Fluid Mechanics

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Lecture No. # 11

Welcome to this lecture number 11 on this N P T E L course on fluid mechanics for chemical engineering undergraduates students. In lecture number 10, we were discussing how to visualize fluid flows using various methods in while discussing flow kinematics. After we derive the derivation; after we derive the expression for substantial derivative of a quantity, we started discussing visualizing flows using various methods and in that context, we discussed the notion of a path line.

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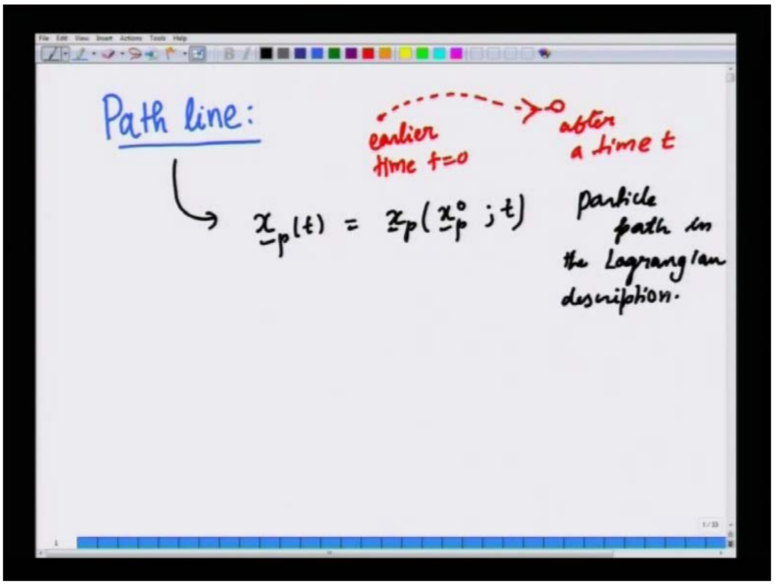

A path line, this essentially obtained by marking a point in fluid with a dye and assuming that the dye does not diffuse within the time scales of interest to us. We can locate the trajectory of the dye as the fluid moves this is earlier time t equals 0 and in general because of fluid flow this particle; this fluid particle may move this dye point which essentially marks a fluid particle may move to another position at after a time t. So, this is path line this is essentially, the path line is what we call the particle trajectory in the Lagrangian coordinate system is a function of the initial position of the particle and time.

So, we call this the particle trajectory or particle path and the Lagrangian description of motion in the last lecture. The path line merely says is way of obtaining that particle path experimentally. So, how this is done is by either introducing a puff up smoke at a given point in space, instantaneously and you visualize how this puff up smoke are a point of dye in a liquid moves as a function of time.

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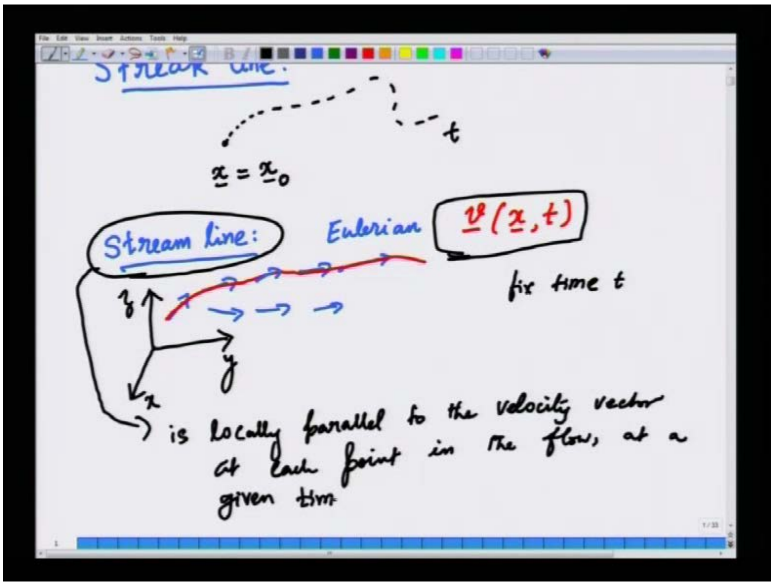

This gives you the particle path or what is called the path line. Another useful concept is what is called the streak line; the streak line is obtained by continuously injecting material at a given spatial location. Let say you are locating a spatial location by position x 0, your continuously introducing dye and this. So, this will eventually give you a trajectory and of particles are dye molecules that are introduced continuously at the same point. So, you will essentially get a visualization of at a later time t of particles that have all positive with the same point, but at various instances of time, that is the streak line. And finally, we have the stream line which is an Inherently Eulerian concept.

In the sense that, in the Eulerian description the fluid velocity is given as a function of spatial locations with respect to a fixed coordinate system and time and in this regard you can visualize. Suppose, you have this vector function, how do you visualize this I said at well let us at a fix time. Let us fix time t to some value; you can plot the velocity vectors at various points in space. For example, it might look like this of course, there are infinitely many points, but you can choose to plot the velocity at some points and points with higher velocity can be denoted with vectors with greater magnitude. And length with some sense and this will give raise to a description of the velocity vector at each and every point in the fluid at a given instance in time.

Now, suppose I take infinitely close points and draw a curve such that at each point the velocity vector is tangential to the curve. For example, as we have shown here in the red line such a description is called a stream line, a stream line, a stream line is locally parallel to the velocity vector, at each point in the flow, at a given time so that is the stream line. The stream line is therefore, instant Inherently Eulerian idea, while the path line is an Inherently Lagrangian concept. And we also started by discussing elementary example which tells how these three descriptions of motion of the ways of visualizing motion how they compare and contrast with each other.

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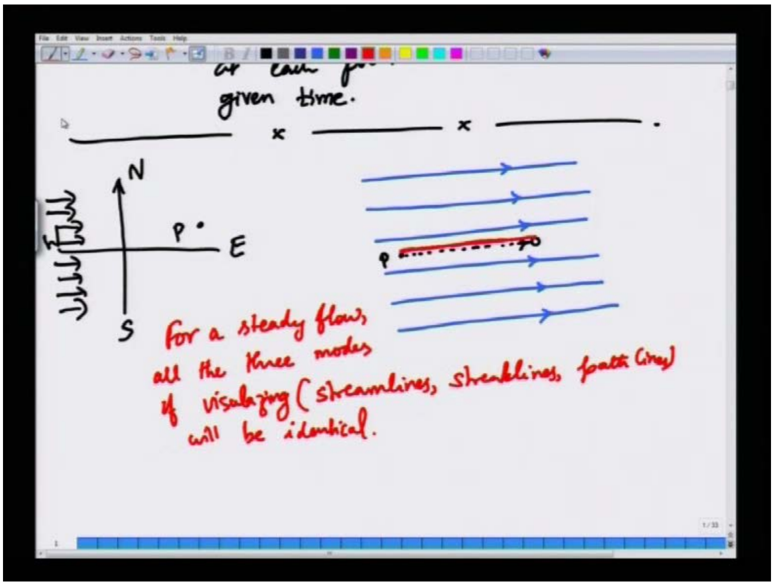

So, this was obtained in a simple setting, where which said that north, south, east, and west. So, imagine that when was blowing from West to East and at some point x, let say p smoke is been continuously introduced at that point. So, we can imagine, how the stream lines are going to look like and this is steady flow in the sense that the velocity vectors are constant at each and every point in the fluid, in the Eulerian sense the flow is steady. So, if you draw the essence it is a constant velocity, if you draw the stream lines they will all be parallel to each other and we can plot many stream lines, as we wish.

Now, if we look at path line suppose, introduce our point of pop up smoke at a point p of the at the point p, this pop up smoke will also move in a straight line and it will also merge with one stream and in the fluid. And how about the streak line well that will also merge I will draw this with red line, red continuous line because, as the dye molecules are continuously being introduced. Since, the flow is steady each molecules that is introduced at a later time also will follow identically the same path, as the path line. So, for a steady flow all the three modes of visualizing will yield the same result, namely stream lines, streak lines and path lines, will be identical.

