Fluid Mechanics

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Lecture No. # 26

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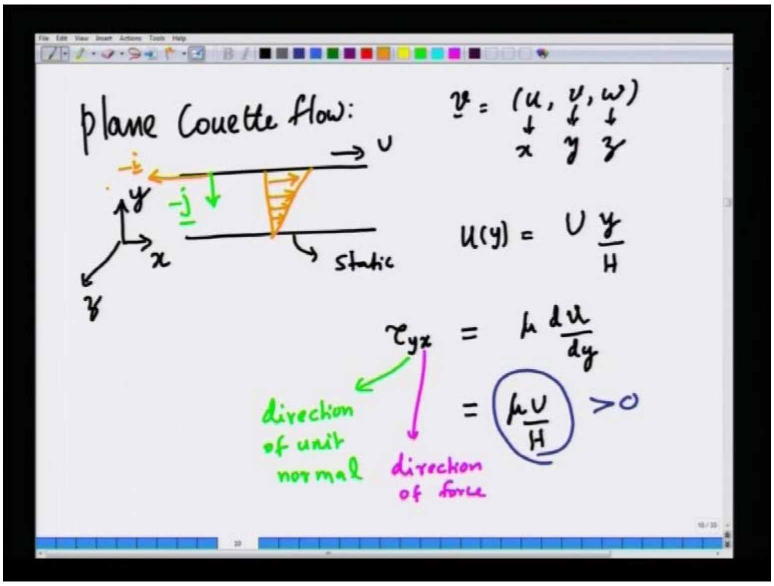

Welcome, to this lecture number twenty six on this NPTEL course on fluid mechanics for undergraduate chemical engineering students. The topic, we were discussing in the last lecture was differential momentum balances. And we applied it to two case studies; one is that of plane couette flow, which was driven by the relative motion between two rigid plates. So, the bottom plate was stationery, and the top plate was moving with a velocity U, and then we found out that after solving the problem, that the velocity profile is linear in in the y direction. Suppose you put coordinate system, the flow direction is x, normal direction is y, and the plane out of the board is z, and we found out that the velocity profile is linear in the variable y.

So, if you want to write the velocity profile v x. So, we write the velocity vector as u, v, w; this is the component the x direction, this is the component in the y direction, this is the component in the z direction. The steady velocity profile that emerges in this gap is given by u of y is capital U times y divided by H; using this you can find, what is the sheer stress exerted by the fluid on the solid surface, that is simply mu d u by d y; this is the force, that is exerted by the fluid on the solid surface. So, this is mu times U divided by H. We also found out that this force has a sign convention. So, this is the direction of the unit normal of the surface on which the force is acting, and this is the x is the direction of the force itself, the direction of the force.

Now in this application, the unit normal is pointing in the minus j direction, that is in the minus y direction. So, as per our sign convention, if you find that all these quantities are positive; mu is positive, mu is a scalar number, it is a magnitude of the velocity of the top plate, and H is positive. So, this is greater than 0, this means that a force of mu U by H x acts on a surface whose unit normal is in the minus j direction, and the direction of the force is in the minus x direction or i direction.

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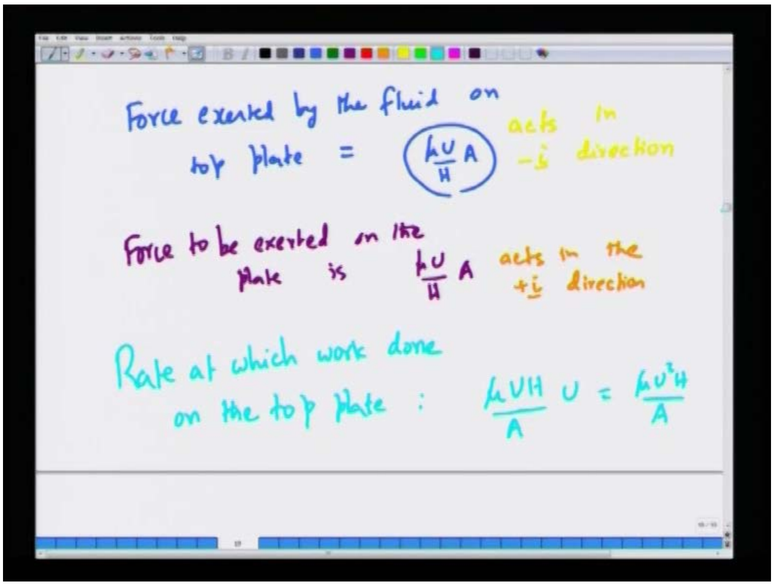

The force - the drag force exerted by the fluid on the top plate is in the minus i direction, if you look at the bottom plate so, we will just go little bit down, and then if you look at the bottom plate, the unit normal is in the plus j direction. Therefore the force exerted by the fluid on the bottom plate; the drag force will be in the plus i direction. So, using this sheer stress, you can find, what is the force exerted by the fluid on the top plate; that is equal to mu U divided by H times the area.

