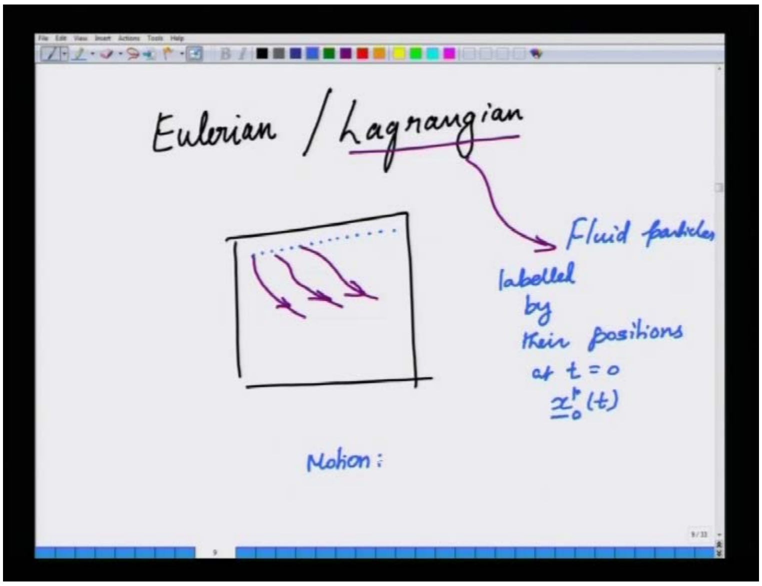
Fluid Mechanics

Prof. Vishwanathan Shankar
Department of Chemical Engineering
Indian Institute of Technology, Kanpur

Lecture No. # 10

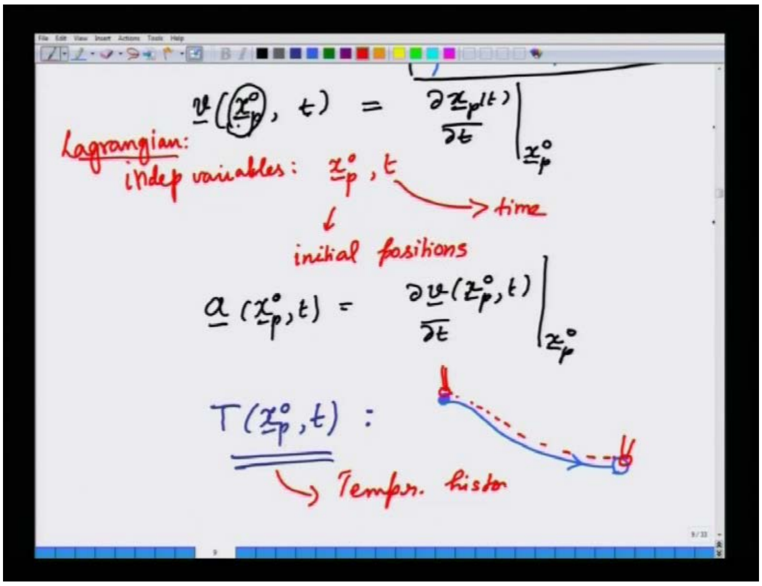
Fluid Mechanics

(Refer Slide Time: 00:28)



Welcome to this lecture number 10, on the NPTEL course on fluid mechanics for under graduate students in chemical engineering. In the last lecture, we discussed in detail the notions of Eulerian versus Lagrangian description of motion. So, just to recall very briefly in the Lagrangian description, we imagine that a fluid is compressed. Suppose, you have a box of fluid, a fluid is compressed of various points, which are called material points or fluid particles. And these points can be followed, as a function of time. The position or location of these various points can be followed as a function of time. Each point in the Lagrangian approach is denoted, is labeled by fluid particles or material points are labeled by their positions at time t is equal to 0, which we called x 0 T, where p stands for particle.

(Refer Slide Time: 01:40)



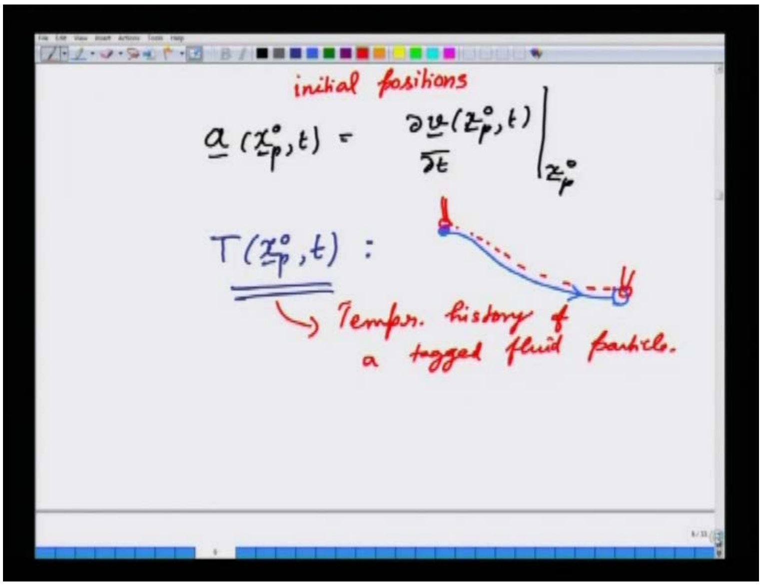
So, the motion of the fluid is described by, how the position of various particles, fluid particles changes function of time. And they are given as the function of that initial positions x 0 p. So, this is there is no time here, where x 0 p is definition, x p let us call this x 0, in the super script x p at time t equal to 0. So, again we will have superscript here 0 and time. So, each point here moves in a specific way and at after this is a time t is equal to 0 after a time T, the trajectory of all these points in principle and infinite set of points are known. And once they are known, you can compute quantity such as velocity, velocity of a fluid particle. Instead of writing v p, we will write velocity of a fluid particle x p of t at a time t or other. Velocity of a fluid particle, which is dented by x p 0, that is the particle which was there at x p 0 and time t equal to 0 that is the label of the particle.

And after time t this particle would have moved to some point here, from here. So, this is the velocity of a fluid particle, which was at x p 0 at time t is equal to 0. The velocity of such particle at time t is given by the rate of change of it is position by keeping, the identity constant the label constant. So, in the Lagrangian a description of motion, the independent variables the Lagrangian description, independent variables are the initial positions and time. This is these are the initial positions of various fluid particles that serve as their identity, they serve as the label and time.

So, not just these are not just restricted to velocity, you can calculate acceleration of a fluid particle at time T, which is nothing but the rate of change of it is velocity as you keep, the identity of the particle the same and so on. So, once you have the trajectory of the motion, as described by this equation. This is the motion, the trajectory of the particles one can compute, kinematic quantities like velocity and accelerations. One can also, extend this to other field, such as temperature field, temperature of a fluid particle at a later time. This is imagine, this is a Lagrangian description, imagine you follow a motion of a particle, the particle was here at time t equal to 0 and it is moving at a later time here.

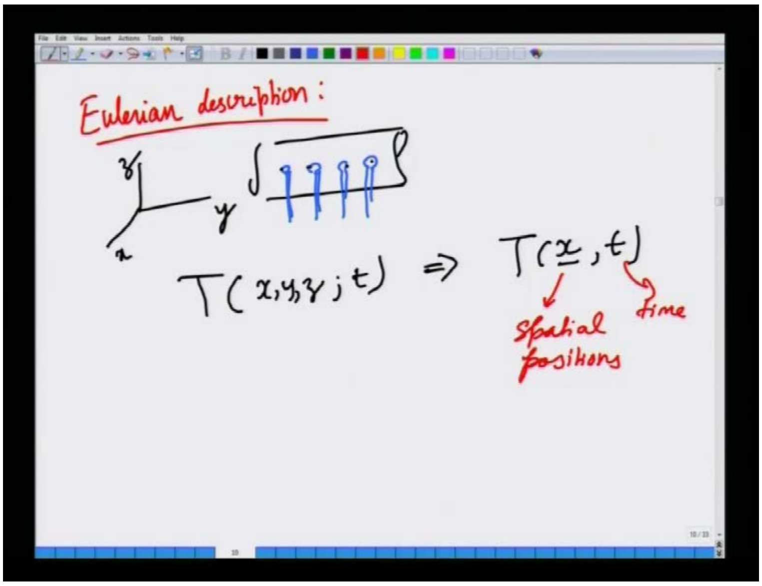
So, imagine putting a thermometer, attaching a thermometer to a particle and then you follow the same particle and then you measure the temperature history of a given fluid particle. So, that is t as a function of time. So, there are Lagrangian function description tells you the historical information, such as temperature of a fluid particle as you follow it or velocity of a fluid particle as you follow it and so on.