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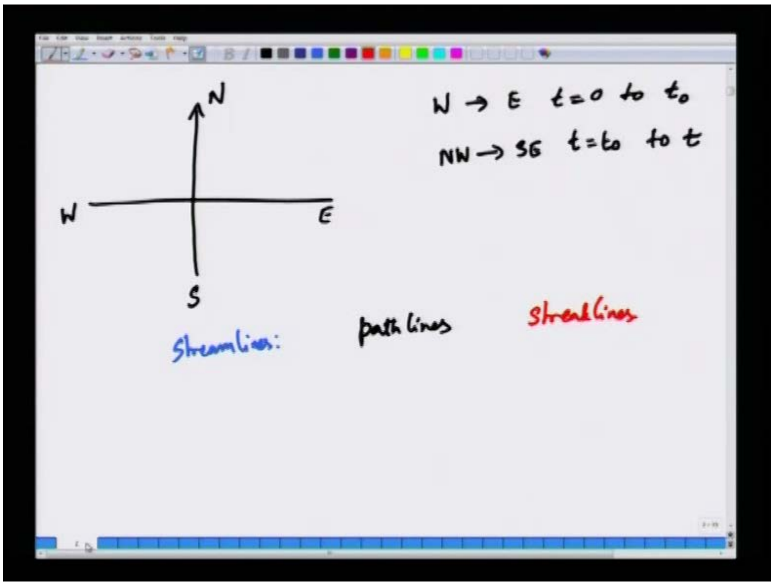

However this is not quite the same for an unsteady flow as we discussed in a related example North, South, East, West. Suppose, the wind is blowing from West to East from time t equals 0 to t equals t not and then North West to South East from t equals t not to current time t. So, this is an inherently unsteady flow, because you have the wind direction changing abruptly at some time t not. So, first we are now going to visualize this flow by stream lines which will I will denote by blue, path lines I will denote by black and streak lines I will denote by red. So, stream lines it will be simple, because at a later time. So, all of this we are going to look at time t greater than t 0.

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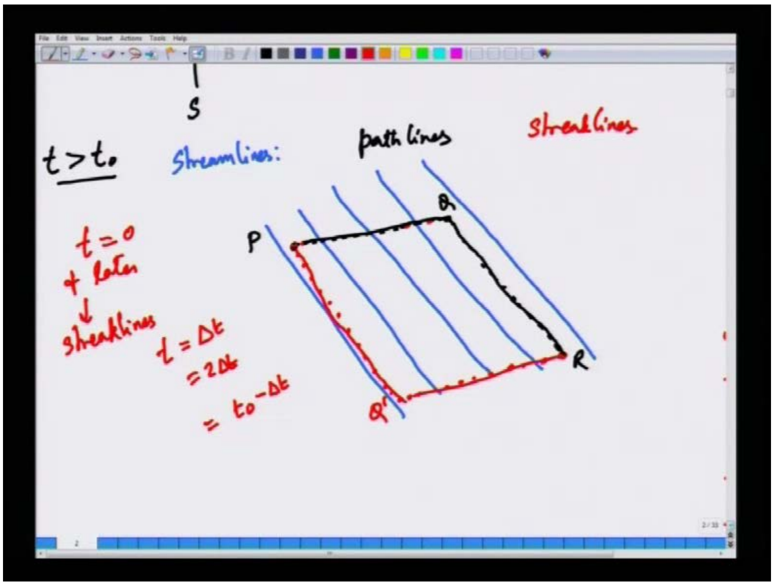

So, where the wind at same stretch direction from West to East to North West South East. Now, stream lines at any later time will all be in the direction North West to South East, because stream lines will nearly depict the instantaneous location of fluid velocity. And since, the fluid velocity for any time t greater than t 0, there are all in the direction North West to South, the South stream lines will look. Suppose, let us look at path line, path line of a particle that was of a pop up smoke, let us say that was introduced at a point P. So, this path this would undergo a motion straight up to time from tie t equal to 0 the t equals t 0 after that the wind will start blowing in this direction.

So, it will take the root Q R straight lines on the other hand are found by particles that are continuously injected from t equal to 0 and later. So, streak lines are formed with dye-molecules here in this case of are you have pops of smoke that are continuously introduced from t equal to 0 and later. So, the pop up smoke that was introduced at t equal to 0, with the identical to the path line it will go up from P to Q up to t equal to t 0 and from Q to R from t 0 to t the current time. A particle that was introduce later, let say t equals delta t would start from P, but it will not reach up to Q, because by the time it reaches here, the wind would have changes this direction. So, from here, this particle would move and it would reach here the wind would have change this direction so from here, this particle would move and it would reach here.

The particle that was introduce at t equals some 2 delta t would go from p to some point here and then it would reach here. So, particles that were introduced from t equals to delta t to t equals to t 0 would essentially, so particle that was just introduced just before take t 0 would have just moved somewhat here and then it would move here. Now, a particle that was introduce at t equal to 0 will directly move here, because the wind has changed direction, the particles that were introduced at t greater than t 0 will all move in this direction. So, the path line will be this red line, where as the sorry the streak line will be the red line, where as the path will be the black line P Q R, well the streak line will be given by P Q prime R. So, for unsteady flows different ways of visualization will be give you different types of information and they do not all the three lines do not coincide with each other.

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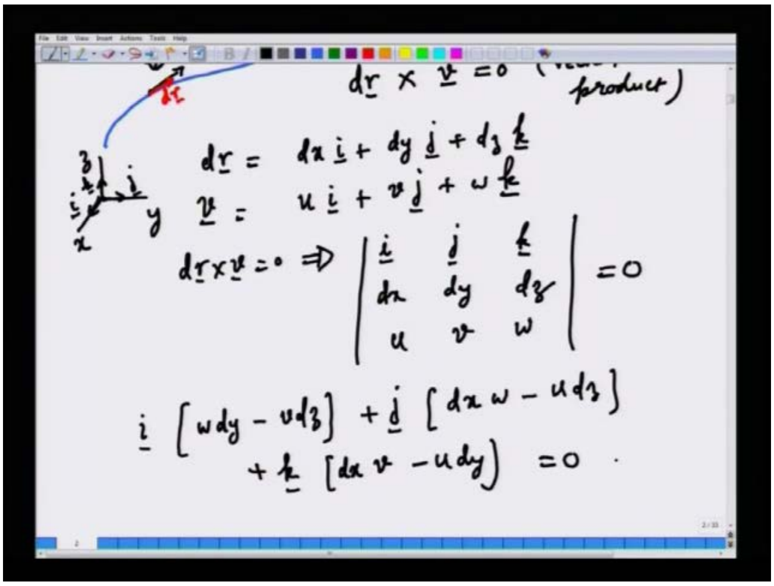

So, we can also derive an expression for mathematical expression for stream line. So, a stream line as I told you, if you take a stream line, this is a stream line, it is parallel to the local velocity vector. So, if this is the point at which your looking the velocity is here, if I take an elemental distance dr at about the same point. So, dr will be parallel to v, that is the definition of stream line at any given point, if I take an infinitesimal step along the stream line that displacement vector will be parallel to the velocity vector. Now, if two vectors are parallel the cross products are 0 so dr cross vector product, this is vector product or cross product. The vector product of two vectors are 0, then I can write the individual vectors has dr as dx i plus dy j.

So, imagine plotting of a coordinate system x y z and with unit vector i j and k. So, the dx i plus dy j plus dz k, this is the displacement in resolved in the three Cartesian directions, the velocity is u i plus v j plus, the vector velocity is written in terms of the three Cartesian components. So, now the cross product of dr cross v is 0 implies that the determinant found by i j k dx dy dz u v w must be 0. This implies that i times w dy minus v dz plus j times dx w minus u dz plus k times u dx v minus u dy is 0. So, if this is 0 this is a victorial quantity of this is 0 individual components along with three directions was be individually 0, because these three are orthogonal directions.

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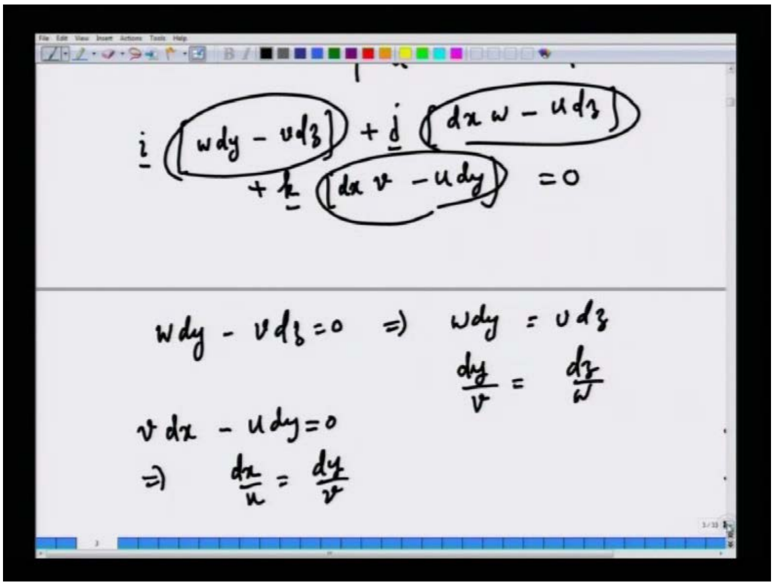

So, w dy minus v dz is 0. Which implies w dy this v dz or dy by v is dz by w. And dx v minus u dy is 0, implies dx by u is dy by v which implies that equation describing a stream line is dx by u is dy by v is equal to dz by w in general in three dimensions. So, this is the equation that describes this stream line. So, this is the equation governing a stream line.

