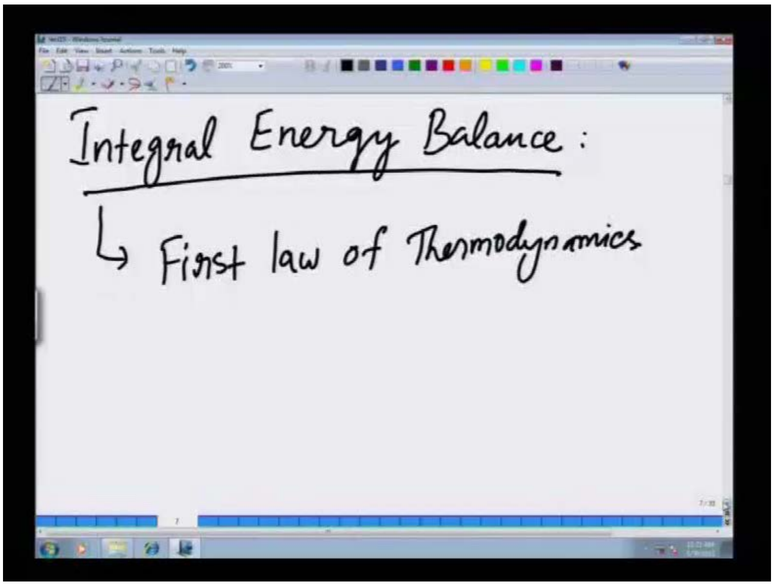
Fluid Mechanics
Prof. Viswanathan Shankar
Department of Chemical Engineering
Indian Institute of Technology, Kanpur

Lecture No. # 16

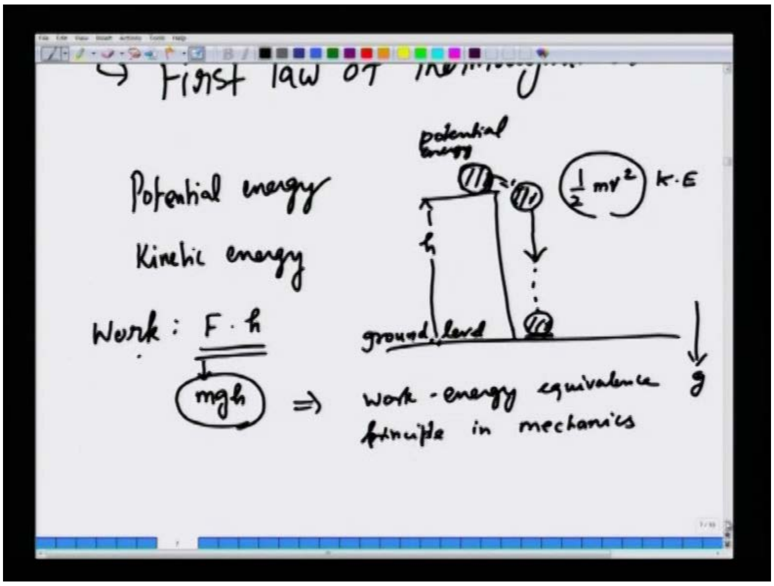
(Refer Slide Time: 00:30)



Welcome to this lecture number 16 of NPTEL course on fluid mechanics for chemical engineering and undergraduate students. Today, we are going to discuss the integral energy balance, and we started this discussion very briefly in the last lecture. And as I told you in the last lecture all the fundamental integral balances in fluid mechanics namely, mass momentum energy they rest on a fundamental physical principle.

For example, the integral mass balance came out as a consequence of the principle of conservation of mass, and integral momentum balance came out as a consequence of the Newton second law of motion. Now, we are ready to discuss the integral energy balance, and the fundamental principle that helps us to derive the integral energy balance is the first law of the thermo dynamics. The first law of thermodynamics is essentially generalization of or the notion of energy to system such as fluids.

(Refer Slide Time: 01:20)

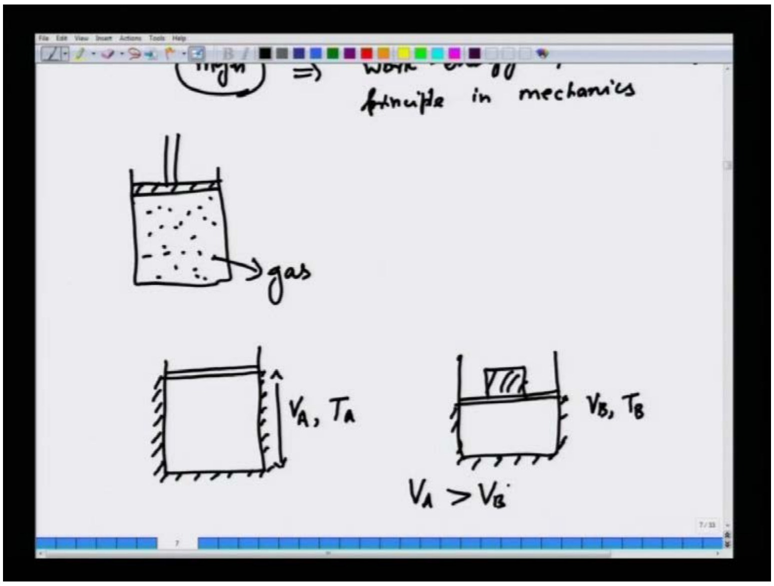


So, the notion of energy is familiar from mechanics where you had potential energy, and kinetic energy of a macroscopic object like a sphere. So for example, you could say that a sphere is at rest if it is on top of a table let us said, and gravity is this acting like this is the ground level. Now, you can tip over this sphere and this sphere acquires it moves, and the motion the energy associated by virtue of its motion is half m v square, where m is the mass and v is the velocity, this is called the kinetic energy of the sphere, which as which is associated by the motion of the sphere.

While the sphere is at rest we say that this sphere has a potential energy by virtue of its elevation from the ground level. So, of you allow if you just tip over this sphere this sphere is going to fall down eventually and it is going to come to the ground level. Now if you ask the question how is I going to bring this sphere back up to the same level you have to do work. So, that is you have to apply a force on this sphere that is we have to apply a force over this distance h, let us call this distance as h and the application of the force over a distance is h will amounts to doing work on this sphere.

So, that you can bring it back to its original elevation and the force that you apply is exactly m g. So, the work that you done in order to bring the system back to its original state, this sphere back to its original level elevation is the same its initial kinetic energy. So, this is the work energy equivalence principle in mechanics principle in Newtonian mechanics which deals with objects such as spheres of macroscopic sizes and so on.

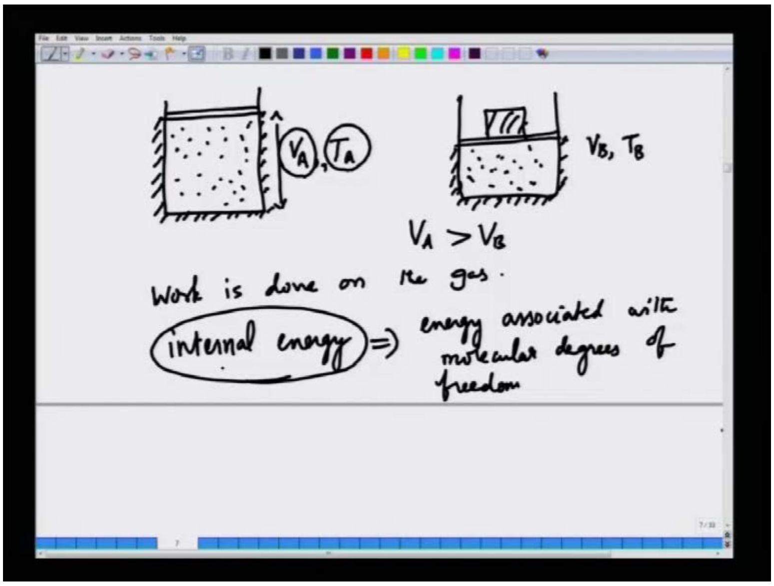
(Refer Slide Time: 03:43)



But now, we are in the domain of discussing energy conservation for systems like beaker of a fluid. For example, you may have a piston cylinder assembly and you may have a gas a dilute gas such as air present in inside in the piston cylinder assembly. Now let us imagine that you have this gas and initially the gas is at a given volume which is denoted by V 1 or V A, let us call it V A and the temperature. Let us say is constant or let us even insulate thermally this piston cylinder assembly and it has a temperature T A. Now imagine push putting in a weight.

