of doing several operations and then finding out.

So, people have found that for coarse reduction power obtained by Kick's law is more reliable. So that is this number is going to be more reliable than this number because Rittinger's law is good for the fine reduction not for the coarse reduction whereas the size in this problem is a kind of in the coarse reduction size, so this is going to be more reliable 8.88KJ/kg is going to be more reliable.

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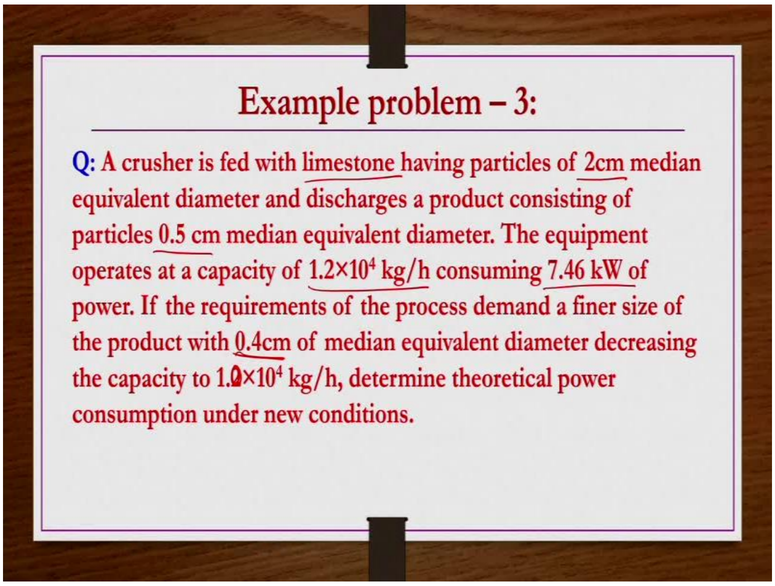

Example problem – 3:

Q: A crusher is fed with limestone having particles of 2cm median equivalent diameter and discharges a product consisting of particles 0.5 cm median equivalent diameter. The equipment operates at a capacity of 1.2×10^4 kg/h consuming 7.46 kW of power. If the requirements of the process demand a finer size of the product with 0.4cm of median equivalent diameter decreasing the capacity to 1.0×10^4 kg/h, determine theoretical power consumption under new conditions.

So, next before going to the law we will take an example problem in Kick's law. You are a crusher is fed with limestone having particles 2 cm median equivalent diameter. What do you mean by median equivalent diameter? That means 50% of if the median equivalent diameter is dm so that means 50% of material would be having says less than dm and then 50% of the material would be having more than dm size. Then the dm is known as the median equivalent diameter of the sample.

So here in the sample 2 centimetres is the median equivalent diameter that means the 50 percent of the material in the sample is having less than 2 centimetre size and then 50 percent of the sample is having more than 2 centimetre. So feed is 2 centimetre size, discharge product is having 0.5 median equivalent size. The capacity the flow rate is the mass rate is 1.2 into 10^4 kg/hour. And under such condition it is consuming 7.46 kilowatt of power if the requirements of the process demand a finest size of the product.

Now, the feed is same but the product if you wanted to further make it fine to 0.4 centimetre of median equivalent diameter by decreasing the capacity 1.0 into, (1.0 x 10^4 kg/hour) then how much power would be required. So, now the product in the second case product size is reduced as well as the capacity is also reduce. So, by that way by changing this operating condition requirement as well as the operating conditions mass rate conditions the power requirement is it going to change or is it going to be same that is what we are going to calculate.

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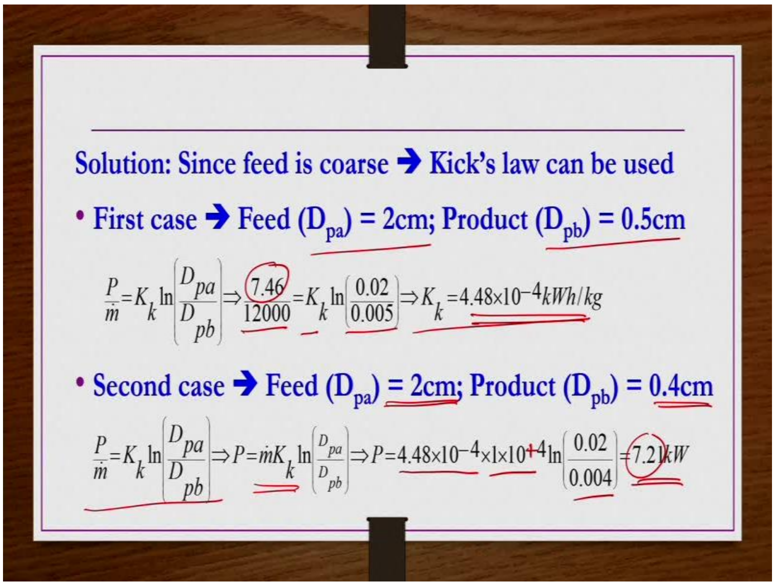

Solution: Since feed is coarse ➔ Kick's law can be used

- First case ➔ Feed (D_{pa}) = 2cm; Product (D_{pb}) = 0.5cm

$$\frac{P}{\dot{m}}=K_k\ln\left(\frac{D_{pa}}{D_{pb}}\right)\Rightarrow\frac{\boxed{7.46}}{12000}=K_k\ln\left(\frac{0.02}{0.005}\right)\Rightarrow K_k=4.48\times10^{-4}kWh/kg$$

- Second case ➔ Feed (D_{pa}) = 2cm; Product (D_{pb}) = 0.4cm

$$\frac{P}{\dot{m}}=K_k\ln\left(\frac{D_{pa}}{D_{pb}}\right)\Rightarrow P=\dot{m}K_k\ln\left(\frac{D_{pa}}{D_{pb}}\right)\Rightarrow P=4.48\times10^{-4}\times1\times10^{+4}\ln\left(\frac{0.02}{0.004}\right)=\boxed{7.21}kW$$

So, in the first case first of all the problem it is not given which law should be used since the size of the feed as well as the product in the centimetre cases what we can say it is going to be in coarse reduction category so for coarse reduction category we have seen that Kick's law is a kind of better option compared to the Rittinger's law. So we are going to use here Kick's law. First case D_{pa} is 2 centimetres the feed size is 2 centimetre product size is 0.5 centimetres.