(Refer Slide Time: 05:15)



So, this gives you the temperature of history, of a tagged fluid particle. In contrast, we also mentioned, this Lagrangian description is not very practical, because measurements in labs are often, done based on keeping probes at fixed location space. For example, we will keep thermometers or various temperature sensing devices at various points in space or rather then follow it along to the fluid particle normally.

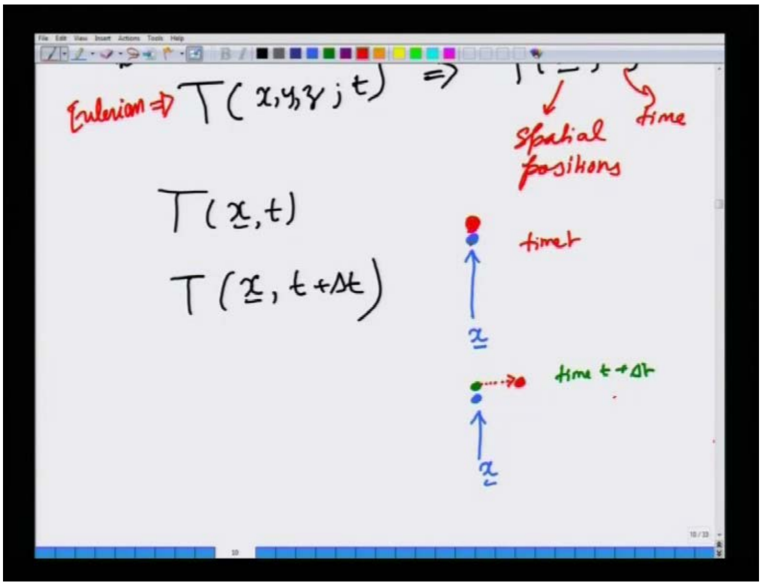
(Refer Slide Time: 05:48)



So, in the other description of a used in the fluid mechanics or the description that is often used in the fluid mechanics is the Eulerian description or the spatial description. The Lagrangian description is also, called as the material description as I mentioned in the last lectures, here you imagine you having a fixed reference frame in your lab. And suppose have a pipe in which fluid I following, you can measure, you keep probes with in the pipe, you can say keep thermometers with in the pipe. And you can measure for example, the temperature as a function of x y z coordinates and time, at fixed at various fixed locations in space.

So, you can measure, so this is often condensed in short hand and t as function of x vector t. So, the independent variables in the Eulerian vector description of the spatial positions and time. So, keeping the differentiates, Eulerian and Lagrangian description of motion is that the independent variables, that are used to characterize the motion. And 1, in 1 case you are following the particles in rather case, in the Lagrangian case you are following in fluid particles, where as in the Eulerian, you are sitting at the various points in space and measuring quantities and function of time.

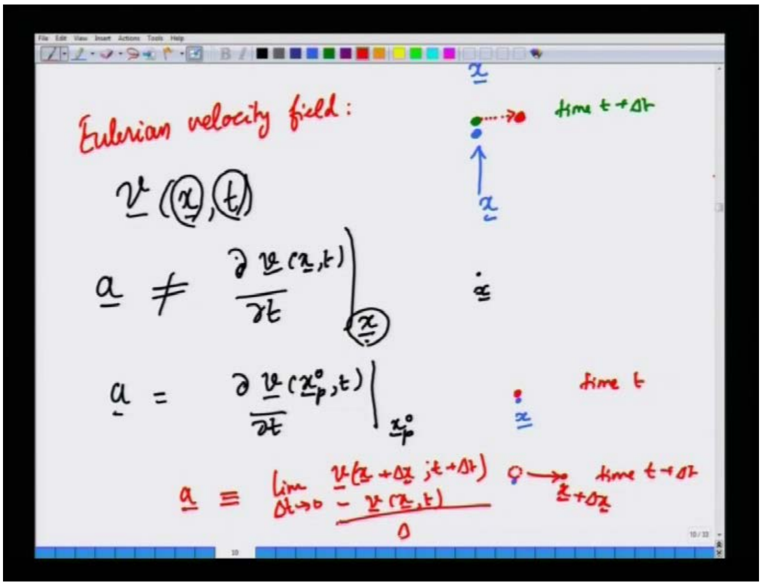
(Refer Slide Time: 07:15)



So, in the Eulerian description, this is the Eulerian description by nature of it is of the description, we will use notion of historical information. That is, if you point the temperature at a given point in space at time t. And, if you find at a later time, t plus delta t. Suppose, you have flow and you measure, suppose you have flow you measure temperature at a point this is the fixed point, this is the point x. You measure temperature using a thermometer and at a time x, this point will be occupied by let us say red particle at time t at a later time. The same point x, special location x will be occupied this red particle, will move at a t time t plus delta t this red particle move elsewhere and may be some other particle will come and occupy.

And t plus delta t, this red particle would have moved from the location x to some other point. And the location x itself, it will be occupied by some other fluid particle. So, what we are merely measuring in the Lagrangian, in the Eulerian description. I am sorry, that at a given point is what are the properties of such as temperature velocity or acceleration or pressure as a function of time without worrying about, which particles, belong I mean without worrying about what is the material particle, a fluid particle that is occupying that location.

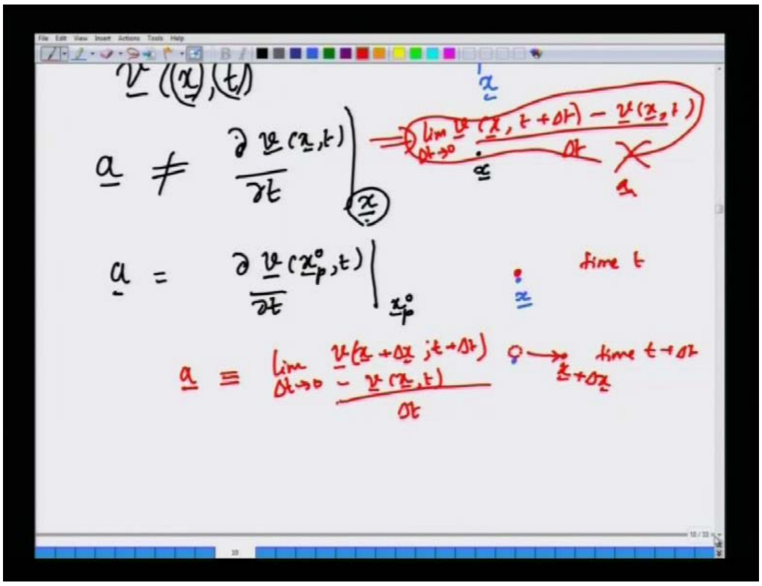
(Refer Slide Time: 09:02)



This is usually ok, except in the case of acceleration. Suppose, the Eulerian velocity field the Eulerian velocity field is given by v as a function of x t. Once you have this information, we cannot compute the partial derivative of v as a function of x t with respect due to t and call it acceleration. Because whenever, we take partial derivative, we keep since the independent variables are x vector and t vector, when you partially differentiate with respect to t, we are keeping the respecter same, that is we are merely taking the at given point x. What is this spatial, what is the variation of velocity at that point as a function of time? Whereas acceleration really speaking is what is the velocity rate of change of velocity of a fluid particle? That is acceleration of a fluid particle is inherently here like Lagrangian quantity.