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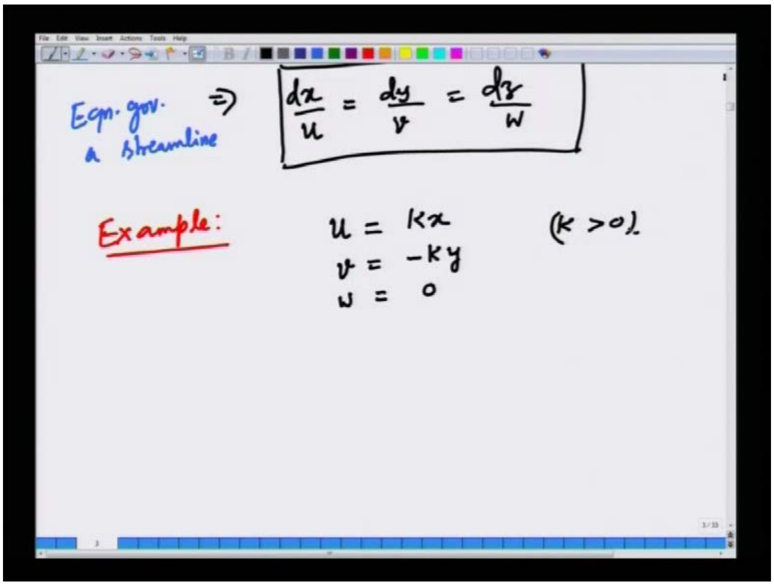

So, if we are given the Eulerian velocity vectors, so I will illustrate with the help of an example: how to calculate an expression for the equation for a stream line. Suppose, I have the u velocity, Eulerian velocity to be some k times x v velocity to be times to be some minus k times y and w velocity is 0. Suppose, this is the case where k let say greater than 0 so positive constant.

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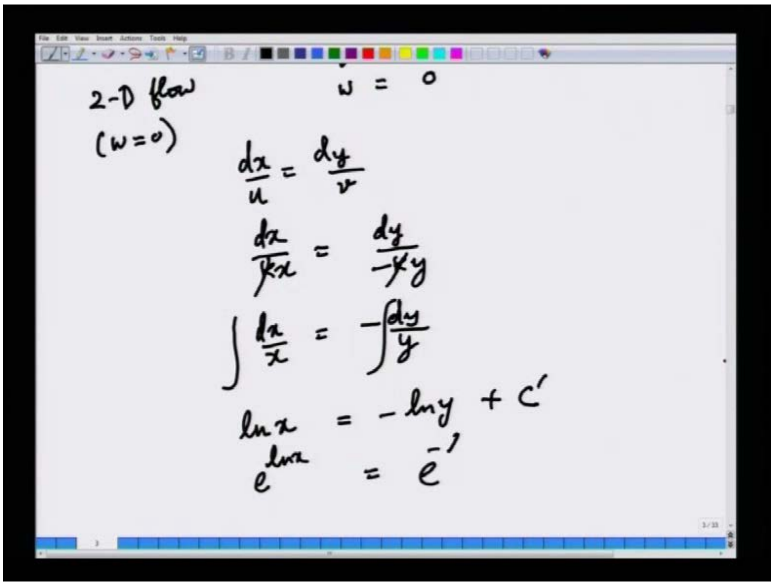

So, constant that greater than 0, so it is a 2 dimensional flow in the sense that w is 0. So, we have to worry only about dx by u is v dx by u is nothing but dx by kx this dy by v is minus ky k k will cancel. So, dx by x is minus dy by y. So, if I integrate both sides so I get log x is equal to minus log y plus some constant c. So, if I exponentiate which call its c prime exponentiate both sides I get e to the log x is e to the minus log y plus c prime,

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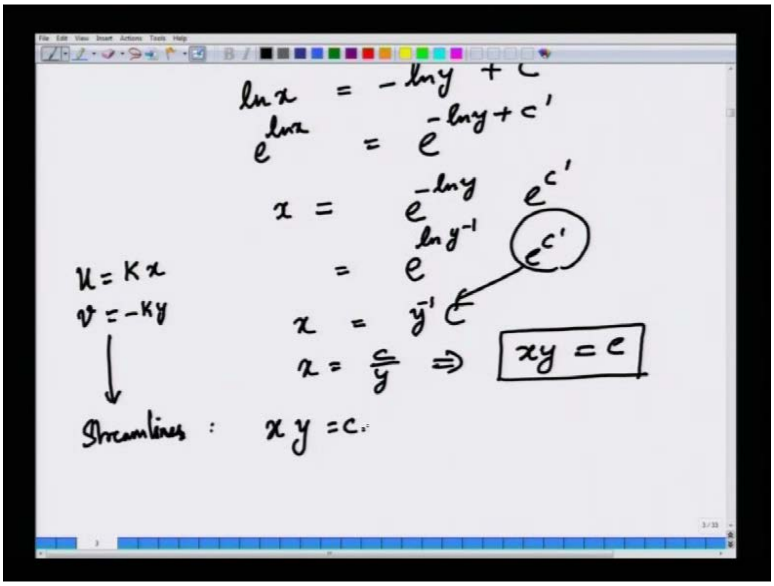

e to the log x is natural of log of x is simply x e to the product of two quantities is e to the minus log y times e to the c prime. So, e to the minus log y is a simply, e to the minus log y can be written as e to the log y to the minus 1 times e to the c prime. So, this is simply y to the minus 1 at e to the c prime is another constant, which i c this is y to the minus 1. So, x is c by y or x y is a constant. So, if I have this velocity profile use kx and v is minus ky, then the equation describing the stream lines is given by x y is a constant,

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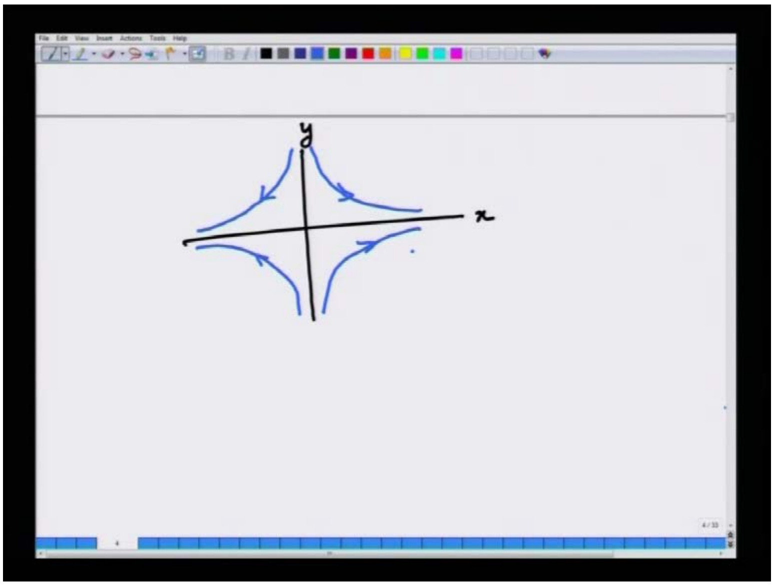

So, if you understand what this means if I plot x y coordinates I will get a hyperbolic kind of behavior. Of course, we are not worried about how this flow is going to happen here, in this in the context of kinematics, we are simply trying to understand how to describe flows and motion. So, in general so we have stream line, streak lines and path lines. So, stream lines and all the three descriptions will merge if the flow is steady, but if the flow is not steady, then all the three descriptions will be different from each other and you will have to a figure out it is not easy to figure out one from the other. And we should also remember that the path line is an Inherently Lagrangian concept, while the stream line is Inherently and Eulerian concept.

