Fluid Mechanics

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Lecture No. # 12

Welcome to this lecture number 12, on this N P T E L course and fluid mechanics for chemical engineering undergraduates students. In the previous lecture, that is lecture number 11 we were discussing an important topic called the Reynolds transport theorem.

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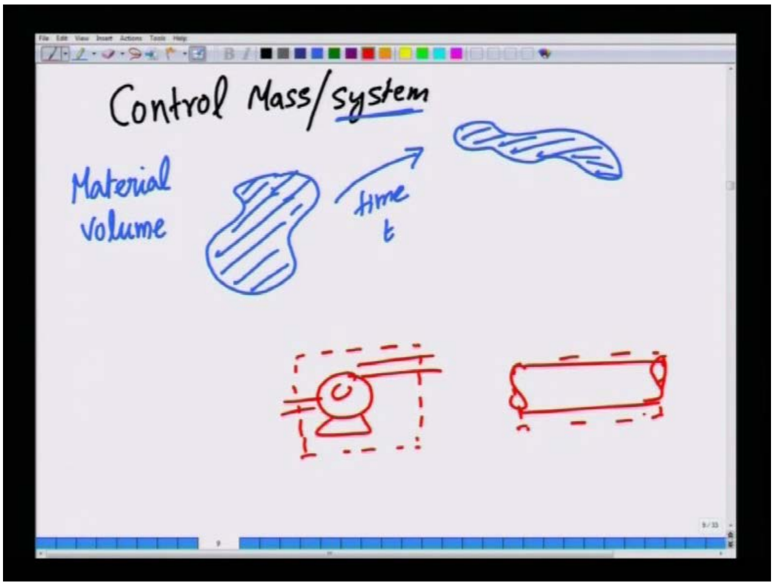

So, just give you a background on what we were discussing. So, essentially there are two ways of analyzing problems in mechanics, one is called the system or control mass approach or system approach. Here, you are following a same set of particles, material particles, fluid particles and as time proceeds this particular a volume this which we call the system, which has the same set of material particles or fluid particles. As a fluid flows after a time t the same volume will be stressed and be found in shape and it will change it is a orientation and so on. It will move as the fluid flows but, essentially you are following the same set of fluid particles as the fluid is moving.

This is not very unlike what one does in a simple particle mechanics, Newtonian mechanics where you identify a mass like a swear and then we worry about the forces that are acting on those fear. And then we can compute that trajectory or motion of the particle, we are applying Newton's laws motion.

So, likewise one can imagine doing that for a fluid. Although fluid is not a discrete entity in the continued hypothesis it is continuous medium. So, a chunk of fluid can be identified and which has the same set of mass points and material points. And as time proceeds due to motion, this control mass are sometimes this is also called as material volume will evolve in time. And we can, once you identify the forces, the body forces and surface that are acting on the same set of mass points you can compute the a motion of this by applying the Newton's second law.

But, the difficulty in fluid mechanics is that we do not want to follow the same set of mass points because a most applications in fluid mechanics. For example, you may have a pump which is delivering the fluid. So, you may want to understand what the power requirement in the pump or you may have a pipe and you may want to worry about what are the drag forces that are exerted by the flow on a particular section in a pipe.

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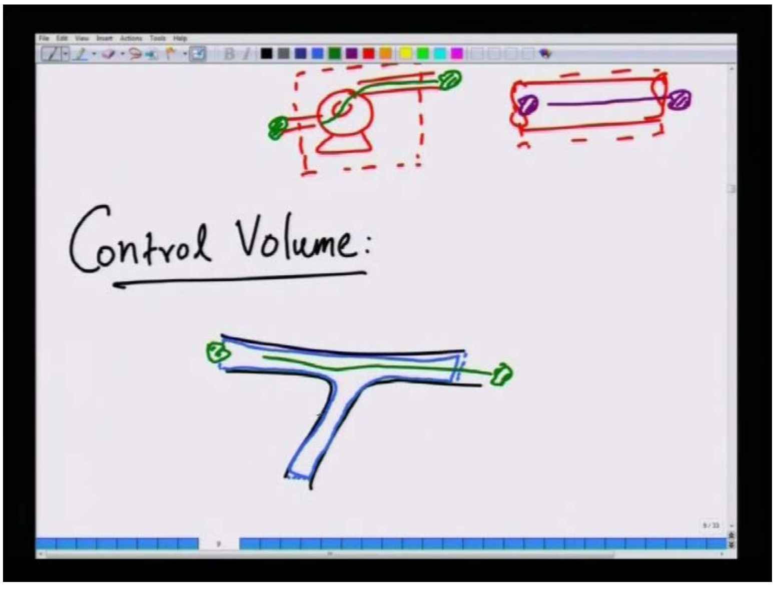

In such instances, you are not worried about the same set of for example, if you identify this green patch. This green patch will move through the pump and it will eventually go away and so will attach in a fluid in a pipe this will not continue to remain in the pipe it will go away.

So, whereas, we are interested in fixed regions and space in most engineering applications. So, it becomes important for us to identify what are called control volumes. Which is a fixed region and space, which can comprise of a device like a pump or pipe and so on. So, for example, you may have a piping network like this and you can identify a fictitious boundary where fluid is entering and leaving. There is it is not a physical boundary but, it is nearly that surface that demarcates the region from outside it is called the control surface.

And once you have this, there is physical boundaries compressed by the wall of the tube and you can identify this as your region of interest but, fluid continuous to come in and leaves. So, if you consider a patch of fluid will come and then it will leave. It may not stay in the same region of space, physical region of space it will continue to move.

But, whereas, this is very convenient because we are interested in understanding properties such as what must be work that we must perform on the fluid in order to make it flow at a given flow rate. So, in a pump for example, so in such cases we want to identify a fixed region and space and we want to analyze flow problems. So, that is call the control volume approach.

So, here we are not following the same set of mass points as in the control volume of system approach. This is the control volume approach this is the preferred approach in fluid mechanism. Once we choose this approach, then again we land up with the same issue. For example, when we did a eulerian versus lagrangians description.

We realize that even if you want work only with eulerian description we cannot compute accelerations by just taking partial derivatives because accelerations correspond to identical piece of a point and point with a fixed identity. And the rate of change of it is velocity not nearly the rate of change of velocity with time when expressed in eulerian coordinates. So, we had a tool called the substantial derivative, which enabled us to calculate the rate at which a particle changes it is acceleration as you follow the particle at a given point in time and space.

So, likewise here we want to extend a similar idea to not just a single point but, a microscopic volume. So, the reason is that eventually we want to write down fundamental principles of a mechanics such as a law of conservation mass, law of conservation of momentum which is momentum, which is Newton's second law of motion and a law of conservation of energy which is first law of thermodynamics.

So, in such instances all these laws are applicable to identifiable piece of matter in their most natural form. So, for example, Newton's law says that mass time acceleration is equal to sum of forces and an identifiable piece of matter not on a fixed region in space.

So, we need again a tool or a vehicle that transforms a control volume information to control mass information. So, that we can naturally apply a quantity a principles such as newtons second law.

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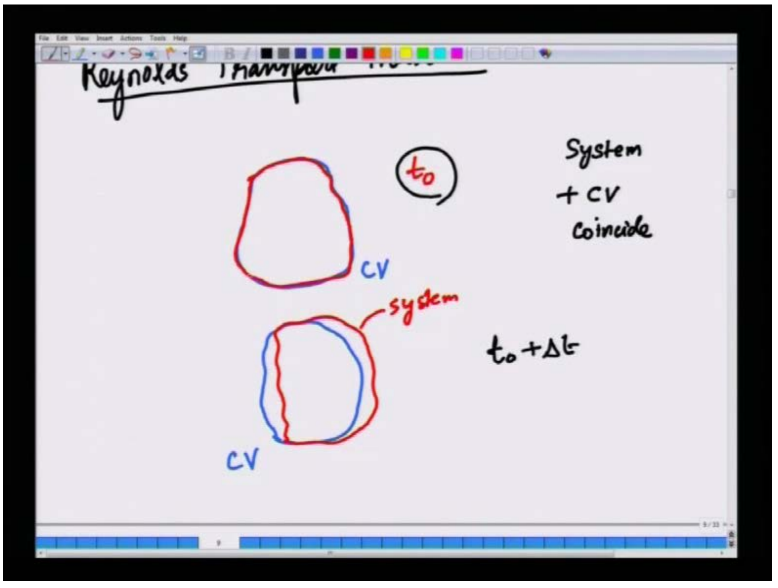

So, this is a done by using what is called the Reynolds transport theorem. This was what we procedure within the last lecture, let me quickly a recapitulate what we did. (No Audio Time: 07:00 to 07:07). The Reynolds transform theorem is way of relating time derivatives of quantity such as mass in a given region of space to that a with associated with the system or the material that is present in the system. For example, we said that you identify a fixed region of space. This is our C V indicated in blue and let us identify our system or the control mass to be that that coincides with the c v at a given instant in a time.

This is time t 0, at time t 0 the system and C V, C V is short form control volume coincide. At a later time you still have the C V is a fixed region in space but, the system would have moved depending on the way the fluid flows I am just showing schematically here something like this.

So, this is at time t 0, this is at time t 0 plus delta t the C V is denoted in blue remains the same. The system that is because of fluid flow if you follow a set of points they will in general move away from the system from the C V.

So, we want to be able to write rate of change of instantaneous rate of change of mass momentum energy or the fluid that present in the C V, of the fluid. That is present in the system in terms of the variables of this C V because we want to follow only the control volume approach. While the fundamental principles are most naturally a applicable to the control mass approach. So, we want to be able to write the derive a relation that is the purpose of reynolds transport theorem.