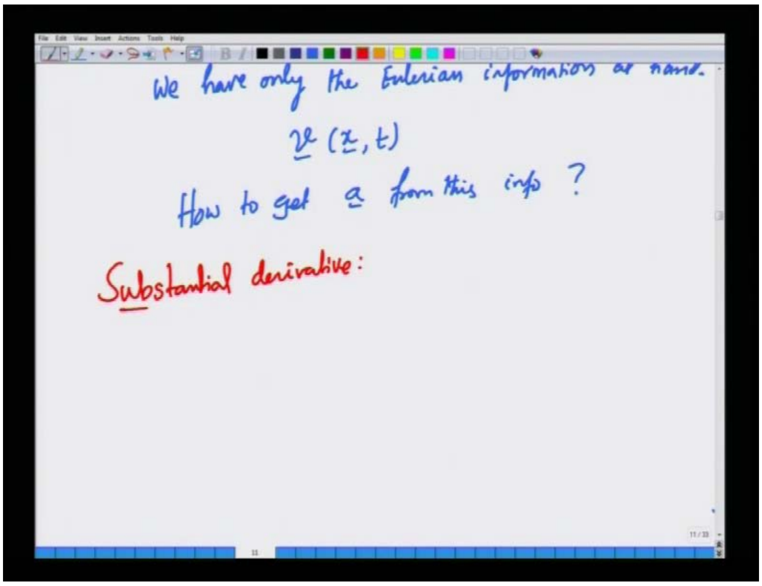
So, in principle, what we must do to compute acceleration? Is that, suppose you are at a point x, let us use blue color to denote the point, this is the physical location x at time t. This red particle is here and at time t plus delta t this red particle, would have in general more away from x. This is the point x, the red particle, which was originally here would have moved more elsewhere. So, the acceleration in principle is defined as limit delta t tending to 0, velocity of the particle. Which has which was that x at time t, sorry velocity of the particle, which is at x plus delta x. At time t plus delta t minus velocity of that particle, which was at x, because this particle was at x time t divided by delta t.

(Refer Slide Time: 11:15)



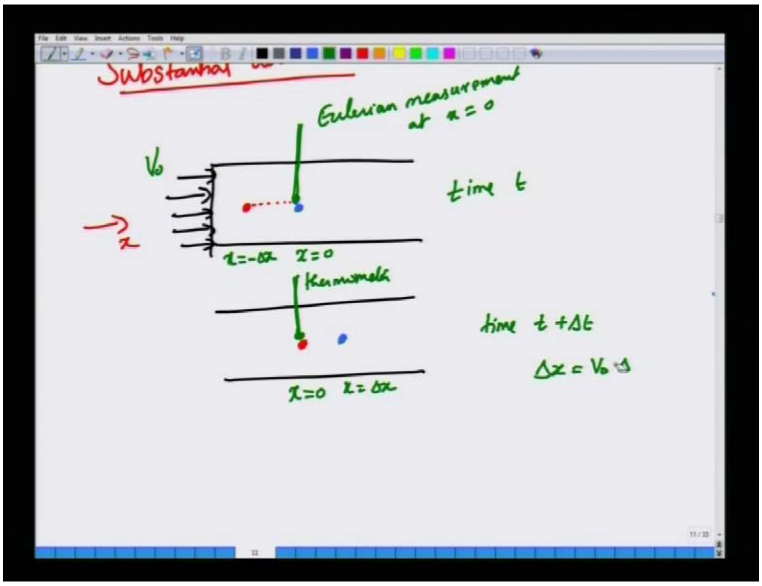
Whereas, normal partial derivative, such as this would merely measure, v at x plus delta sorry, v at x t plus delta t minus v at x t divided by delta t as limit delta t going to 0. So, this is clearly not acceleration, because this is not acceleration because we are not following the same particle, whereas, here we are recognizing explicitly the fact in this definition that, we are following the same particle. And you have to realizing the fact that this particle, which was here at x would have moved to some other location at a later time.

(Refer Slide Time: 12:00)



Now, while this is all fine, normally in fluid mechanics, we do not have the, we have only the Eulerian information at hand. That is we have only v as a function of spatial coordinates and time. Now, the question is how to get accelerations from this information, well we saw this notion of substantial derivative that will help us to do this, we said this in the context of we explained in the context of temperature. Because the temperature is the scalar and it is much simpler.

(Refer Slide Time: 12:49)

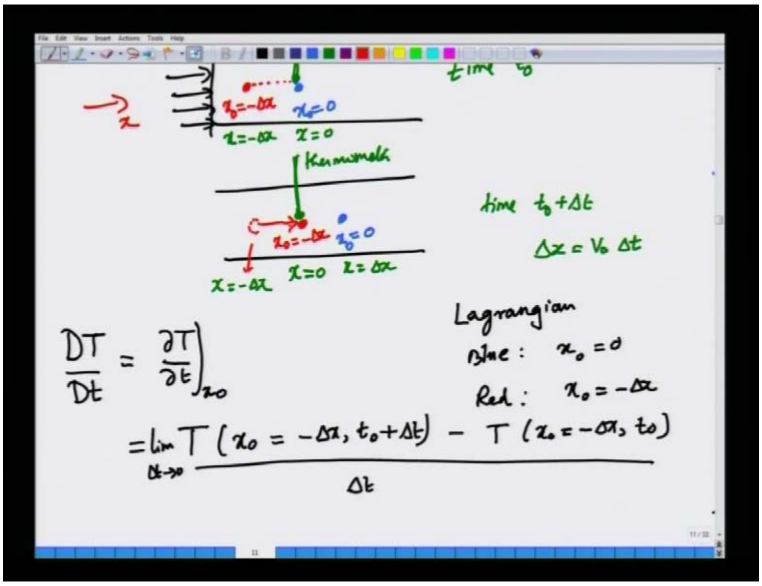


So, imagine you have a channel and fluid is flowing at constant velocity, uniform velocity, constant velocity and you are at a point x equal to 0. And this is the point x equal to 0. And at this point, we had a particle that was occupied, we had the blue particle and at an earlier location, x equal to minus delta x, we had a red particle. So, x is minus delta x. Now, the distance is minus delta x now since for simplicity, we will consider, only 1 dimension that is direction. So, we will call this x, we will call it x equal to 0, here x equal to minus delta x. So, the distance separates delta x this particles behind, because the co ordinate axis increases in this direction. So, we have minus delta x here.

Now, imagine measuring temperature using a probe such as a thermometer at this point. So, you are doing an Eulerian measurement, that x equal to 0, we are putting a thermometer x equal to 0. And we are measuring temperature as a function of time, this is a time t. At a later time t plus delta t, this point blue point would have moved. So, I am going to draw it roughly at the same locations. So, the red point would have moved to blue point, where the blue point was x equal to 0. And the blue point would have moved somewhere else. So, this distance is still this is the location x equal to 0, let me call this, let we write this in green ink the spatial location x equal to 0. Now, this becomes x equal to plus delta x.

So, let us call this spatial location x equal to 0 x equal to minus delta x this x equal to 0, this is now this is time t plus delta t. The thermometer is keep still at x equal to 0, this is a thermometer fluid is flowing. So, fluid motion takes the point, blue point which was at x equal to 0 to x equal to delta x. Since the velocity is constant v 0 delta x will be v 0 times delta t. delta x is uniform motion velocity is constant at each and every point in space in time. So, it is a constant velocity. So, velocity is the displacement delta x is v 0 delta t. Now, what is a thermometer measuring at let us also, before I proceed further, let us also denote the Eulerian labels of this particles. So, this particle is denoted by it is position at time t 0. Let us call it to be consistent with previous lecture, let us call it t 0. So, all points are marked by the positions at time t 0.

(Refer Slide Time: 16:13)

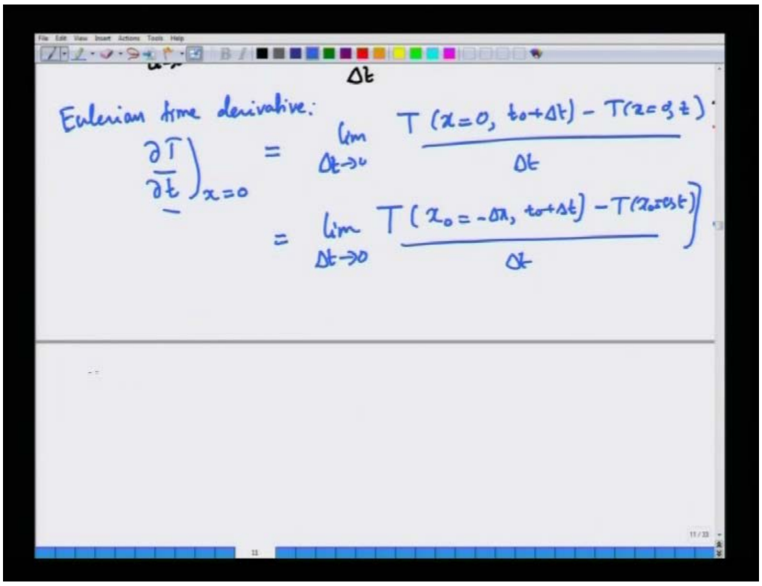


So, this point, the Lagragian coordinate is x equal to 0, we need not have the vector substitutes, because we are just worrying about one direction here, this is x equal to minus delta x. Now, even at a later time the Lagangian coordinate of this point is simply, still x equal to 0 or let us to be specific, let us call it x naught equal to 0, this is x naught. And here, the Lagrangian position of this particle is still, x naught is minus delta x, delta x is 0 delta t that is the magnitude of the displacement. So, the labels of these particles are simply the blue particle, the Lagrangian labels. Lagrangian label of blue particles is x 0 equal to 0. So, I am sorry, I do not need the vectors symbols and the Lagrangian label for the red particle, is simply x 0 is minus delta x.