So, this a sort of sets the ground for analyzing fluid flows, because we are now discuss the elementary principles of kinematics. Now, I am going to go to another major topic how to analyze fluid motion. (No audio from 19:53 to 20:02).

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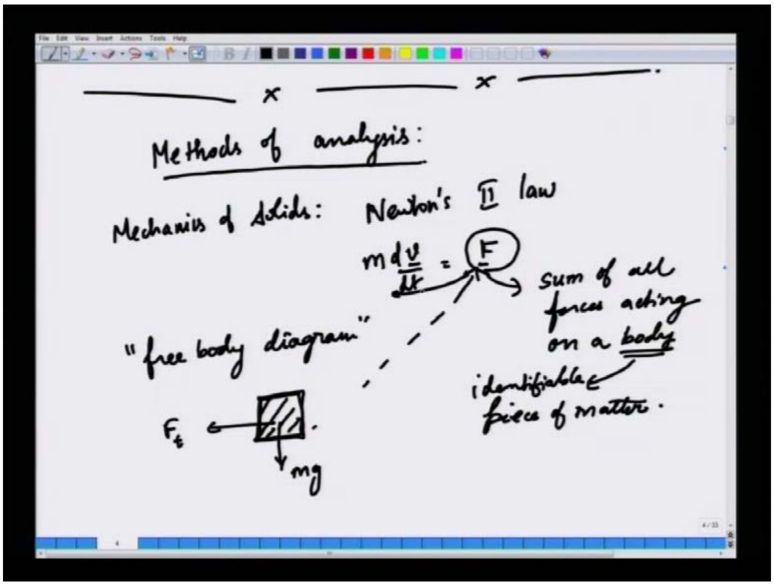

Methods of analysis, so in solid mechanics, in elementary mechanics of solids of point particles and such we simply apply Newton second law. So, which simply says that the rate change of momentum m d v d t is sum of all the external forces acting on a body. A body is essentially an identifiable piece of matter in mechanics that is we can identify a set of points and we can always make sure that the same set of points will constitute the body piece of matter in mechanics. So, the way in which problems are dimensionally do a free body diagram. In mechanics identify the body let say you have a block of food a something solid block, we identify the forces this is our body, you may identify forces such as gravitational force and then may be some other tangential force and so on.

And then, we will try to compute the motion, why identifying by isolating the body and asking question as to what are the forces and once you figure out what are the forces and plug in here and then, you can compute the velocities and how they change with time with the help of an initial condition.

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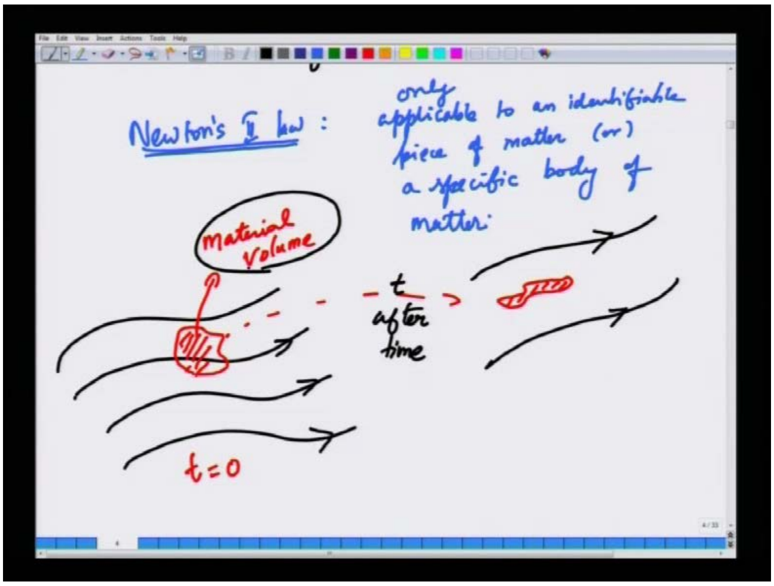

But the key in all of this is Newton's second law which is the fundamental law of motion of material, that we have in classical physics this applicable to an identifiable is only applicable identifiable piece of matter or a is specific object; specific body of matter. So, whereas in fluid mechanics, we mentioning that it is not feasible for us to identify the same set of point, a same point or same set of points, because of fact that in fluid flows, we are not generally interested in the motion of a given point or given set of points. So, there is a fundamental sought of dichotomy here, that while Newton's second law as we know in mechanics is applicable to a specific body. In fluid mechanics we never, we rarely actually worry about motion of a specific set points in a fluid rather, we are interested as I mentioning the last couple of lectures.

We have been; we are interested in what is the force on a surface and we are not really worried about, which particular fluid particle or which set of particles are going to exert that force. But in principle at least in fluid also, we can identify a suppose, you have a fluid flow like this, these are the stream lines. In principle, we can identify a set of points by imagining by putting a blob of dye not a point of dye but the colored blob of dry dye drop of dye let us say of some shape at initial time. And if the dye molecules do not diffuse much in the time scales of interest to us in the flow, then this particular piece of fluid that is being identified by a blob of dye is a specific body it is a material body or material volume.

A material volume in fluid mechanics contains the same set of material points or fluid particles. So, earlier in kinematics, we discuss the notion of a single fluid particle which was identified by its initial location a material volume is nearly a collection of various such fluid particles. And we can look at their motion collectively together as the fluid flows this material volume will moves stretch deform and so forth. Now, it is in principle possible to do this or in practice this can be idealized by imagining that you put a blob of dye colored dye in a fluid like water. And assume that the dye molecules do not diffuse much within the time scale of interest to us in the flow.

So, this is at time t equal to 0 let us say, at a later time this blob of fluid may. So, this blob of fluid which was like this here, may deforms stretch move in space and so on. So, this is after some time t, then this particle may this material volume which we imagine contains a same set of material points will stretch deforms and moves.

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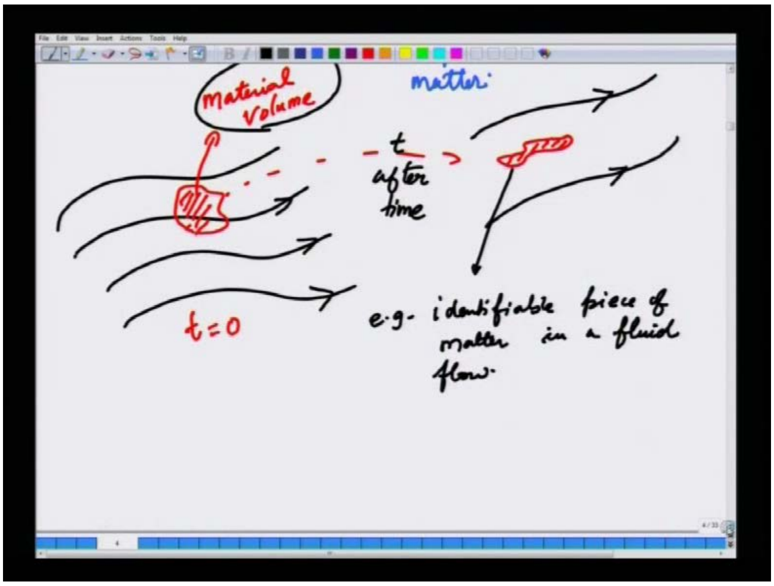

So, this is an example of an identifiable piece of matter in a fluid flow (No Audio From: 26:18 to 26:24). So, this is one way in which you can identify a same set of a fluid particles or material particle.

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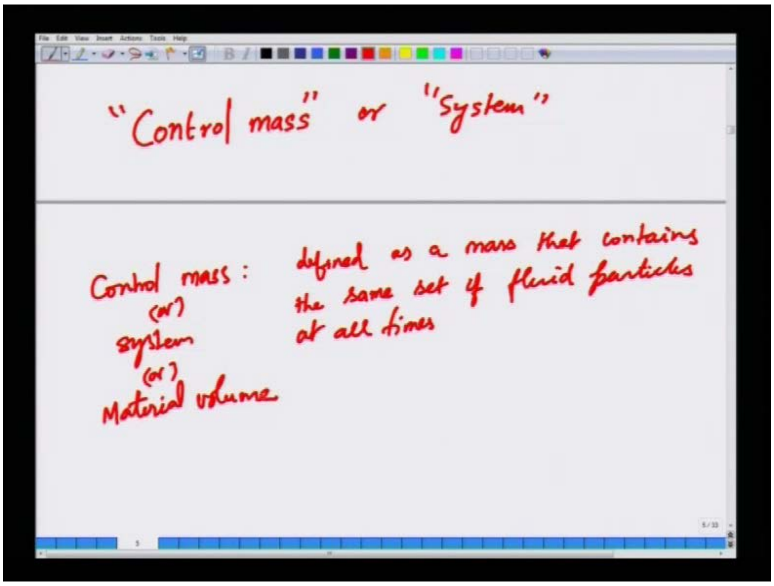

Such are description is called control mass or the system approach. A control mass contains by definition is defined as a mass that contains the same set of fluid particles at all times, this is called control mass or system or sometimes it is refers to as material volume.