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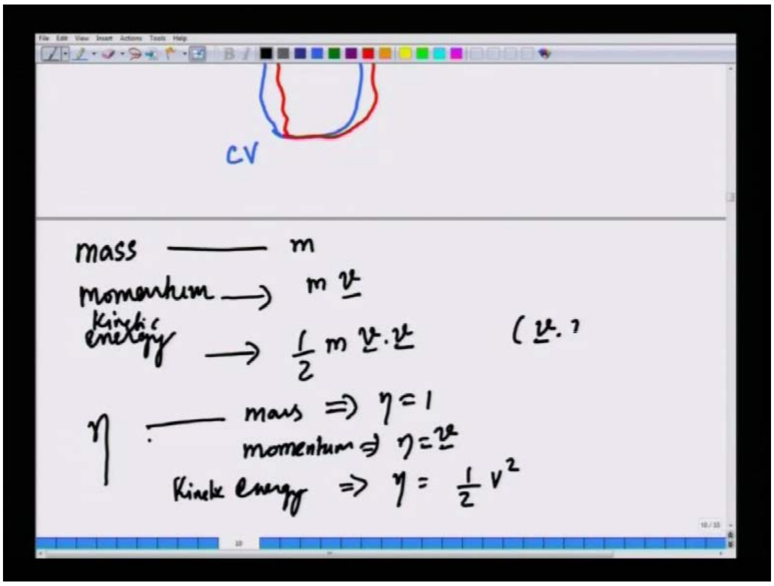

To this end so, we want to derive apply conservation of mass, momentum and energy to the system that is present in the C V at time t, not although the system is moving away from the c v at a later time. So, mass is simply the mass of fluid is m, momentum is m times v. So, small it is lower case m and energy is half m v dot v.

Now, we define a generic quantity eta for which we will derive the reynolds transport theorem. This could be any quantity of a upper unit mass. For example, for mass eta is 1 for momentum, this is mass per unit mass for momentum eta is m v divided by m eta is v. For energy kinetic energy, this is kinetic energy motion in the fluid eta becomes half v square, v dot v denote as v square.

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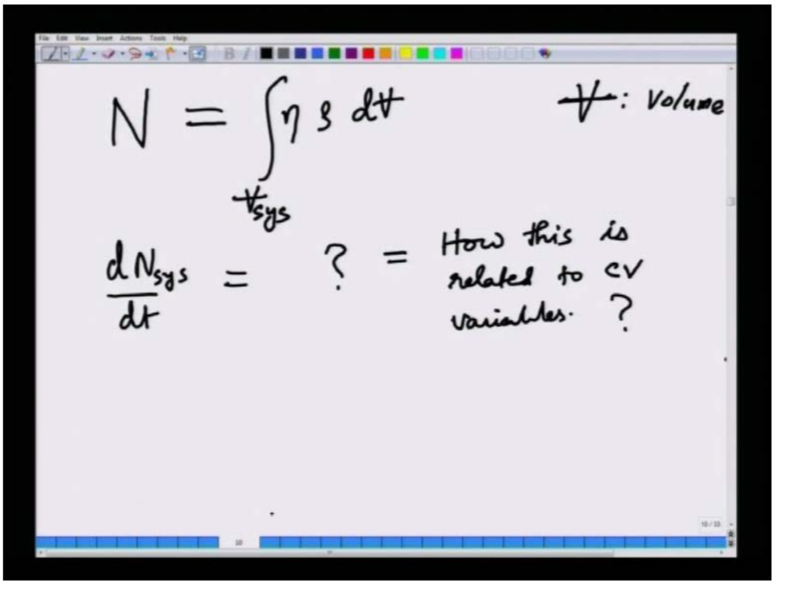

So, once we have this then we are interested in the total amount of quantity that present in the system of mass momentum or energy. That is obtained by taking a volume integral over the system. So, in my notation v with a cross is the volume because the v without a cross will be denoted for velocity will be result for velocity.

So, to distinguish volume from velocity I will use a cross. So, this volume over the system eta which is quantity per unit mass time is rho, which is mass per unit volume integrated over the volume. This will give you the total amount of either mass momentum or energy depending on what you put for eta. So, this is a volume integral and we are interested in d N by d t of the system. And how this is related to C V variables. That is the net outcome of the reynolds transport theorem. So, what we do is first go back to the figure. So, let us keep reference on this figure.

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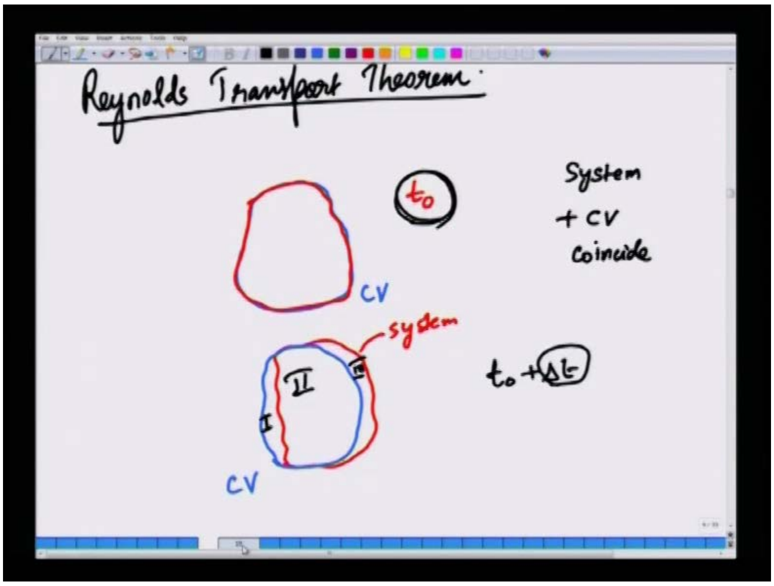

So, imagine that at time at t 0, the system and C V are coinciding but, a later time the system as left the C V but, there is some since we are only looking at a infinitival time difference. There will be some region II, where the system and C V still are coinciding. There will be a region I where the system has left the C V and there will be region III, which it is system as created a fresh because of it is motion.

So, there will be 3 volumes one can identify in this composite thing and will try to simplify based on. So, will simply look at this diagram and will try to simplify our time derivative of all quantities a in terms of this particular picture that we have.

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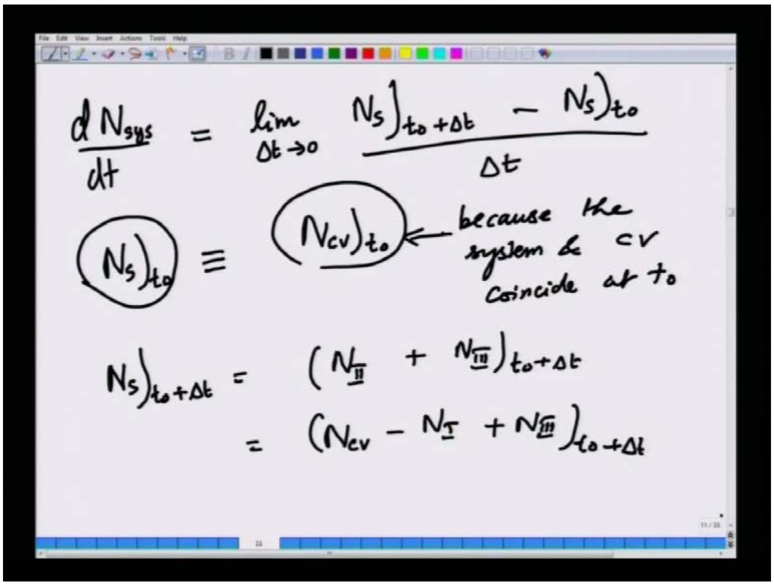

So, by fundamental definition the rate of change of the total quantity of either mass momentum or energy present in the system is limit delta t going to 0. The instantaneous rate of change is the total amount present in the system at t 0 plus delta t minus total amount present in the system at t 0 divided by delta t. So, that is the fundamental definition.

So, from the figure that I just drew at t 0, the system coincides with C V because the system and C V coincide at time t 0 but, at a later time so the amount of quantity present in the system and C V are identical because if this region.

But, at a later time t 0 plus delta t, the system is comprised of N II, which is the common region between the system and C V at time t 0 plus delta t plus N III at time t 0 plus delta t.

But, N II is nothing but, N C V minus N I. So, recall in our picture here that N II is basically, N II is that region which is the total C V volume minus the volume that is vacated by the system at a later time. So, N II is N C V which is this entire thing minus N I and N III, is the new space that is created by the a system due it is motion from the C V.

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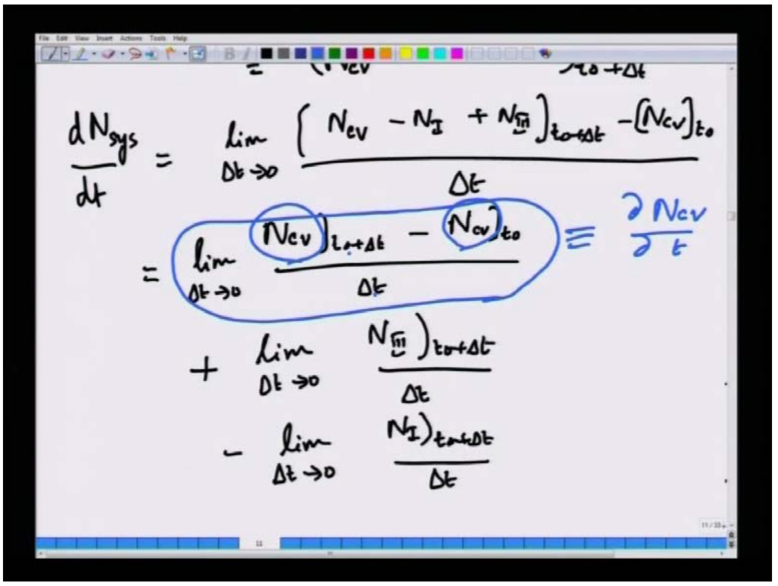

So, N II can be written as N C V minus N I plus N III will remain as such at t 0 plus delta t. So, I am going to substitute all this in the fundamental definition of time derivative. So, rate of change of the total amount of something present in the system is limit delta t going to 0, N C V minus N I plus N III at t 0 plus delta t minus N C V at t 0 divided by delta t.

So, this can be split into 2 or 3 terms so, limit delta t going to 0 N C V at t 0 plus delta t minus the amount of various quantities present in the C V at t 0 divided by delta t. This is one term and the second term is plus limit delta t going to 0, N III t 0 plus delta t by delta t minus limit delta t going to 0, N I t 0 plus delta t divided by delta t.

So, if you look at this expression, here we are just looking at the rate of change of the quantity present in the C V at two different times divided by delta t. So, from fundamental definition of calculus this is d N C V divided by d t.