So, that this piston now moves. So, there is a weight that is acting there is a force that is acting. So, you compress the piston. So, that you are now in a new state B with volume V B and temperature T B. Now you have done work on the gas because you have applied a force and you have moved the piston over a distance. So, the volume of the gas initially which was V A is now greater than the final volume V B. So, you have certainly done work on the gas. Now initially the gas is stationary now after doing work if you leave the gas for some time it will again come to rest that is there will be no visible macroscopic motion of the fluid that is present inside piston cylinder assembly.

(Refer Slide Time: 05:30)



So, this means that you have done work was certainly done on the gas by virtue of the weight or the force that you apply over a distance. Now we want to generalize the principle of energy work equivalence to systems like this. In the previous case, when we did work on this sphere by lifting it up we said that the work done has gone on to increasing the kinetic energy of the sphere. Now what has happened to the work done? So, this system a system of gas present in a piston cylinder assembly is not a single sphere it has many many many many fluid molecules.

So, in the continuum description we think of this system as we think of this collection of fluid molecules gas molecules as a system, which is prescribed by a variable such as its volume and its temperature. Now you have done work on this system and you have compressed it what has happened to the work that you have done well. Since you have completely insulate the system this work that has been supplied to this system clearly has not gone to increase the potential energy of the system nor has it increased the macroscopic kinetic energy of motion of the fluid molecules of the fluid, because initially the fluid at rest and finally, also the fluid is clearly at rest.

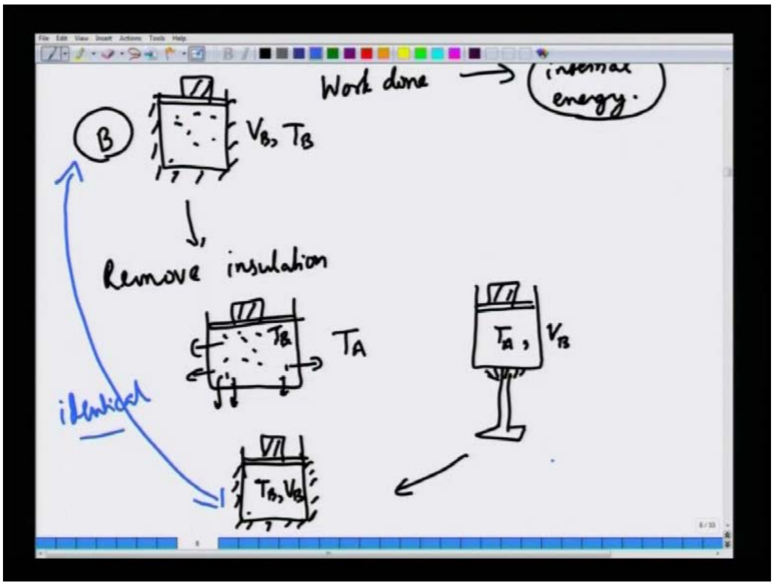
So, what has happened to the work is that it has gone on to increasing the internal energy of the system and internal energy is a new concept that comes from thermo dynamics and it has a role to play even in fluid mechanics because we will see that these notions or principles are helpful while understanding the principles of conservation of energy and energy balance even in fluid flows, internal energy is that part of energy associated with microscopic or molecular degrees of freedom.

So, what has happened is that when you do work initially the gases at a temperature T A. Now from elementary kinetic theory that you may be aware from physical chemistry or some physics courses earlier, you know that the temperature of a gas is related to the average kinetic energy of motion of the molecules gases. Of course, compressed ultimately of molecules and these molecules are always in constant motion even though they even though on the average gas appears stationary or static, but microscopically at a molecular level all the molecules are moving at a very high velocity and the temperature is a measure of the average kinetic energy of motion of the molecules.

When you apply a force over a distance there by doing work on the piston, you are compressing the gas and the work that you have done has been has gone on to increase the average kinetic energy of motion of the molecules and therefore, it has gone on to increase the temperature.

So, this energy of motion kinetic energy of motion associated with molecular degrees of freedom is essentially characterized as the internal energy that is the energy that is internal to the fluid that does not manifest in macroscopic senses such as potential energy or kinetic energy, but it is actually characteristic of the molecules that comprise the fluid is called the internal energy.

(Refer Slide Time: 09:07)



So, what we have said so far is that when you have an insulated container when you do work on a gas by placing a weight, the work that you do has increased the internal energy of the system. Because, clearly the gravitational potential energy has not change nor has the kinetic energy changed. Because, the gas is stationary macroscopically speaking there is no visible bulk motion of the molecules. So, internal energy work done can change internal energy this is an experimental fact of the molecules of the gas. Now is this the only way to change internal energy the answer is no, we can also change internal energy through an entirely different route.

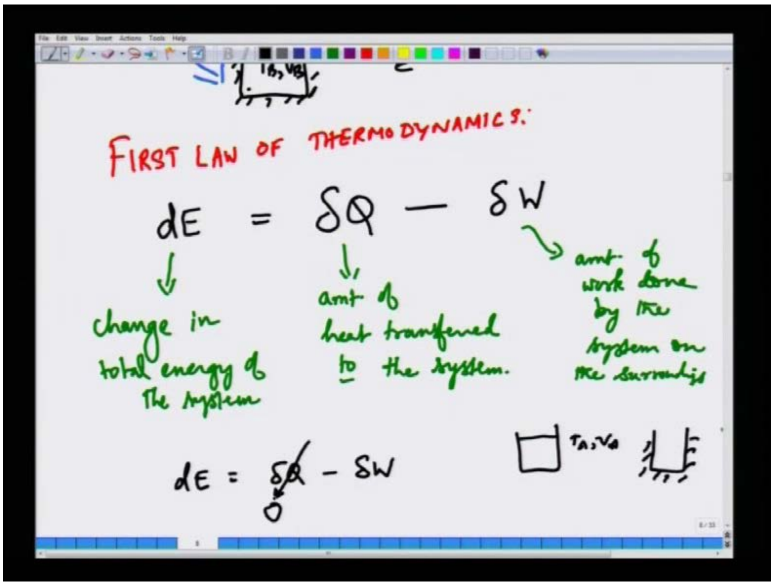
So, imagine in state B now the gas is in state B with volume V B temperature T B. Now imagine removing the insulation the thermal insulation there by you are now allowing the beaker or the piston cylinder assembly with gas present at temperature T B to transact energy with surroundings by virtue of transfer of heat. We know from experience that whenever you have a hot body in present in a cold atmosphere, you know that that hot body will eventually cool down and there is a transfer of energy that is heat transfer that happens from the hot body to cold body.

That is what will happen eventually you will have the system with temperature T B and volume V sorry temperature T A because, it has lost all the energy to the surroundings, but volume is still V B. Now, imagine heating the system using a burner such that you have provided enough energy. So, that eventually you are now reach the state where you have reach the state where you have temperature T B and again volume V B. Now once you are reached temperature T B you can insulate the system and this will be identical to the state B so the state B that you had here and the state B that you had here are identical.