But we also realize that at time t equal to t naught, it also coincides the blue particle coincides with the spatial location, that x equal to 0. The red particle coincides with the spatial location x equal to minus delta x. Now, what is the substantial derivative of temperature? We mentioned in the last lecture, that substantial derivative is detonated by a special symbol capital D D t, is simply D D, the rate of change of temperature with time as we keep x naught constant. So, that is the key difference. So, this is nothing but when we keep x naught constant T of x 0 is minus v 0 delta t minus delta x T naught plus delta t minus t of x naught is minus delta x at t 0, divided by delta t limit delta t going to 0. This is by definition, what is substantial derivative? The substantial derivative is the time derivative, as you follow; let us say the red particle.

The red particle is currently time T naught plus delta t at a position x 0, x equal to 0. But at the earlier time, it was a position here. So, let me just draw the red particle figure it this is the earlier location of the red particle x equals. Let me just write it in green color this spatial location, x is minus delta x, this red particle has moved from here to here. As you follow the red particle, how was the temperature changing, that is the definition of substantial derivative, but the thermometer measures the completely different.

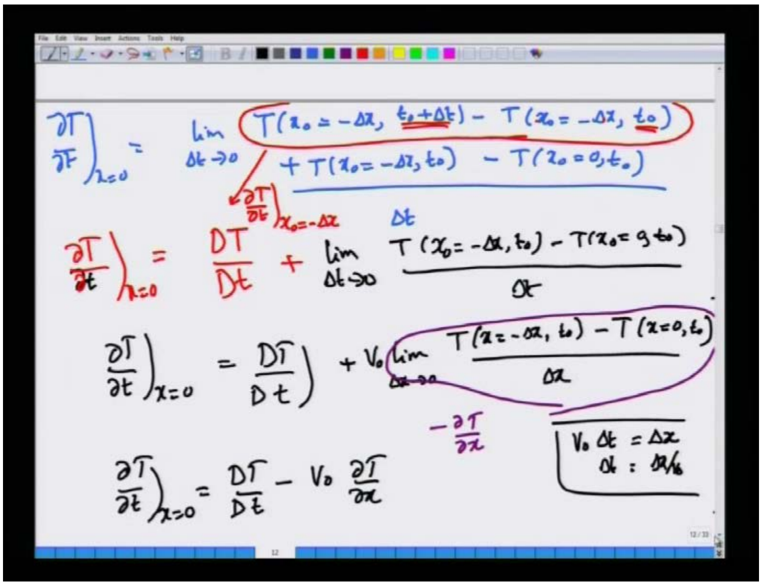
(Refer Slide Time: 19:13)



What the Eulerian time derivative as measured by the thermometer, because it is sitting at a same spoice, same point on space and so it simply, measuring at a given x, lets x equal to 0. What is this? This is, limit t delta going to 0, T of x equal to 0, t naught plus delta t minus T of x equal, to 0 t divided by delta t. So, somehow, we should get this information this is the substantial derivative as we follow the same material particle from this information, from this description, the spatial description. So, how do we do that very simple terms, we simply take this and then add and subtract the following quantity?

First, we realize that x equal to 0 t 0 plus delta t is nothing but so we could also do it something similar instead of doing it at x equal to 0, we can call it x not x that is so this is perfectly fine. So, I am going to re label the Eulerian independent coordinates with Lagrangian independent coordinates T, let us just go to the figure. So, in this figure at x equal to 0, at time t plus delta t it is the blue particle that is let me just go here. At x equal to 0 at time t plus delta t, it is the red particle that is present. So, I am going to change the label to the Lagrangian label, limit x naught equal to minus delta x t naught plus delta t at x equal to 0 at time T. It was the particle that was present at time t or t naught, that is at x equal to 0 is the blue particle with identity the x naught equal to 0. So, I am going to simply write this as, x naught equal to 0 T, and then close the bracket divided by delta t ok. So, this is the partial derivative.

(Refer Slide Time: 21:46)



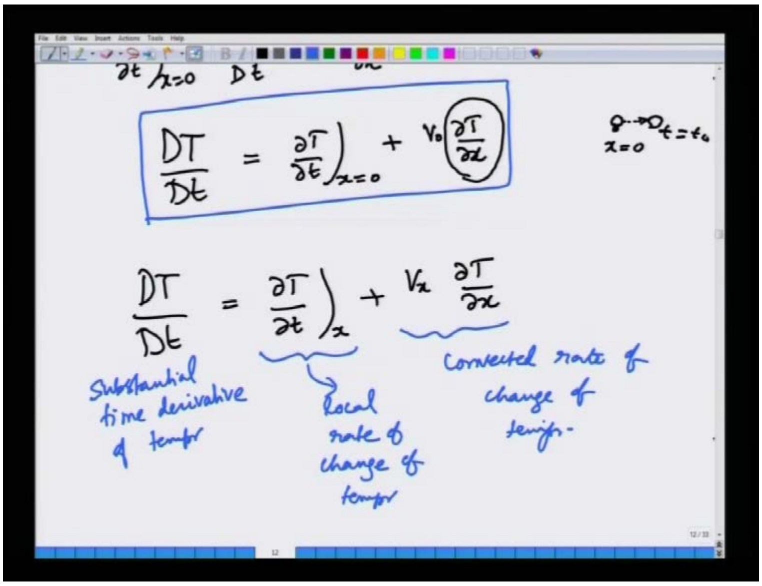
So, the partial; normal partial derivative at x equal to 0 is given by limit delta t going to 0. Now, I am going to add subtract a term, which is help us identity the relation delta t minus T x naught is minus delta x t 0 plus T x naught is minus delta x t 0 minus. I am adding and subtracting this same quantity minus T x naught is 0 t, t 0 divided by delta t. Now, we can identify the following, that is at this, if you look at this combination here this is nothing but here you are identifying the same material particle, the x naught is minus delta x, but the time is different. So, this is nothing but this term divided by delta t as time delta t going to 0.

So, the left side remains normal partial derivative. So, one of the terms on the right side is the substantial derivative that, we want to calculate this is nothing but partial T partial t as x naught is kept constant, which is minus delta x here and that essentially, the substantial derivative plus we have and additional term. Let me write it, which we will simplify now T of x not is minus delta x t 0 hence, T of x naught is 0 t 0 divided by delta t. So, sorry this is partial by partial t here. At x equal to 0 is the substantial derivative plus instead of writing at as delta t, delta t v 0 delta t is delta x. So, instead of writing it as delta t, I am going to write this as so it is delta t, I will write it as delta x by v 0. So, I will get v 0 limit instead of delta t going to 0, I will get delta x going to 0 T of now x naught equal to minus delta x.

So, let us go to the figure x naught is minus delta x is x equal to 0 and x naught is minus delta x at time t 0 x naught is minus delta x at time t 0 corresponds to the Eulerian location x equal to minus delta x. So, I am going to write this as, x equal to minus delta x at time t 0 minus x naught equal to 0 at time t 0 is T, sorry, we remove the bracket here, this is T and x equal to 0 time is 0. So, we are converting this Eulerian labels to Lagrangian labels here to Eulerian labels, just by looking, where this point x naught was this is x naught, the x not equal to minus delta x was exactly at time t equal to 0 at the spatial location, x equal to minus delta x, x naught equal to 0. The point, the Eulerian label, the Lagrangian label corresponds to the location at time t equal to 0 x equal to 0 divided by delta x.

This is nothing but so let us forget this term is nothing but minus partial T by partial x. So, partial temperature by partial time x equal to 0, is the substantial derivative. As you follow I am sorry, as you follow a material particle, which was at x equal to 0 at time t naught minus v 0 partial T partial x r.

