Fluid Mechanics

Prof. Viswanathan Shankar
Department of Chemical Engineering
Indian Institute of Technology, Kanpur

Lecture No. # 22

Welcome to this lecture number twenty two on the NPTEL course on fluid mechanics for undergraduate chemical engineering students. The topic of our discussion today is differential balance of linear momentum. In the last couple of lecture's we have started discussing differential balances and first we derived the differential balance of mass and that went by the name, of that goes by the name of continuity equation. The key thing about differential balances is that they are valid at each and every point in the fluid and the form they take are by way of differential equations. That is why they are called differential balances. And today we are going to focus on the differential balance of momentum.

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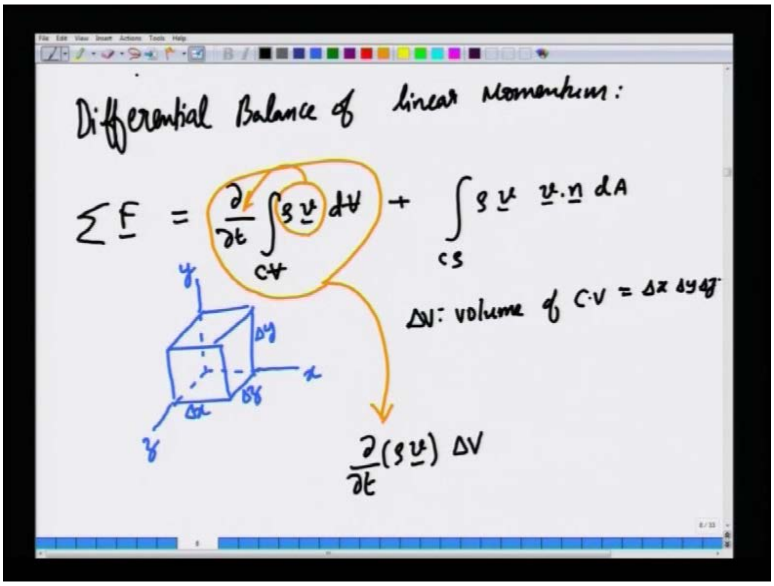

Specifically, we are going to look at linear momentum. The starting point for this deriving the differential balances essentially you take the integral balance that we derive for c v for a control volume. And we are going to apply to a control volume that is of infinitesimal size that is in the limit as the control volume shrinks to a point.

So, recall that the differential balance sorry the integral balance for momentum for control volume is sum of forces acting on the control volume is equal to the rate of change of momentum present inside the control volume, plus the momentum flux that goes in and out of the control volume by virtue of fluid flow. Now, we are going to apply this to an infinitesimal cubic control volume, just as before we are done for mass. So x, y and z. So this is delta x, delta y, delta z so these are the three sides of the control volume and we are going to take the limit as this control volume shrinks to a point. Now, as I mention before for a mass conservation the chief simplification that arises out of an infinitesimal control volume is that this integrals can be simplified in the following manner. Now, in the limit as the control volume shrinks to a point the control volume itself becomes so small that this quantity rho v can be taken outside the interval because it is the control volume is so small the domain of integration volume integration is so tiny that, you can pull the integral outside the integrand outside the integral.

So that is our approximation that we do so d d t of rho v times delta v, where delta v is the volume of the control, infinitesimal control volume this equal to delta x times delta y times delta z.

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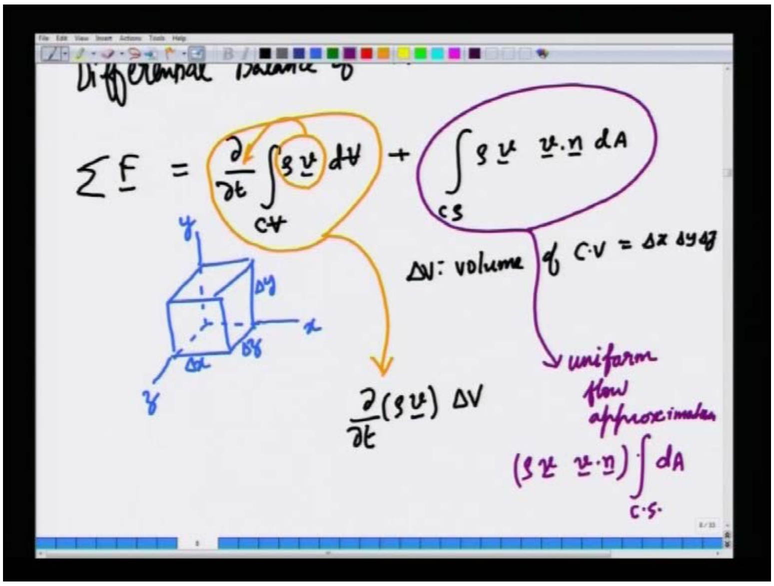

So that is the first term. The second term simplifies because of fact that here since the control volume is so tiny, you can assume uniform flow approximation. That is the velocities v rho times v dot n can be pulled out of the area integral because they vary so little across the cross section.

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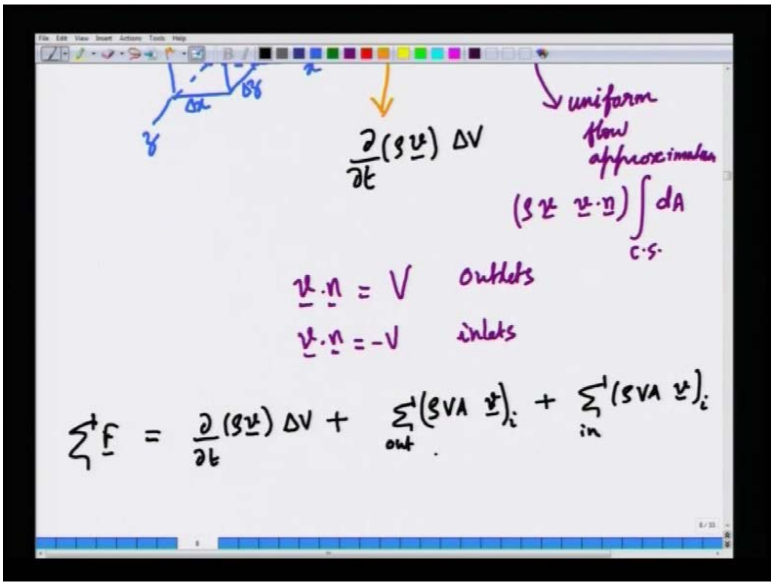

Now we also know that v dot n is V plus V for outlets and v dot n is minus V for inlets. Outlets minus V for inlets. Using this we can write this equation as summation of all the external forces acting on the control volume is equal to d d t of rho v times delta v plus, summation over all outlets times rho V A times v dot n sorry rho V A times v plus summation over all inlets rho V A ok, times v where i is the index that goes out various outlets.

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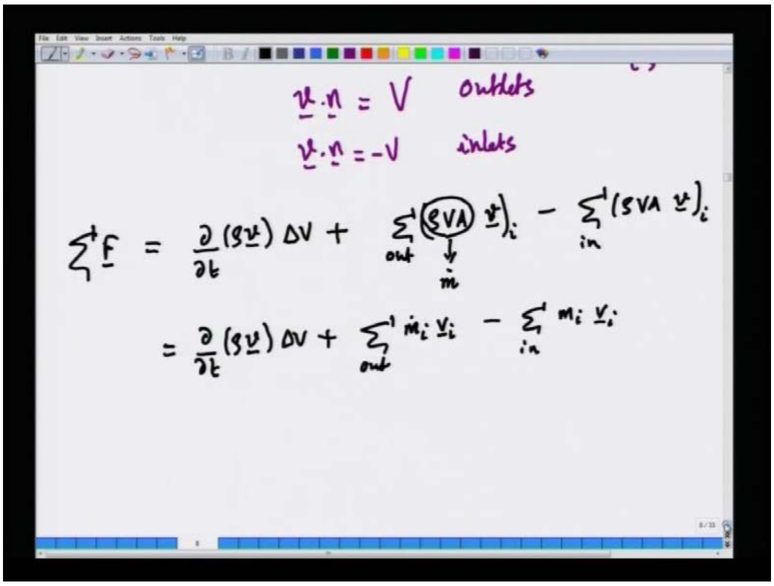

So there is a minus sign because you know that we do not end is minus for inlet now rho V A is nothing but, the mass flow rate. So I can simplify this further as times delta v plus summation over all outlets times m dot i times v i minus summation over all inlets times m dot, i times v i where the index goes over various inlets and outlets.