Now, the rate at which work you must so, essentially you have to apply an force; this force acts in the direction in minus i direction; this is the magnitude of the force; the direction of the force is minus i. Now, if you want to move the plate at a particular velocity so, the force to be exerted on the plate is mu U divided by H times A, and this force should act in the plus i direction. The force an external agency that should - which should exert on the plate; so, that the plate moves at a constant velocity is plus mu U divided by H A, and an equal, and opposite force is exerted by the fluid below the plate on the plate in the minus direction - minus i direction or minus x direction. The sum of forces cancel out so, that the plate moves with a constant velocity U. So, this is the idea, we also found that we can calculate, what is the rate at which at which work must be done on the top plate; that was simply rate of work is simply the force times A velocity. So, it is mu U H divided by A times mu so, mu U square H divided by A. This is the rate at which the work must be done on top plate, in order that the plate continuously moves at a velocity U.

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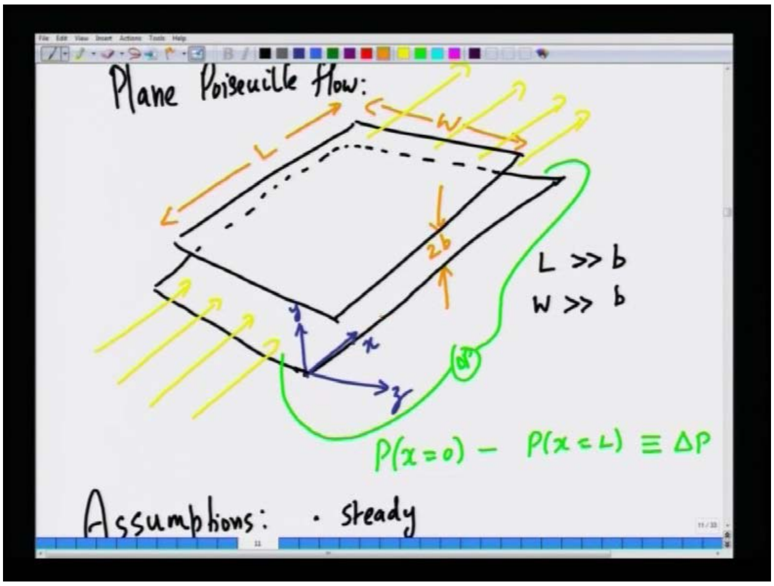

Now the next problem, we turn to the plane poiseuille flow; now this flow is the flow between the space - gap between two long and wide plates. You imagine, you have a slit like distance between two long plates. So, let the length of the plate be L, and the width of the plate be w, and we want to assume that the flow is in the gap between these two long and wide plates, the flow comes in like this, and goes out like this. So, it is convenient to put a coordinate system to begin with the direction of the flow is x; the direction normal to the flow is y; and the direction out in the in the width, the direction along the width of the plate is z.

Now, we are going to further assume, we said in the last lecture, L is a very - very large compared to H. Let us, call the width sorry of the plate as two b, here the gap between the two plates through which the fluid is flowing is twice b. So, b is essentially the half width. So, L is large compared to b, and we also said that the plates are very wide, w is large compared to b, and the reason, why fluid is flowing in the plus x direction is, that there is a pressure difference between these two between entry and exits. So, there is a delta P, there is a pressure difference. So, P at x equal to 0 minus P at x equals L is, what we call as delta P, the pressure difference from between the beginning of the gap, and end of the gap, and that pressure difference drives the flow. We made several assumptions in order to proceed further, those assumptions were steady flow - flow steady, that is reasonable, because if the applied pressure difference is in variant with time; you would expect that the velocity, that flows in between the two plates should also be steady, that is independent of time. Flow is fully developed that is, there is no variation of the fluid velocity profile in the direction of the flow.

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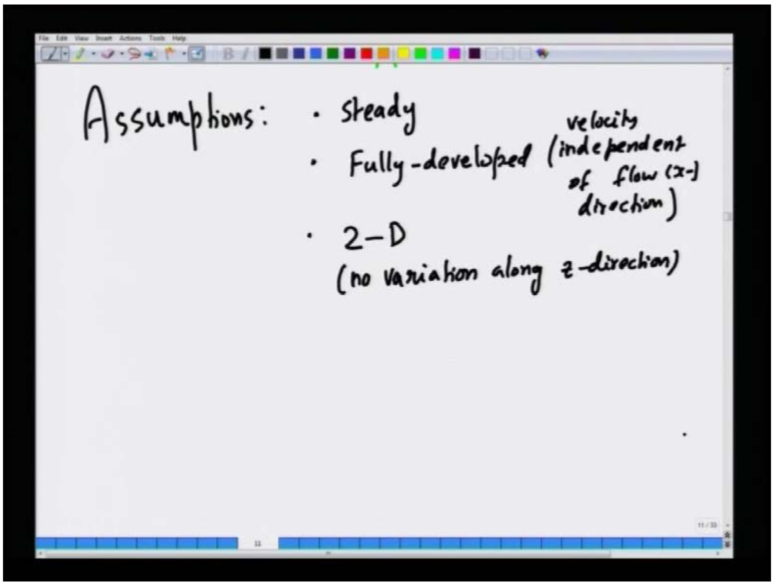