So

$\frac{P}{\dot{m}} = K_K \ln\left(\frac{D_{pa}}{D_{pb}}\right)$

$\frac{P}{\dot{m}}$ we substitute here, Kk we do not know, $D_{pa} = 2cm$ and $D_{pb} = 0.5cm$ so from here what we get Kk as 4.48 into 10^{-4} KWh/kg, you can see this is dimensional quantity as I mentioned it is not dimensionless.

$$\frac{P}{\dot{m}} = K_K \ln\left(\frac{D_{pa}}{D_{pb}}\right) \Rightarrow \frac{7.46}{1200} = K_K \ln\left(\frac{0.02}{0.005}\right) = 4.48 * 10^{-4}\, {}^{kWh}\!/_{kg}$$

But now in the second case feed is again remaining same 2 centimetre median equivalent size but product is 0.4 size the product is becoming slightly finer and then capacity is decreased to 1×10^4 kg per hour.

Then we apply the same law what we have $\frac{P}{\dot{m}} = K_K \ln\left(\frac{D_{pa}}{D_{pb}}\right)$

, m we can take to the right hand side $P = \dot{m} K_K \ln\left(\frac{D_{pa}}{D_{pb}}\right)$ so then So from this conclusion what we understand for this problem not generalised one specific $P = 4.48 \times 10^{-4} \times 1 \times 10^4 \ln\left(\frac{0.02}{0.004}\right) = 7.2kW$, to this problem.

When you reduce the or when you make the product slightly finer and at the same time if you reduce the capacity of the equipment whatever the mass rate if you slightly reduce the power requirement is not going to affect much because in the first case 7.46, in the second case 7.21 kilowatt, similar kind of thing. But this is specific to this problem, we cannot generalise this conclusion for all kind of situations.

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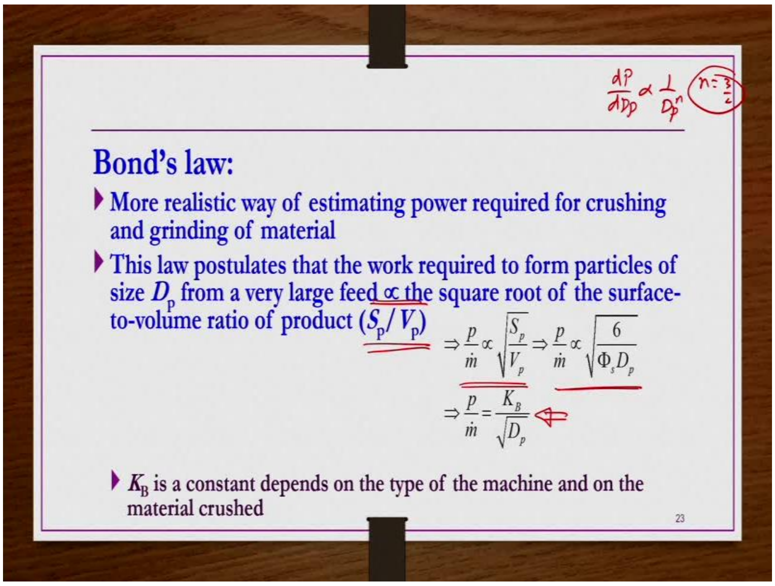

$$\frac{dP}{dD_p} \propto \frac{1}{D_p^n} \quad \left(n = \frac{3}{2}\right)$$

Bond's law:

- More realistic way of estimating power required for crushing and grinding of material
- This law postulates that the work required to form particles of size D_p from a very large feed ∝ the square root of the surface-to-volume ratio of product (S_p/V_p)

$$\Rightarrow \frac{P}{\dot{m}} \propto \sqrt{\frac{S_p}{V_p}} \Rightarrow \frac{P}{\dot{m}} \propto \sqrt{\frac{6}{\Phi_s D_p}}$$

$$\Rightarrow \frac{P}{\dot{m}} = \frac{K_B}{\sqrt{D_p}}$$

- K_B is a constant depends on the type of the machine and on the material crushed

Next one is the Bond's law there are two different ways of doing it, let us say if we have

$$\frac{dP}{dD_p} \propto \frac{1}{D_p^n}$$

, if you take n equal to 3/2 here and then do simplifications you get the Bond's law. That is one way other way is you know you can find the other ways also that we are going to see. So it has been found that this Bond's law is a kind of more realistic way of estimating power required for crushing and then grinding of material because this law postulates that work required to form particles of size dp from a very large feed is proportional to the square root of the surface to volume ratio of the product.

So whatever the power requirement is there that is proportional to the square root of sp by vp of the product. So, that is $\frac{P}{\dot{m}} \propto \sqrt{\frac{S_p}{V_p}}$ of the product that is going to be formed and then we have seen previously this $\frac{S_p}{V_p} = \frac{6}{\emptyset_s D_p}$. Dp is the nominal size of the product now, so from here if you take out the proportionality and then whatever the proportionality constant combined with 6 pi etc. all those things you can write this equation as $\frac{P}{\dot{m}} = \frac{K_b}{\sqrt{D_p}}$. K_b is Bond's constant.

$$\frac{P}{\dot{m}} \propto \sqrt{\frac{S_p}{V_p}} \Rightarrow \frac{P}{\dot{m}} \propto \sqrt{\frac{6}{\emptyset_s D_p}} \Rightarrow \frac{P}{\dot{m}} = \frac{K_b}{\sqrt{D_p}}$$

And then it is function of the type of machine used for reduction and then as well as the type of material that is been crushed. Okay, so this is the Bond's law but we are trying to use dressed in the same Bond’s law in a different form right. For that we need to define a kind of work index.

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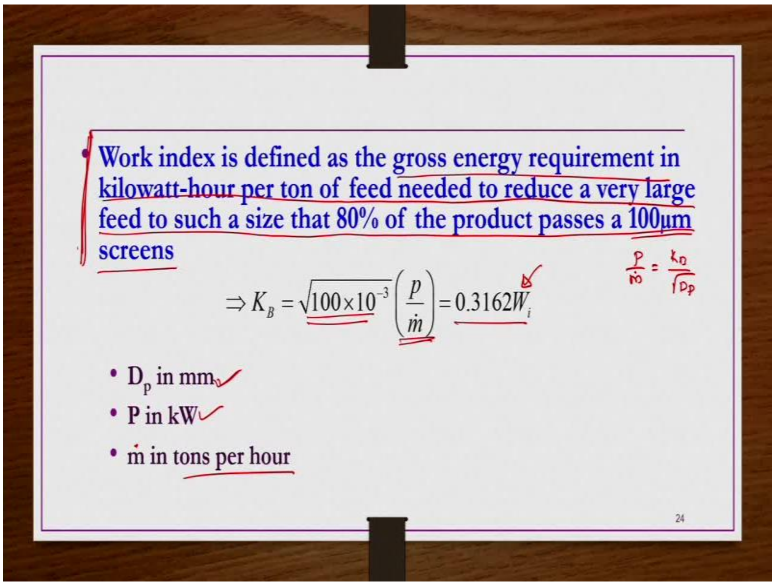