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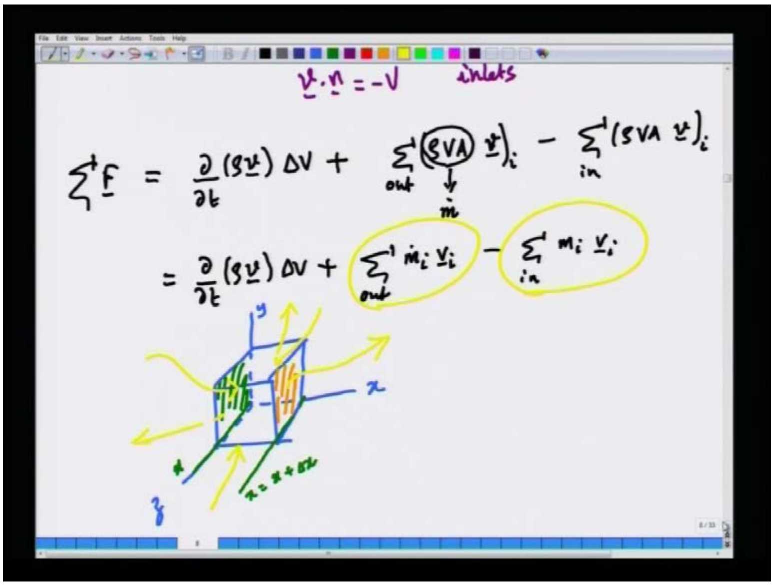

Now if you look at the control volume control volume is at tiny cube. We have x y and z so you have faces at x equal to zero faces at this is at x equal to zero. This is the face at x equal to x plus, delta x so we can call it any x so just to be general.

So there are two faces one is the left face, the face at x the other is a right face the face at x plus delta x. So in general momentum in the x direction along the x direction momentum can come in through the face at x and go out through the face at x plus delta x or it may go otherwise also, But just for a sake of clarity we will assume that momentum is coming in like this and momentum is going out like that. And similarly, momentum can come in at to the through the face at y and go out through the face at delta y y plus delta y. And like wise momentum can come in through the face at z and go out through the face at delta z. So there are various contribution to the in and out terms of the momentum flux.

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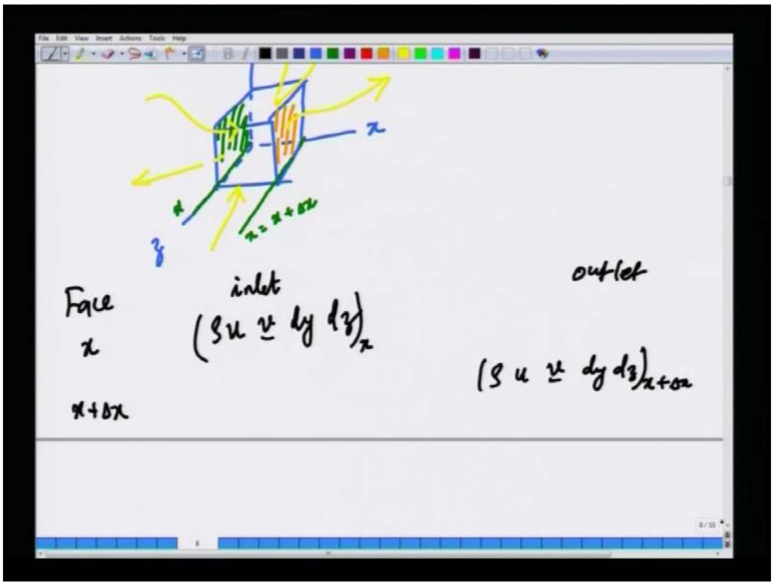

Ok so we will further write this as so the momentum flux at face x the momentum enters at face x so the momentum flux is simply rho u times v. So I am writing this term this term this is m dot i times v i, rho u times v times d y d z, and at the outlet all evaluated at x..The outlet is at face x plus delta x so you will have rho u times v d y d z at x plus delta x.

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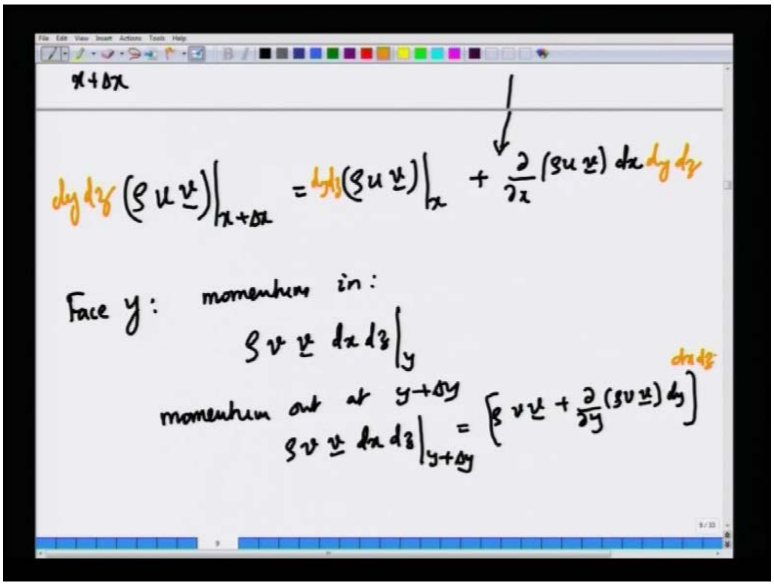

Remember that we are going to use Taylor expansion for this term so rho u v at x plus, delta x is rho u v at x plus, d d x of rho u v times d x. But you have to multiply the entire thing by d y d z so we add d y d z sorryd y by d z (Refer Slide Time: 08:47) that is a cross sectional area through which momentum is flowing. So d y d z d x d y d z. So similarly, you will have a face at y momentum in the y component of the momentum enters, Sorry (Refer Slide Time: 09:14) the momentum enters that the momentum that enters through face y is simply rho v times v times d x d z evaluated at y. And momentum out at y plus delta y is rho v v d x d z evaluated at y plus delta y which is nothing but, d sorry (Refer Slide Time: 09:43) rho v v plus d d y of rho v rho v v times d y times d x d z, let us say area.

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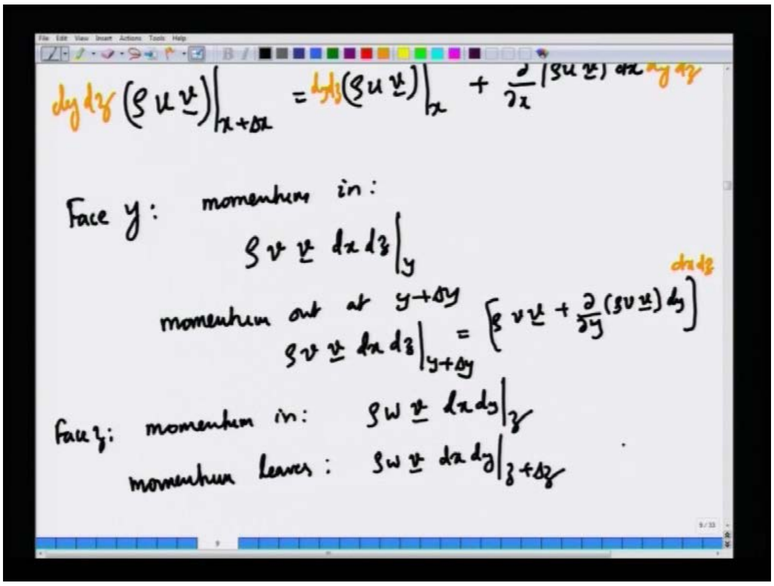

Similarly, the face z momentum enters that enters is rho W v d x d y and the momentum that leaves is rho W v d x d y at z plus, delta z. Ok so having done all this we can substitute all these terms into the balance the sum of all forces acting are on this c v.

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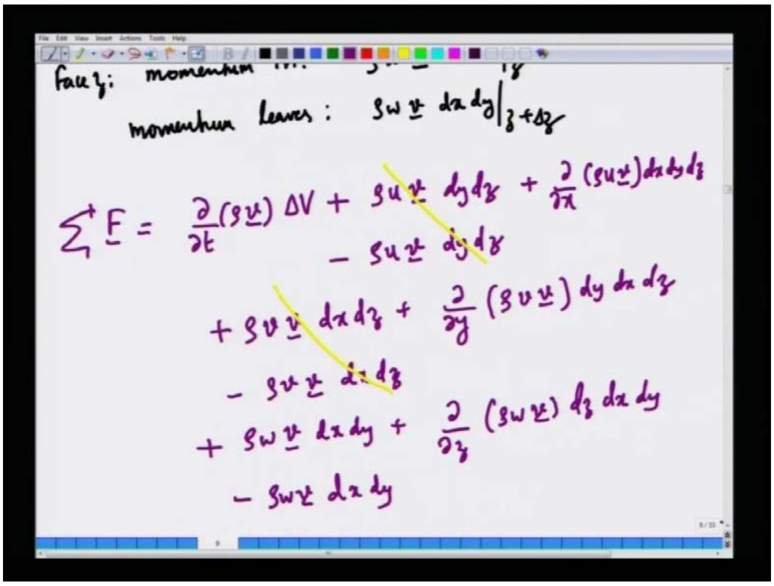

Is equal to d d t of rho v times delta v plus, rho u v times d y d z this is the momentum that exists at face x plus, delta x plus d d x of rho u v d x d y d z minus momentum entry at x simply d y d z. Ok these two terms are going to cancel out.