If you grow along the flow direction U, the velocity profile in velocity independent of flow direction. This is the fully developed assumption; the third assumption is the flow is two dimensional; in the sense, that there is no variation of the velocity U in the third direction along the width of the plates. So, there is no variation along the z direction. The flow is strictly 2 dimensional; this means, that no variation along z direction, because z direction the width of the plate is so, the flow direction is x the width of direction along with width is z, and the direction along the gap is y.

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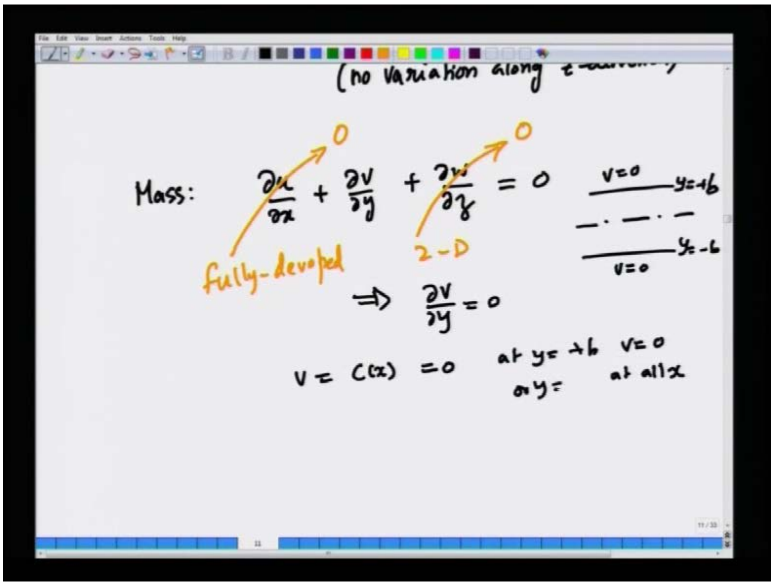

After making these assumptions, and we found that the continuity equation or the mass conservation equation, partial u partial x plus partial v partial y plus partial w partial z is 0, because the third direction is so, large compared to other two directions, and there is no driving force for the fluid to move into z direction; we can say that the term vanish, there is no w velocity; there is no velocity in z direction. We also said that there is flow is fully developed. So, fully developed flow means that the d u d x is 0; 2 dimensional flow means that d w d z is 0, then the mass conservation equation simply implies d v d y is 0.

Now, if we look at the flow geometry at y equals minus b, and plus b; the velocity v has to be 0, because these are rigid walls. So, the fluid cannot flow penetrate into the rigid walls, they are impregnable rigid walls; so v, if you partial integrate this v should be a constant with respect to x, because d v partial v partial y is 0, but that constant should also be 0, because at y equals plus b, v is 0 at all x plus or minus b at either of the two plates plus b or y equals minus b; the velocity is 0 - the vertical velocity is 0. So, it has to be 0 right across the entire channel.

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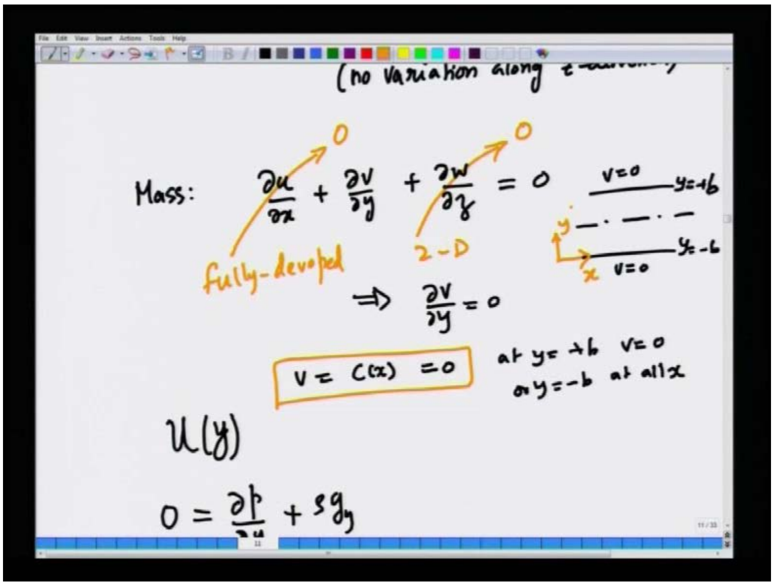