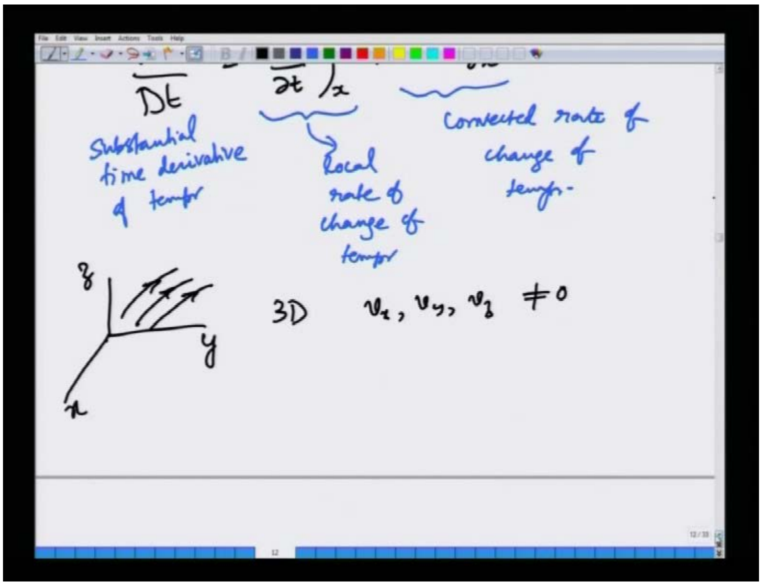
(Refer Slide Time: 26:05)



In other words, the substantial derivative of a particle, that was at time t equal to 0 at x equal to 0 substantial derivative of temperature as you follow. The particle which was at time t equal to t 0, x equal to 0 as this particle is moving, it is rate of change of temperature is because of the local rate of change of temperature at the fixed location in space at the x equal to 0 plus. A convicted rate of change of temperature, which happens because of fact at this particle is moving, probably moving presumably moving from a region of lower to higher temperature. So, it is temperature will change not just, because of the inherent rate of change at a given point also, because of gradient. In gradient space and temperature as you follow the same point.

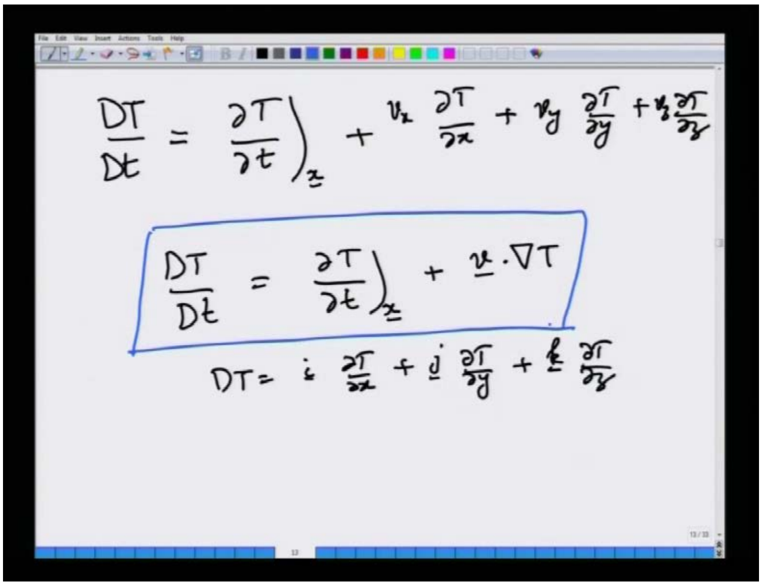
So, that is the substantial derivative, this is the very important result. So, we can generalize this for any arbitrary velocity, instead of having V 0, we can generally write this as partial T and constant x plus V x is a direction of velocity in the direction plus the gradient of temperature in the x direction. So, this is called, the local, this is the substantial time derivative of temperature, this is the local rate of change of temperature, this is the convicted rate of change of temperature.

(Refer Slide Time: 28:01)



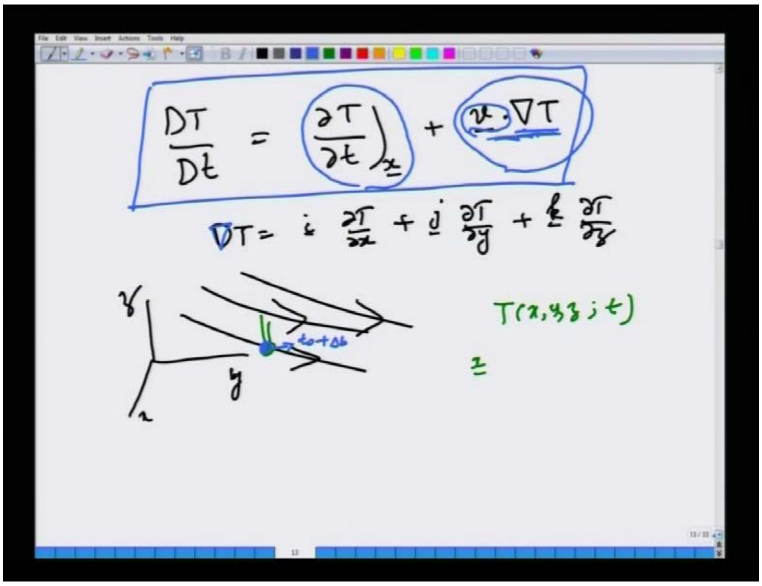
So, we can also generalize this to 3 dimensions, when velocity you have velocity vector in all the 3 direction. So, fluid is flowing in arbitrary 3 D, this is called 3 D motion, where V x, V y, V z are not equal to 0, in which case you can analyze this.

(Refer Slide Time: 28:22)



So, the substantial derivative in 3 dimension is given by substantial derivative of scalar filed like temperature is given by the local partial derivative plus v x just by generalization partial T partial x, plus v y, partial T partial y, plus v Z, partial T partial z, we can also use, the symbol from vector calculus this is nothing but partial rate of change of temperature at a fixed location plus v dot the gradient of temperature. Grad t is of course, i partial T partial x plus j partial T partial y plus k partial T partial Z this is the gradient of the temperature. Familiar from gradient of any scalar field is familiar from vector calculus like this. So, this is the substantial derivative of temperature.

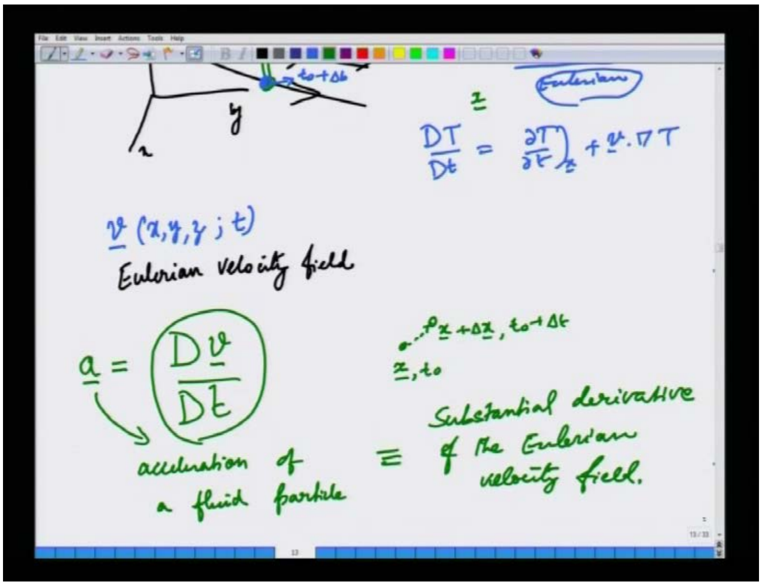
(Refer Slide Time: 29:22)



Again what this means in the Eulerian context is the suppose, you have a frame of reference Eulerian frame of reference in the lab. And let say fluid is flowing in some orbit limit and you are sitting at a point in the fluid aerodynamic, you are putting a thermometer and measuring temperature. And you get this information as a function of time, temperature as function x y z you have to keep many thermometers and get this information. Suppose, you are interested in point, you are fixing x and you are asking. Suppose, I have a particle, that was here a time t 0, as I follow this particle by time t 0 what is the rate of change of temperature? As I follow the particle that is the meaning of substantial derivative, this is gradient.

So, the answer is you have a local rate of change plus a convicted rate of change as you follow the particle. The particle may go from regions of 1 temperature to higher temperature or lower temperature that will cause a gradient of temperature. And when you dot that the velocity vector that will give you the convicted rate of change while, this is the local rate of change.