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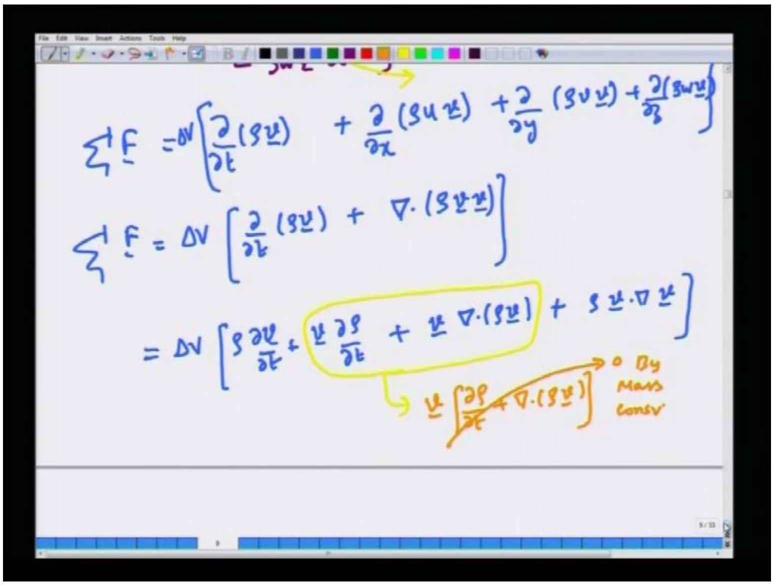

Similarly, there are other two faces plus rho v v d x d z plus, d d y of rho v v d y d x d z minus, rho v v d x d z. These two terms are going to cancel out and then the final the plus the through the momentum that enters and exits through the z face is simply rho w v d x d y plus d d z of rho W v d z d x d y minus rho W v d x d y so these two terms will cancel leaving us with the following simplified equation: Sum of all the forces acting on the control volume is d d t of rho v times delta v plus d d x of rho u v times delta v. So I can take delta v common plus d d y of rho v v plus d d z of rho w v. So I can pull delta v entirely from the and I can take it common so that will give us,

This is sum of all forces is equal to delta v times d d t of rho v plus. This can be written as del dot rho v v. So that is the sorry (Refer Slide Time: 13:42) del dot rho v v this can be simplified further as follows, rho v plus v dot del rho v plus rho so let me simplify the entire thing. We can write this as rho d v d t plus, v d rho d t plus, v times del dot rho v plus rho v dot del v. Ok now these twothings are essentially this is essentially v times d rho d t plus del dot rho v that is identically 0 by continuity equation conservation of mass .The differential balance of mass has already given us the same equation by continuity equation by mass conservation.

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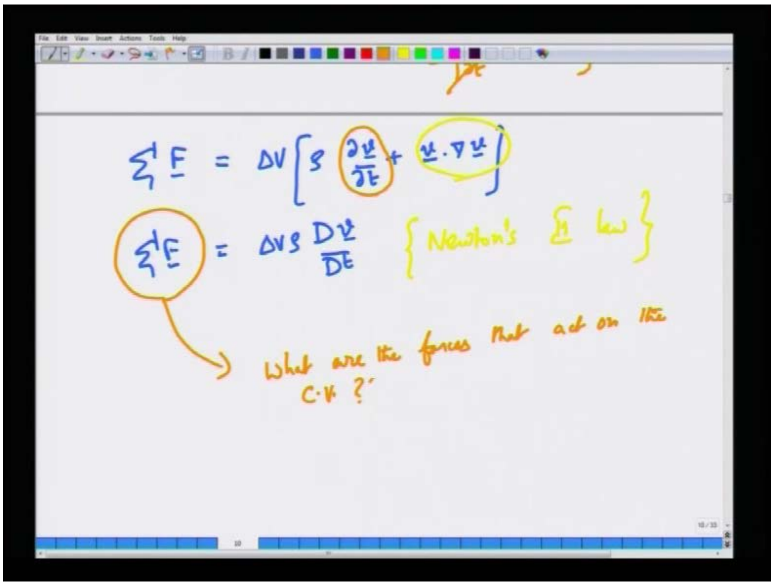

So, we are left with the following term summation of all forces that act on the c v external forces that act on the c v del v times rho d v d t plus v dot del v. This is nothing but, del v times rho times the substantial derivative of velocity. Ok so this is a local acceleration as we have seen several times, this is the convective acceleration this is the local acceleration this is the convective acceleration combined together you get the substantial derivative. There is a total acceleration of fluid particle. Now rho is mass per unit volume times the tiny volume is the mass. So, mass times acceleration is equal to sum of forces this is essentially Newton's second law, but applied to a very very tiny piece of fluid that is present in the c v. Now, we have to worry about this term, what are the forces that act what are the forces that are acting on.

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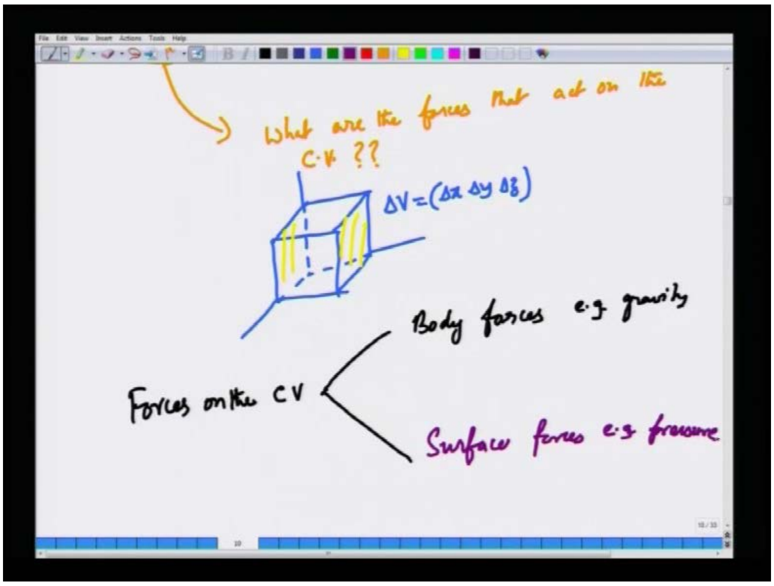

That act on the surfaces of on the c v. These are the external forces that act on so to remind you our c v is simply a tiny cubic volume element, an infinitesimal cubic volume element. Ok so this is our c v with volume delta v equals to delta x times delta y times delta z. So, what are the forces acting on the c v in general one distinguishes two types of forces. Forces on the c v the forces that act through the entire volume of the c v are called the body forces. And one common example is gravitational force that acts on the entire volume of the fluid. The forces that act only on the surface of the c v that are exerted by the neighboring fluid elements they are called surface forces and pressure is one example of a surface force it.

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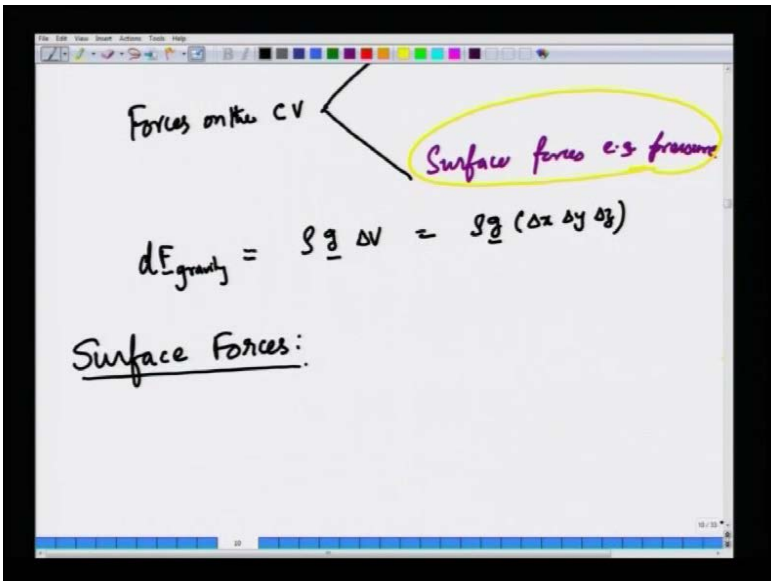

Ok, it acts only on the surface. First so we can worry about the body force very easily. The body force that acts on this tiny control volume is nothing but rho times g, times delta v is rho g, times delta x, times delta y, times delta z. This is rho, times delta v is a mass per unit volume, times acceleration due to gravity is the gravitational force acting on this control volume. Ok, so that is simple now we have to focus on surface forces that is slightly more involved so we will go through this a little slowly.