Because, they have the same volume and same temperature, but what is important to understand is that the way you have reached the state B is entirely different in this two cases. In the first case you compressed an insulate a gas present in an insulated cylinder from an initially state of T temperature T A and then you apply it you did work by applying a force and that resulted in a new state with temperature T B and volume V B. Now, you removed the insulation thermal insulation and allowed the gas to cool down to the ambient temperature T A. Again you reach T B the previous state B through an entirely different process namely, by transacting heat with by exchanging heat from surroundings by let us say having a burner.

So, what is important and this is the important lesson from first law of thermo dynamics is that fluid systems such as piston cylinder assembly containing gas, macroscopic system which innumerable molecules. They are characterized by not just by their gravitational potential energy or there macroscopic kinetic energy of motion, but they are also characterized by their internal energy and this internal energy can be changed by doing work as we have seen or by exchanging heat.

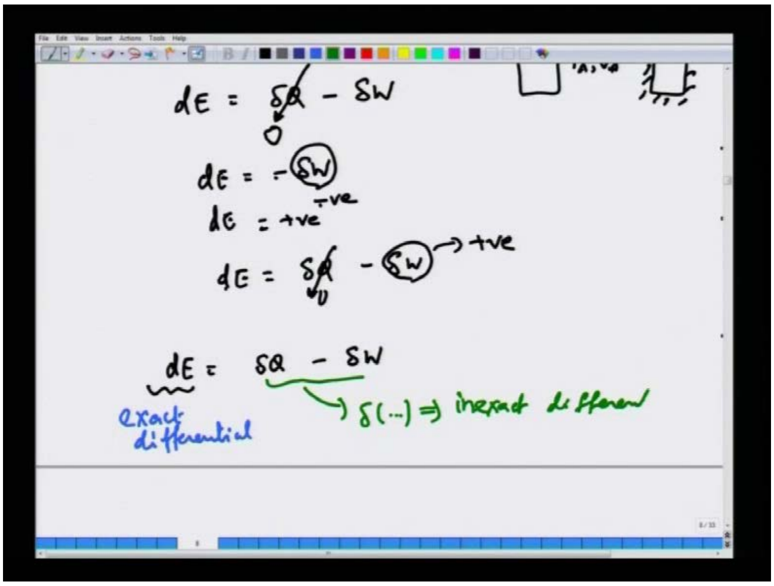
(Refer Slide Time: 13:11)



So, this is the statement of first law of thermo dynamics, that first law of thermo dynamics it states that the change in energy of a system is equal to the rate at which heat is transferred to the system and sorry the amount of heat transferred to the system and amount of work done by the system. So, let me write down all these things in words and then we will discuss the sign convention. So, this is change in total energy of the system is equal to amount of heat transferred to the system, this is the amount of work done by the system on the surroundings.

Let us understand the implication of this equation from the example that we just did. Initially we said that so let me write this equation again dE is small delta Q minus small delta W. In the first case we said that we had a system where you had T A V A and you applied you insulate the system. When you insulate the system then there is no heat transfer with the surroundings. So, small delta Q is 0.

(Refer Slide Time: 15:02)



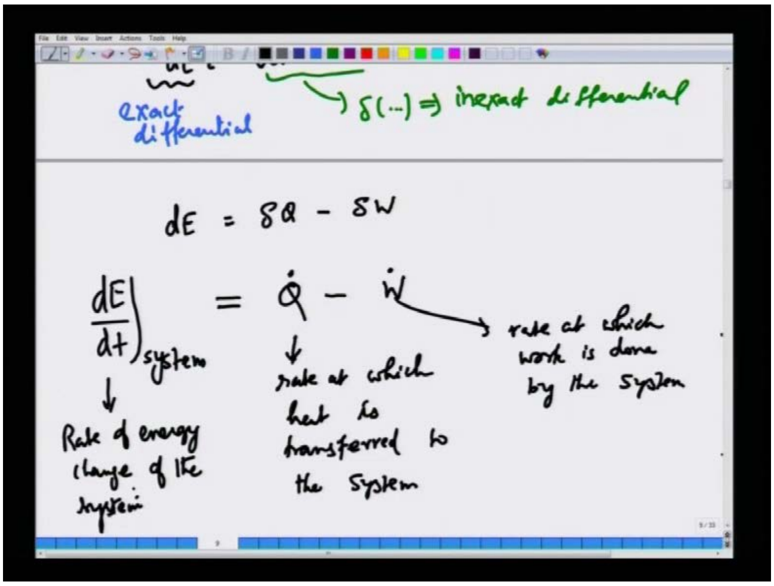
So, the change in energy is equal to minus small delta W, if you do work on the system then small delta W is negative. So, negative of negative is positive. So, the energy increase positive that is the system has increased its energy which is what we said and it makes intuitive sense as well now if the system does work. So, let us imagine another case again insulator if the system does work on the surroundings it is going to lose energy. So, dE will be negative. So, if work done is positive if small delta W positive the system is doing work on the surroundings then the system will lose its energy.

So, that is the meaning of the sign convention. So, there can be other sign convention. So, you must be careful about other sign conventions. The other common sign convention is that heat transfer to the system is positive as we said here, but also work done on the system is positive in which case the first law will read as dE is dQ plus dW small delta Q plus small delta W. Another point I will just mention in passing, but not spend too much time on is the nature of the differentials here dE in thermo dynamics is called an exact differential whereas, small delta Q and small delta W are called delta of something is n in exact differential.

This is because in a these quantities such as work and heat they are not functions of, I mean the amount of that work you transact depends on the path the way in which you go from state one to state two whereas, energy is a function only of the initial and final states. So, quantities like energy temperature pressure they are called state functions because, they just once you know the thermo dynamic state of the system, you have let us say you have temperature and pressure you can say what its density is, what its energy is, regardless of how the system was prepared. It is independent of the history of the process by which you have reached that particular state.

So, quantity such as energy, pressure and temperature they are called state functions in thermo dynamics whereas, work and heat clearly you can reach the same state by doing completely work or by doing heat. So, clearly work and heat are not state functions it depends on how you carry out particular process. So, they are called path functions. Therefore, you cannot you cannot write work or heat as an exact differential. So, they are denoted by a delta the infinitesimal work done is denoted by delta rather than d because it is not an exact differential, but more of this will be done in the thermo dynamics course, but we will not do this right now.

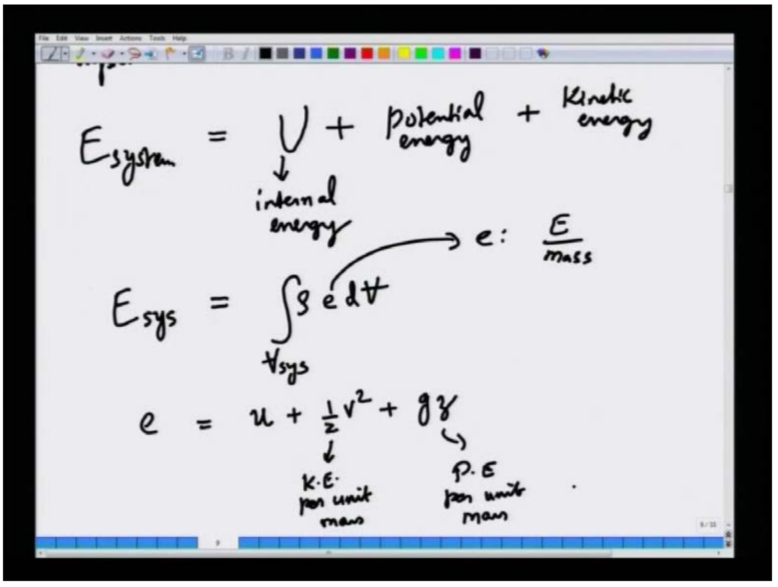
(Refer Slide Time: 18:07)



Now, we will rather proceed to generalize this first law which we initially wrote for a finite amount of heat or work that is transfer to or from the system, and we related that to energy changes in the system. But in fluid mechanics, we do not look at we do not look at systems where you look at a finite amount of heat transferred, and then you take out a work in one shot. Things happen continuously in many engineering operations. So, we need to generalize this to a form in which we talk about the rate at which energy is changing in the system to the rate at which energy heat is transferred to the system and to the rate at which work is done by the system.