- Work index is defined as the gross energy requirement in kilowatt-hour per ton of feed needed to reduce a very large feed to such a size that 80% of the product passes a 100μm screens

$$\Rightarrow K_B = \sqrt{100\times 10^{-3}}\left(\frac{P}{\dot{m}}\right) = 0.3162W_i$$

$$\frac{P}{\dot{m}} = \frac{K_B}{\sqrt{D_p}}$$

- D_p in mm
- P in kW
- $\dot{m}$ in tons per hour

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Overall index is defined as the gross energy requirement in kilowatt power bars are done for feed needed to reduce a very large feed to reduce a size that 80 percent of the product passes and 10 micron screen. So, this walk index W_i is defined as, is nothing but $\frac{P}{\dot{m}}$ for a case where the large material large size particle you know crushed such a way that 80 percent of the product should pass through 100 microns screen, so that is Dp is going to be 100 microns, so whatever the $\frac{P}{\dot{m}} = \frac{K_b}{\sqrt{D_p}}$

So, here now dp is you take 100 microns, in this law actually we have already seen that p you know kind of previous case kilowatts and Dp is in mm it has to be taken in mm. So then what you get kb is equals to square root of Dp is nothing but 100 mircons, that is 100 into 10 power minus 3 mm $\frac{P}{\dot{m}}$ So this $\sqrt{100 \times 10^{-3}}$. This value is going to be 0.3162, so 0.3162 and then $\frac{P}{\dot{m}}$ is nothing but Wi.

$$\Rightarrow K_b = \sqrt{100 \times 10^{-3}}\left(\frac{P}{\dot{m}}\right) = 0.3162W_i$$

This Wi is a kind of work index. So here in this equation whatever the Dp that we have taken that should be in mm and then power should be in kilowatts and then m dot the mass rate should be in tons per hour. So that is work index how you define. This is going to be very important Bond’s law because when you need to do analysis power size calculation sometimes this D_{pa} D_{pb} sizes are not given, they may be given in kind of screen analysis something like that. So you have to find out what is D_{pa} D_{pb} from the screen analysis data. That is also becomes essential sometimes.

So how to find out? You have to select a size you know through which 80 percent is passing through a kind a kind of required size if it is a feed. If it is a product similarly what is this size? 80% of the product is passing through that should be taken as the average size of the product size. We are going to see the problem anyway so that is the reason the definition of this work index is kind of very important in understanding and solving the problems. So work index is defined as gross energy requirement in kilowatt hour per ton of feed needed to reduce a very large feed to search a size that 80% of the product causes 100 micron screen.

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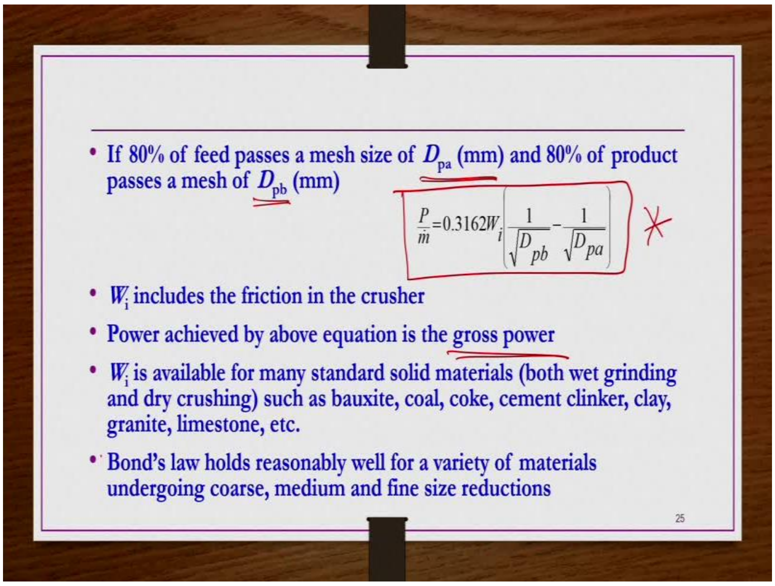

- If 80% of feed passes a mesh size of D_{pa} (mm) and 80% of product passes a mesh of D_{pb} (mm)

$$\boxed{\frac{P}{\dot{m}}=0.3162W_i\left(\frac{1}{\sqrt{D_{pb}}}-\frac{1}{\sqrt{D_{pa}}}\right)}$$

- W_i includes the friction in the crusher
- Power achieved by above equation is the gross power
- W_i is available for many standard solid materials (both wet grinding and dry crushing) such as bauxite, coal, coke, cement clinker, clay, granite, limestone, etc.
- Bond's law holds reasonably well for a variety of materials undergoing coarse, medium and fine size reductions

If 80 percent of the feed passes a mesh size D_{pa} and then 80% of the product passes a mesh size D_{pb} so then D_{pa} should be feed size and D_{pb} should be product size. So this is how you need to find out if D_{pa} D_{pb} is not given right, because for that Bond's law it is required that you know when you know when you use a work index so then 80% concept should come out that 80% of the material passing through you know whichever size is there that should be taken as a kind of size required size of the calculations.

Then if you write in terms of both feed and product product form the feed size and product size so then the same Bond's law we can write now then $\frac{P}{\dot{m}} = 0.3162W_i$

so and then this remaining $\left(\frac{1}{\sqrt{D_{pb}}}-\frac{1}{\sqrt{D_{pa}}}\right)$ that you get. So this is the other form of the Bond's law when you are taking the reference of two sizes feed size as well as the product size. So, $\frac{P}{\dot{m}}=0.3162W_i\left(\frac{1}{\sqrt{D_{pb}}}-\frac{1}{\sqrt{D_{pa}}}\right)$

This Wi work index includes the friction in the crusher and then power achieved by this above equation is the gross power which includes the friction etc. the value of W_i is available for many standard solid material both wet grinding and dry crushing such as bauxite material, coal, coke, cement clinker clay granite limestone etc. for several materials this W_i information is available. As we have seen that kb is function of the equipment used and then what type of material have been crushed.

So is the W_i also so because W_i is related to K_B directly so that is the reason is function of again the equipment used for the crushing as well as the material that has been used for the crushing and then material that has been crushed. So both wet grinding and dry crushing for the several kind of material this W_i information is available.

And then fortunately this Bond's law holds reasonably well for variety of material undergoing coarse, medium and fine reduction unlike the other two laws where Rittinger's law is good, holds good only for fine reduction and then Kick's law holds good only for coarse reduction but Bond's law holds good for all 3 types of reduction, coarse, medium and fine reduction.