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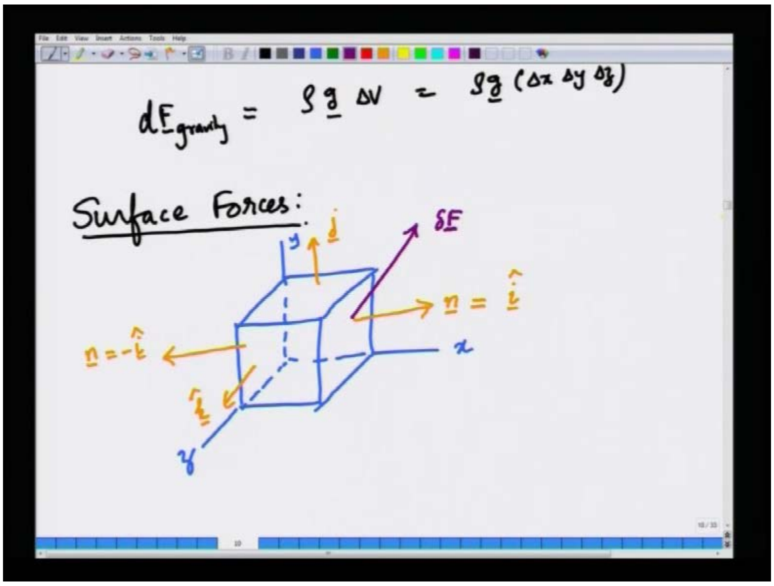

So, surface forces now if you look at this cubic element lets draw the cube again. Ok the cube has 6 faces x y z now each faces denoted by, so this is the face force unit outward normal. So each face is now characterized by n unit outward normal for this outward normal is simply in the plus i direction. Whereas, for this face the unit outward normal is in the minus same direction for this face unit outward normal is in the j direction and for this face unit outward normal is in the k direction and so on. So there are 6 faces and therefore, there are 6 unit normal that represents the orientation of the 6 faces. Now, at any face the force exerted by the fluid on the neighboring side of the control volume on the, on this face can be in any direction. This is thus surface force so let us call that surface force as delta f. We are calling everything as delta because small because we are worrying about a tiny control surface.

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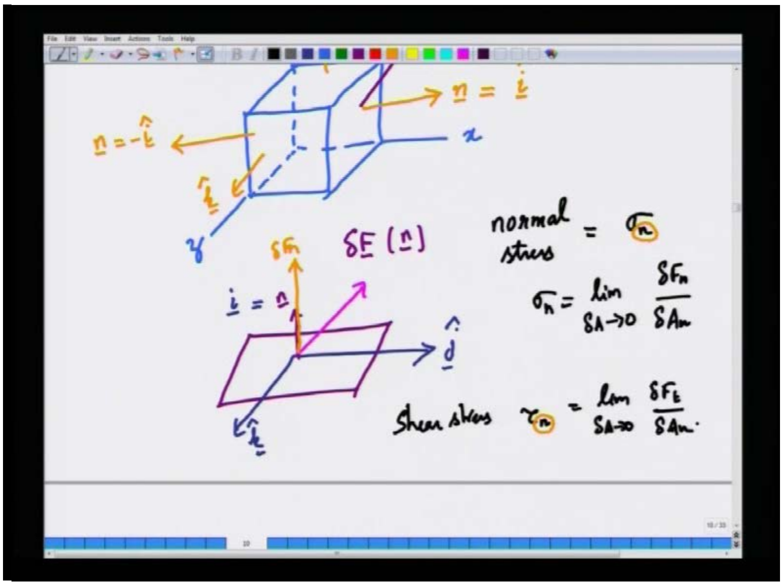

So, in general this force that acts on a surface so this is the force that acts on a surfaces whose unit normal is in a particular orientation. For example in this particular case this is i this will have two contributions. So let us now draw a generic surface which has a unit normal. Ok, and this force can in general at in a particular way delta f n, this will have a component in the direction of the unit normal that is called the normal component of the force. Ok delta f n and there will be a component in the plane and there will be two for any surface there will be two tangential components. So, this n is equal to i in this particular example so the two tangential components will be in the direction of j and k.

Ok, so the force acting on any given plane any given surface can be resolved into the three components along the three coordinate directions one along the normal to the along the direction of normal to this outward normal to the surface. The other along the direction parallel to the direction of the along the two directions parallel to the direction of the surface So accordingly these two forces are called the normal stress stresses force divided by unit area on which the forces acting so the normal stress is sigma denoted by sigma n. Sigma n is limit delta a tending to zero. Delta f n divided by delta a n that let delta n be the area of the surface on which the force is acting and in the limit as this area shrinks to a point then you will have a force per unit area acting at each and every point in the fluid.

Similar the shear stress is denoted by tau n, n stands for the unit normal the subscripts n stands for the unit normal of the orientation of the surface on which the force is acting. Ok tau and is limit delta a tending to zero delta f tangential which in the acts the direction of the in the direction parallel to the surface divided by delta a. Ok I will give you this is the general definition, ok. We can of course resolve it in many concrete ways as follows

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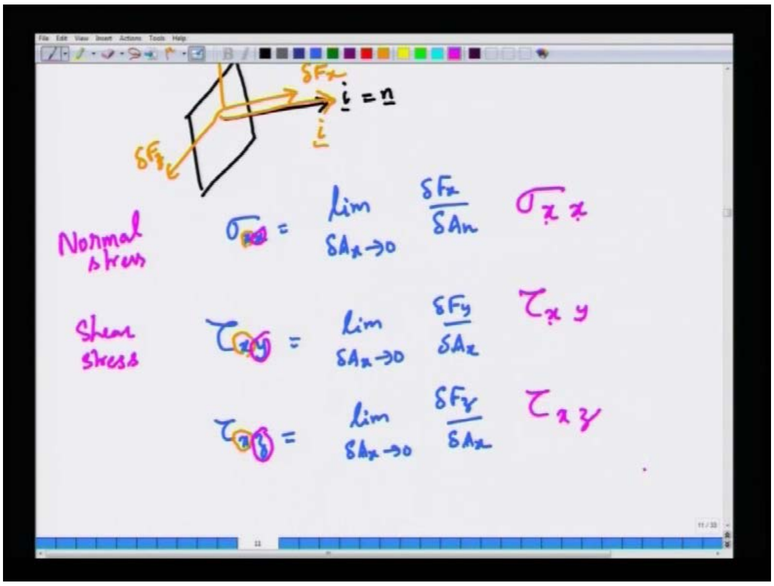

For this particular surface at, you are now considering a surface whose unit normal is in t plus i direction.

This force can be resolved into a normal component which is a delta f x a component along y delta f y and the component along z delta f z. So, correspondingly you can define stresses like follows. Sigma x x is limit delta a x tending to 0 in the limit as the area shrinks to a point you divide the force acting on the surface in the normal direction x, direction divided by the area. Now, you can also define tangential or shear stresses this is the normal stress, ok. These are shearing or tangential stresses shear stress, now this can be written as tau. Now, there are two componential tangential stresses one is in the direction of y so limit delta a x tending to 0 the y component of the force divided by area delta a x.

So likewise you can have tau x z this limit delta a x tending to 0 delta f z by delta a x. So, notice that there are two subscripts to the stresses. The first subscript in our convention x denotes the plane on which the force is acting here it is in the direction of the unit normal i. So, it is the direction of the unit normal outward normal to the surface on which the force is acting so the unit outward normal on this surface is in the x direction so, this x points to that. The next direction sorry the next subscripts denotes the component of the force. So, sigma x x is the force per unit area acting on a surface whose unit normal is in the x direction and the force is in the x direction.

Tau x y is the force in the y direction acting on a surface whose unit normal is in the x direction. Tau x z is the force that acts in the z direction on a surface whose unit normal is in the x direction plus x direction. So there are two subscripts associated with this stresses.