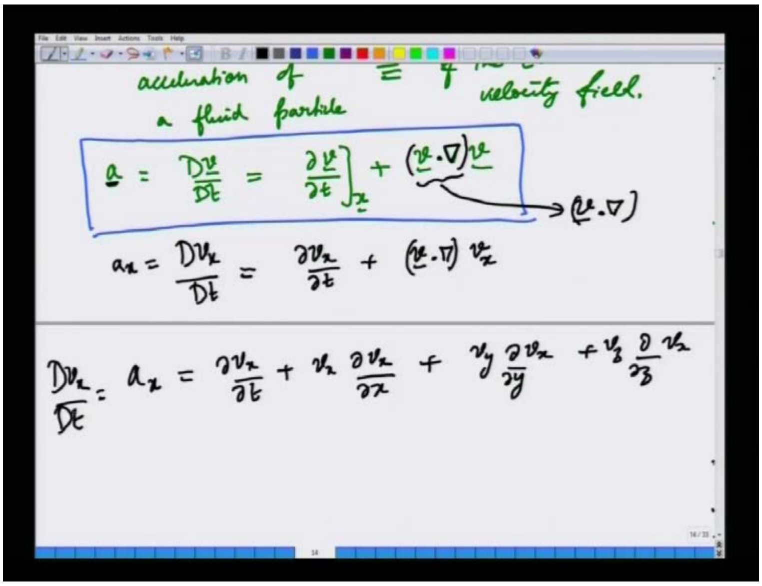
(Refer Slide time: 30:38)



So, this is the notion of substantial derivative, where in you can actually get information from Eulerian description, this is a Eulerian description. You can get on this Eulerian description of temperature as the function of 3 coordinate directions in time. And how temperature will change as you follow particle, which is at a given spatial location at the given time, this is the meaning of the substantial derivative. Now, we can also generalize these 2 more complicated objects, such as velocity temperature was a skill. So, that was much simpler. So, Dt, Dt was partial t partial t and constant x, plus v dot del t. So, what is the substantial, if I have the velocity Eulerian velocity filed, this is the Eulerian velocity filed.

Suppose, I have the Eulerian velocity filed, how do I complete the substantial derivative of the velocity, what is the meaning, physical meaning of this? If I have a particle at x at time t 0, this particle would in general move to x plus delta x at time t 0 plus delta t. So, what is the rate of change of velocity as I follow the particle. And that physically is the acceleration, in the Eulerian description, the acceleration of a fluid particle is obtained by the substantial, this is the acceleration of a fluid particle is equal to the substantial derivative of the Eulerian velocity filed.

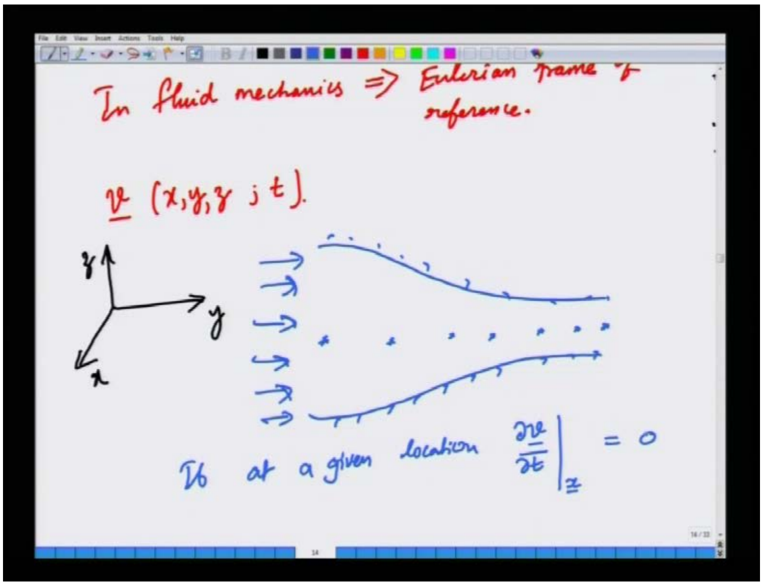
(Refer Slide Time: 32:40)



(No audio from 32:21 to 32:34) Now, the idea is very similar, a is Dv Dt, just by looking at the previous formula that was written here, we can write by partial v, the local rate of change of velocity at a given spatial location plus v dot grad instead of temperature here, which was a scalar and here we wrote v dot grad t well we wrote grad t. So, this is the Eulerian acceleration field obtained from the Eulerian velocity field, nearly by taking the substantial derivative of the velocity. Now, acceleration is a vector, so we will have to take individual components. So, for example, the x component is given by D vx the substantial derivative of the x component of the fluid velocity.

So, this is partial v x by partial t plus v dot grad. So, remember that v dot grad is the scalar operator, while velocity is the vector and grad in the vector there is a dot product. So, this makes it a scalar operator v dot grad. So, v dot grad will remain as such and since, we are taking the x component of this vector a x here. So, you will have v x here. There is no any need for taking component here, because v dot grad is already a scalar operator. Because both v and dell or vector operators and if you take a dot product you will get a scalar operator. So, ax is partial, vx partial t D vx by Dt. This ax plus vx partial vx by partial x plus vy partial vx by partial y plus v z partial v x by partial v z. So, this is how accelerations can be computed whenever, you have Eulerian field velocity information.

(Refer Slide Time: 34:50)

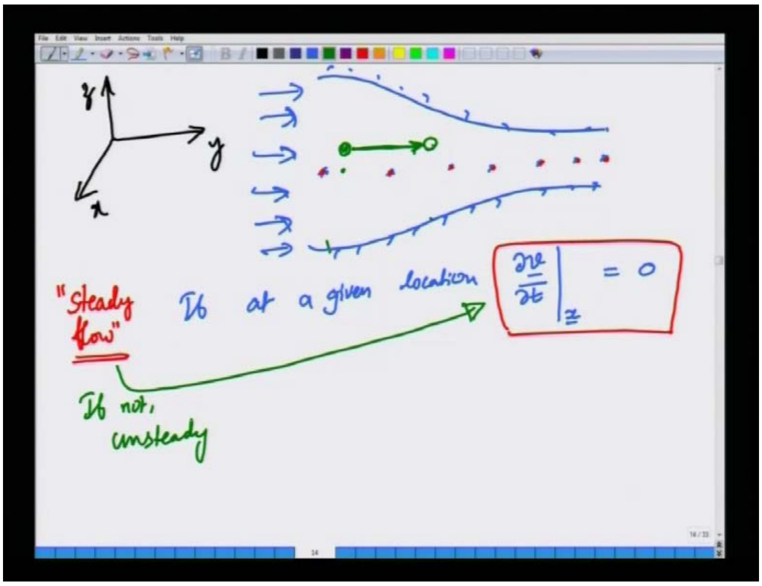


Now, the next topic that we are going to discuss is the following. Suppose, how to describe the notion of flow further? So, we are going to in this course, in fluid mechanics and in most fluid mechanics courses are researched one normally, uses the Eulerian frame of reference. That is the most convenient from an experimental point of view and that is the most useful from a practical view point also. So, Eulerian frame of reference will be what is used in this course as well as most applications, in fluid mechanics will encounter. But the connection between Eulerian and Lagrangian are always important, because thats what helps us to arrive at the notion of the substantial derivative.

Because even, if you want to work with in the Eulerian description, it is always useful to have the notion of substantial derivative, because only then we can compute quantities such as acceleration. So, in fluid mechanics, we will stick to Eulerian frame of reference and within the Eulerian frame of reference, the velocity is the vector is denoted as the function of three spatial coordinates and time. In fluid mechanics, we will use the Eulerian frame of reference and the Eulerian velocity field is given by v, the velocity vector. Remember that the velocity is a vector. In general in fluid mechanics all the three components of velocity will be present. So, we need to preserve this, v as a vector and it is a function of three spatial locations, that you chose to work with and it is also function of time.

So, imagine you have the situation, where you have a channel like this. And let us say that at a give point in space, the velocity is independent of time. So, at various locations in this fluid is flowing like this. This is the channel valve and fluid is flowing like this at various locations. Let us assume that for an experimental realization, that velocity is independent of time. So, at a given location, velocity, partial derivative of velocity at any given location is 0.