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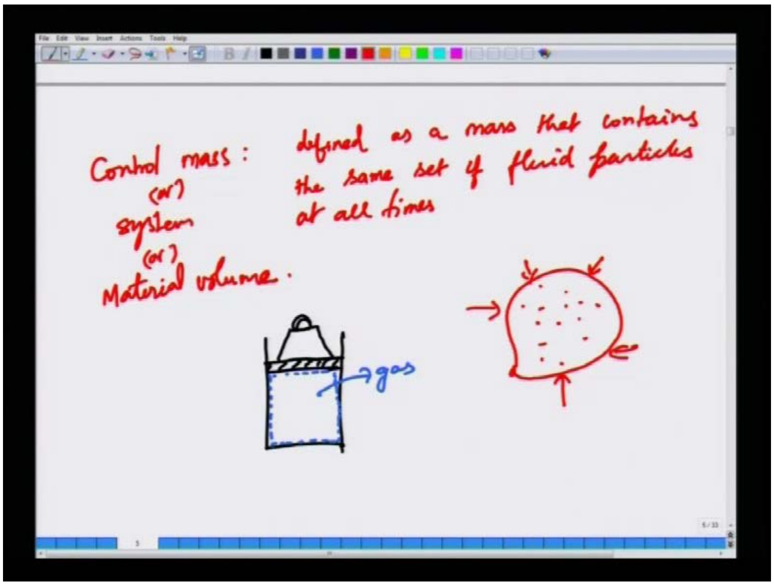

Now, examples of control mass are the following how to realize this in a real experiment imagine you have a piston cylinder assembly, let us common thermodynamics. So, imagine you have a mass, now this dotted line inside contains gas and as you increase the wait as you compress this gas, this physical volume will shrink, but the material volume will have the same set of gas molecules or fluid particles. So, this is an example of a control mass it has the same material. Another example is that of a balloon which is filled with gas and you can stretch deform the balloon by applying forces, but this will have the same set of material points all though be they may be moving about within that particular volume, but they will all be present within the same a enclosure.

So, it contains, the control mass always contains the same set of fluid particles or material particles. But in fluid mechanics it is not feasible to do it, this is always good in thermodynamics of closed system, because the system is completely closed the sense that there is no entry or exit of mass into the system into the area of interest.

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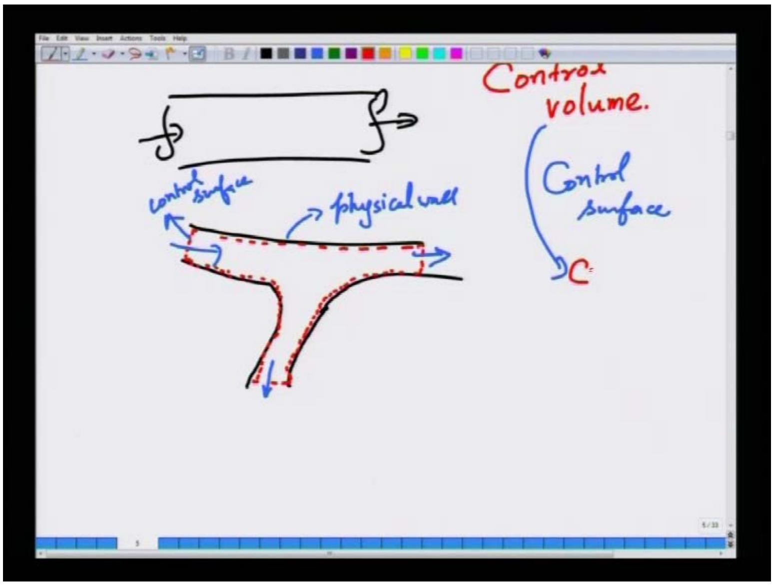

But in fluid mechanism it is not the case, because we are usually dealing with open systems like pipe of width, enters, leads, the pipe and so on. So, it is not feasible to follow a single point or the same set of points as the fluid is moving in a device such as a pipe or a pump over or anything that we have. So, it is not easy for us to a follow a specific body in fluid mechanics. So, what is normally done fluid mechanics? Is to use, what is call the control volume approach, the control volume is a fixed location fixed region in space of interest to us. So, for example, you may have a manifold like this, a flow manifold like this so control volume, we may choose this something like this. It is completely imaginary it just aids our problem solving you may just choose it, so that it is present just inside surface of the conduit of this t shape conduit.

So, there are surfaces where fluid comes in and there is a surfaces, where fluid lives, these are imaginary surfaces these are called control surfaces. May need not be always real in the sense of a real surface that is separating the inside and outside the control volume, sometimes they can be simply imaginary. For example, this is a physical wall where fluid cannot enter early whereas this is a control surface where fluid can enter or live its only in our mind that the there is a surface. Because this is a region that we have chosen to work with in the control volume.

So, a control volume is very very suited for problems in fluid mechanics for analyzing problems in fluid mechanics, because of the fact that usually we have open a systems in the sense that fluid will come in and go out of a given device or a given pipe and so on. So, it is not possible for us to follow the same set of points therefore, we have to a isolate a region of interest which is called the control volume and the boundaries of the region is called control surface. But, the boundaries can be real boundaries such as walls of a tube or imaginary boundaries where in you just draw the boundary. So, as to demark it the region of interest control volume with outside and fluid can enter or leave the control volume by virtue of the control surfaces which are not true surfaces that bound the fluid.

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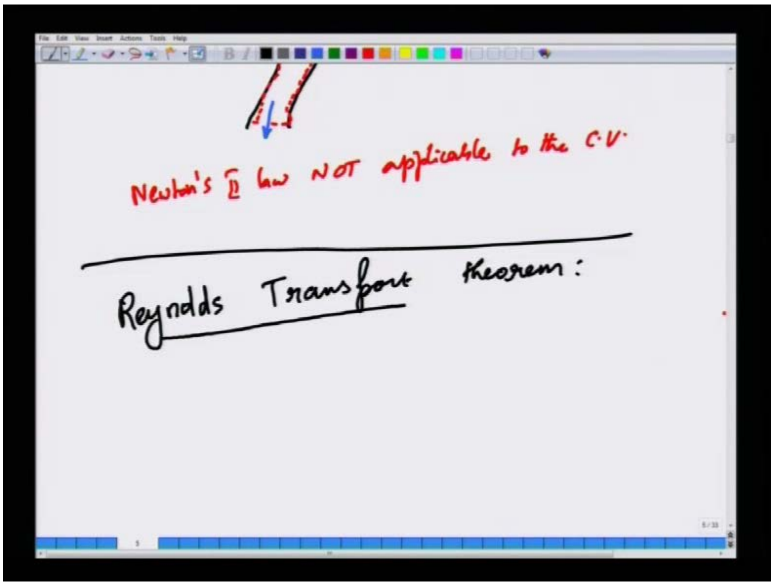

But, this control volume is normally denoted by the short hand form c v, c v approaches is has its own disadvantages. Because Newton's second law is not applicable to c v, directly applicable to a c v, because a c v is not an identifiable piece of matter so the Newton second law is not directly applicable to the c v. So, therefore, we need to find a way to relate quantities, such as acceleration of a mass present in the c v, two quantities that are present in the c v. So, we need to find a way to rate of changes in the c v, to rate of changes of the system, because the system is an identifiable piece of matter, it is a collection of material point while c v is not. So, this is achieved by what is called the Reynolds transport theorem (No audio from 33:14 to 33:22).

The Reynolds transform theorem is a tool to convert derivatives, time derivatives of system in terms of time derivative of the variables that are present in the c v. This is not very unlike the conversion of a Eulerian time I mean time derivative, to the Lagrangian time derivative, by virtue of the substantial derivative. So, this is very similar to that only that, we are not talking about a single point in space or a single material point but here we are considering a microscopic volume called the control volume. And we would like to behavioral derive a result, which relates the rate of change of a various quantities related to the control volume. Such as mass present in the control volume or momentum present in the control volume or energy present in the control volume, in terms of the variables that are represented in terms of the system. So, let us just start the derivation and we will; we write down the formula for a control volume as we go along.