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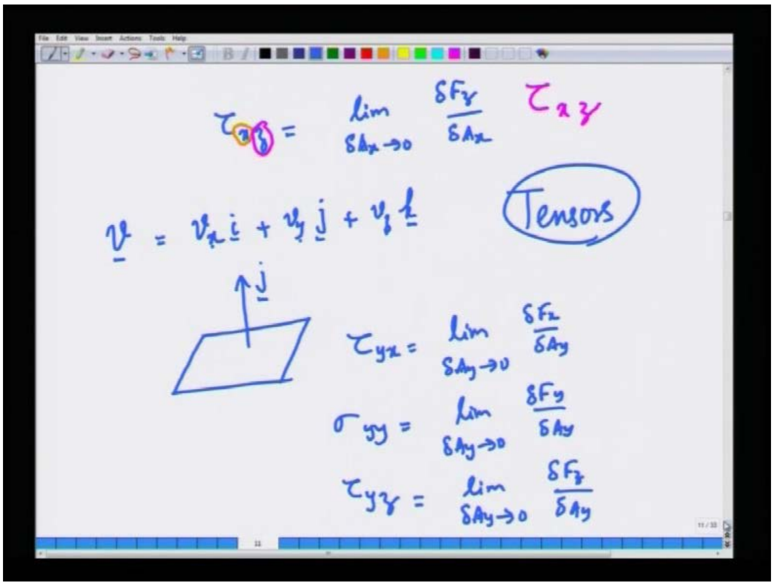
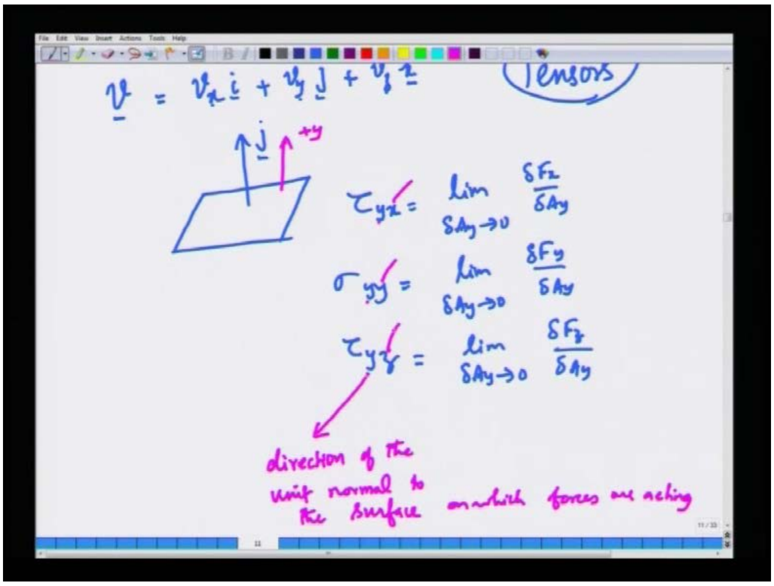


So, if you notice a vector like velocity can be written as v x times i plus v y times j plus v z times k. The components along the three coordinate directions times the unit vectors. So, there is only one subscript x y and z because there is only one direction associated with velocity namely the direction of the velocity. But if you consider a concept like stress there are two directions associated with it. One is the direction of the force stress is force divided by unit area so force itself is a vector. So, it has a direction associated with it and so does the a area because any area has unit outward normal. So, the orientation of the area provides you another direction. So, there are two directions associated with a quantities such as stress.

So these are called tensors. So the stress in a fluid is a tensor because it has two directions associated with it. One is the direction of the force that is acting on a given surface and the other is the direction of the unit outward normal to the surface. So these are the two directions. So, vectors like velocity have only one physical direction associated with it namely the direction of motion is the direction of the velocity. Ok but, quantities like stress has quantities such as stress have two directions associated with them. So they are called tensors specifically they are second order tensors because there are only two direction. There are also higher order tensors which are, which can have more direction associated with them.

Ok, so now just as we defined stresses on a plane whose unit normal is in plus x direction. We can also consider a plane whose unit normal is in the plus y direction. So, we will get tau y x is limit delta a y tending to 0 delta f x by delta a y. Tau y y is limit delta a y tending to 0 delta f y by delta a y so this is sigma because it is a normal stress, so its denoted as sigma.



Tau y z is limit delta a y tending to 0 delta f z by delta a y. Now, here again the two subscripts have the following meaning. The first subscript denotes the direction of the unit outward normal. So, the unit outward normal to the surface is in the plus y direction. That is why this first subscript denotes the direction of the unit outward normal to the plane to the surface over which forces are acting on which forces are acting with the neighboring fluid by the adjacent fluid.

The second subscript tells you the direction of the force because the force acting on a surface with a given unit normal, itself need not be exactly in the direction of the unit normal or the direction perpendicular to the unit normal it can in general act in any direction. Therefore, you need to resolve that force into three components. So, you have these three components and you also have the force acting on the z plane.

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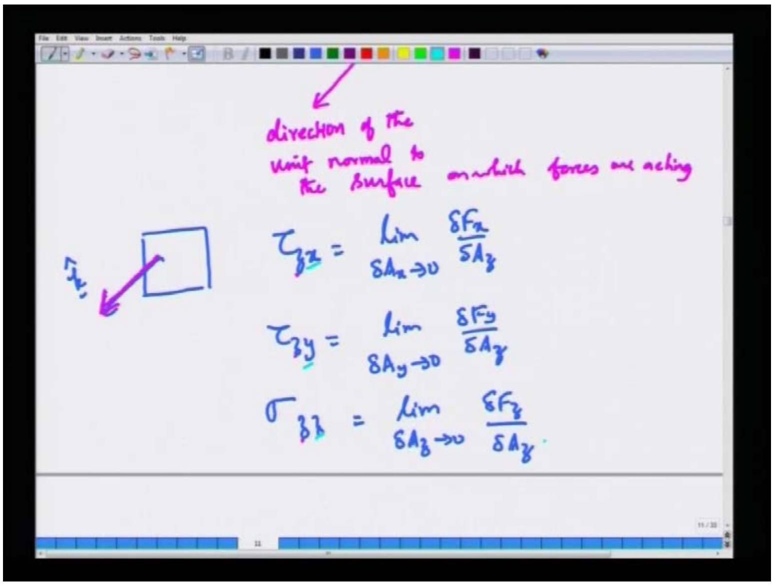

Ok, so tau x sorry tau z x is limit delta a z tau z y is limit a y tending to 0 tau z z. So, it is a normal so we denote normal stresses by the Greek letter sigma is limit delta a z tending to 0. Ok, so again z is the direction of the unit outward normal to the stress surface over which forces are acting. And the second subscript x y z denotes a direction of the force on which the force on the surface direction of the force which is acting on a given test surface.

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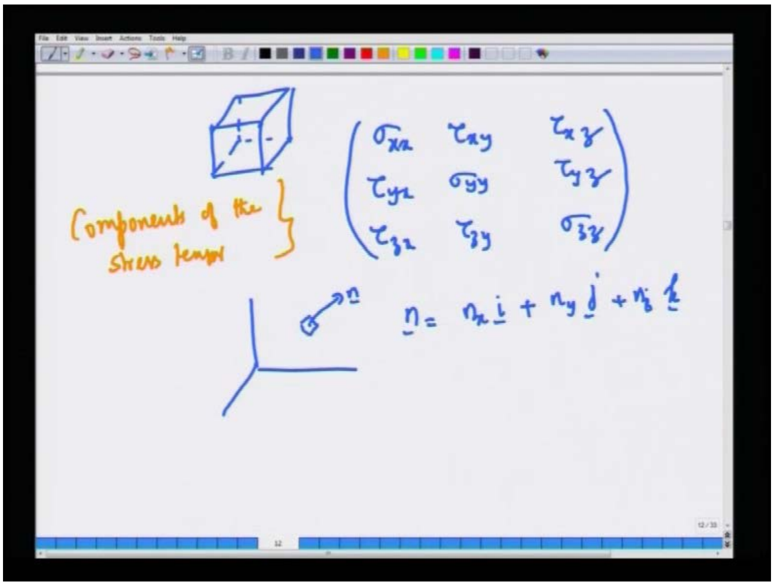

So, as the control volume over which we are writing this balance we are writing. We are taking a cube at a centroid about a point with infinitesimal size. As this control volume shrinks to a point the state of stress in a fluid is denoted by the set of nine components. So, we have sigma x x, tau x y, tau x z, tau y x, sigma y y, tau y z ,tau z x, tau z y, sigma z z. So, this looks like a three by three matrix. These are so the components of the stress tensor are order in a matrix form components of the stress tension. Suppose you may ask what is the use of knowing the stress tension? If I have a coordinate system, if I consider a point in the fluid. I can construct a surface on this point with any arbitrary orientation.

And there are infinitely such many orientations about a point, you can keep a tiny plane and you can orient it any way you want and there are infinite number of orientations. So, what is a stress state of stress on any arbitrary orientation n. So, far we have just discussed about the direction the stresses acting on planes on surfaces which have well defined unit normals. That is either they are in the plus x direction, i plus y direction, j plus z direction k. But nothing forces us to have only orientations which are aligned along the coordinate directions. In general you can have any arbitrary orientation n x n which is denoted by the unit vector n. This is n x i times y j plus n z times k.

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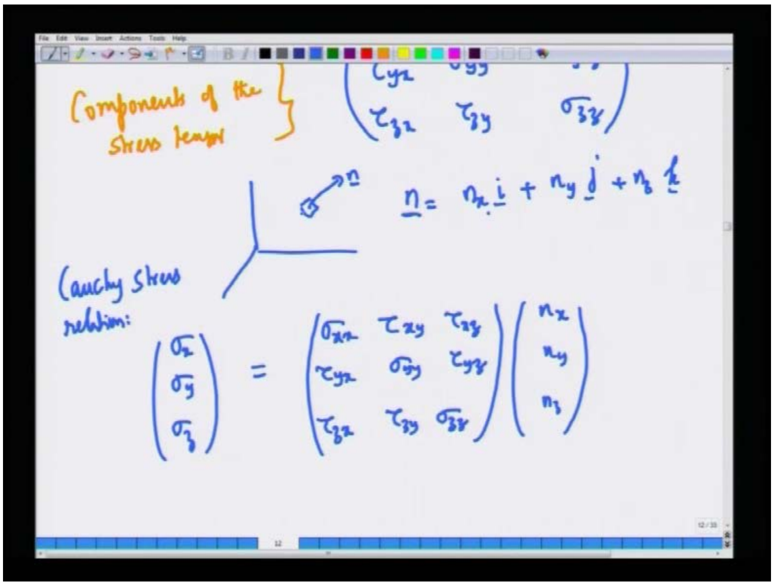

Now, if you have such tiny area element with arbitrary orientation, what is the stress the force per unit area acting on that such a surface. It is called that sigma x, sigma y, sigma z, acting on the surface this is called the Cauchy’s stress relation which will not prove. It turns out that one can show that you can relate the stress on an arbitrary plane with an arbitrary orientation at a given point to the components of the stress tensor, which tell you about the forces that act on well-defined coordinate directions through a simple matrix multiplication. You take the stress tensor right down the components as three by three matrix. Do a matrix multiplication with the three components of the unit vector.