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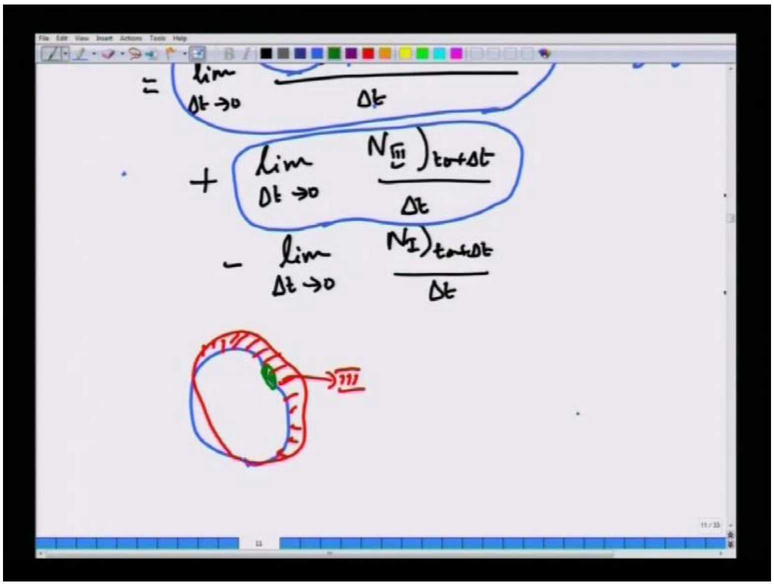

So, that is essentially the rate of change of the quantity that is present in the C V that is very very simple. So, this precise we can put partial derivative. So, now we have to simplify these two terms, let us call this term, let just look at this term. So, if you recall the system was here, the C V was here the system has moved and this was region III. Region III is the newly created region. So, what is this? This is the amount of change that is of in any quantity like mass momentum or energy because of the fact that this system as moved to this new region in space.

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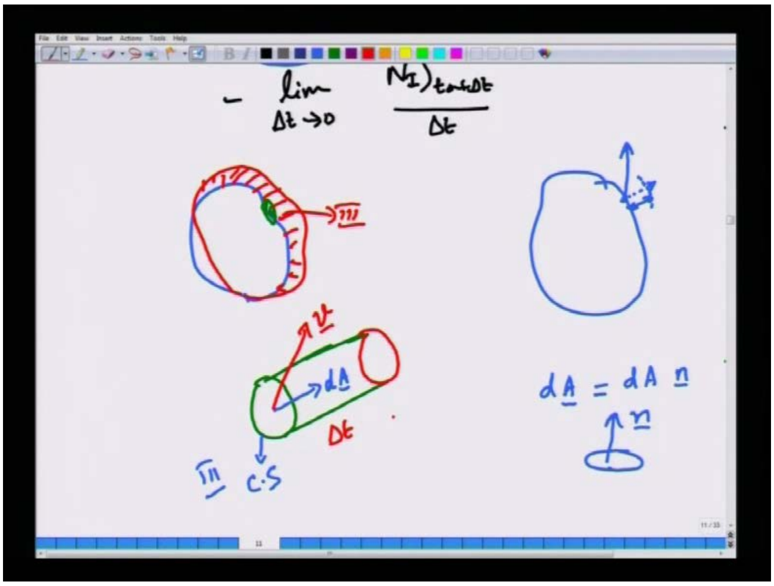

So, how do we a calculate this? So, why is the system or control mass moving? The control mass is moving because if you take a patch of surface on the C V. Let us say, you take a patch of surface on the C V. Now, that patch is moving to a newer region because of the fact that the fluid on the surface of the C V has some normal velocities, has a component of velocity in the normal direction. If you consider the C V to be like this if the fluid is purely tangentially moving on the surface that cannot be any motion in this direction.

But, if there is a normal motion if at a point the velocity is like this then there will be a component that is moving in this direction. That means that the system will change it is location at a later time by that amount.

So, you take this is the surface of the C V this is at you know at all times the C V is remaining fixed in space. That is called the C S, surface of the C V is called control surface. And this is the region III and why is this movement? This moving because this is the unit area, this is the area patch. Area is denoted by a magnitude and direction because whenever you have area, you can denote it is direction by telling the way in which the normal is pointing. You can say the normal is pointing from in to out or out to in will normally take in to out is called out ward unit normal.

So, this area is a vector. Area element is a vector and the velocity in general can point in somewhere direction. So, this volume that is created over a time delta t must be because of the fact that the fluid element on this surface has a component in the normal direction.

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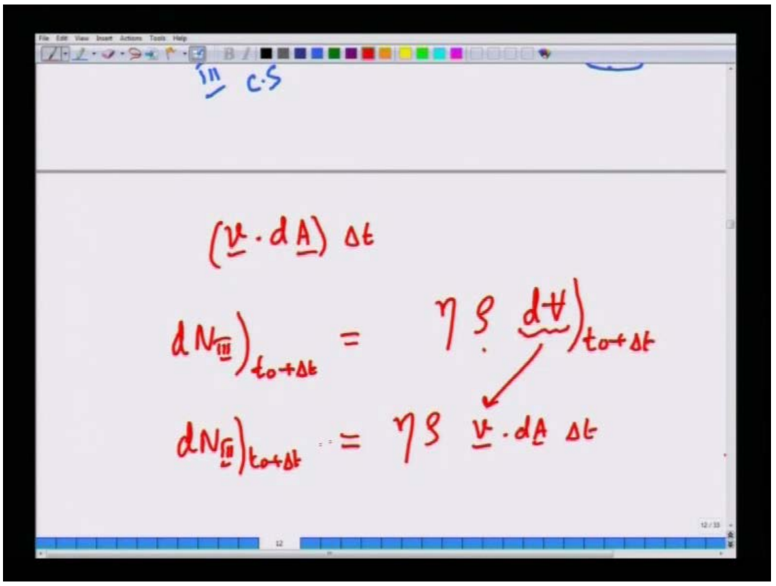

So, v dot d A is essentially times delta t will be the volume. That travels by a tiny patch of area on the control surface of the C V. As the system proceeds to leave the C V and occupy a new nearby location.

So, this length is mod v times delta t, if the magnitude of the vector v a times delta t. So, to calculate the volume. So, d the amount of material, as amount of the extent of change in any quantity like mass, momentum or energy in a tiny volume due to the motion of the system from the C V is given by the volume is given by essentially the amount of quantity per unit mass times mass per unit volume times the volume at t 0 plus delta t.

But, we know what the volume is it is eta times rho d v is something that we just calculated as v dot d A times delta t. This is d N III t 0 plus delta t.

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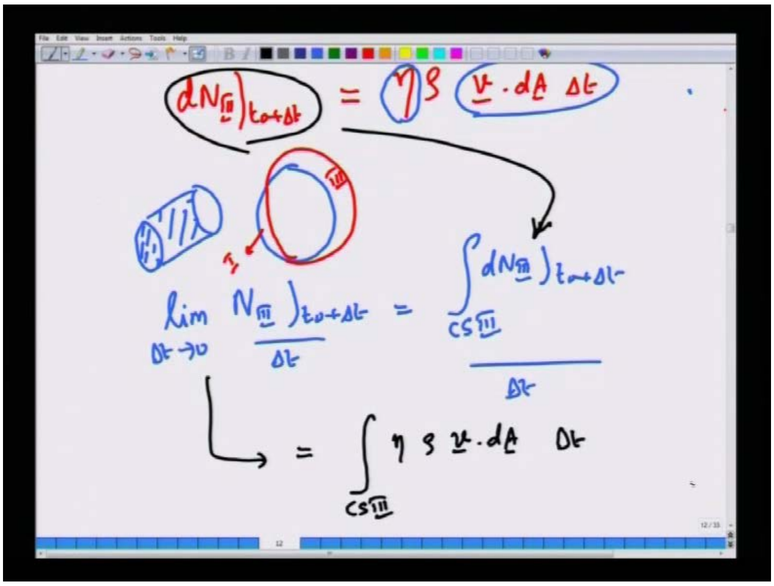

This is the extent of change that you will encounter because of the motion of the fluid through a tiny patch of surface. And therefore, there is a new tiny volume that is created and what is the change in mass momentum of energy due to this volume. That is this term so, you multiply you take the quantity either mass momentum or energy multiplied by the per unit mass that is eta, multiplied by rho that gives you per unit volume times the volume itself which is this.

Now, the total amount is, so, we are interested in if you remember quantity limit delta t going t 0 N III of t 0 plus delta t divided by delta t. That is the quantity that appeared as one of the terms if you remember previous slide so, you had this term. So, we are now trying to simplify this term.

So, what is that term? That term is nothing but, we have eta and so, this is d N. In order to get N you will do an integral over the control surface, the entire control surface III plus now if you recall so, the C V and the system this is the region III which is been created this is the region I, which is been deleted by the system due it is motion.

So, will do the integral over the entire control surface III of d N III at 0 plus delta t divided by delta t. This will be our result and in order to a simplify this further substitute d N from here to here. So, this term which we are after is simply integral C S III eta rho v dot d A delta t divided by delta t the delta t will cancel out to give limit delta t going to 0

N III divided by delta t is nothing but, integral over the control surface III eta rho v dot d A.

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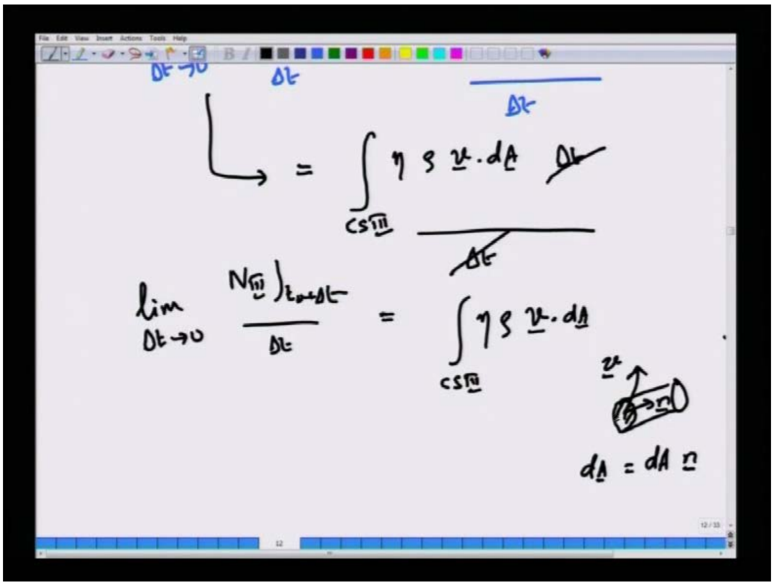

Now, what this means physical is that so, v dot d A is the volumetric flow rate. Suppose, you consider tiny patch on the control surface III d A and n is d A is the unit area. So, d A is given by vector is given by this magnitude times direction the outward unit normal is velocity is like this. The volume that is created per unit time, the volumetric flow rate is v dot d A times rho will give you the mass that is flowing per unit time in this volume times eta will give you the rate at which a quantity like mass, momentum or energy is changing per unit time by virtue of the motion of the fluid out of the C V.