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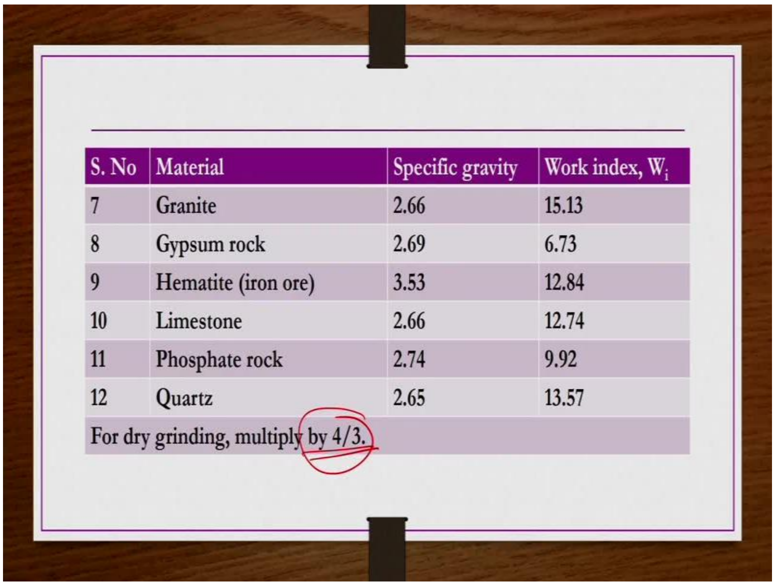

S. No	Material	Specific gravity	Work index, W_i
7	Granite	2.66	15.13
8	Gypsum rock	2.69	6.73
9	Hematite (iron ore)	3.53	12.84
10	Limestone	2.66	12.74
11	Phosphate rock	2.74	9.92
12	Quartz	2.65	13.57
For dry grinding, multiply by 4/3.			

We will see some numbers for W_i for dry crushing and grinding different types of material this W_i numbers are given here. They are given for wet grinding, so whatever the materials given here that is given for the wet grinding. If you are doing the dry grinding so these numbers whatever the W_i numbers given in this tables they should be multiplied by 4/3. In the table whatever the W_i numbers are given are given for the weight reduction size reduction. If you are doing the dry size reduction by dry grinding then these number should be multiplied by 4/3.

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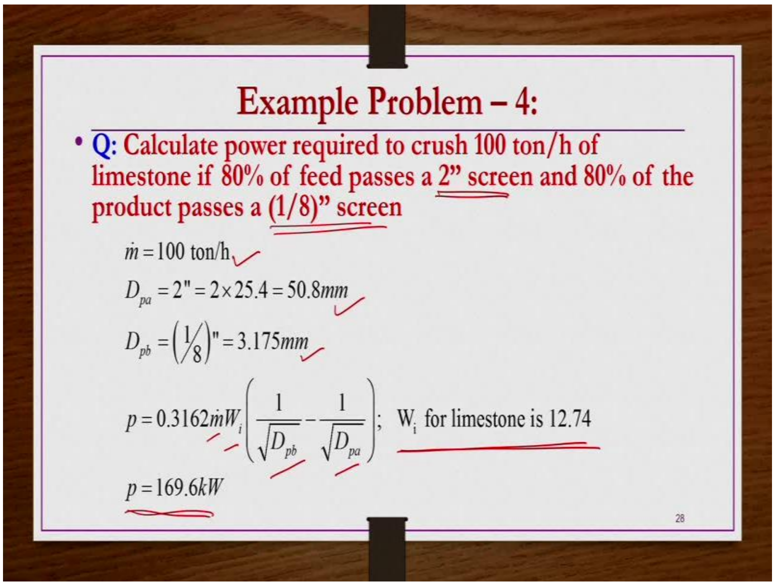

Example Problem – 4:

- **Q:** Calculate power required to crush 100 ton/h of limestone if 80% of feed passes a 2" screen and 80% of the product passes a (1/8)" screen

$$\dot{m} = 100 \text{ ton/h}$$

$$D_{pa} = 2" = 2 \times 25.4 = 50.8mm$$

$$D_{pb} = \left(\tfrac{1}{8}\right)" = 3.175mm$$

$$p = 0.3162\dot{m}W_i\left(\frac{1}{\sqrt{D_{pb}}} - \frac{1}{\sqrt{D_{pa}}}\right); \quad W_i \text{ for limestone is } 12.74$$

$$p = 169.6kW$$

Now, we take an example problem so that we can understand how to apply this Bond's law for power requirement for a given size reduction operation. So, calculate power required to crush 100 tons/hour of limestone, if 80 percent of feed passes a 2 inch screen and 80 percent of the product passes 1/8 inch screen, so it is clearly given that 80% of feed is passing 2 screen. So that means D_{pa} is nothing but 2 inch screen, so D_{pa} is nothing by 2 inch, and then D_{pb} is nothing but 1/8 inch because product 80 percent of the product passes through 1/8 inch screen.

So, this D_{pa} is nothing but 2 inches that you have to convert into mm and D_{pb} is nothing but 1/8 inches that you have to convert into mm as per the requirement of the problem. So, m dot is given 100 ton/hour, so we need m dot in tons per hour so that is as it is. D_{pa} is 2 inches so you convert into the mm. D_{pb} is 1/8 inches that also you convert into the mm. Then we have

$$\frac{P}{\dot{m}} = 0.3162W_i\left(\frac{1}{\sqrt{D_{pb}}} - \frac{1}{\sqrt{D_{pa}}}\right)$$

and then from the tables we can understand the limestone for limestone. Wi is nothing but 12.74, 12.74.

So you substitute D_{pa} D_{pb} m and W_i here in this equation so you will get power requirement as 169.6 kilowatts.

$$\dot{m} = 100\ {}^{ton}/_{h}$$

$$D_{pa} = 2^n = 2 * 25.4 = 50.8mm$$

$$D_{pb} = \left({}^{1}/_{8}\right)^n = 3.175mm$$

$$\frac{P}{\dot{m}} = 0.3162W_i$$

W_i for limestone is 12.74

$$P = 169.6kW$$

Okay, so compared to the previous class what is the additional thing that we need to give attention here, what is the D_{pa} size D_{pb} size. You have to take based on the 80 percent of the material passing through which screen that you have to take as the size of that particular sample whether it is feed or product. So, here 80% of feed is passing through 2 inch that is the reason D_{pa} is 2 inches. Similarly, 80% of the product is passing through 1/8 inch screen that is the reason here D_{pb} is 1/8 inches.