So, if you tell me what is the orientation of the vector and if you also tell me what is the state of the stress on the system. By telling this components of the stress tensor referred with respect to a cartesian coordinate system. Then I can tell you what is the stress at any arbitrary orientation. Now, this is a very very powerful result because in principle you might have imagine that to find the stress at a point, And since you have to specify also the orientation of the surface you have infinitely many orientations. You may imagine that you have to tell a whole lot of information because you have to keep on changing the orientations to find what is the stress, but the Cauchy relation tells us that it is not necessary to keep on measuring or calculating at infinitely large number of orientations.

All you have to do is to measure the force per unit area at three along the three coordinate directions. And once you have it you can find what is the force that acts per unit area, on any on a surface with any arbitrary orientation about that point. This is the power of the Cauchy stress principle. Now, whenever we define things like stress or work there is always sign convention.

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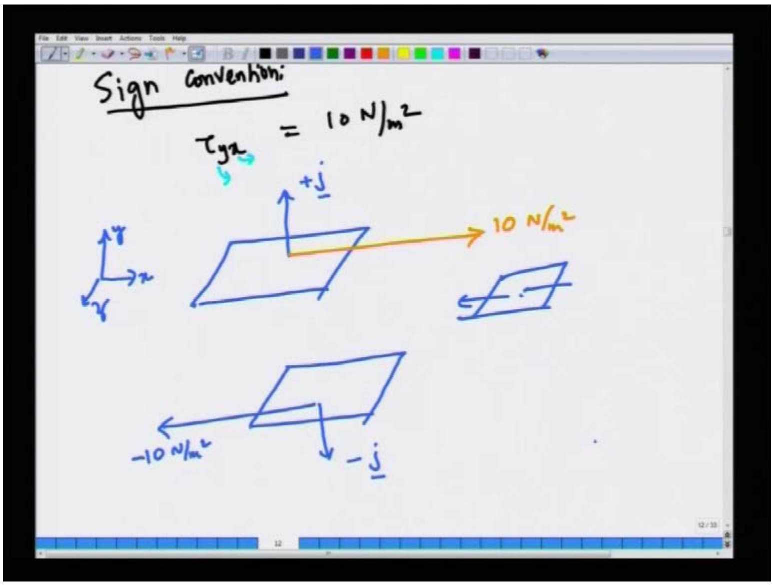

So, you have to be little clear about the sign convention. If I say that if I say that the tau y x is positive is let us say equal to ten Newton per meter square . For example, what does this mean first of all we have to understand the two subscripts y and x Y is the direction of the unit normal. So you are essentially thinking of a surface whose unit normal is in the plus j direction and x is the direction of the force. So, we are we are now saying that on a surface of 0 normal is the plus j direction there is a force which acts per unit area of the surface with a magnitude of 10.Now, it also turns out that if I say tau y x is ten Newton per meter square. Now, at the same point you can also construct a surface whose unit normal is in the minus j direction. And the force is acting in in the negative x direction, so this is x y and z.

So, if I say tau y, tau y x is 10 Newton per meter square. It either means that a force of plus 10 Newton per meter square is acting on a surface whose unit normal is in the plus y direction. Or it also means that the force of magnitude 10 Newton per meter square is acting in minus x direction on a surface whose unit normal is in the minus j direction. But you can see that this is actually a consequence of Newton’s third law because if you take the same point the sum of forces acting on the surface equal and opposite. Therefore this is essentially consequence of Newton’s third law of motion.

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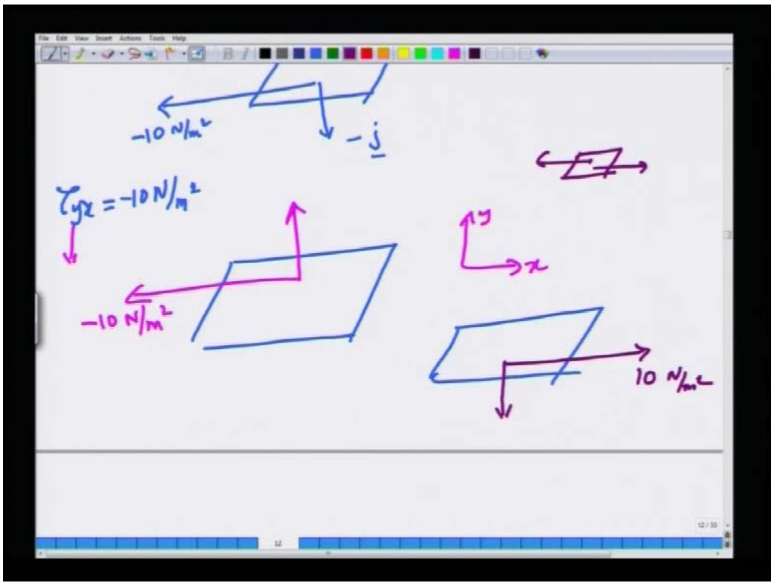

So, if instead force if I say tau y x is minus 10 Newton per meter square. Then what it means is that according to the sign convention. So y means the first subscript is so draw this slightly below. The first subscript y means that you are considering a unit normal in either, I mean let us say it is in the plus y direction. Then the magnitude of force is in the minus x direction so this is x this is y. So, the force of the force is acting in the minus x direction. Instead you could also choose at the same point the direction of unit normal to be in the minus y direction. Then the force will be acting in the plus x direction this is because of Newton's third law because we consider a given point and you consider a surface about that point.

There are equal and opposite forces acting on that surface. So this sign convention tells you basically the following. If I say that a force the magnitude of the value the force is positive then, either a positive direction of force is directed is acting on a surface unit normal is in the positive direction. So, where the first subscript is in direction of unit normal and the second subscript is in direction of force. So, if I tell you tau y x is plus 10 newton per meter square that means that you can tell say that if I construct a unit normal to the surface in the plus y direction then the force is acting in the plus x direction.

Or if you construct a force unit normal is minus y direction ion the force is in the minus y direction. So, the direction of the force on the direction on unit normal is either both positive or negative. If I tell you the value of the force to be positive. If you tell you the value of the stress component tau x to be positive. If I tend say that it is negative then if you construct the unit normal to be in the plus direction y direction. The force is in minus x direction vice versa if we construct the force unit normal is in the minus y direction then the force is in the plus x direction so if the value of the stress component is negative, Then the direction of the force in the unit normal are either one is positive the other is negative and vice versa. So that is the sign convention that is followed in our course and most of the most of the text books.

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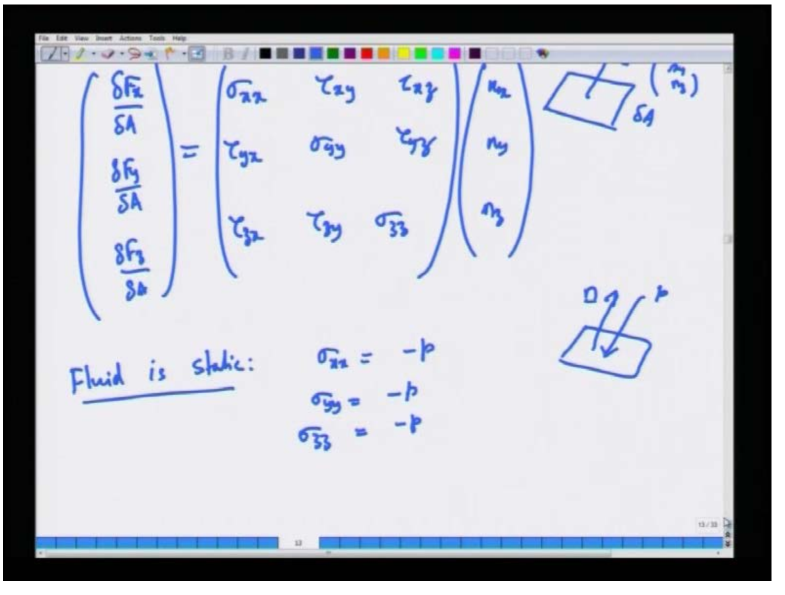

Ok, so essentially what we are now saying is that if you tell me the if I take a point arbitrary point and with arbitrary unit normal n with components n x n y n z. Then what we are saying is that if you have a surface if you have a tiny area a then the force acting in the x y and z direction per unit area delta a a is delta is the magnitude of the area. It IS nothing but, the value of the stress components that are acting on the three cartesian directions times the unit components of the unit normal to the arbitrary plane.