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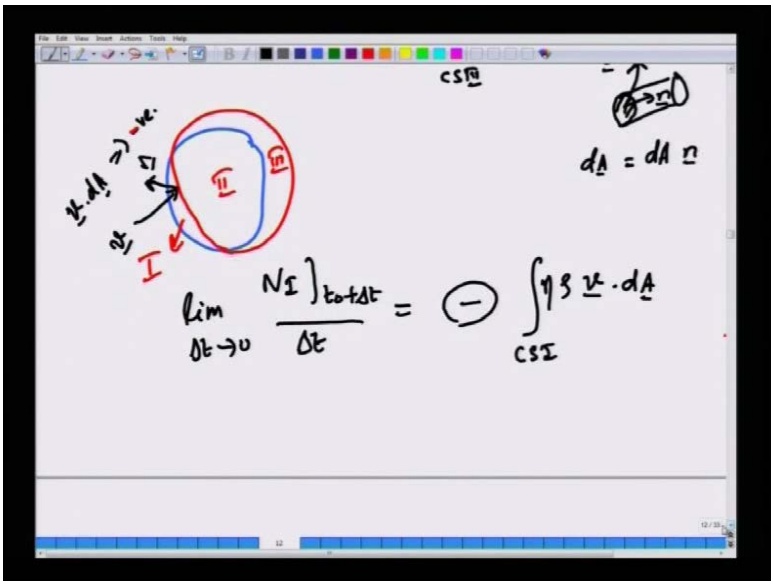

This is essentially of the physical interpretation of this term. Now, we can do same thing for region I, recall that you had the system, the control volume to be fixed, system has moving at a later time this was region III which was just computed, region II, which we already calculated and region I is again the stuff that was deleted by the motion of the system or control mass from the C V.

So, we can write a similar expression because the ideas are very similar. Limit delta t tending to 0 N 1 t 0 plus delta t divided by delta t is equal to.

Now, the key thing is that here the unit outward normal is in this direction but, we have assumed that the fluid is moving this direction; the velocity is in this direction. So, v dot d A will be a negative quantity now but, we have already introduced a negative sign here in our earlier slide here. Earlier, we assume that this quantity is already we have taken into account a negative term. So, we have to put an explicit negative sign because we have already taken the negative signs there because we have assumed that fluid is the system is deleting this volume. So, since v dot d A is negative, we will put an explicit negative sign. So, v dot d A is negative rho over eta rho v dot d A.

If we did not put the negative sign in the previous slide, then we can simply forget about negative sign here because the v dot d A itself is negative but, since we explicitly took into account the negative sign their. We have to take care of an additional negative sign here, that is idea.

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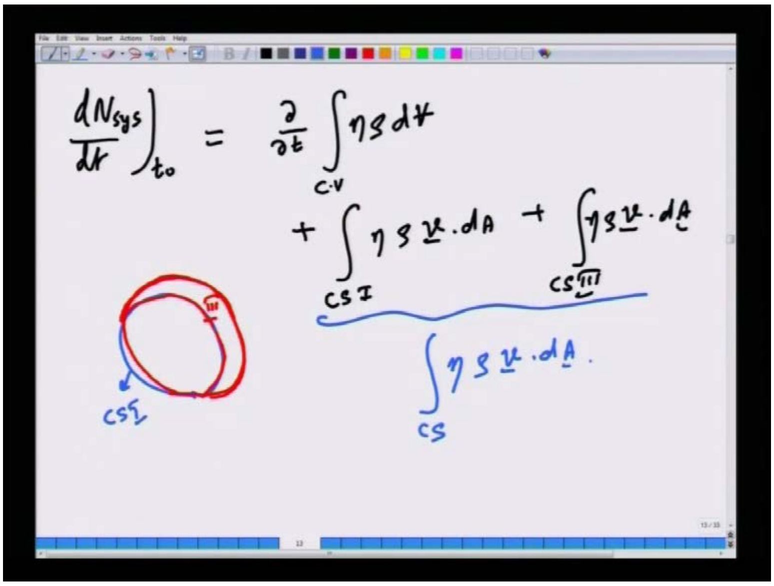

So, if we had all the terms that we have. Therefore, we have all the terms that we have then d N d t of the system at a given time t 0 is equal to d d t over the C V of eta rho d v plus integral over C S I. Now, the 2 negative signs will add to give a positive sign eta rho v dot d A plus integral over C S III eta rho v dot d A.

Now, this is C S I and C S III will form the entire control surface because this was the system C V and this was the system. So, this was control surface III and this is control surface I which was deleted. This is the one that is created so; this will form the entire control surface of the C V. So, this is simply eta rho v dot d A.

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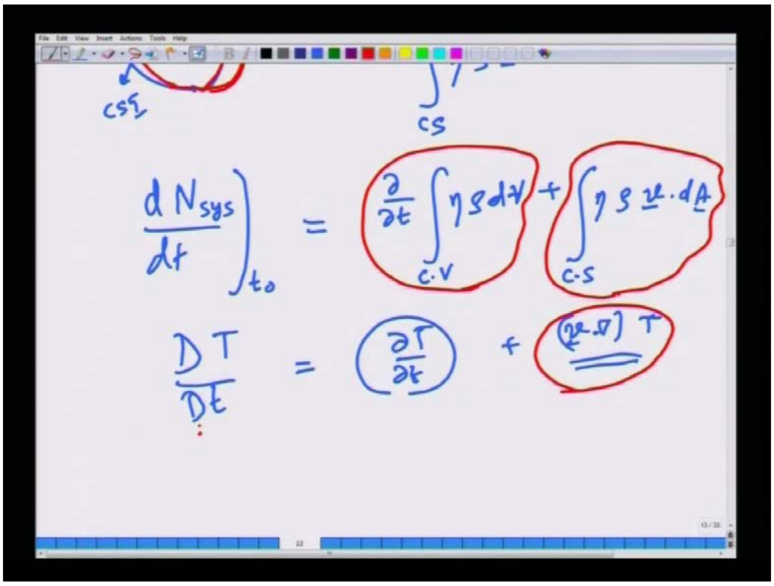

So, the rate of change of a quantity like mass, momentum or energy that present in the system at a given time is equal to, if the system and C V coincide at that time is equal to there are two terms one contribution is the bulk rate of change of quantities in the C V itself eta rho d v. The next contribution is due to the motion of fluid in and out of the C V due to fluid flow that is the flux term.

So, just if you recall in the substantial derivative of a quantity like temperature if you recall was d T d t plus v dot del t. This was the convicted rate of change is the local rate of change. Likewise, if you take a finite volume of C V, then you ask the question, what is the rate of change of mass, momentum or energy of the fluid material that present in the C V? As you follow the material there will be two contributions, one is the local rate of change present in the C V itself a due to various reasons and the other is because of the fact that the fluid is flowing in or there will be a net flux a fluid in or out depending on the problem. And this will lead to either a flux of energy momentum or mass or an influx of energy momentum or mass.

So, therefore, this contributes to the convection or convicted, this similar to this convicted contribution to the substantial derivative this because of the flow that takes fluid away. And hence therefore, it takes mass away momentum away as well as energy away or into the control volume.

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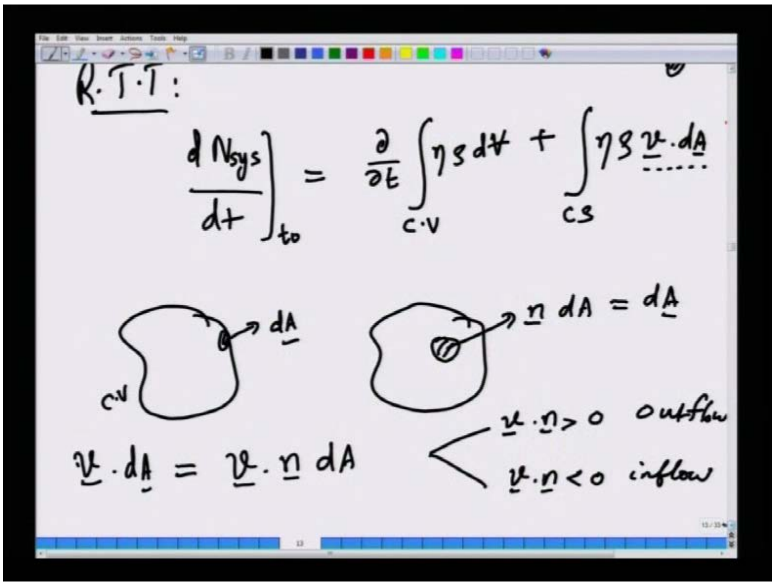

So, this is the Reynolds transport theorem. So, the Reynolds transport theorem sometimes shortened as R T T says, that the rate of change of any quantity such as mass, momentum or energy present in the system at a time t naught is comprised of two parts assuming that the system and C V coincide at time t 0 is equal to the inherent rate of change of such quantities within this C V plus a surface contribution eta rho v dot d A. v dot d A is the volumetric flow rate on a tiny patch, about a tiny surface patch rho times that will be the mass flow rate and eta times is eta is quantities per unit mass.

So, this will be the rate at which certain quantities such as mass, momentum or energy or either carried away or carried into the control surface, control volume depending on the problem nature of the problem. So, the key thing that we must note here is that, if you have a C V and you take a patch of surface so, that is what I called d A here. Now, this d A is normally written as n times d A, the unit outward normal from the C V is pointing outside.