So, we are going to generalize this to flowing system where things change with time continuously. So, the rate of change of total energy of the system is equal to the rate at which heat is transferred to the system minus the rate at which work is done by the system. So, Q dot is the rate at which heat is transferred to the system and W dot is rate at which work is done by the system, this is the rate at which energy of the system changes with time rate of change of energy of the system.

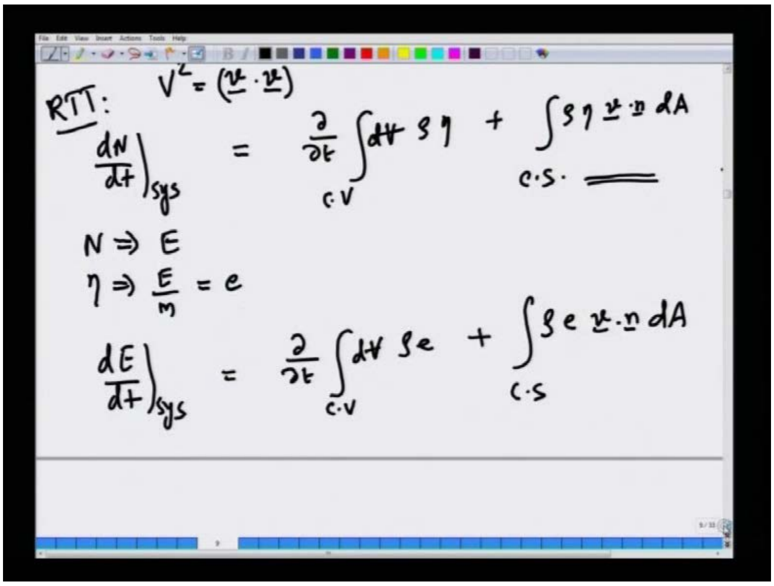
(Refer Slide Time: 20:03)



Now, what is the energy of the system? The energy of the system of course, comprises of the internal energy plus potential energy macroscopic potential energy by virtue of its elevation with respect to datum or a ground level plus macroscopic kinetic energy. Now we want to use Reynolds transport theorem to convert the rate of change of energy of the system to the rate of change of energy in the control volume. So, remember in Reynolds transport theorem we always write E system as volume of the system rho e dV where e is e divided by mass energy per unit mass small e is the total energy per unit mass.

So, small e is nothing, but the internal energy plus unit mass plus the kinetic energy plus unit mass kinetic energy is half m v square, if you divided buy mass you will get half v square plus the potential energy per unit mass the potential energy is m g z, z is the elevation. So, these are this is k E per unit mass; this is potential energy per unit mass.

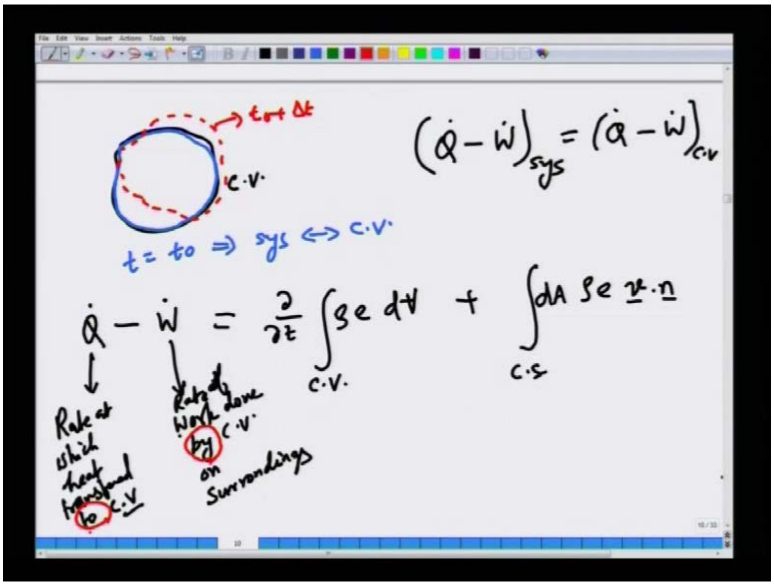
(Refer Slide Time: 21:33)



So, this is and V square is the magnitude square of the magnitude of the velocity is given by the vector velocity v dot v. So, we want to now generalize this to control volume kind of a kind of a situation formulation. In order to use the Reynolds transport theorem we have to say remember the Reynolds transport theorem, we have to use dN dt of the system. This is nothing, but d dT this is the Reynolds transport theorem of cV dV rho eta plus integral over c s rho eta v dot n dA over all the control surface. This is the rate at which a quantity like momentum energy changes within the control volume.

This is the flux term that takes mass momentum or energy inside or outside the control volume by virtue of flow within flow to into and out of the control volume. So, now we are going to say N is nothing, but E sorry n is capital E and eta is e by mass which is e small e. So, the rate of change of energy of the system as written in terms of the variables relevant to a control volume is rho e this is the local rate of change present in the control volume plus the flux term rho e dA. This is something that we have seen frequently before for mass conservation and momentum balance.

(Refer Slide Time: 23:24)

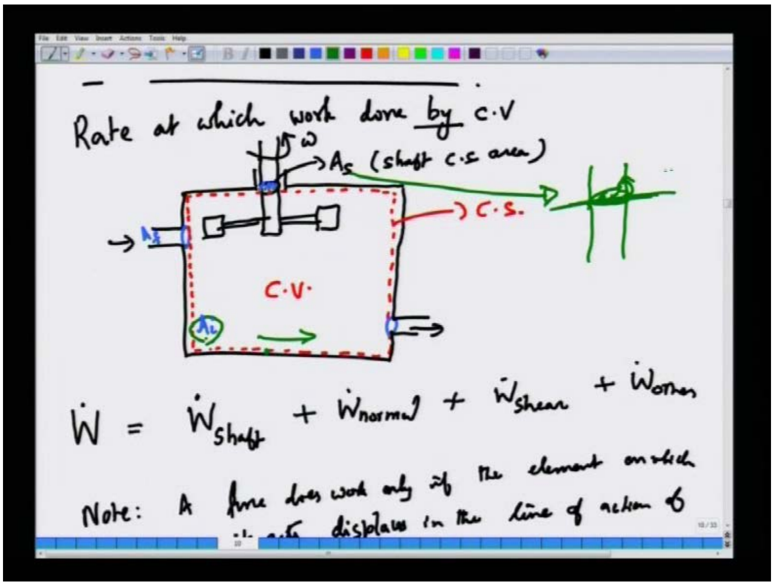


So, it is same here now remember, when you do the Reynolds transport theorem, you identify the control volume and you say that the system and control volume coincide at time t 0 system at time equals to t 0 system is the same as control volume, but at a later time you know that the system is going to move out of the control volume. So, this is t naught plus delta t system is moved out of the control volume, but we are now looking at the instantaneous rate of change of energy of the system.

When the system and the control volume are the same at some time t naught. So, if you want to apply the first law of thermo dynamics at that instant, when the system and control volume of the same are the same, then the rate at which Q dot minus W dot is the rate at which work heat is transferred and the rate at which work is done for the system is the same as that for the C V, because the system and the C V coincide at the time t equals t 0. So, the first law of thermo dynamics the rate form of the first law of thermo dynamics becomes this is equal to d dT integral over C V rho e dV plus integral over the C S rho e this is a flux term.