That is the simple meaning of of 0 stress tensor normally what is done is that the normals suppose you look at a static fluid. Then the forces act purely normally to any surface so for static fluids sigma x x is minus p sigma y y is minus p sigma z z is minus p because the pressure acts to compress. So, pressure acts out as we have been telling several times pressure the direction of minus n.

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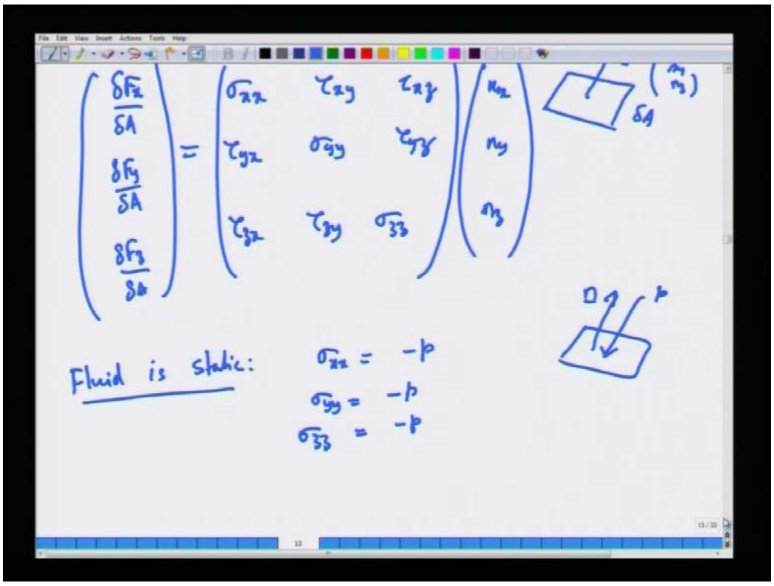

So, you have minus p so in a static fluid the state of stress is given by simply minus p zero, zero, zero minus p zero, zero, zero minus p. In a flowing fluid in motion it is normally, it is a usual convention to write the total normal stress as something that is existed even in a static fluid plus, the normal viscous stress.

This is called the normal component of the viscous stress or the shear stress. Its more appropriate to call them as viscous stress because they are acting purely on normal direction. So sigma y y is minus p tau y y sigma z z is minus p plus tau z z. So therefore, we have this spilt as toward the normal stresses are they are compressed of the pressure force, which is already there in a static fluid. And then plus the normal component to the viscous forces.

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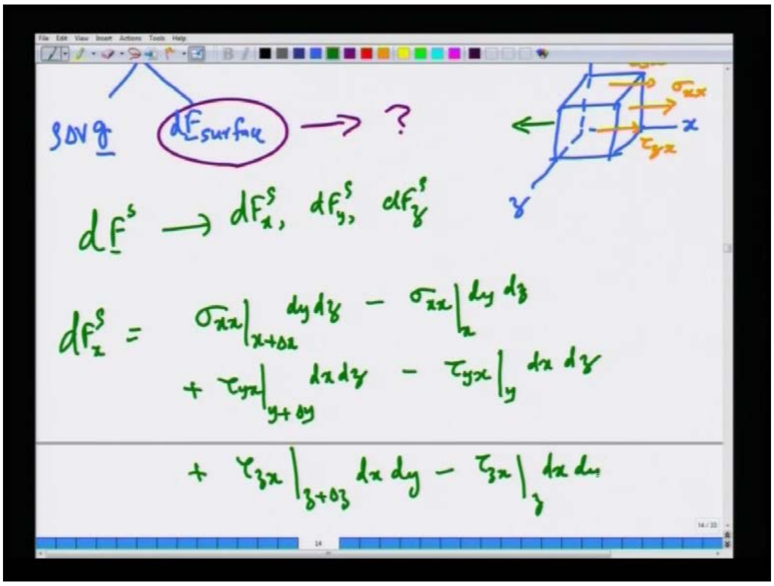

So, now recall that the momentum balance that we wrote was summation of all the forces is rho delta v times D v t. Now, this force has two components the body force which is rho delta v times g plus the surface force and it is in the quest of what the surface forces are that we went into the notion of the stress tensor in detail. So, now we have to write down the net surface force in the x direction y direction and z direction. So, let us call this since it is an elemental area we call it d surface, it is a vector so we have to now worry about the surface forces in the various coordinate directions d f s is the surface force so this has components d f s x d f s y d f s z the three along the three directions as coordinate directions.

So, what is d f s x it is nothing but, so what we are saying is that now we are considering the force in the x direction. So, we are considering the c v so we have to be a little more careful about this. We are considering the c v And if you know this is x y z this is the, if you recall this is what we did, now we are going to consider all the forces in x direction acting on the c v. The force acting on this plane is sigma x x, the force acting on this plane in the x direction is tau y x. And the force acting on the plane is unit normal so this side plane is tau z x because the unit normal is in the plus z direction, and the force is acting in the plus direction and so on. So, this and similarly, the forces are acting also in the other three directions which we will write down.

So, the total force in x direction is nothing but, you have sigma x x that is acting here. this is at x plus delta x times d y d z times the area of the surface over this forces acting. Now, you have to add all the forces but, these forces acting the unit normal to this is in the minus x direction, So you will have a negative sign force. Sigma so in principle you have to add forces but, since the unit outward normal is in this direction, the force will also be in the negative direction. So plus tau y x at x plus, delta x d y d z sorry Now we have to be little careful times d x d z this is the area over which things are acting minus, tau y x at x d x d z plus tau z x like this is a sorry

this is a y plus delta y z y minus tau z x at z times so this is d x d y.

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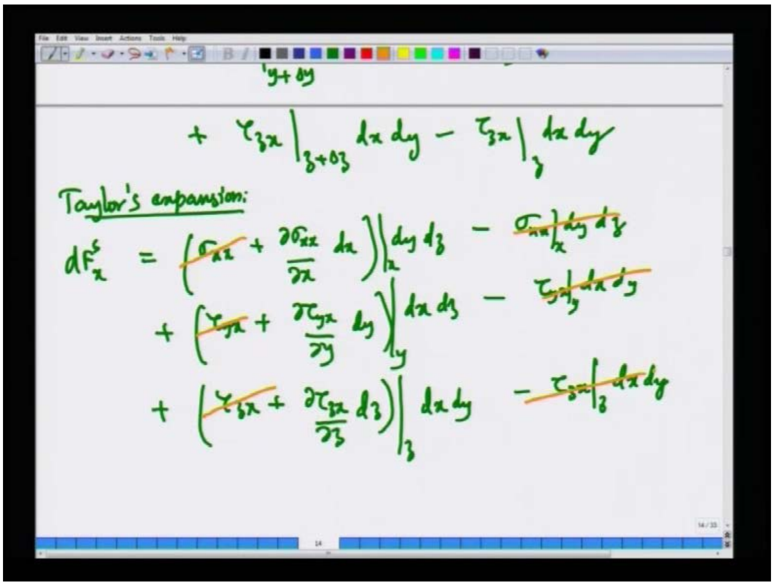

Now, we have to use the Taylor series expansion we have to use Taylor’s expansion, again to retreat this negative sign we have to simply add all the forces in the x directions. So, but this negative sign is because of the fact that this unit normal is acting in the minus x direction. Similarly, this unit normal on this face is the minus y direction and this unit normal on that face is in the minus z direction. After we doing taylor expansion you will find that d f s x is sigma x x plus, d sigma x x d x times d x times d y d z minus, sigma x x d y d z plus, tau y x plus, d tau y x d y d y d x d z minus, tau y x d x d z evaluated at x.

After doing Taylor expansion plus tau z x plus this all evaluated at x I mean this is at y so on d tau z x by d z times d z evaluated at z d x d y minus tau z x evaluated at d x d y you can anticipates the calculation of several terms as follows. So this term will cancel this term this term will cancel this term this term will cancel this term leaving us with essentially d f s x is d sigma x x by d x plus d tau y x by d y plus d tau d z x by d z times d x d y d z.