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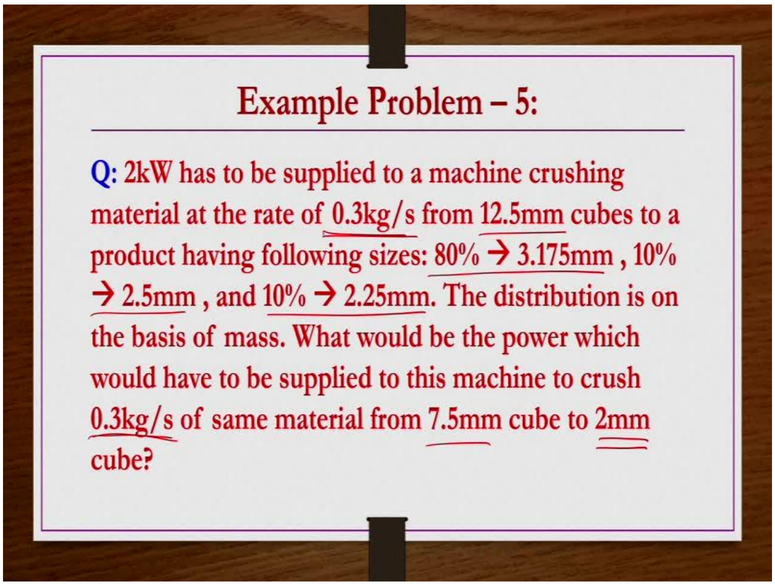

Example Problem – 5:

Q: 2kW has to be supplied to a machine crushing material at the rate of 0.3kg/s from 12.5mm cubes to a product having following sizes: 80% → 3.175mm, 10% → 2.5mm, and 10% → 2.25mm. The distribution is on the basis of mass. What would be the power which would have to be supplied to this machine to crush 0.3kg/s of same material from 7.5mm cube to 2mm cube?

Okay, so now we take one more example problem so that we can understand clearly here. So 2 kilowatt has to be supplied to a machine crushing a material at the rate of 0.3 kg per sec from 12.5 mm^3 to a product having 80 percent of the product having 3.175 mm. 10 percent of the product is having 2.5 mm size and then remaining 10% is having 2.25 mm size. So product size distribution is given, 3 sizes it is given 80% is having 3.175 so D_{pb} is 3.175, the distribution is based on the mass basis what would be the power, which would have to be supplied to this machine to crush 0.3 kg per second of the same material from 7.5 mm^3 to 2 mm^3. So, in the first case the all the details are given in the second case power details you need to find out.

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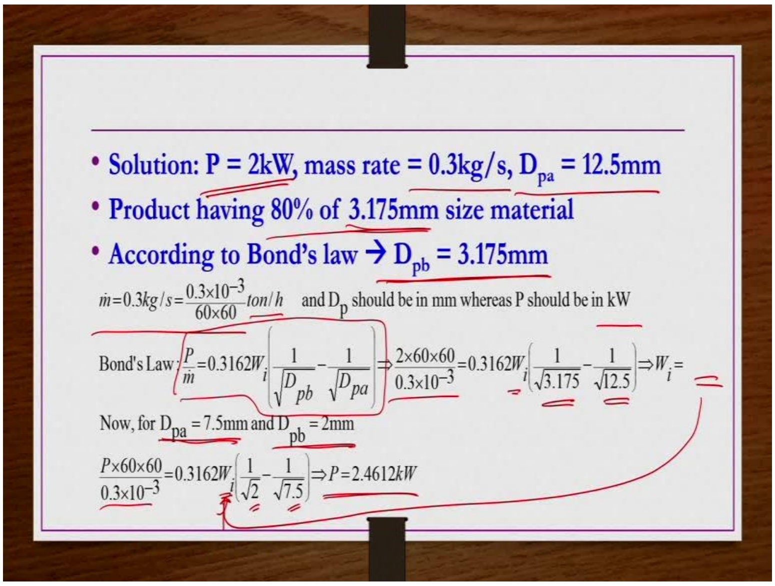

- Solution: P = 2kW, mass rate = 0.3kg/s, D_{pa} = 12.5mm
- Product having 80% of 3.175mm size material
- According to Bond's law → D_{pb} = 3.175mm

$\dot{m}=0.3kg/s=\frac{0.3\times10^{-3}}{60\times60}ton/h$ and D_p should be in mm whereas P should be in kW

Bond's Law: $\frac{P}{\dot{m}}=0.3162W_i\left(\frac{1}{\sqrt{D_{pb}}}-\frac{1}{\sqrt{D_{pa}}}\right)\Rightarrow\frac{2\times60\times60}{0.3\times10^{-3}}=0.3162W_i\left(\frac{1}{\sqrt{3.175}}-\frac{1}{\sqrt{12.5}}\right)\Rightarrow W_i=$

Now, for D_{pa} = 7.5mm and D_{pb} = 2mm

$\frac{P\times60\times60}{0.3\times10^{-3}}=0.3162W_i\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{7.5}}\right)\Rightarrow P=2.4612kW$

So, in the first case p is 2 kilowatt and then mass rate is 0.3 kg per sec you have to convert it into tons per hour. Dpa is 12.5mm it is given, product is having 80% of 3.175 mm size material. In the product 3 values are given, some material is having 80 percent material is having 3.175mm and then 10% of material is having 2.5 mm and then remaining 10% of material having 2.25mm so which one should be taken. So as per the Bond's law, whatever the 80% of material passing through the screen size that size should be taken a product as a kind of size required size.

$$\dot{m} = \frac{0.3 * 10^{-3}}{60 * 60} {}^{ton}/_{h}$$

So in the product 80 percent of the material is having 3.175 mm size so that means dpb is 3.175 one has to take. You cannot take 2.5 mm or 2.25 of remaining 10 percent material.

Okay so according to Bond's law we can say now D_{pb} is 3.175mm this is the only difference compared to the other crushing laws. One should be careful while applying these things. So now $\dot{m}$ 0.3 kg per sec so that you convert into tons per hour and then p should be in kilowatts that is anyways given so Bond's law

$$\frac{P}{\dot{m}} = 0.3162W_i\left(\frac{1}{\sqrt{D_{pb}}} - \frac{1}{\sqrt{D_{pa}}}\right) \Rightarrow \frac{2 * 60 * 60}{0.3 * 10^{-3}} = 0.3162W_i\left(\frac{1}{\sqrt{3.175}} - \frac{1}{\sqrt{12.5}}\right)$$

So, you substitute all the values here the P is given here $\dot{m}$ is given W_i is not known. D_{pa} is 12.5 D_{pb} is 3.175 you do the simplification you get some number for W_i work index. So from the first case of the data we find out the W_i now in the second case if D_{pa} becomes 7.5 mm and D_{pb} becomes 2.5mm how much power required that is, that is you need to find out. So, here in the second case just now we calculated $\frac{P}{\dot{m}} = 0.3162W_i$ so that you can substitute here. D_{pb} is 2mm D_{pa} is 7.5mm so here in this equation everything is known except P. W_i is also know because just now we calculated this Wi that you can substitute here. So you get P as 2.4612 kilowatts.

$$D_{pa} = 7.5mm\ and\ D_{pb} = 2mm$$

$$\frac{P * 60 * 60}{0.3 * 10^{-3}} = 0.3162W_i\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{7.5}}\right) \Rightarrow P = 2.4612kW$$