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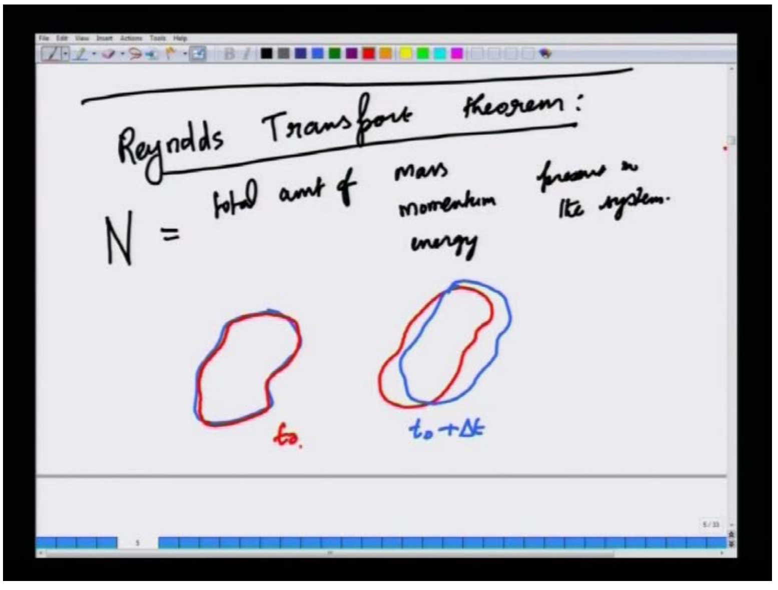

Suppose, let N, capital N be the total amount of mass or momentum or energy present in the system. We imagine that the system and c v coincide at a given time but at a later time, while the c v remains the same the system would have moved the blue thing has a system where as the red thing has a C V. Suppose, at a given instant of time, we identify all the fluid particles that are present in the c v as our system. These set of points will not be staying put in the c v, because if the fluid is in general moving this points will move away. So, this blue line is not no longer occupying where it the; where the red line was at time t equal to t 0 at a later time t 0 plus delta t, this is time t equals time 0 the later time the blue lines and red lines will separate, because the fluid chunk of molecules the chunk of fluid particles have moved elsewhere.

So, the Reynolds transform theorem is a tool to that enables us to relate the time derivatives of an identifiable piece of matter, which is what we call the system with the properties that are relevant or the properties that relate to C V, which is a fixed region in space. This is not very unlike the notion of a substantial derivative where in we a used the Eulerian information, which relates to various fixed points in space to find the instantaneous rate of change of a property like a velocity or temperature of a fluid particle, which happens to be at a given spatial location as we follow the particle. Here, we are nearly generalizing that notion not to just one fluid particle, but to a collection of identifiable piece of fluid particle called a system.

So, if you follow the system, you are following a same set of a large collection of a identifiable fluid particles and as the fluid flows this fluid particles, the collection of fluid particles will keep moving they will deforms stretch and so on. And you are following the same set of particles, this is space control a mass or the system approach. Whereas the control volume is a fixed region in space and a for example, you may have a section of a pipe which; in which we are interested in calculating the force exerted by the fluid on the solid surface the drag force. And in that case, we will just a isolate a region of interest that namely a section of the pipe or pump or compressor and so on. And we will try to analyze that particular region of interest towards.

But none the less even though the control volume approach is more suitable in practical applications of fluid for problems, we still need a to be able to relate the rate of change of a quantity like a velocity. As you follow the same set of particles, because a physical law such as Newton’s second law they are not apply to a fixed space rather they are applied to an identifiable piece of matter namely the control mass. So, we need the tool to be a which translates the information from the control volume approach to the control mass of approach and that is basically the Reynolds transfer theorem. So, once we have the Reynolds transform theorem, which we are going to derive now, then we will able to very easily write all the rates of change of or changes of a quantity. Such as mass of a fluid or momentum present in a fluid or energy of the fluid, that corresponds to a system or an identical piece of matter with the variable set correspond to a c v which is a fixed region in space.

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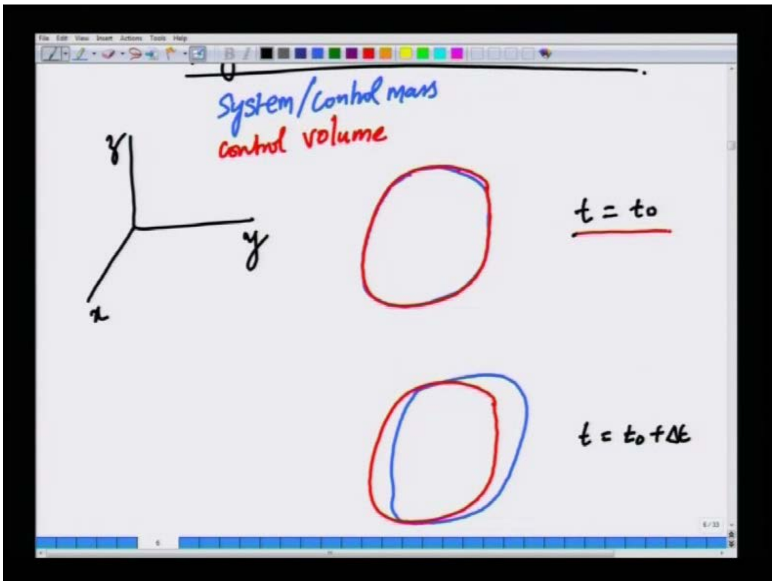

So, in order to help visualizing, you imagine that the system is essentially denoted by the blue color, system or control mass is denoted by blue color, while the control volume is denoted by red color. Imagine that the system on the control mass both the blue color and red color regions, they coincide at time t equal to 0, that is what I shown here a to the best I can of course, no all these are identically over overlapping, the both the blue color and the red color at some time t equal to t 0. At a later time t equal to t 0 plus delta t the c v is the fixed region and space respective to a coordinate system x y and z, the c v will remain the same, but the system the blue line would have moved elsewhere, because the fluid is continuously moving.

So, if you mark the set of fluid particles by a blob of dye that will in general not be located in the same spatial region, because as the fluid flows it will stretch the blob of dye and it will deform it will rotate it will translate and so on. So, in general this blob of dye, which is what I call control mass, but I moved elsewhere to a some other location immediately after a time delta t.

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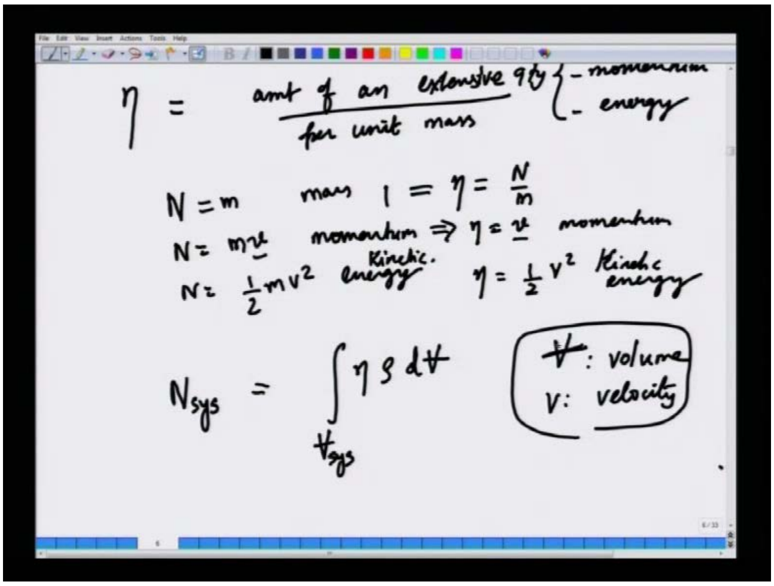

So, to derive the Reynolds transport theorem, let us call a quantity eta is the amount of an extensive quantity, such as mass, momentum and energy of the fluid that is presenting in the control mass, per unit volume Sorry per unit mass. So, N is m for mass, N is m v for momentum, N is half m v square for energy so for kinetic energy for the fluid. So, eta is per unit mass N divided by m, so eta becomes 1 for mass, so eta becomes v for momentum, because m v is the momentum per unit mass is simply v and eta becomes half v square for kinetic energy of the microscopic kinetic energy of the fluid.