Now this is the rate at which work is done by the system or the C V, rate of work done by C V on surroundings rate at which heat is transferred to the C V. So, this key thing is you should notice by C V on surroundings this is to C V on surroundings. This is very important when we do the problems in energy using energy balance. Now that completes the statement the formal statement of energy balance, but now of course, we have to sort of write this in terms of many contexts that are relevant to fluid flow.

(Refer Slide Time: 25:51)



So, first is what is the rate at which work is done by C V, what is the rate at which work is done by C V on the surroundings that is positive when work is done by the C V on surroundings, when work is done on the C V it becomes negative. What are the various types of work? So, in order to understand and appreciate this let us imagine typical process that will happen in much fluid flow equipments. So, let me draw a container in which fluid is coming in and going out and there may be a shaft and which has rotary blades which rotate by virtue of the rotation of the shaft and this is the area cross sectional area of the shaft C S area cross section area.

So, fluid is coming in like this and going out like this and let us now draw what the C V is sorry let us now draw the C V by demarcating the C V form the surroundings using the control surface. So, the line marked in red the dotted line is my control surface and whatever is presenting whatever is present inside is my control volume fluid is continuously coming in and going out. Now, what are the various types of work? So, there is an area here where the control surface is cutting across where fluid is flowing in that. Let us call that area A f, where the flow is happening the rest of the area.

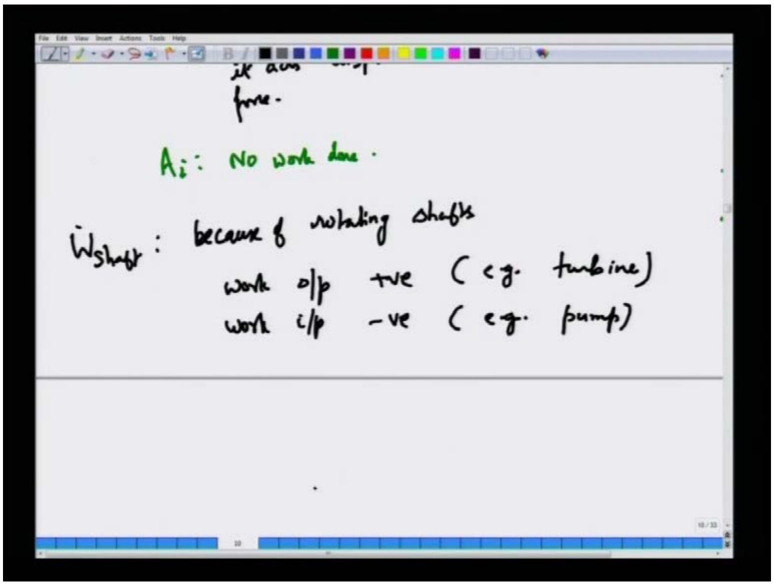
Let us call it A i and of course, this area where the control surface is cutting across the shaft which is rotating is A S. These are the three different types of areas that we can distinguish. So, the rate at which work is done is due to various reasons work done by the shaft work. So, whenever you have a shaft that is rotating and when the C V is cutting across the shaft, there is a stress on the solid surface and there is a force therefore, and since there is the shaft is also rotating; that means, that there is a force that acts with a velocity and therefore, there is some work done.

Because of this rotation of the shaft, if the shaft rotates and does work on the C V that becomes negative; on the other hand if the C V rotates the shaft and therefore, the work is done by the C V on the surroundings delivering useful work for the surrounding, then it becomes positive quantity as per our convention. Now, the rate at which work is done by normal forces on the C V on the C S that is control surface the rate at which work is done is by shear forces and if there are other contributions the rate at which work is done by other reasons.

Now, remember one important fact is that a force does work only if the element on which it acts moves in the direction of force displaces in the line of action of force. What this means is that, if you look at there could be fluid flow here by virtue of it there could be a stress here due to this fluid flow, but this control surface is stationary it is a rigid stationary boundary in which case there is even though there is a force acting there is no motion therefore, there is no work done. Since, no work done on the internal area A i that is what we try to conclude from this simple idea that force does work only if the element over which it acts moves in the line of action of force displaces.

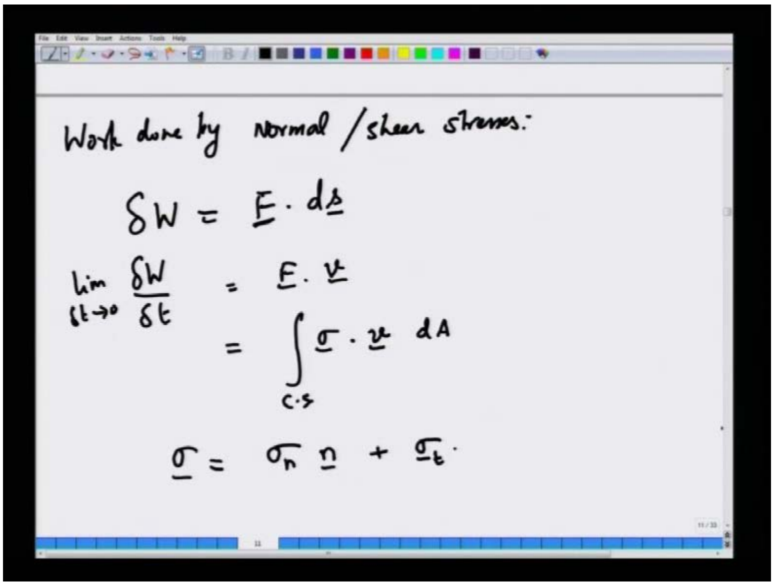
So, there is no work done in A i the internal area. Now if you look at the shaft work if you look at A S then there will be a stress I am just sort of. So, this is the C S that is cutting there will be a stress in the solid surface over which the control surface is cutting across and therefore, there is a shear stress and there is an associated motion rotatory motion. So, there are forces associated with there is work rate at which work is done either by the system on the surroundings or by the surroundings on the system whenever the control surface cuts across a rotating shaft. So, there is a rate at which work is done and that is called the shaft work. So, that shaft work is what is denoted as W dot S or W dot shaft.

(Refer Slide Time: 31:10)



This is because of the rotating shafts, which may either deliver work to the system or which may take work away from the system. So, work output from the C V is considered positive, this is an example of a turbine which delivers work and work input to the C V is negative is an example of a pump, where you have to expend external energy to increase a pressure head of a fluid. Now the shaft work and then you have work done by normal and shear stresses.

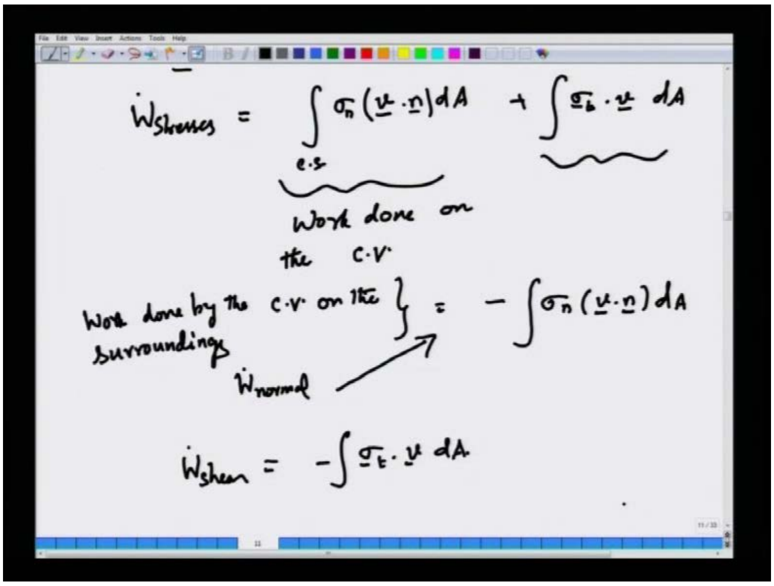
(Refer Slide Time: 31:51)



So, we will look at this case by case work done by normal and shear stresses. So, the rate at which sorry so work done is simply F dot ds. The rate at which work done is small delta W small delta T limit small delta T going to 0 is nothing, but F dot v. So, let me denote all vectors with an underscore F dot v. So, F is nothing, but the stress vector that is acting on the surface dotted with v, but you have to integrate it over the entire control surface, where there is flow as well as there is motion.