So, if v dot n times d A so, v dot vector d A is v dot n times d A if v dot n is positive that is v is in the same direction as n. There are two cases if v dot n is positive, greater than 0 that means there is a net outflow of fluid. If v dot n is negative there is a inflow, at a fluid around that patch and if you integrate over the entire patch if you either find v dot n to be, I mean entire integral to be positive that means there is a net flow out and if you find that it is negative it is net flow in.

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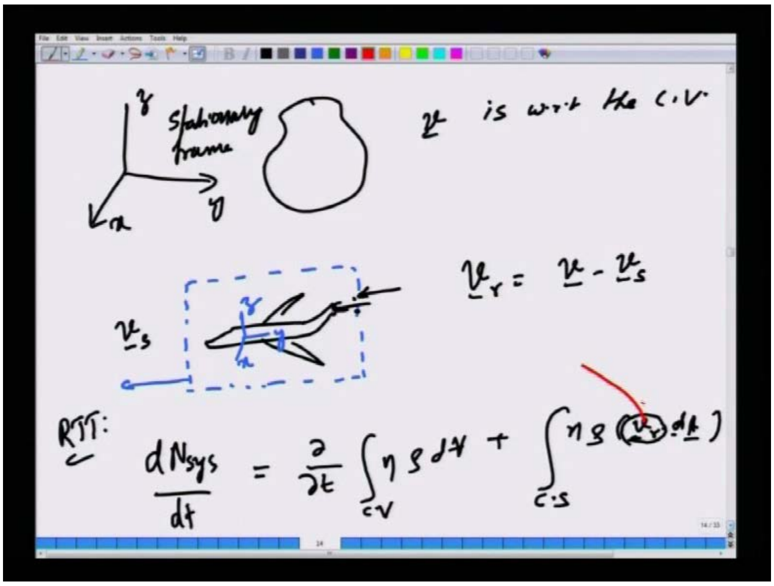

And also notice that the velocity is measured with respect to the C V, that is you put a coordinate system and the velocity is with respect to this coordinate system. The reason I am mentioning this is that suppose you have a moving C V. This is stationary frame of reference. Suppose I have an airplane is moving so, I can construct at a constant velocity. So, I can construct a moving frame of reference where I put my coordinate system on a the plane itself. In which case the C V itself is moving at a constant velocity your c v is not the fixed region in space it is following the airplane. So, it is moving with the constant velocity.

So, we can define relative velocity as v minus v s. Let say the system this the C V is moving with a velocity v s. Now the Reynolds transport theorem for the moving frame of reference is d N system by d t is d d t over C V eta rho d v, that now as the same plus integral over C S eta rho v r dot d A. Instead of having absolute velocities, you have relative velocities because only with the relative velocity is that between the fluid and a control, I mean and the volume and the velocity at which the C V itself is moving will there be a net inflow or outflow. If the fluid is moving at the same speed as the C V then there is obviously no net inflow or outflow.

So, the Reynolds transport theorem changes in the sense that here for moving frame it is the relative velocity that constant the flux term otherwise it is the same.

So, this is the very important result in a fluid mechanics. There is a Reynolds transport theorem. Now, once we have this tool we can apply it to the fundamental laws of physics that are as applicable to a fluid.

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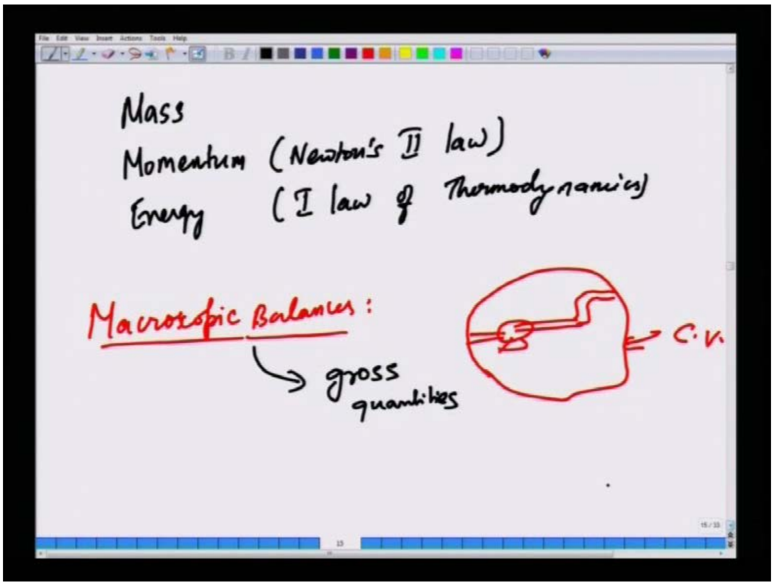

So, essentially we will apply a law of conservation of mass to the C V then law of conservation of momentum which is Newton's second law and conservation of energy which is first law of thermodynamics. I will give you enough background as we go along.

But, essentially will apply the Reynolds transport theorem to this fundamental principles and then those will be the at starting point for us to solve problems in fluid mechanism. Now there are two ways to go about it, the first approach is what is called the macroscopic approach. The macroscopic approach says that, you apply the principles or macroscopic balances over larger regions of volume. So, those pieces of regions of volume could be so large that they can include a pump, then a pipe and then you have whatever that you have in your problem you can include all of this inside a huge C V. And then you can apply the principles of mass conservation momentum balance and energy balance and you can do problems, you can try to address problems.

Such balances are useful but, they are very they give only grows quantities. They do not give or they cannot give detailed information like what is the velocity at each and every point in place, what is the shear stress at a point in a wall and so on.

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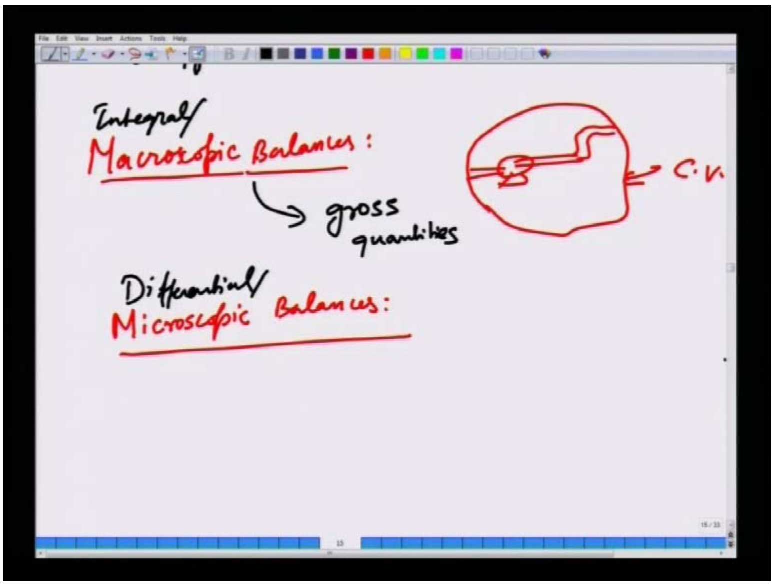

But, you can also apply the fundamental principles to; instead of microscopic volumes you can apply to differential volumes which will lead to what are called Microscopic balances. Sometimes, macroscopic balances are also called Integral balances because the equations will be in the form of integrals. While the microscopic balances are sometimes called the Differential balances because the equations that we get will be in the form of differential equations.

So, we will first apply the principles of mass conservation momentum balance and energy balance to mass conservation. First to macroscopic C V and then we will proceed to microscopic balances. So, both these approaches have their own advantages and they have their own difficulties so, we will try to indicate them as we go along.

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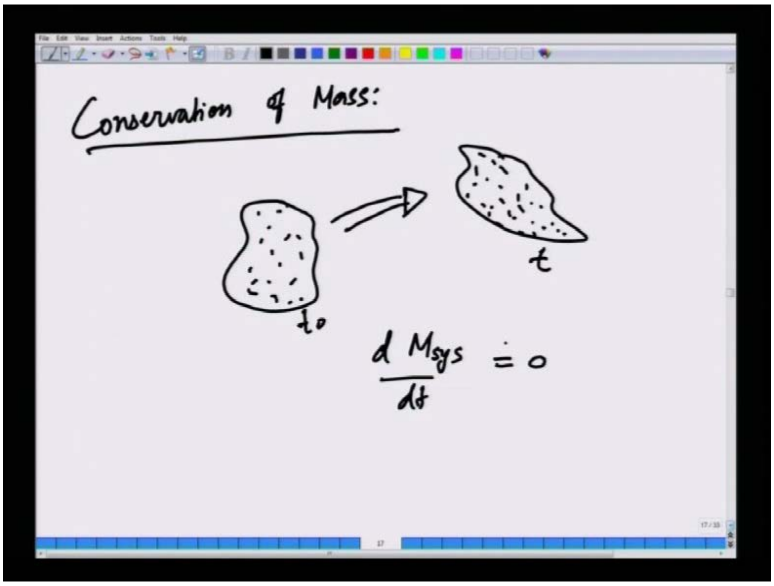

Now, first is conservation of mass (No Audio Time: 35:23to 35:33). Now, conservation of mass, law of conservation of mass is that mass is neither created nor destroyed. When we apply this to a system suppose you follow a system or a control volume at time t 0 and at a later time, this may undergo change like this.

But, if you are following the same set of mass points because that is the definition of a control mass. That is your identity you are following the same identical set of fluid particles or material particles. So, by definition the rate of change of mass of this system or a control mass is 0 because we are following the same set of mass points, as we follow the material volume or the control mass.