Now, we are interested in the quantity that present in the system, the total amount of quantity such as mass momentum or energy is present in the system. So, this is integral or volume of eta rho d v, I am going to use v with which is struck like this, which is cross like this for volume, because V is normally used for velocity. So, in order to distribute volume from velocity will use v with a cross line like this, for volume and just we are v for velocity. Now, this is the total amount of let say mass momentum or energy that present in the system.

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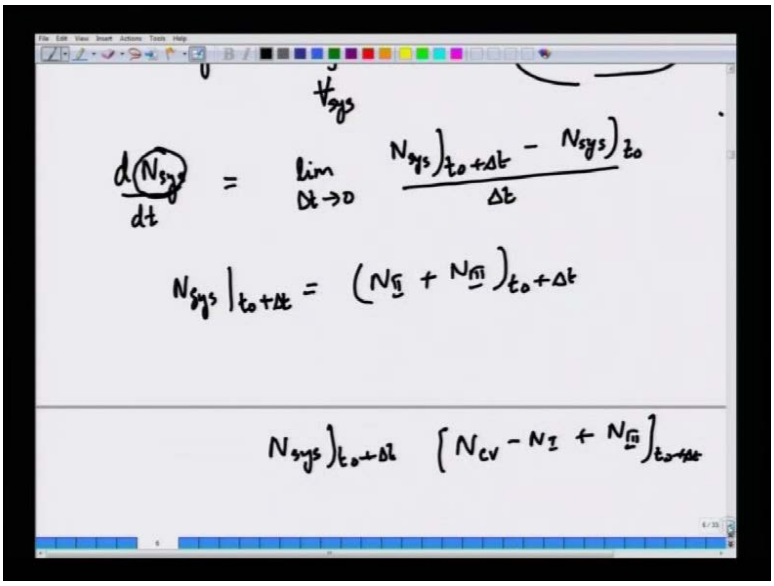

Now, what we are interested in obtaining is the rate of change of; the total amount of quantity such as mass momentum and energy present in the system is nothing but by fundamental definition limit delta t going to 0 N system at time t 0 plus delta t minus N system at times t 0 divided by delta t. Now, we have to take help of the figure, let me just do that, in this figure we can identify three regions. Region 1, which is essentially this region, which is vacated by the system at a later time t 0 plus delta t. So, this is the c v the help the portion that is present in the C V, where the system as vacated that is the fluid particles of left this region at a later time t 0 plus delta t. Region 2 is the common region between system and c v at a later time and region 3 is the new set of volume that is been created by the material particle set a movie. So, will call this three regions 1 2 3 as shown in this figure.

So, by the fundamental definition, so the late of; rate of change of quantity such as mass momentum or energy present in the system is d and system by d t. Is as limit delta goes to 0 and system t 0 plus delta t minus N system at t 0 divided by delta t. So, N system at times at time t 0 plus delta t is nothing but from the figure at time t 0 plus delta t is nothing but N 2 plus N 3 in this example. N system is therefore, limit delta t going to 0, N system at t 0 plus delta t is nothing but N 2 plus m 3 at t 0 plus delta t this is nothing but this is N system at t 0 at delta t is N 2 plus N 3.

N system at t 0 plus delta t, N 2 is nothing but N c v minus N 1 plus N 3 at t 0 plus delta t to do this a understand this or appropriate this better, we look at the figure. So, N 2 which is just this region is nothing but N 2 N sorry N system at time 0 plus delta t which is the blue line is nothing but N 2 which is as hatch by black lines and N 3 which is hatched by violet lines. So, N system at time t 0 plus delta t is nothing but N c v and N 2 is nothing but N c v minus N 1 so N c v minus N 1. So, let me just go here N 2 is nothing but N c v which is a red minus N 1 which is; minus N 1 is this yellow shaded region. So, if you subtract this you will get N 2 at time t 0 plus delta t. So, if you subtract N c v, which is total red line from this you will get N 2.

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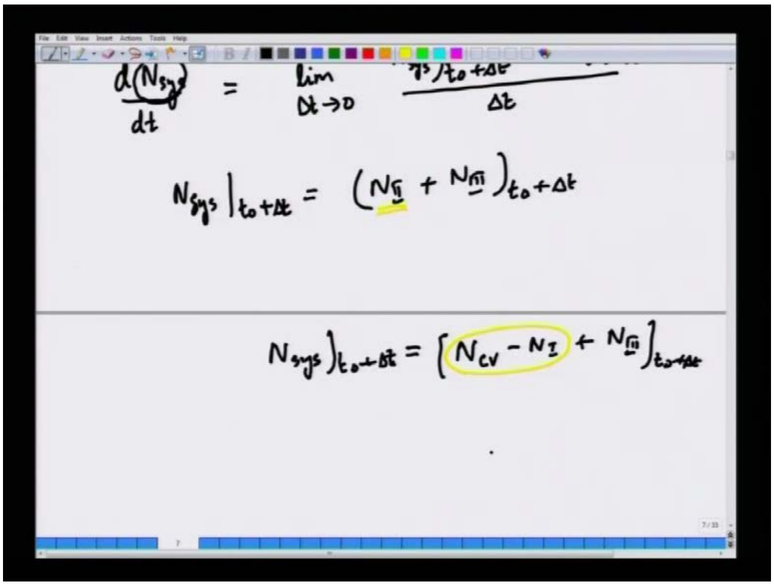

So, if you subtract that you will get N 2 at time t 0 plus delta t, because that is N c v minus N 1, N 1 is the region that was vacated by the system. So, this will completely give us N system at time t 0 plus delta t.

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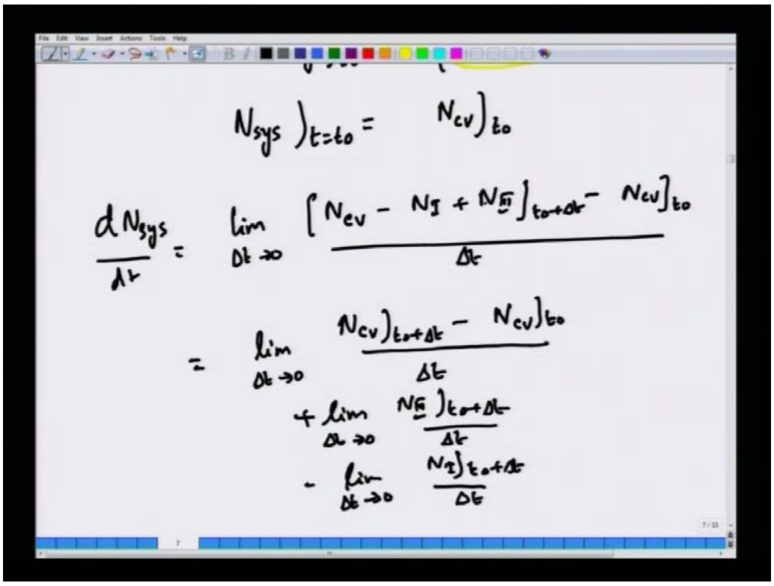

And N system at time t equals t 0 is simply N c v at time t 0. So, system on c v collapse or they identical or coincident at time t 0. So, the rate of change of the quantity N in the system with respective time is nothing but limit delta t going to 0 N c v minus N 1 plus N 3 t 0 plus delta t minus N c v t 0 divided by delta t. So, this is nothing but limit delta t going to 0 N c v t 0 plus delta t minus N c v t 0 divided by delta t, plus limit delta t going to 0 N 3 t 0 plus delta t divided by delta t, plus limit delta t goes to 0 or minus N 1 at t 0 plus delta t divided by delta t. So, this is nothing but the various terms that are present in this equation. So, N c v at t 0 plus delta t minus N c v at t 0 this is the first minus N 3 then minus sorry plus N 3 minus N 1.

These are the various terms that are expanded out from the basic definition of the rate of change of the property that is present in a system. So, what is; what are the various terms what is this term.