So, remember that there is a work done only if there is a motion on the motion of the element on which stresses are acting in the line of action of the force as we have been repeating. Now, sigma is nothing, but sigma normal which is in the direction of the unit normal plus sigma tangential or shear which is in the direction perpendicular to the normal.

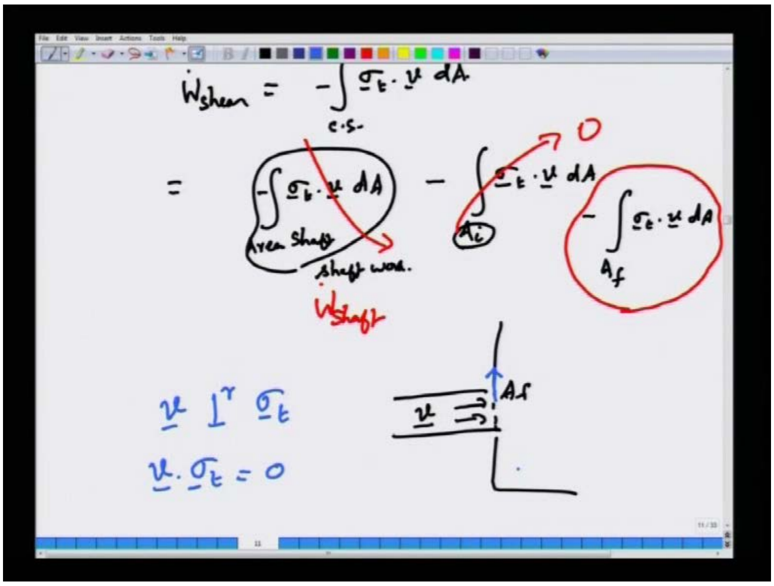
(Refer Slide Time: 33:14)



So, the rate at which work is done W dot due to stresses fluid stresses is nothing, but sigma n times v dot n dA over the control surface plus integral sigma tangential dotted with v dA. Now this is the normal rate at which work is done by normal component to the stresses this is the rate at which work is done by shear or tangential component to the stresses. Now this is the work done by on the entire control surface. So, this is the work done on the entire control surface by the shear stress. So, work done by the control surface on surroundings is the negative.

So, this is the work done on the C V. I am sorry So, this has to be this is the work done on the C V by the, this is the work done on the C V, if you want to know what is the work done by the C V on the surroundings. So, work done by the C V on the surroundings is nothing, but minus. So, the normal component of the work W dot normal is minus the negative of whatever we have written because, this is basically the work done on the C V, but the work done by the C V on the surroundings is the normal component, due to the normal component of the stresses is this and due to shear is nothing but minus sigma t dot v dA.

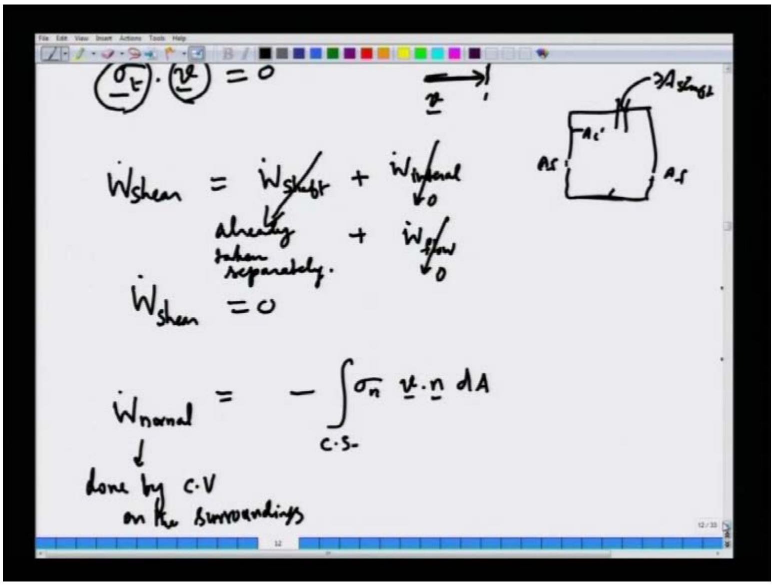
(Refer Slide Time: 35:13)



Now, what are the various components to the shearing forces? There are three types of areas. One is the area over the shafts sigma t dot v, this we already I have accounted it as the shaft work. So, this entire thing is called the shaft work minus integral sigma t dot v dA over solid surfaces, this is the A i part minus A flow part sigma t dot v dA. Now over the entire A i internal area surface, which we just if you look at the general figure that we drew in the over the entire internal area surface. Velocity is 0, because the fluid is stationary.

So, even though there is a stress that is acting there is no work done because the velocity is identically zero. So, this drops out this was already taken into account as shaft work W dot shaft. So, the only that is left for us to understand is this, but imagine that if you have. So, where the flow area is where the inlet or outlet is actually present this is A flow. So, if velocity is acting like this and the tangential stress act in the direction perpendicular to the velocity, if you choose your control surface such that the velocity is normal to the control surface. Then v is perpendicular to the sigma stress tangential stress. So, v dot sigma t is 0. So, by carefully choosing the control surface you can actually have the tangential stress on the inlets and exits to be zero.

(Refer Slide Time: 37:05)

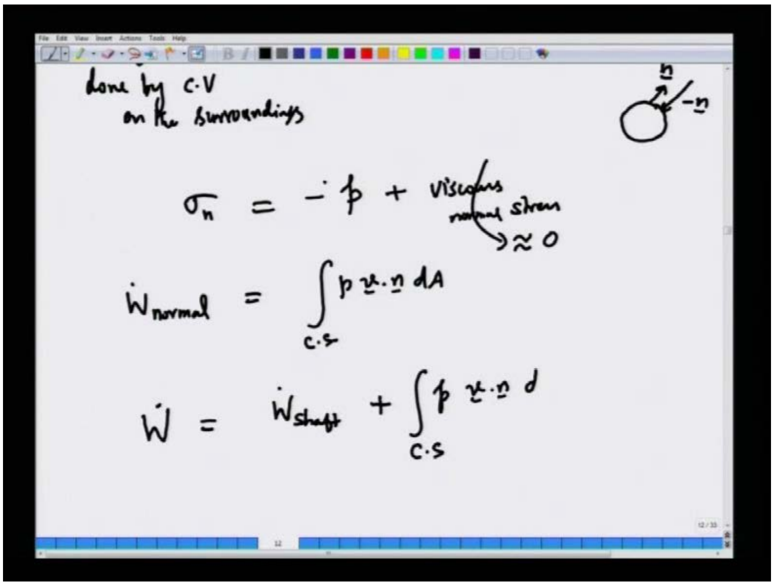


So, we can by carefully choosing the control surface such that the velocity is normal to the control surface, then the tangential component to the sigma t stress is acting perpendicular to the velocity. So, the stresses acting like this velocity simultaneously the perpendicular. So, there is no the dot product is 0 of this two vectors. So, the shear component of the work, there are three types of areas that we said one is, let me just draw the internal area, the flow area through which fluid entering and exiting and the shaft area, which is cutting across the control surface.

So, there are three contributions to the shear work, one is due to the shaft work which we have already separated or so that we do not have to include one is due to the work done at the internal areas that we said is zero because even though there is a stress there is no velocity. So, sigma t dot v is identically 0 and in the third case we had the work done at the flow areas and there it is 0, because the stress and the force and the velocity are orthogonal or perpendicular to each other. So, W dot shear is 0 because, we have already taken into account the shaft work separately.