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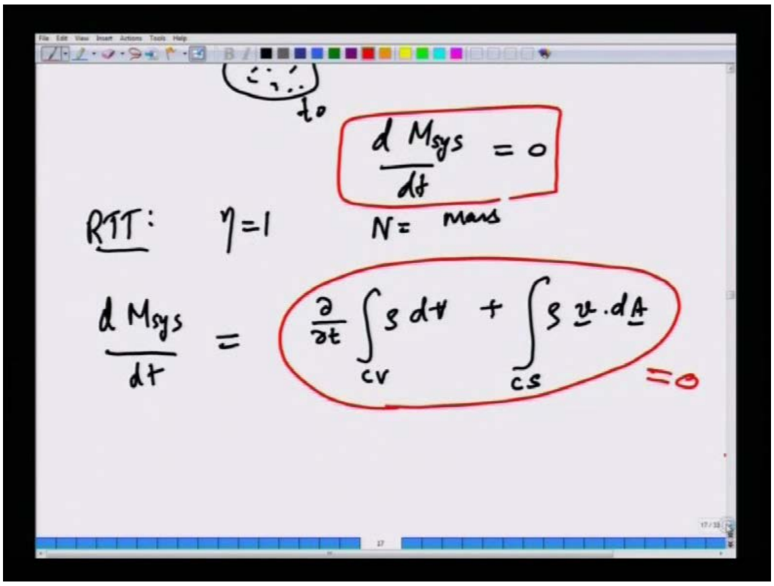

So, the rate of change of mass of the system is 0. Now, we will use Reynolds transport theorem with eta equals once because n is the mass that present in the system. So, mass divided by mass becomes unity so, eta becomes unity eta is any quantity per unit mass. So, the rate of change of mass in the system per unit time from Reynolds transport theorem is; There are two contributions, one is the volumetric contribution C V rho d v plus the other is the surface contribution rho v dot d A but, conservation of mass says that this must be 0.

So, this entire quantity must be 0 because there is why principle of conservation of mass. So, we have d d t integral rho d v plus C V plus integral C S rho v dot A to be 0. So, we can rewrite this slightly differently.

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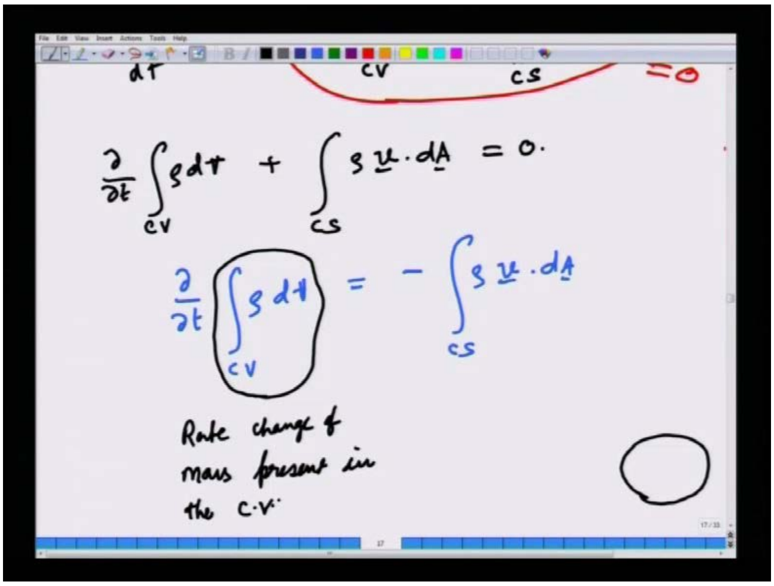

Take this to the other side. Now, if once we rewrite this we can interpret this physically in a very simple way. What is the what is the term on the left side integral suppose you consider a C V integral rho d v over the C V is the total mass present in the C V. So, this is the rate of change of mass d d t of that is rate of change of mass present in the C V in the control volume.

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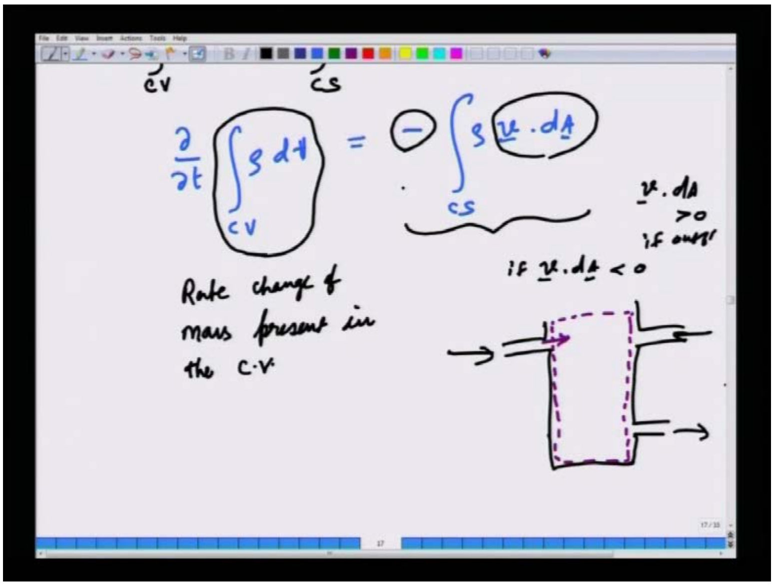

Now, why is the mass; why should the mass change in the control volume? Well it is because of the fact that in a real application for example, real C V could be something like a tank in which there are in flows and out flows. So, that could be multiple inlets and outlets.

So, this is a this is the way real application that could be inlet see here and outlet see here and so on. So, if you take this as on your C V then why is the rate, why is the mass present in the C V, why should it change it should change, because there it should change with time because there is a net imbalance between the inflow and out flow.

So, let us look at the right side what is it mean i told you that v dot d A is positive v dot d A is greater than 0. If there is an out flow, if entire surface integral is greater than 0 that means there is a net out flow of fluid. Now, this negative sign means that if there is a net out flow of fluid from a C V then of course, the mass of fluid present in the C V must decrease, that is what this means.

If v dot d A is negative, if the surface integral is negative, the negative of negative is positive. If v dot d A is negative then there is a net inflow. So, if there is a net inflow through all the flows into the C V that means that mass present in the C V must increase as a function of time. So, this is a positive quantity so, that is all it means.

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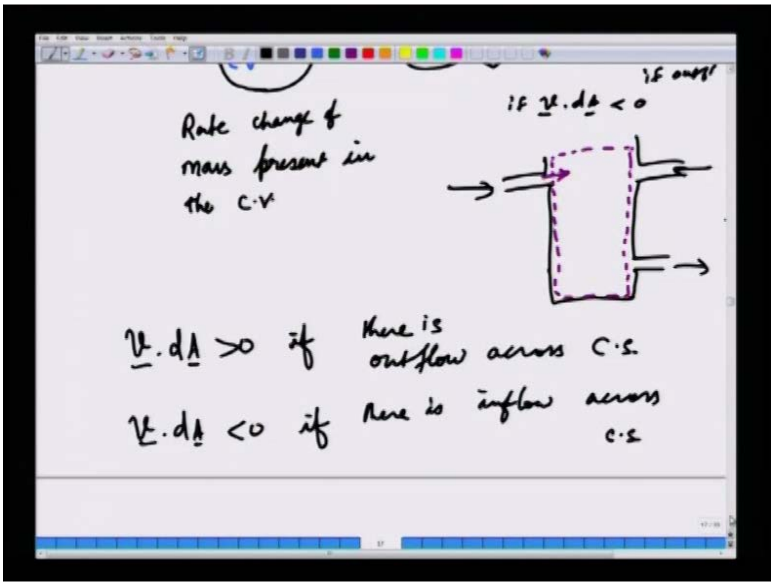

So, the again the sign convention is such that v dot d A is greater than 0 if there is out flow across a control surface, across the C S and v dot d A is less than 0 if there is a inflow across C S.

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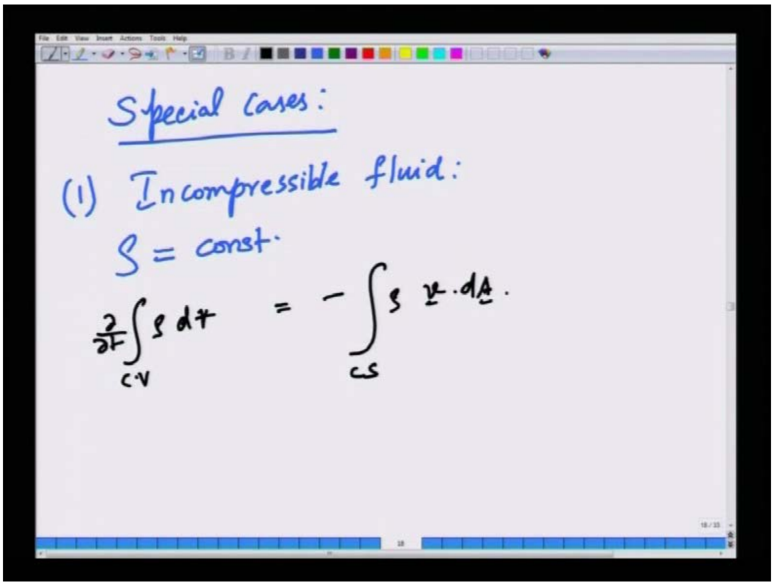

Now, let us try to simplify this in some special cases so special cases where that is the most general form of mass conservation equation for a C V. Now, let us simplify it for some special cases, first let simplify this for an incompressible fluid (No Audio Time: 40:38 to 40:46). For an incompressible fluid by definition means that the density of the fluid is a constant it is not changing due to the flow.

So, the most general form of mass conservation is integral rho d v d d t of C V is minus integral C S rho v dot d A. Now, since the C V is generally fixed in space it is not a function of time, you can pull this time derivative in if you choose to.

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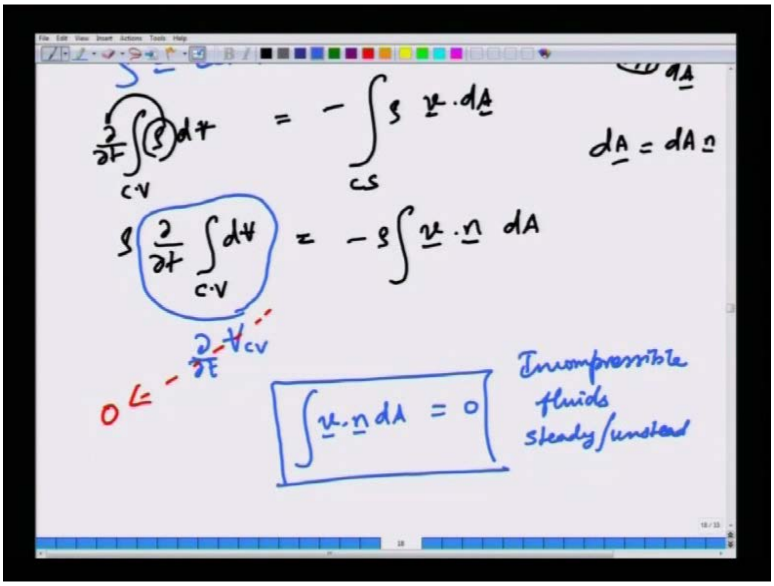

But, let just keep it like that but, since rho is a constant rho can be pulled out. So, rho d d t of integral C V d v is minus integral rho because rho is a constant, v dot d A can be written as v dot n d A because d A the unit area, it is a area element is a vector. It is magnitude times a unit output normal.