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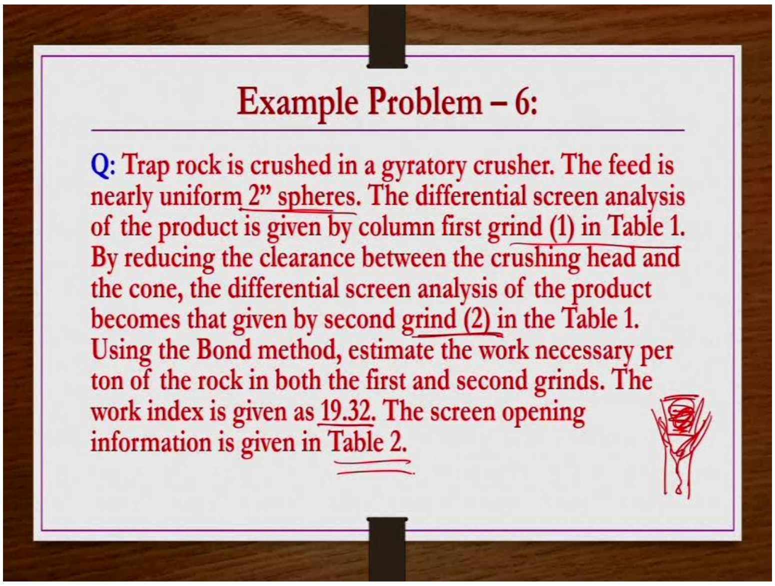

Example Problem – 6:

Q: Trap rock is crushed in a gyratory crusher. The feed is nearly uniform 2" spheres. The differential screen analysis of the product is given by column first grind (1) in Table 1. By reducing the clearance between the crushing head and the cone, the differential screen analysis of the product becomes that given by second grind (2) in the Table 1. Using the Bond method, estimate the work necessary per ton of the rock in both the first and second grinds. The work index is given as 19.32. The screen opening information is given in Table 2.

Now, we take a last example problem where we can understand the things much better. So here trap rock it crushed in a gyratory crusher. The feed nearly uniform 2 inches spheres. The differential screen analysis of the product is given by first grind 1 in table 1 in the next slide we will see. By reducing the clearance between the crushing head and cone in this gyratory crushers we can reduce the clearance between the crushing head and cone so that clearance is reduced and then again the crushing has been done.

So, whatever the differential screen analysis of the product that we get by reducing the clearance between the head and cone of the gyratory crusher that is levelled as grind 2. So, in the same material same equipment two times grinding has been done. Initially, whatever the clearance between the crushing head and then cone was there that is reduced in the second case that is the only thing. That space is reduce actually gyratory crusher that equipment we are having something like this so this this you know this is the crushing head and then this is the kind of cone.

So whatever this clearance you know that you can adjust. So in the second case the clearance is reduced so that after reducing the clearance whatever the product you get and then do the size analysis it's differential screening analysis is given as grind (2) in table one. So using the bond method estimate the work necessary per turn off the work in both the first and second grinds. W_i=19.32 the screen opening information is given in table two.

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Table-1

Mesh	First Grind (1)	Second Grind (2)	Mesh	First Grind (1)	Second Grind (2)
4/6	3.1	-	28/35	9.5	19.5
6/8	10.3	3.3	35/48	6.5	13.5
8/10	20	8.2	48/65	4.3	8.5
10/14	18.6	11.2	-65	0.5	-
14/20	15.2	12.3	65/100	-	6.2
20/28	12	13	100/150	-	4
			-150	-	0.3

Table-2

Mesh	4	6	8	10	14	20	28	35	48	65	100	150
D_{pi}, mm	4.699	3.327	2.362	1.651	1.168	0.833	0.589	0.417	0.295	0.208	0.147	0.104

So table one this is actually mesh size first grind, second grind is given. Similarly, mesh size first grind, second grind is given, actually 4/6, 6/8, 8/10, 10/14, 14/20, 20/18, 28/35, 35/ 48, 48/65, -65, 65/100, 100/150, -150. In the first grinding you know only up to 65 mesh size it is having because you know there are not much finers. So in the second grinding the more finers are there so further more screens are attached at the bottom. So this first grinding 3.1 is nothing but that is mass basis.

So 3.1% of the mass, solid mass is retained on 4 mesh, 6/18, 10.3 that means 10.3 percent of solid mass fraction percent of the solids is written on mesh 6. Like that all the information this mass fractions are given here for both first and second grind right. So that is given here. So you need to know this size of mesh 4 and 6 like 8 and 10 and all these meshers. The sizes are need to be known, so mesh 4,6,8,10,14,20,28,35,48,65,100,150 their mesh openings dpa in mm’s are also given here in the table.

So this is table one where it is giving the differential analysis for first grind and second grind, this is table 2 where it is giving information about the mesh opening of all the screen increments that are been used for differential analysis. Whatever the differential analysis process that we have seen previously that data is given here from here we are going to calculate the power requirement. So the same thing we are going to tablet it.

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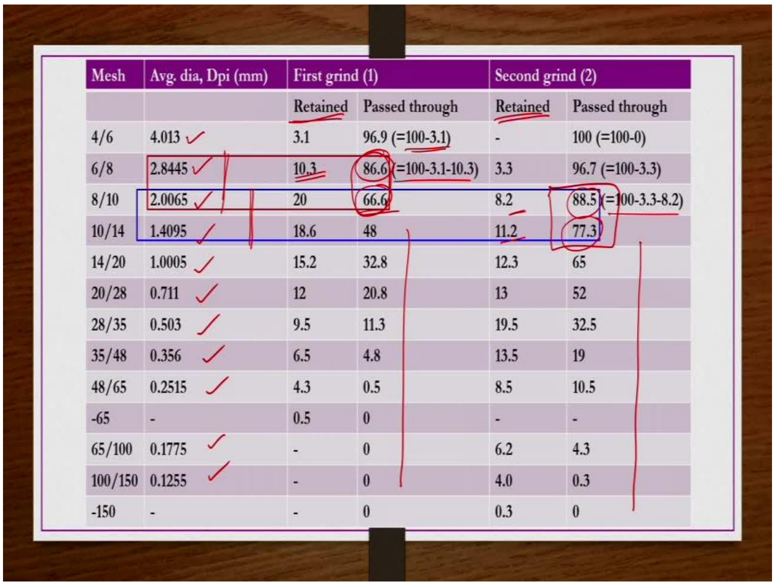