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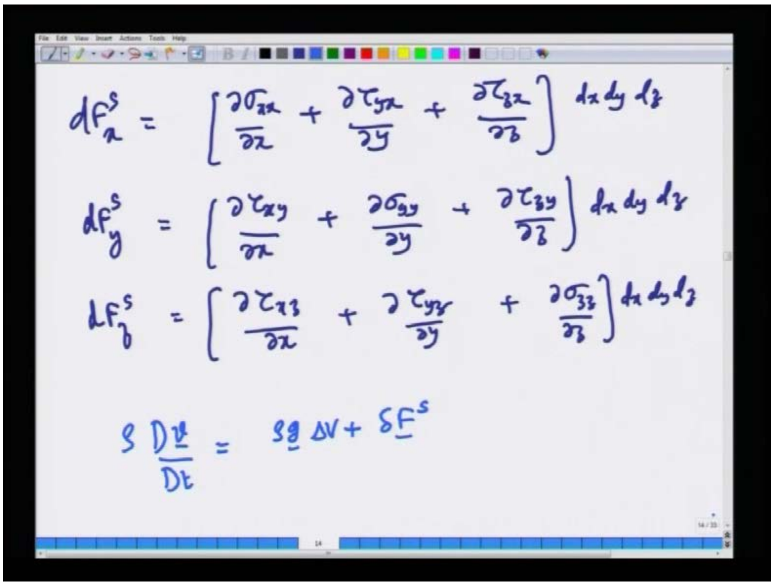

So, and likewise d f s y is d tau y x by d x plus, d sigma y y by d y d tau x y will be x plus, d tau z y by d z times d x d y d z d f z as this. Likewise you can write this as d tau x z by d x plus, d tau y z by d y plus, d tau sorry d sigma because the normal stress d y d z. So, we had this momentum balance to be rho D v by D t is rho g delta v plus the surface force acting on this differential control volume.

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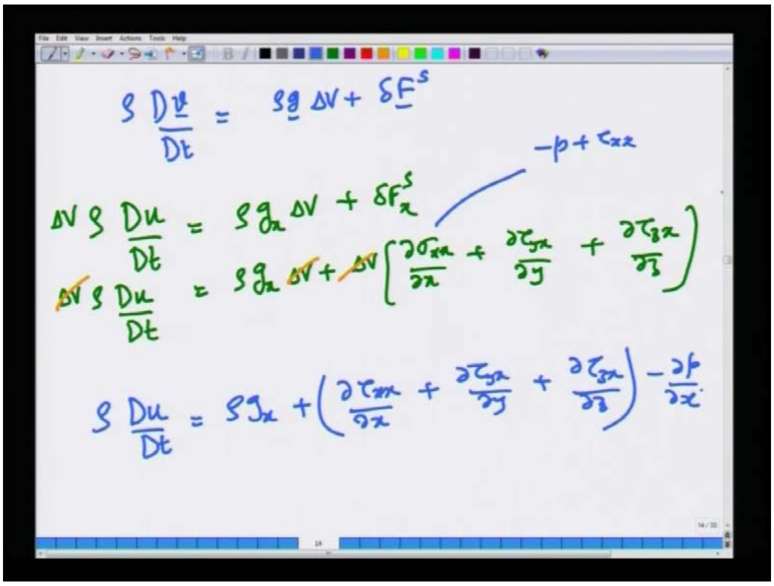

This will become if I now write various components for the sake of clarity. Rho D u D t is rho g x x component of the gravity plus delta f s x, but that is nothing but rho g x delta v plus we just derived this as delta f x. If you recall is this entire thing so we have to simply use that plus delta v times d sigma x x d x plus d tau y x d y plus d tau z x d z times delta v. So, you can see that this is the x component of the momentum balance differential momentum balance, you can see that this delta v will cancel out uniformly throughout the equation leaving us with the simple equation rho D u t is rho g x plus for sigma x. I am going to substitute as minus p plus, tau x x plus, d tau x x by d x plus, d tau y x by d y plus, d tau z x by d z minus, d p d x.

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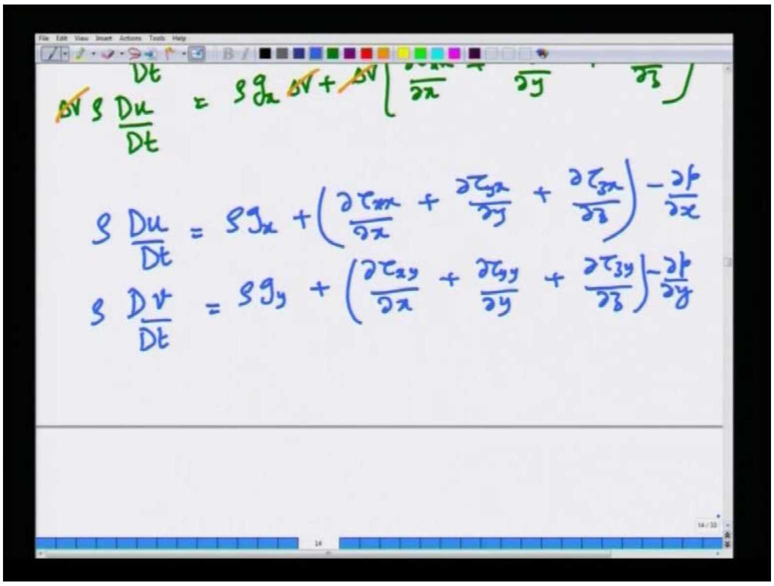

Likewise, I can do the y and z component of the momentum balance I will not do the algebra separately, but you can easily see that you can do it by a just analogy. Now, I am splitting the normal stress into pressure plus normal viscous stress.

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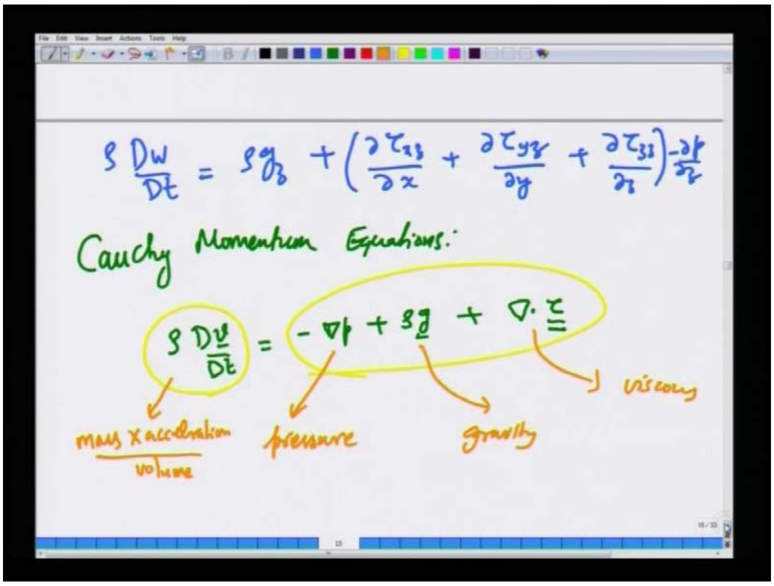

Then the z component of the differential momentum balance, So this completes the derivation of integral sorry differential momentum balance. These are called the Cauchy stress balances or the Cauchy momentum equations.

The interpretation of this is very simple. I can also write it in a vector form rho D v by D t is minus del p plus rho g plus divergence of the stress tensor. So just as you had a divergence of a vector which gave you a scalar quantity divergence of a second order tensor will give you a vector. So this is mass times acceleration acting per unit volume of the fluid that is and that is equal to sum of all forces. What are the various forces acting there are pressure forces acting per unit volume this is a gravity forces acting per unit volume and these are the viscous stresses acting per unit volume.

This is the rate of change of momentum mass times acceleration per unit volume acceleration per unit volume of the fluid. So, the final interpretation is very very simple and it is very analogous to Newton's second law of motion.

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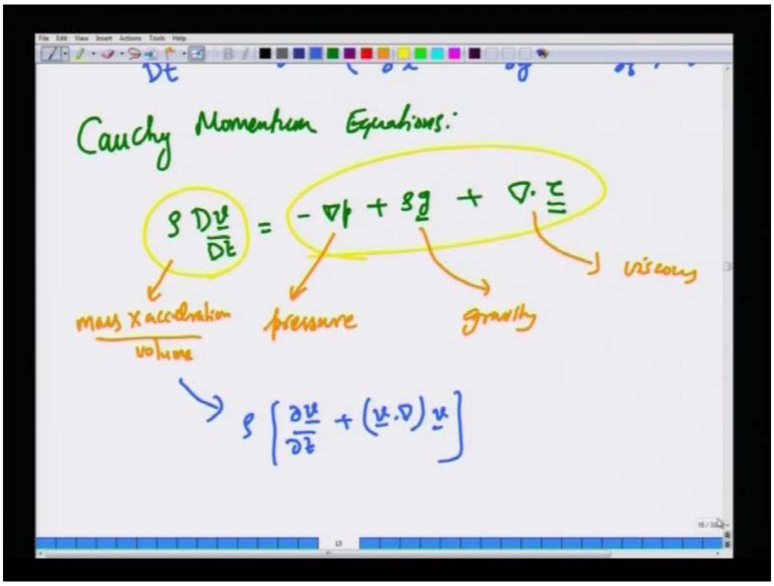

But also although the form of the equation is slightly more complicated and remember that the substantial derivative disguises more complex term its d v d t plus v dot del v. So, this is the Cauchy momentum balance we will stop at this point and then we will continue from here in the next lecture. And we will develop what are called the Navier stokes equation for a Newtonian fluid.