So, whenever you have a small patch of area it is denoted by a magnitude and a direction the normal that is pointing from in to out. That is called the out ward unit normal, unit vector normal to the surface that is why it is called out ward unit normal. Now, let us try to see what this term means. This is integral over C V of d v that means this is d d t of the volume of this C V itself.

But, the C V is the fixed region in space so; it is volume does not change with time unlike a material volume which changes with time. So, this becomes 0 so, this left side is 0. So, the right side for an incompressible fluid simplifies to integral v dot and d A is 0.

So, this is the most general form of mass conservation equation for incompressible fluids. Notice that we are not placing any restriction on whether the flow is steady or not. So, it is valid for valid for both study as well as unsteady. The only stipulation we make for an incompressible fluid is that the density is a constant. So, the time derivative vanishes because C V is fixed in space not because flow is steady.

Still the quantity is inside the C V could change with you know so, still the velocity can change with time. But, that does not mean that this is valid only for steady cases. Even if you have unsteady cases if the fluid is incompressible then v dot n must be integral v dot n d A must be 0.

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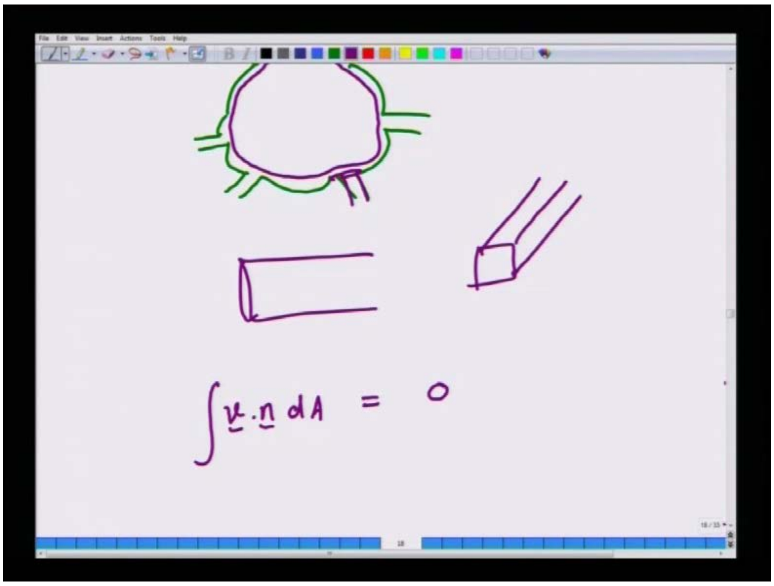

Now, let us try to think of a reasonably concrete examples, typically what you have C V with various inlets and outlets, a multiple inlets and outlets. So, this is let say some sort of a container and this is your C V, there are various inlets and outlets. Now, typically this inlets and outlets are pipes or of some cross section. So, you can have pipes or some rectangular conduits that is how inlets and outlets will be in real applications.

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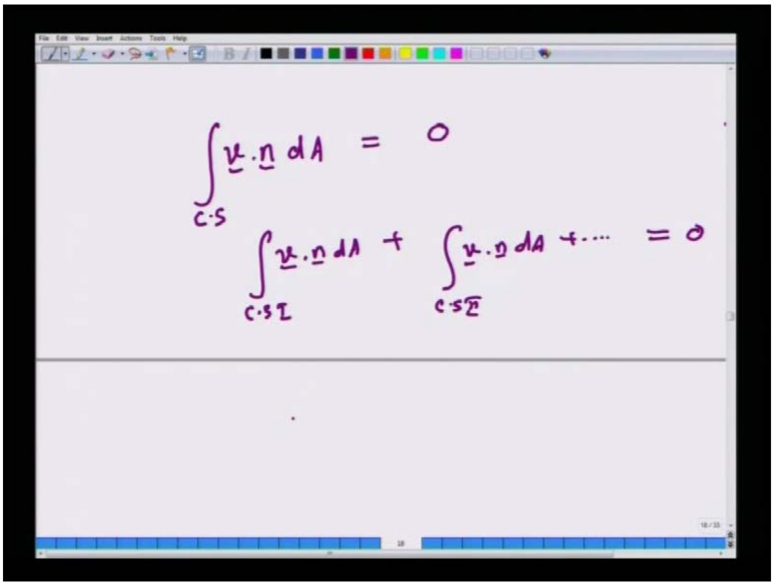

In general if you want to say what is integral v dot n d A 0. You have to; this is over the entire control surface you have to split this integral into various surfaces. So, integral over C S I v dot n d A plus because there are only few inlets and outlets over which fluid is flowing. So, you have to assume that we have to apply this two various inlets and outlets. At each inlet and outlet therefore, suppose I take a given inlet. Suppose, this is my control surface.

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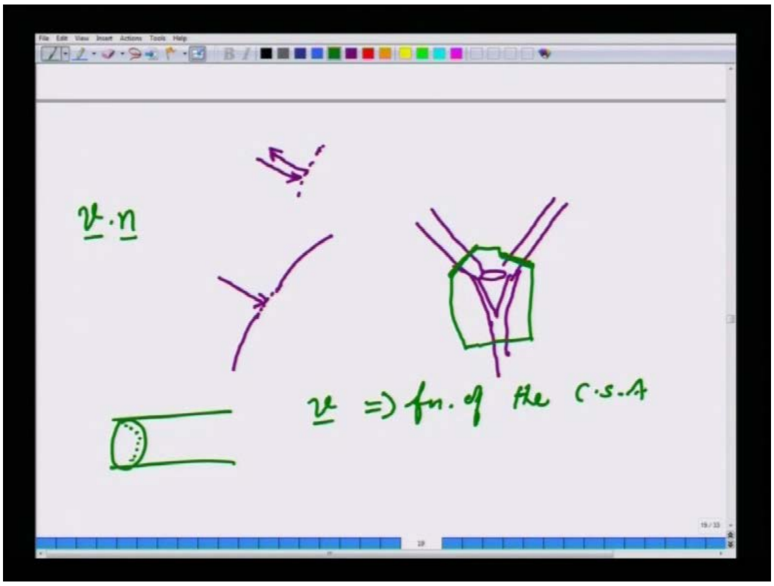

Now, it is useful to take the velocity, the control surface such that it is perpendicular to the; that is it is the normal of the control surface is parallel to the velocity vector that is entering the C V. For example, suppose you have a control surface like this, let us try to suppose you have C V like this.

It is useful to keep the control surface and let say fluid is flowing perpendicular in this manner. It is useful to keep the control surface like this. So, even if you have a C V in which suppose you have a jet that is flowing like this. That is impinging on surface and it is flowing like this. Suppose, you have conical surface and jet is impinging and it is leaving.

Suppose, you want to keep, you draw a C V with the C S. It is useful to keep this C S to be perpendicular to the outflow of or inflow of the fluid because so, this could be your C V, where the control surface is drawn such that they are perpendicular to the outflow because only then this v dot n becomes an easier quantity to evaluate. So, that is one simplification that one can achieve. The other thing is whenever you have flow in pipes and channels. We will see later that in general v is a function of the coordinates of the cross sectional area. That is if you have a pipe the velocity at various locations in the pipe, the point wise velocity need not be the same. They are in general different.

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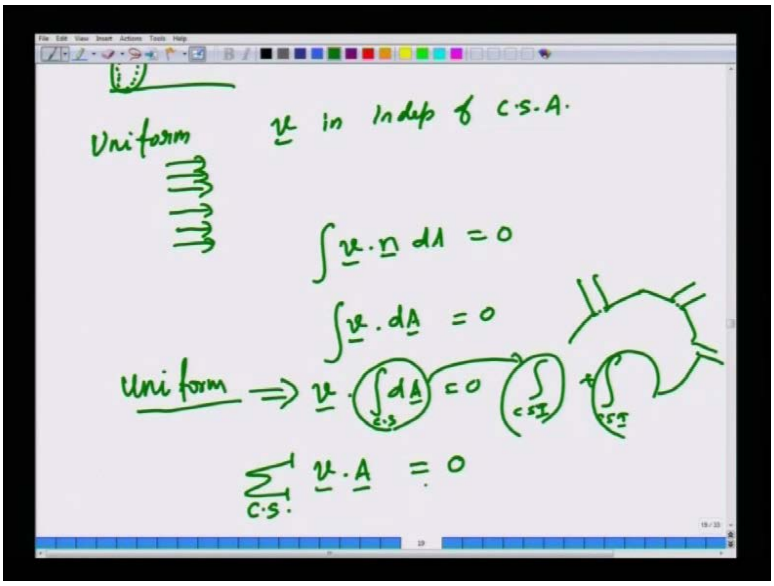

But, if you assume the flow to be uniform then v is independent of the cross section area. That is that means that at each and every point the velocity vector is pointing in a constant direction and the magnitude is also a constant. If so, we have v dot n d A is 0 if v is a constant and if you are assuming that this can also be written as v dot d A is 0. If you are assuming that v is a constant, it can be pulled out of the integral this is only for uniform flow approximation.

Uniform flow approximation only in that thing we are velocity is independent of the cross sectional coordinates you can pull it out. So, integral over d A of the C S is nothing but, the area of the C S.

So, v dot d A is 0 but, remember that the integral will have various inlets and outlets. So, we will have to count over all the inlets and outlets. So, the as I just mention a genetic problem could have many inlets and outlets.