Mesh	Avg. dia, Dpi (mm)	First grind (1)		Second grind (2)	
		Retained	Passed through	Retained	Passed through
4/6	4.013 ✓	3.1	96.9 (=100-3.1)	-	100 (=100-0)
6/8	2.8445 ✓	10.3	86.6 (=100-3.1-10.3)	3.3	96.7 (=100-3.3)
8/10	2.0065 ✓	20	66.6	8.2	88.5 (=100-3.3-8.2)
10/14	1.4095 ✓	18.6	48	11.2	77.3
14/20	1.0005 ✓	15.2	32.8	12.3	65
20/28	0.711 ✓	12	20.8	13	52
28/35	0.503 ✓	9.5	11.3	19.5	32.5
35/48	0.356 ✓	6.5	4.8	13.5	19
48/65	0.2515 ✓	4.3	0.5	8.5	10.5
-65	-	0.5	0	-	-
65/100	0.1775 ✓	-	0	6.2	4.3
100/150	0.1255 ✓	-	0	4.0	0.3
-150	-	-	0	0.3	0

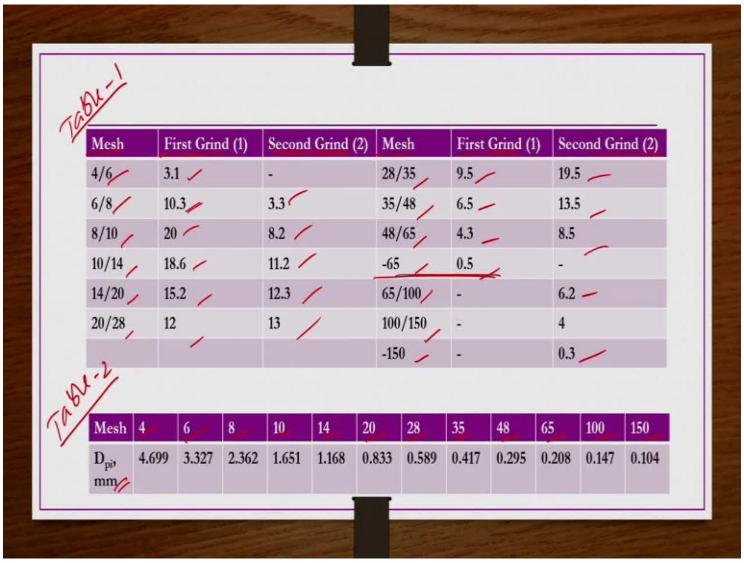

Table-1

Mesh	First Grind (1)	Second Grind (2)	Mesh	First Grind (1)	Second Grind (2)
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8/10	20	8.2	48/65	4.3	8.5
10/14	18.6	11.2	-65	0.5	-
14/20	15.2	12.3	65/100	-	6.2
20/28	12	13	100/150	-	4
			-150	-	0.3

Table-2

Mesh	4	6	8	10	14	20	28	35	48	65	100	150
D_{pi}, mm	4.699	3.327	2.362	1.651	1.168	0.833	0.589	0.417	0.295	0.208	0.147	0.104

The mesh size so average size of 4/6 is nothing by the average of the 4.699+3.327/2 =4.013 like that average diameter for all the these things are given here calculated and provided here. So in the table first grind, second grind whatever the fractions are given that is the material retained, the percentage of material retained is given because that is how the differential analysis that we know. 4/6 corresponding x_i is 3.1 x_i % = 3.1, that means 3.1% of material retained on the screen 6 passing through 4 and then returning on 6 like that.

Passing through 6 returning on 4 like that, so like that all these things are there. So you need to find out how much material has passed through overall is 100%. So from 100-3.1=96.9% of the material is passed through screen 4. Only 3.1 is retained 3.1% is retained on screen 4 and 96.4% passed through screen 4 or mesh 4. Similarly, in the second case 10.3 that means you know this whatever the 3.1+10.3 that material that amount of material is retained up to 4 and 6 screens and rest all material is passed through.

That is 100- 3.1-10.3 =86.6% material has passed through screens mesh 6 like that. Similarly, here in the case 100 - 3.1-10.3 - 20 that is 66.6% of material that is passed through screen 8 that is passed through screen 8. Like that you know one can construct for all the screens how much percentage of material passed through for a given mesh increment. Similarly, one can do the same thing for the second grind here also. In the first grind no material is retained on size mesh 4 that means all the material is passed through mesh 4 100 – 0 is 100.

Next case only 3.3 is retained on 6 mesh that means 100 -3.3 that is 96.7% of material is passing through the 6 mesh. Similarly next case you know 3 mesh. 8.2 is only being retained on 8 mesh that means 100 - 3.3 -8.2 or the amount that is passed through that is 88.5% is passing through the 8 mesh. Like that here 77.3 that is 100 - 3.3 - 8.2 -11.2 that is 77.3 % of material is passing through 8 mesh. Why we are doing this passed through kind of things because from here only we get respective number.

How much 80 percent 80% of the material passing through that size we have to take a reference size for the product. Feed size is given that is the reason we are making this tabular column passed through column as well in addition to the given written column. So that means so that means in the first grind so 80 percent is between these two right 6/ 8 and 8/10. So in between these two somewhere it is having 80% of material passed through.

So you can do the you can do the graphical method material passed through versus the average size D_{pa} and then you have the curve for this passed through versus average size so 80 corresponding to 80 percent, whatever the Dpa is that you are getting that D_{pa} you should be taken as a D_{pb} for the first grind. Similarly, for the second grind also the material passed through fraction versus D_{pa} you plot and then corresponding to 80 percent whatever the D_{pa} is there that should be taken as D_{pb} for the second grade.

So we are not going to do the, I mean you can do the interpolation also we are not going to do the graphical methodology. Similarly, here in the second case in between these 88.5 and 77.3 somewhere 80 percent is passing through corresponding D_{pa} average D_{pa} we should take as a product D_{pb}. So, that is somewhere in this one. So, for the first case D_{pb} would be somewhere between 2.8445 and 2.0065 and then for the second case D_{pb} would be somewhere between 2.0065 and 1.4095 so those things only we are going to find out and simply substituting in the Bond's law rest everything is known.