(Refer Slide Time: 37:18)

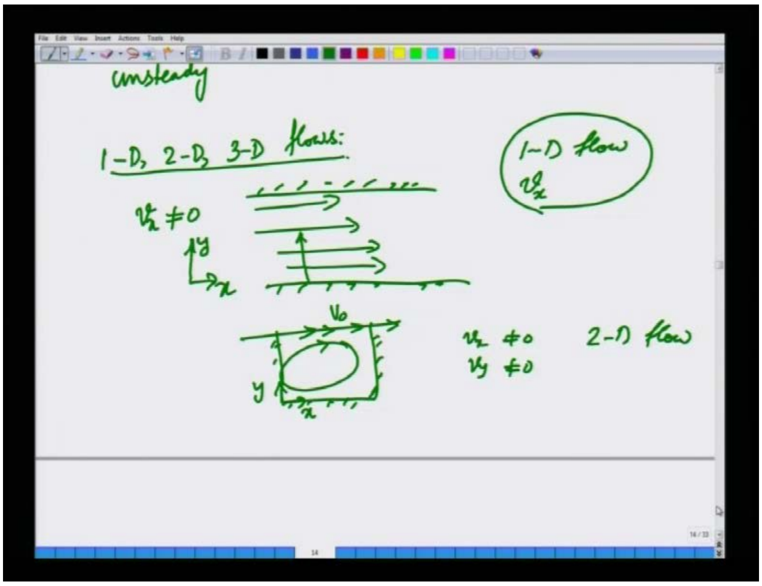


Then the Eulerian; this is on within the Eulerian description. This is the Eulerian description of velocity field, this is the Eulerian description where velocity is given as function of spatial positions and time. If at a given location the partial derivative of velocity with respect to time 0, that means the velocity of independent time any location x such flows are denoted as steady flows. So, in a steady flow in the Eulerian sense, if you look at various points and space measure the velocity at each point of the velocity will be independent of time. So, take the partial derivative of the velocity it will be 0 at each point. You fix a point and then measure the velocity and take it is time derivative, it will not change. But this does not mean, study in the Eulerian and it does not mean, that the velocity of the given fluid particle is not changing.

For example, if you follow this green particle from here to here, it is going from the region of higher cross sectional area to lower cross sectional area, if mass, if the fluid is incompressible, then the fluid is incompressible, then the amount of fluid is flowing here must be the same amount of flowing in here. As the cross section area as more here, compare to here, this fluid particle will get accelerated as it goes from here to here. So, even though at given fixed location, the velocity does not change the time, as you flow fluid particle it will it can accelerate in general. So, this is what we mean by the convicted contribution to the substantial derivative. Even if there are local even, if it locally studied, that is study in Eulerian scenes.

Given fluid particle can acquire acceleration or deceleration by the virtue of moving from regions of higher velocity to lower velocity to higher velocity or higher velocity to lower velocity. So, but in fluid mechanics by steady flow, we mean that the Eulerian velocity field is independent of time. Otherwise it is called unsteady, if not flows are called unsteady, that is velocity is indeed a function of time at various points of space, then the flow is unsteady.

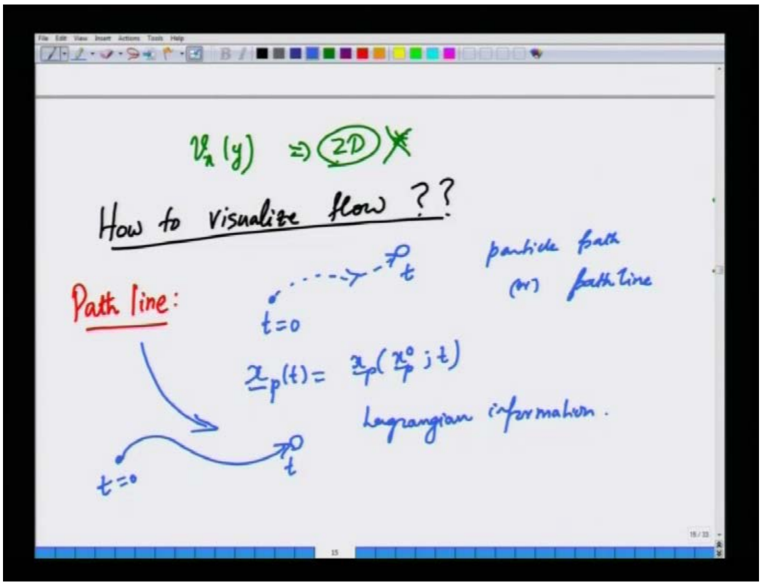
(Refer Slide Time: 39:40)



People also use the classification; 1-D in kinematics, 2-D and 3-D flows and this is in the following sense, if you have only 1 velocity field that is the fluid is flowing in only 1 direction. So, imagine you have a channel and the fluid is flowing only 1 direction, this is x and this is y and the flow is in the x direction. So, let us say only v velocity is none 0 and this vx can in general be a function of the normal direction y, but the flow is only in 1 direction. So, we can call this 1-D flow, because there is only 1 velocity component that is not 0.

And, if the flow is there in 2 direction for example, if you have a flow like in a square like cavity and it is moving this is x y, the top plate is moving with some velocity v naught the bottom side plates are stationary, then the fluid will go like this. So, both vx and vy are not 0 and we can call it 2-D flow and if all 3 velocities are there here, sometimes, we can call it as 3-D flow, but there is not unanimity among various text books and the nomenclature of such things.

(Refer Slide Time: 41:12)



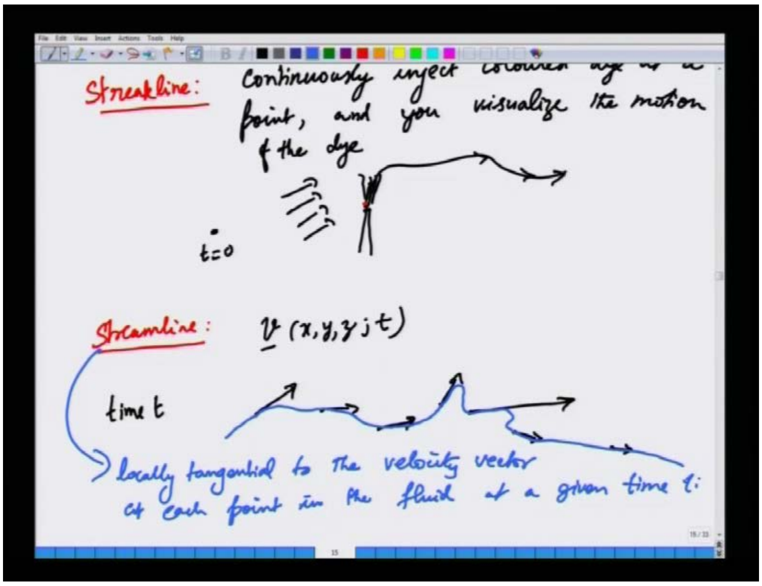
Because in certain conventions, even if vx is a function of y as in this case, even though in this 0 velocity, they will call it 2D flow, because you need 2 directions x velocity and the y direction to describe flow. But this is commonly used, but you may find some text books use this or some conventions use this. But it useful to just think of single velocity field being a function of, if you 1 velocity that is none 0 that we call it 1D flow and so on. Although, it is clear from the context what we mean. So, the next thing is how to visualize fluid motion, how to visualize flow?

In kinematics, we are worried about we are interested in, how to describe flow, how to measure various quantities? So, one of the fundamental descriptions of motion is what is called the path line. Imagine you have a liquid and at you can mark a point in a liquid with a colored dye and let us assume that, the dye does not diffuse then at time at some time t is 0. And then this particle will in general will move to some other location at a later time t, this is called the particle path or the path line. This is called the path line. So, how do we do this experimentally, well we imagine putting a dye at a point in space. And then just look at the motion of the point have been function of time that is the path line.

So, this is what, we formally wrote as p x of t as x p as a function of x p naught and t. This essential what are the Lagrangian description, can be obtained from experiments by putting a dot of dye or you can introduce a puff of smoke in a gas. And you can use the smoke to visualize the flow and the puff of smoke will serve as the identity of the particle, which was at location at time t equal to 0. And assuming, that the smoke does not defuse too much within the time scales of interest, then you can identify, you can visualize this motion of particle a fluid particle from time t equal to time t this is called the path line.

This is inherently a Lagrangian motion, this has Lagrangian information, the path line has Lagrangian information. Because you are following a point identified by it is visual location through by means of for colored dye puff of smoke. Then you are viewing it is evolution as the function of time, special evolution it is function of time. Now, another useful notion is called as streak line.