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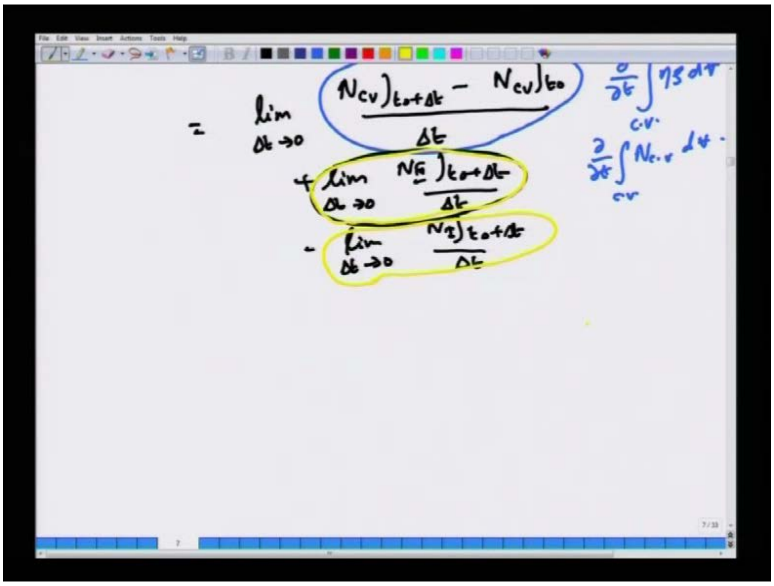

N c v at t 0 plus delta t minus N c v at t 0 divided by delta t, this is nothing but the normal partial derivative of integral c v eta rho d v. So, you could also, well will just keep it like this so this is the usual partial derivative of N c v, N c v d v this is the normal partial derivative, the rate of change with respect to time of the amount that is present in c v of the quantity N which could be mass momentum or energy. So, the second term and the third term which is so this is N 2. So, the first term is simple, the second term and third term need some work so in order do that we will take, we will consider term two which is system this is a term two and while this is term three.

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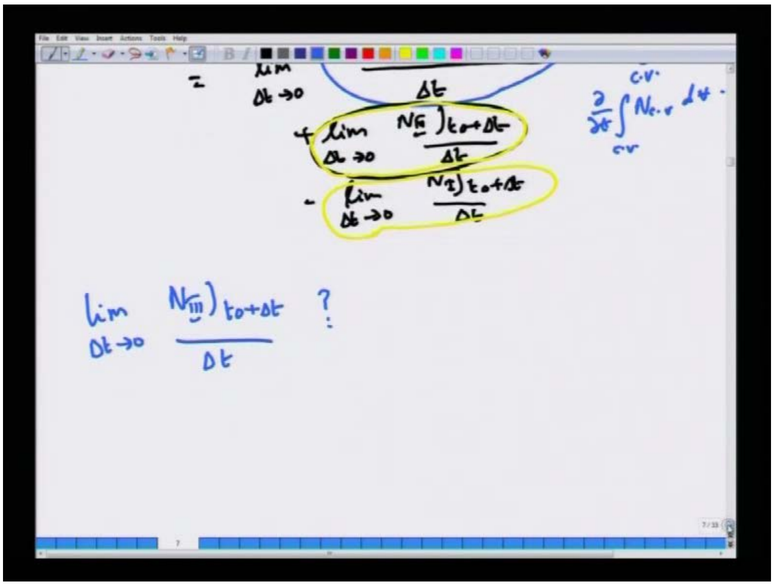

So, how do we calculate limit N 3 t 0 plus delta t divided by delta t as delta t going to 0. However we going to calculate this, this can be calculated by realizing that this, the material volume is moving by virtue of the velocity in the of the fluid that was residing at time t equal to t 0 within the c v. If you look at a sub domain in the region where, if you look, let us just go back to this diagram, if you look at any place here, this place as moved here by virtue of the normal motion of the fluid is present inside and that leads to the motion of this entire object and therefore, that leads to a relation between these two surfaces.

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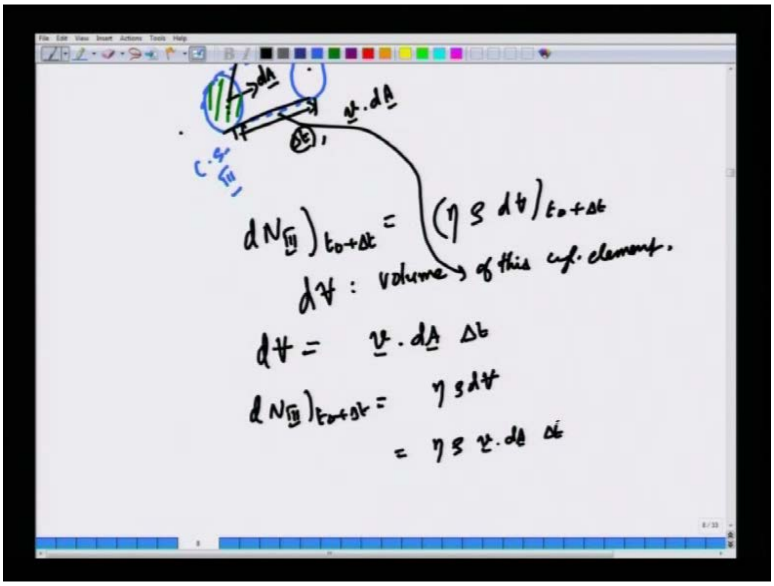

Now, let us imagine that on the surface; on such a surface, this is the control surface, the patch on the control surface in region 3 and this control surface if there is particular velocity that is going to take such a control surface to some other location. So, this is the surface, this is the patch on the surface, patch on the surface of the control volume, control mass or control volume. So, within this patch, this patch is denoted by a vector unit outward normal this is inside, this is outside d A and in general the fluid velocity on that patch will not be the same as same direction as d A in it will be at a direction theta.

So, d N 3 at t 0 plus delta t is nothing but why is this volume, why is this surface moving by virtue of the velocity. So, the change in the quantity such as mass momentum and energy, because of the fact that the surface is moved from here to here is nothing but eta rho d v at time t 0 plus delta t. Where d v is the volume of the cylindrical element of this cylindrical element. (No audio from 54:16 to 54:23) That is the volume of the cylindrical element. So, what is d v? d v is nothing but v dot d A times delta t, because we are looking at a time delta, so this must be proportional to delta t and it that must also be proportional to v dot d A. So, it is v dot d A times delta t that gives rite to the change that volume that is traverse by this surface as it left the control volume and then it reach the new surface at a later time so that volume is v dot d A times delta t. So, for d N 3 at t 0 plus delta t is nothing but eta rho d v is nothing but eta rho v dot d A times delta t.

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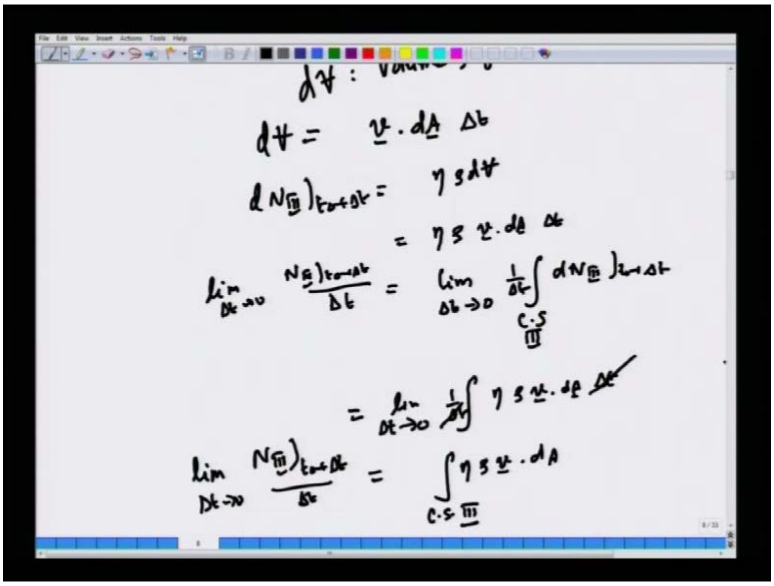

So, if you want for the entire region, we can integrate. So, limit delta t tend into 0 N 3 divided by delta t is nothing but limit delta t going to 0 integral over entire control surface 3 of d N 3 t 0 plus delta t 1 over delta t. So, this is nothing but limit delta t going to 0 integral 1 over delta t rho v dot d A delta t divided by you already have the delta t here. So, this is limits delta t going to 0 rho v dot d A times delta t divided by delta t. So, delta t delta t will cancel to give integral, so limit delta t going to 0 N 3 t 0 plus delta t divided by delta t is nothing but integral eta rho v dot d A over the control surface at t 3. Will stop at this point and will continue in the next lecture.