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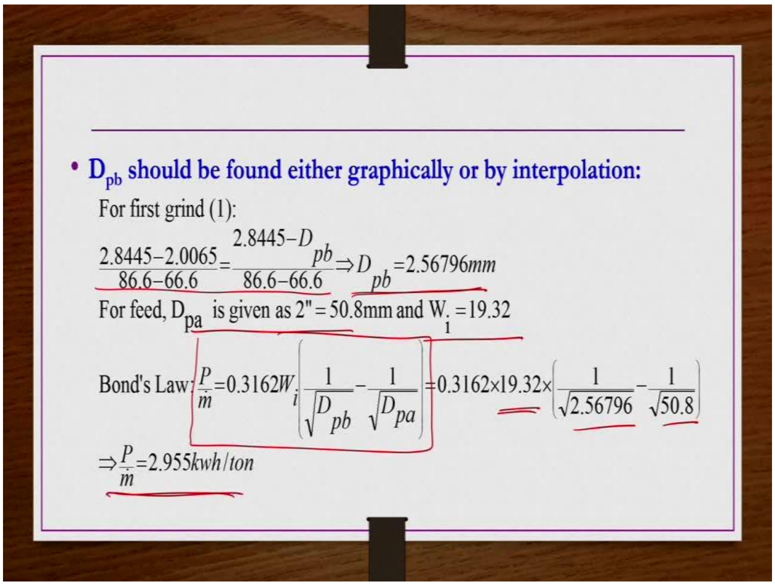

- D_{pb} should be found either graphically or by interpolation:

For first grind (1):

$$\frac{2.8445-2.0065}{86.6-66.6}=\frac{2.8445-D_{pb}}{86.6-66.6}\Rightarrow D_{pb}=2.56796mm$$

For feed, D_{pa} is given as 2" = 50.8mm and $W_i = 19.32$

Bond's Law: $$\boxed{\frac{P}{\dot{m}}=0.3162W_i\left(\frac{1}{\sqrt{D_{pb}}}-\frac{1}{\sqrt{D_{pa}}}\right)}=0.3162\times19.32\times\left(\frac{1}{\sqrt{2.56796}}-\frac{1}{\sqrt{50.8}}\right)$$

$$\Rightarrow\frac{P}{\dot{m}}=2.955kwh/ton$$

D_{pb} should be found either graphically or by interpolation for the first grind, I am doing the linear interpolation assuming there is no much deviation. So D_{pb} when you do the interpolation for the first grind you will get 2.56796 mm and D_{pa} is given as 2 inches, W_i is given. So, once D_{pb} is known everything is known everything is known in this Bond's law or except p by m dot, so that p by m dot you can find out. So W_i is also given as 19.32 this is dpb just now you find out and dpa it is given feed in the problem so p by m. You are going to get this.

$$\frac{2.8445 - 2.0065}{86.6 - 66.6} = \frac{2.8445 - D_{pb}}{86.6 - 66.6} \Rightarrow D_{pb} = 2.56796mm$$

For feed, $D_{pa} = 2^n = 50.8mm\ and\ W_i = 19.32$

Bond's Law;

$$\frac{P}{\dot{m}} = 0.3162W_i\left(\frac{1}{\sqrt{D_{pb}}} - \frac{1}{\sqrt{D_{pa}}}\right) = 0.3162 * 19.32 * \left(\frac{1}{\sqrt{2.56796}} - \frac{1}{\sqrt{50.8}}\right)$$

$$\frac{P}{\dot{m}} = 2.955\ kwh/ton$$

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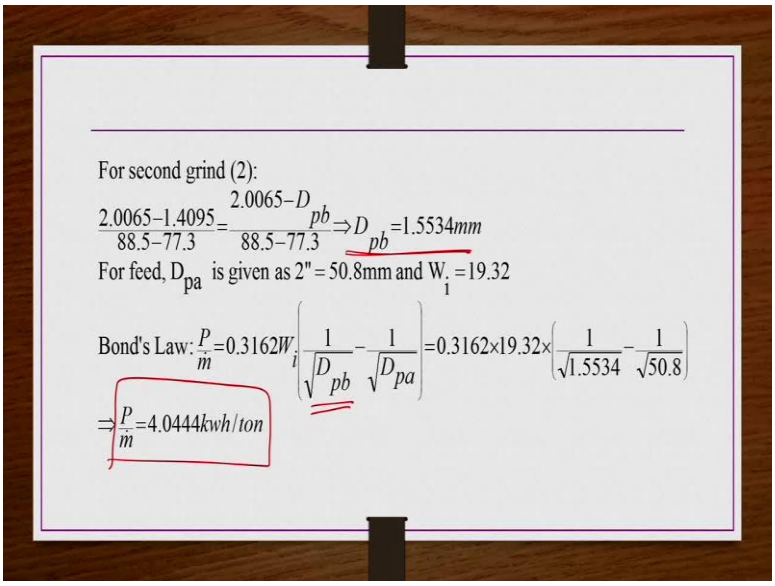

For second grind (2):

$$\frac{2.0065-1.4095}{88.5-77.3} = \frac{2.0065-D_{pb}}{88.5-77.3} \Rightarrow \underline{D_{pb}=1.5534mm}$$

For feed, D_{pa} is given as 2" = 50.8mm and W_i = 19.32

Bond's Law: $\frac{P}{\dot{m}} = 0.3162W_i\left(\frac{1}{\sqrt{D_{pb}}} - \frac{1}{\sqrt{D_{pa}}}\right) = 0.3162\times19.32\times\left(\frac{1}{\sqrt{1.5534}} - \frac{1}{\sqrt{50.8}}\right)$

$$\Rightarrow \boxed{\frac{P}{\dot{m}} = 4.0444kwh/ton}$$

So the exactly the same thing second one we have to second grind we have to do. For the second grind if you do the interpolation D_{pb} you are going to find it out as 1.5534mm, so that you substitute here in this equation D_{pa} W_i rest everything m dot all these things are remaining same so you can find out P. So m dot is not given so $\frac{P}{\dot{m}}$ you can find it out for this case also as 4.044 kilowatt hour per ton.

$$\frac{2.0065 - 1.4095}{88.5 - 77.3} = \frac{2.0065 - D_{pb}}{88.5 - 77.3} \Rightarrow D_{pb} = 1.5534mm$$

For feed, $D_{pa} = 2^n = 50.8mm\ and\ W_i = 19.32$

Bond's Law;

$$\frac{P}{\dot{m}} = 0.3162W_i\left(\frac{1}{\sqrt{D_{pb}}} - \frac{1}{\sqrt{D_{pa}}}\right) = 0.3162 * 19.32 * \left(\frac{1}{\sqrt{1.5534}} - \frac{1}{\sqrt{50.8}}\right)$$

$$\frac{P}{\dot{m}} = 0.444\ kwh/ton$$

So that means you know in the first case the power requirement in the first grind power requirement is less close to 3 KWh/ton whereas in the second case where by reducing the clearance between the crushing head and cone you are getting more fine so in such case you know you are you need to provide more power. So almost 4 kilowatt hour per ton previous case in the first grind it is almost close to 3 KWh/ton.

Now here by reducing the clearance between the crushing head and cone whatever the operation you do you get you need more power to be given that is 4 KWh/ton and then you are going to get a more finer product. So, this is how problems we have to solve using these crushing laws in order to get the power requirement for a given size reduction operations. These are the reference book that I have followed for this particular lecture.

(Refer Time Slide: 74:08)

References

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The problems that I have discussed may be seen in this first 3 reference books, exercise as well as the solved examples you can find from these examples. Some of them have been taken here in this particular lecture. Thank you.