Once we understand this and the normal component to the work is nothing, but integral minus integral sigma n this is the work done by the C V on the surroundings. C S sigma n v dot n dA. This is the work done by rate at which work is done by C V on the surroundings due to the normal component of the stresses. We will see a little later that the normal component of the stress comprises of both a pressure which is acting compressively if you know.

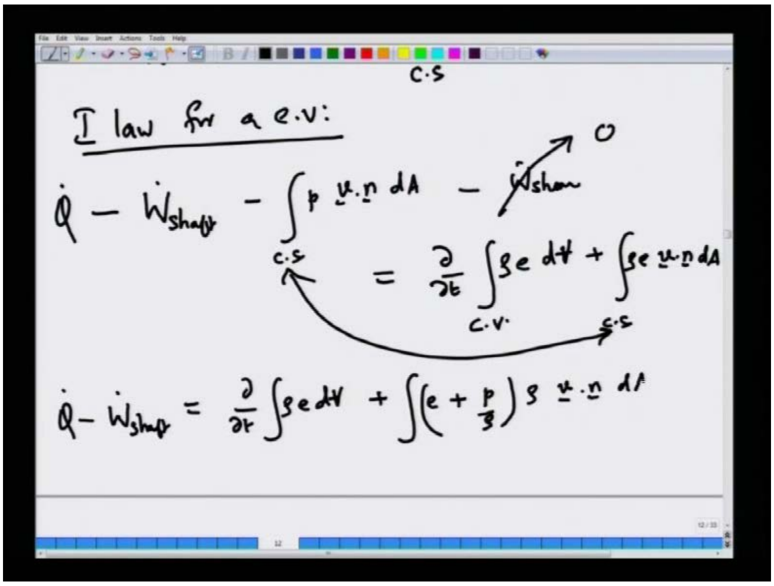
(Refer Slide Time: 39:18)



So, sigma n is minus p because, if you take a surface with unit ward normal n pressure is acting in the direction of minus n p the pressure acts is the direction minus n. So, sigma n is p minus p and then you also have some viscous shear stresses acting in the normal direction viscous, we should not call shear stresses viscous normal stresses, but these are usually negligible they are neglected.

So, sigma n is minus p to a good approximation. So, W dot normal is nothing, but if I substitute this here the two negative signs will cancel out. So, p v dot n dA over the control surface. So, the total rate at which work is done is nothing, but the shaft work when the control surface cuts across rotating element such as shafts, which will typically convey energy in or out of the system convey work in or out of the system times p v dot n dA.

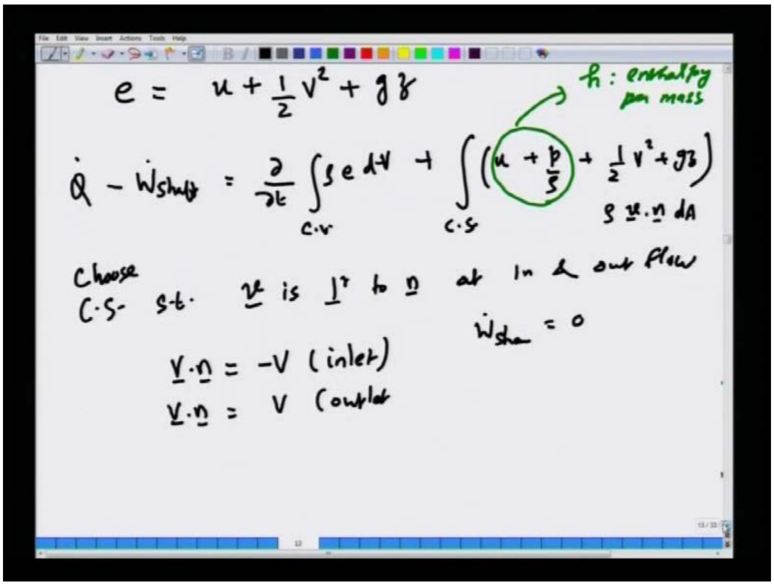
(Refer Slide Time: 40:38)



So, the first law of thermo dynamics for a control volume eventually becomes. So, eventually becomes Q dot minus W dot shaft minus integral p v dot n dA C S and minus W dot shear in general, we said that can be carefully by choosing the control surface appropriately we can set it to 0 exactly. So, this is equal to the rate at which energy is changing and that is the rate at which energy is changing within the control volume plus the rate at which energy is entering or exiting the control volume by virtue of flow, this is on application of Reynolds transport theorem.

Now, I can take if you look at these two terms they are identical. So, I can bring this term out here to write Q dot minus W dot shaft is d dT integral rho e dV plus integral e plus p by rho times rho v dot n dA. this is something I can do by just multiplying and dividing by rho I can do this very easily.

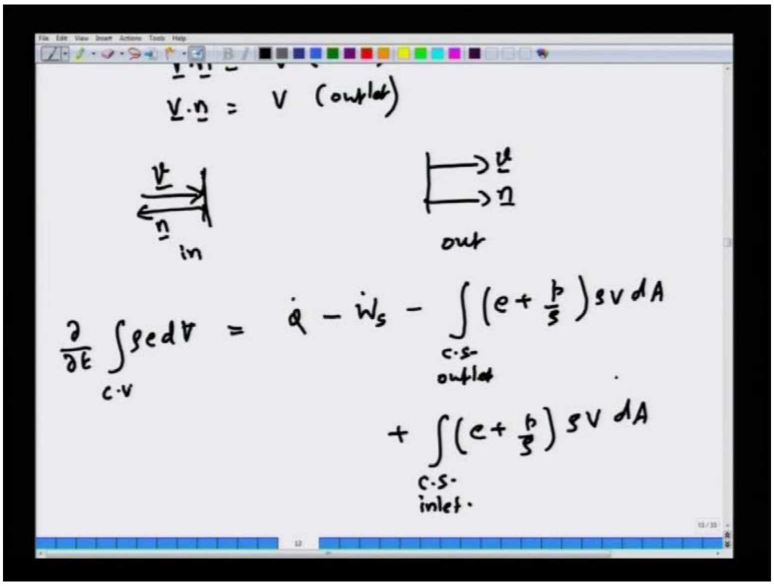
(Refer Slide Time: 42:11)



So, where e if you remember is the specific energy that is energy total energy per unit mass, which is the internal energy plus unit mass plus the macroscopic kinetic energy plus unit mass plus the macroscopic potential energy per unit mass. So, we further write this in the following manner, Q dot minus W dot shaft is d dT integral C V, this is C S, C V rho e dV plus integral C S, I am going to write it as u plus then I also have a p by rho p by rho plus half v square plus g z rho v dot n dA. Now in thermo dynamics this quantity is called the enthalpy per unit mass specific enthalpy, p plus u plus p by rho is called the specific enthalpy p enthalpy p per unit mass of the fluid, but that is in thermo dynamics.

But in fluid mechanics, we normally do not do this for a specific reason. So, I will come to that in a minute. So, remember that we are choosing the control surface choose control surface such that, v is perpendicular to n at in and out flow such that W dot shear is identically zero. That is at the inlet V dot n is minus V and V dot n is V at the outlet. You should always choose your control surface, the velocity vector is perpendicular exactly to the or the velocity vector is exactly perpendicular to the control surface.