(Refer Slide Time: 44:18)



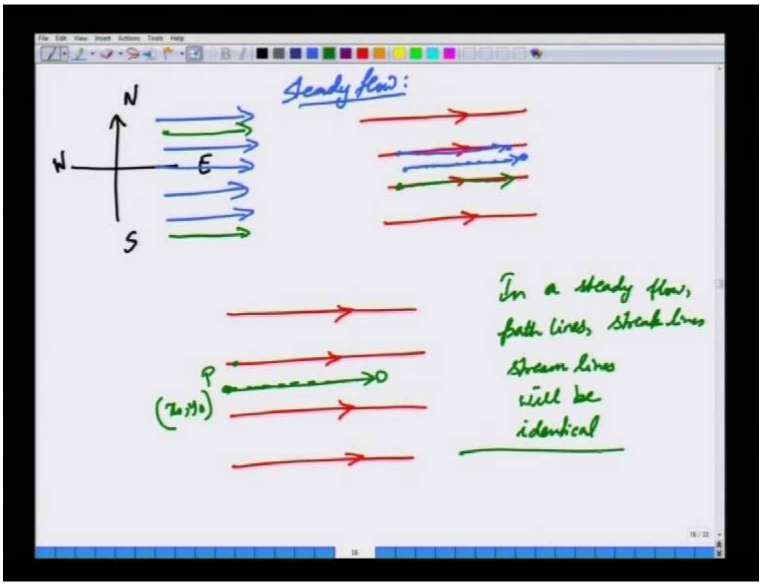
(No audio from 44:17 to 44:22) A Strike line is a, what you will get, if you continuously inject a colored dye at a point. And you try to the motion of the dye, I will give an example after, I am finished with the definitions. So, what the streak line does is, you are fixing point and space and you are continuously introduce dye of that point. And the streak line is the instantaneous locus of all the fluid particles, that have been ejected at the same point at some earlier a time. So, you are continuously ejecting fluid particles from time t equal to 0 at the same location and you are trying to see, what are the various locations of particles of that are being introduce from time t equal to 0 at a later time.

So, again simple example is suppose you have a smoke chimney, from which smoke is coming you are consciously injecting smoke and let say air is moving. So, the path that is taken by the smoke is an example of a streak line. Because you are continuously injecting a black or brownish colored smoke from a chimney and you are trying to locate and you are trying to visualize motion of this colored, you know particle in air. Another very useful notion, this is again this has Eulerian information, because you are not you are basically, worrying about what stuff being introduced at a point, but various particles will come and by occupying at that point. So, this as in some sense Eulerian information, finally, we have stream line, stream line is a mathematical idea. It is we will have to see how it is visualize experimentally, but it is concept.

The concept is that suppose, you imagine in the Eulerian description, you have the velocity vector as a function of three special coordinates in time. Let us look at a given time at a given time t at various spatial locations, you can plot how the velocity vector is going to look like. And you can plot the magnitude by showing a larger arrow and the direction by the direction of the arrow. And if the particle, the various points of the velocity field has different values, you can show it by the both direction as well as the magnitude. Stream line is a line that is instantaneously, tangential to the local fluid velocity vectors. So, let me try it, try to draw this as tangential as possible as.

So, stream line is a line, that is locally tangential to each, to the velocity vector at each and every point on the at a given instant of time, each point in the fluid at an instant of time at the given time. So, stream line is basically an idea, but it comes to we have to understand, how it is we have to prescribe, how this is measured experimentally or how it is visualized experimentally. So, we will illustrate this through an example. So, imagine.

(Refer Slide Time: 48:45)

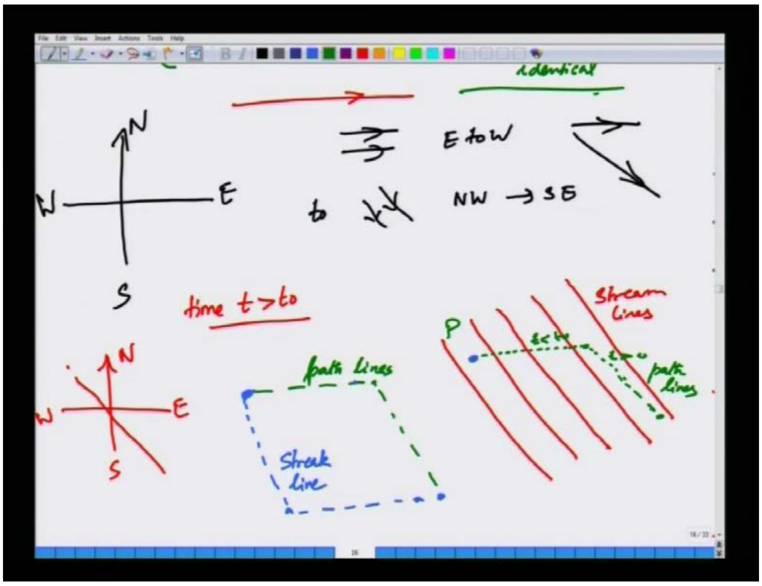


So, I am going to illustrate, the notion of path line, stream line and streak lines through an example. Imagine you have, North four directions, East, south and west. Let us say wind is blowing from west to east, let us say air is blowing or being blown from west to east. So, it is completely parallel to the east direction. So, if you look at the stream lines, they look completely parallel and assume that the flow is that steady. It is each and every point that the velocity vector is not changing with respect to time. So, flow is steady in the stream lines will look like this. Let us look at path lines, path lines you take any point and you inject a dye at that point, time t equal to 0 and local and watch it is motion is at later time.

So, this point would have moved at a later time, but it would also be line that is parallel to the stream line. Now, so it will be identical to the stream line, because you can inject a particle here, it will move exactly parallel on that stream line itself. So, in the steady flow, in the Eulerian sense, the path line and steam lines are the same and the streak line will also be the same. Because if you inject continuously inject smoke or a point then of course, this will if you continuously keep injecting smoke at this point this will keep move. So, simple realization is that, you have a chimney, let me rewrite this. So, you have this stream line, so they are parallel air is blowing from west to east. Imagine that you have a chimney at a location, x naught y naught at point P.

So, the path lines are the trajectory of point that was released at time t equal to 0 at this location. So, that will also be parallel in a steady flow, this you just keep going. Streak line will also be just identical to path line, because the flow steady it will continue to move in the same direction. And all this will be identical to stream lines, you can introduce instead of imagining here, you can draw another streak line here also. Berceuse streak lines are completely parallel in this simple example, because the velocity vectors are completely parallel to each other. So, in a steady flow, the path lines, streak lines and stream lines will merge, will be identical, what happens if the flow becomes unsteady.

(Refer Slide Time: 51:47)



We will illustrate with the same example, imagine that at this is north, east, south and west. So, imagine that at some time initially, the flow is from East to West. Up to time t naught at some time t naught the flow changes, from North-West to South East. So, at some later time, the flow instead of it is being like this. And are continuously, injecting smoke from a chimney, that is what we would imagine. And we are looking at a later time t, greater than t not what is the status of the path line and stream line and streak line. Well stream lines at a later time t greater than t naught. Steam lines are instantaneous descriptions of lines that are parallel to fluid velocity vectors. If the velocity vectors are all parallel to the North-West to South-East direction, you will simply see that the stream lines will be at an angle to the ok. There will be at an angle like this.

So, these are the stream lines. So, stream lines are in red. What about the path line? The path line, I am going to show it in green. So, you take a point P in which you have introduced, we have introduced a point at time t equal to 0. So, this point will be moving from in from the North, West to East direction and then at time t is 0, you are changing that the air is change in the direction from West to East to North-West to South-East. So, this trajectory at this is t less than 0, t greater than t 0, it will come here. So, the green lines are path lines, the red lines are stream lines.

Now, what about streak lines, which I am going to plot in blue. You are continuously injecting material or dye or smoke from at this point, the dye that was introduced at t equal to 0, it would have a trajectory that identical to path line. So, I am going to draw the motion of streak lines here, because it can be confusing this. So, I am going to so you are introducing continuously and for reference to plot this path line. The path lines are clear and time t equal to 0 and injecting something, it will travel up to t 0 here and then t 0 to t, it will go in the North-West to South-East direction. What about streak lines? Streak lines, will be slightly different, I will plot in blue, that point which was introduced at t equal to 0. It will go all the way here, and it will reach here, let me use blue color ok.

But the one, it is introduce at time delta t greater than t equal to 0, It will not have reached up to here, it would have reached up here and it would have changed, it is direction, because of the change in direction even it would have reached here. And like wise things that are introduced before t 0, they would go up to here and they would and the stuff that is introduced just before t 0 will be here and it will reach here. The stuff that was introduced after t 0 would directly follow this line. So, this is the streak line, while this is the path line. So, the path line and streak lines and stream lines will not agree for the unsteady flows. So, we will stop here and we will continue from here in the next lecture and we will see you in the next lecture.