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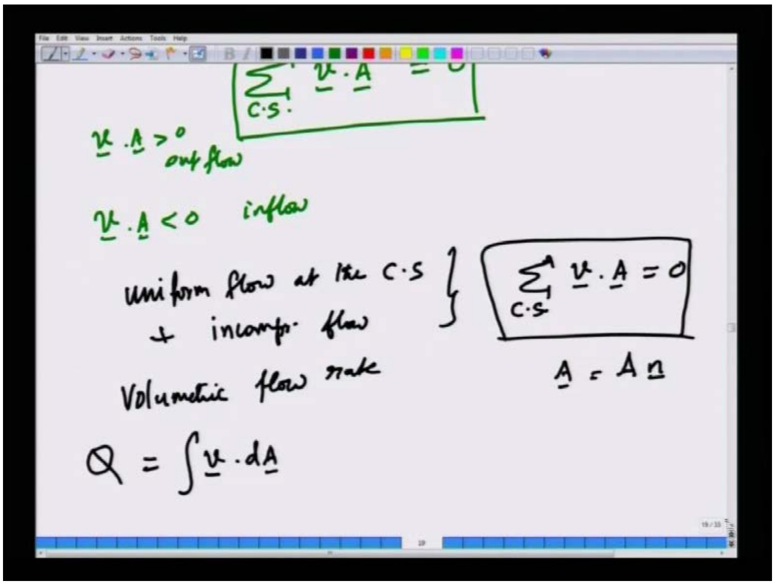

So, the integral over C S over various so, you can split this integral into C S 1, C S 2 and so on. All this will therefore, will become sum over various cross sectional, control surfaces of v dot d A is 0, v dot a is 0 where a is the area of that particular control surface.

And of course, there is a vectorian nature to the area because it depends on whether there is an inflow or outflow. If v dot A is positive then this outflow or if v dot A is negative, it is an inflow because n is an outward normal. So, the outflow normal to the area points from in to out. If the flow is from out to in then therefore, the product of these two quantities will be negative. If the velocity is parallel to the area that is outward normal then it will be positive.

So, for incompressible, for uniform flow at the control surfaces plus incompressible flow whether it is steady or not it does not matter. So, you have integral over all control surfaces, velocity dotted with the area. The area is basically the scalar magnitude times the unit outward normal to each C S.

The physical interpretation is v dot A is the volumetric flow rate. (No Audio Time: 49:50to 49:58). So, integral over v dot d A over any control surface is the net rate at which volume flows per unit time out of the C V, if it is positive or into the C V if it is negative. So, the unit is meter cube per second.

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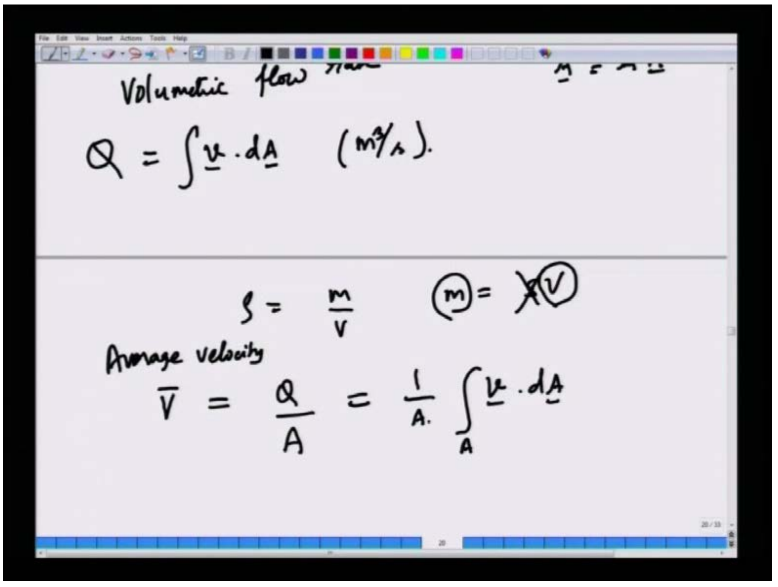

So, essentially for an incompressible fluid the mass conservation boils down to volume conservation because density is a constant. Density is mass per unit volume, density is a constant and since mass conservation so, rho is mass per volume or mass is rho times volume. If rho is a constant then mass conservation amounts to volume conservation in incompressible fluids that is what we are seeing.

Now, sometimes it is useful to think of an average velocity integrated over the averaged over the cross section so, this is Q, the volumetric flow rate divided by the area of cross section 1 over A integral area v dot d A; v dot d a is Q. So, Q by A is one over A integral v dot A. That is the average velocity at a cross section, a average velocity averaged over the cross section.

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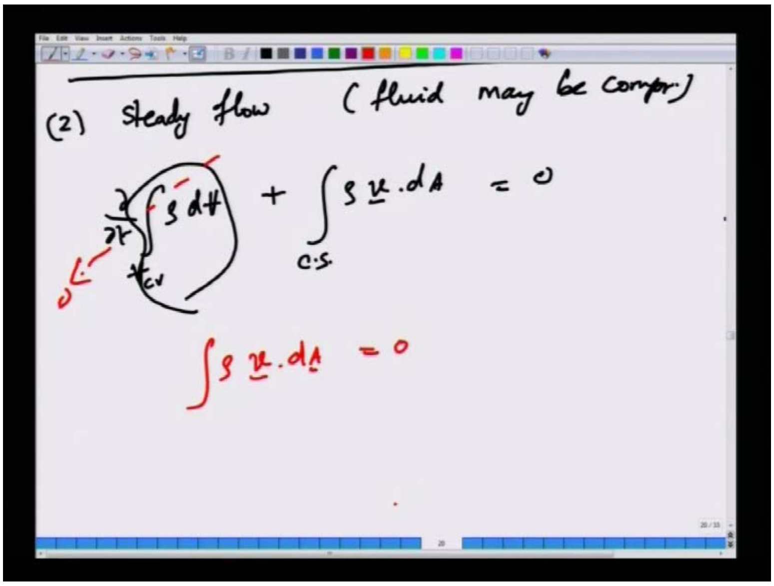

Now, the next simplification we will do is the steady flow. Now, the fluid may be compressible. So, the most general form of the mass conservation equation is integral rho d v plus integral rho v dot d A. This is over volume of C V, lower control surface is 0. By steady, we mean d d t of the quantity is 0.

So, steady means this entire quantity the rate of change of mass is 0 because you are assuming things to be steady in the control volume. So, here you get whether it is a incompressible or compressible fluid. So, rho will occur inside the integral rho v dot d A is 0.

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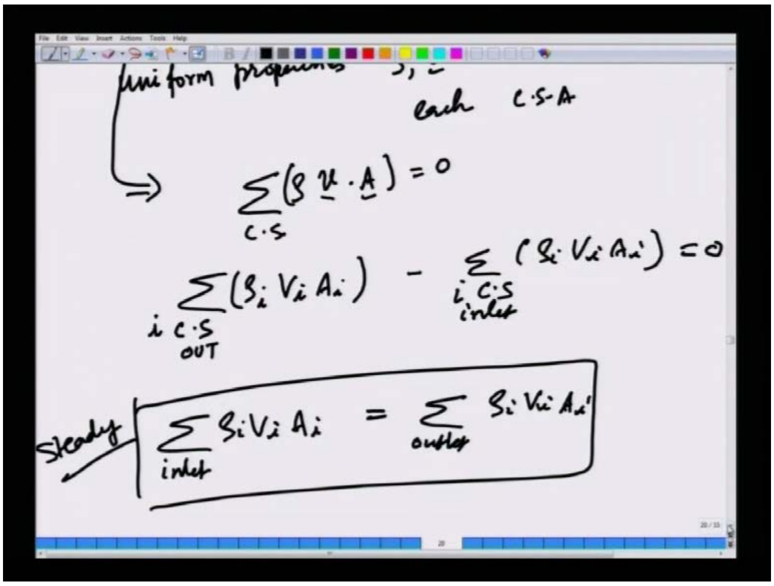

So, if you assume a uniform property that is if rho and velocity do not vary with cross sections. Although they may vary across different cross sections, if at each cross section if rho is a constant, rho v are constant at each cross section although they may vary across different cross sections. If each cross section with of the control surface then this area integral is trivial so, it becomes summation or all control surfaces. So, this becomes rho will still happen inside because in general rho at different control surfaces can be different v dot A is 0.

So, if you assume uniform velocity at all inlets and outlets. So, in general this can be control surfaces outlets spilt into outlets rho v dot A. So, at outlets v dot A is positive so rho i V i A i where i is the index that sums over all outlets minus i is the index over all inlets since v dot A is negative. So, this becomes rho i V i A i over all index is 0 or this simply translates to summation over all inlets, rho i V i A i is summation over all outlets rho i V i A i this is for a steady flow. The mass conservation equation as applied for a steady flow.

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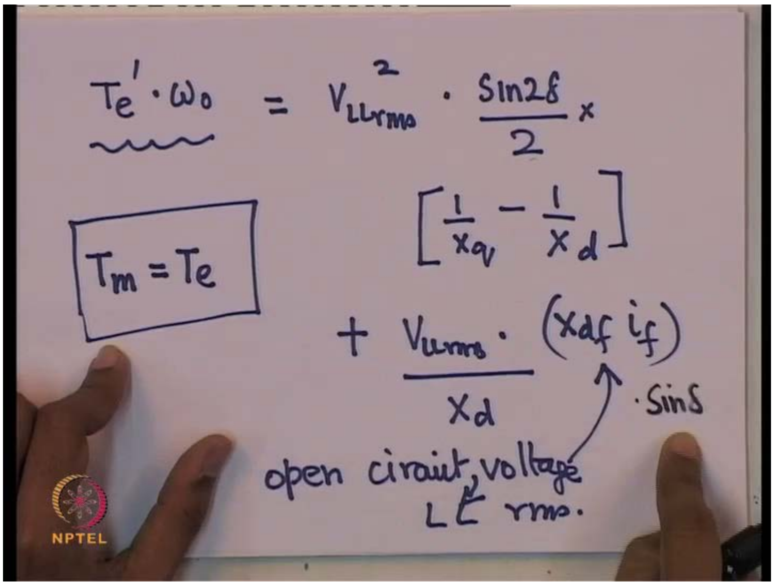

If you just have single inputs single output that is only one inlet and single inlet and single outlet. We have rho in V in A in is rho out V out A out, this is for a steady flow. So, will stop here on the mass conservation balance, mass balance for a C V will continue in the next lecture.