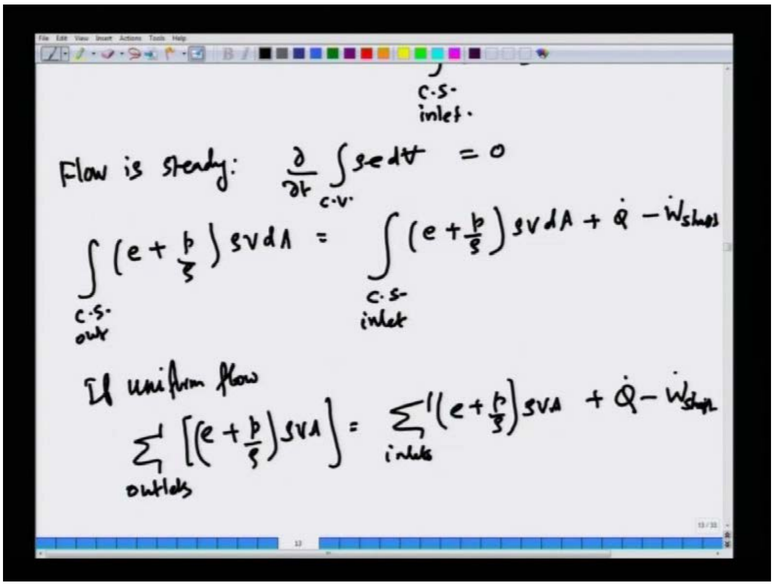
(Refer Slide Time: 44:14)



So, there is no component in the direction parallel to the surface. So, that is something we can always do when we choose our control surface, this is v, this is n at the inlet. So, v dot n minus v this is V this is n at the outlet. So, v dot n is plus v both the vectors are pointing in the same direction at the outlet, where both the vectors are are pointing in the opposite direction at the inlet.

So, we can write further d dT of integral rho e dV over the C V is nothing, but rate at which heat is transferred to the system minus rate at which shaft work is done by the system minus integral over all outlets. I am separating outlets and inlets, now e plus p by rho rho V dA plus integral over all inlets e plus p by rho rho V dA am taking into account this minus v dot n is minus v at the inlet and plus v at the outlet, that is why you are getting two different signs at the inlet and the outlet.

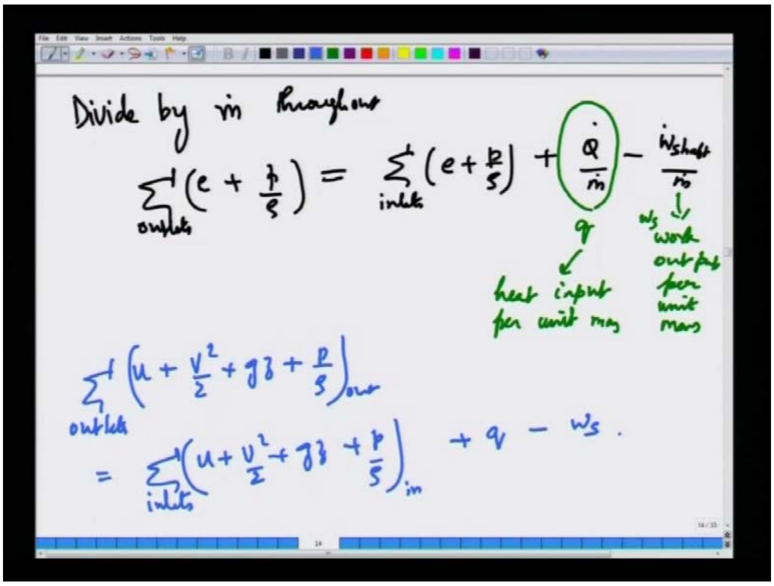
(Refer Slide Time: 45:37)



Now, I am going to do one more thing I am going to assume the flow is steady, I am going to simplify this flow is steady which means that d dT of rho e dV is 0, because nothing changes with respect to time within the control volume. So, all you will get is integral over C S outlet e plus p by rho rho V dA is integral over C S inlet rho V d A plus Q dot minus W dot shaft.

If the flow is if you have uniform flow at the inlet and the outlet and uniform property such as energy and so on, then I can trivially do the area integral to write summation over all outlets e plus p by rho times rho V A is summation over all inlets e plus p by rho times rho V A plus Q dot minus w dot shaft. In a steady flow rho V A in is rho V A out that is mass conservation that is equal to rate at which mass is entering or leaving the system.

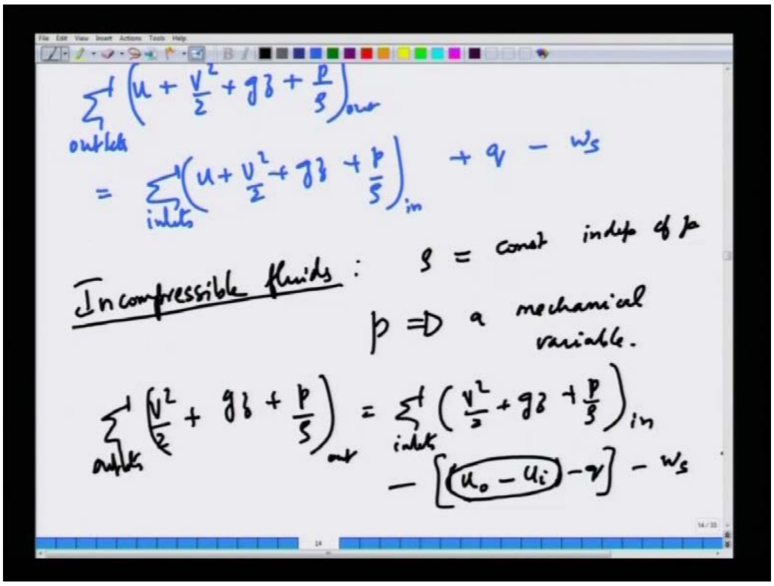
(Refer Slide Time: 47:17)



So, if I divide by m dot it is throughout I get summation over outlets e plus p by rho is summation over inlets is equal to e plus by rho plus Q dot by m dot minus W s shaft by m dot. Now this is the rate at which heat is transferred to the system to the C V by the surroundings this is the rate at which mass is flowing in. So, this combination is denoted by small q this is the heat input to the C V per unit mass and this is the shaft work output per unit mass. So, that is the these are given special names small q and small there is no rate associated with it, because you are dividing the rate at which heat is transferred by the mass flow rate and likewise the rate at which work is done by the system of the C V by the mass flow rate.

So, they will be a we are not no longer rates, this is the rate at which heat is transferred per this is the heat transferred per unit mass and the work done by the C V per unit mass there is no rate associated with their anymore. So, summation over all outlets u plus V square by 2 plus g z plus p by rho at all outlets is summation over all inlets the same quantity this is at outlet this is at inlet plus small q minus small w s. Now this is the energy of the fluid at the outlet is in some sense this is the energy of the fluid at the inlet why is the energy of the fluid at the outlet changes, it could change from energy of the fluid at the inlet, because you are transferring heat in the C V or you are taking out work from the C V.

(Refer Slide Time: 49:58)



Now this is true for any fluid, but in fluid mechanics at least in this course, we will now restrict ourselves to incompressible fluids. Fluids, where rho is a constant that is it is independent of pressure there is no change in the density of the fluid, because of the fact that there are pressure changes in the flow. So, the pressure becomes a mechanical variable it is no longer a thermo dynamic variable that is the pressure. Since the pressure changes do not alter the change in density do not alter the density of the fluid.

That means you cannot take the pressure to be a thermo dynamic pressure it is simply mechanical pressure. So, it is often convenient to keep all the mechanical quantities together and to move the thermo dynamic quantities summation V square by 2 over all outlets plus g z plus p by rho is summation over all inlets V square by 2 plus g z plus p by rho, this is at outlet this is at inlet plus q plus u out minus u in or let us put it minus q minus small w s, this is not a rate.

So, it is minus u and minus u and minus q. So, this is valid for incompressible fluids, now because we are now setting thermo dynamic variable aside from the mechanical variable. And we will stop at this point, and we will try to give a physical meaning for all these terms in the next lecture of our course.