So, there is only one velocity U in the flow direction, and it can be a function only of the normal coordinate y. So, this is the information; we have so, far obtained from the continuity equation. Now, we will go to the x momentum equation, the y momentum equation, will simply tell you that partial p partial y plus rho g y is 0. If you align the gravity vector in this fashion; let us, this say is x, this is y, if gravity is acting in this minus y direction, then g y is minus g.

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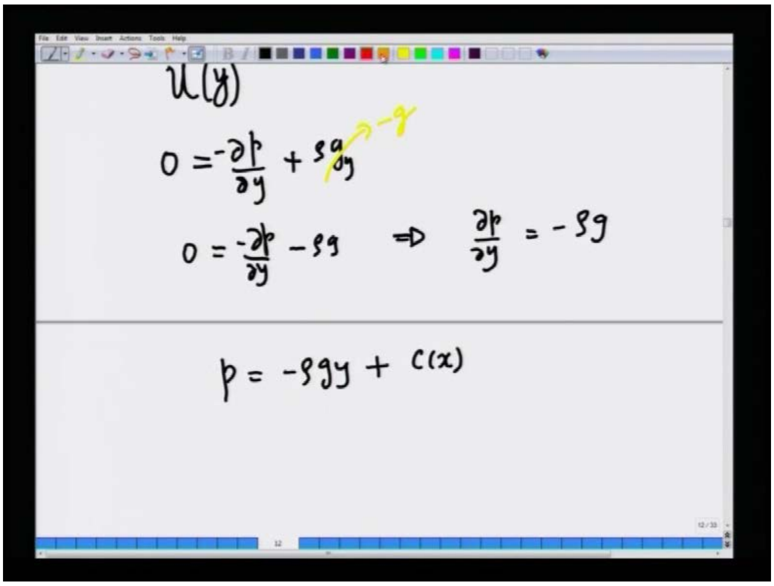

So, 0 is d p minus d p d y I am sorry minus rho g, this implies that d p d y is minus rho g or P is minus rho g y plus some constant P naught, and that P naught can in general some - some constant c could be a function of x, because we are only partially integrating with respect to y.

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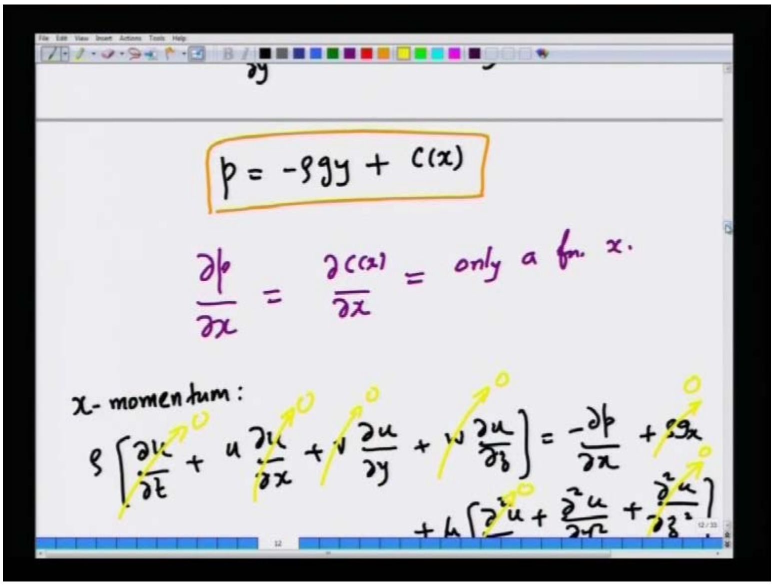

So, this is the information, we get from the y momentum balance; what we can conclude from here is that, if I take the partial derivate of this with respect to x, the first term vanishes, because we need this in the x momentum equation. So, we will simply get partial c of x by partial x; it is only a function of x, It is only a function of x it cannot be a function of y from the y momentum equation. So, the x momentum equation simplifies to so, let me directly quickly write down the entire momentum equation plus rho g x plus mu, the laplacian of the x component of the velocity.

Now, we are going to knock off terms, steady flow means this is 0; fully developed means, this is 0 no normal velocity means this is 0; two dimensional flow means, w is 0; and rho g x is 0, because the channel is aligned horizontally. So, there is component of the gravity in the direction of the channel, and this term is 0, because flow is fully developed, this is term is 0, because flow velocity is independent of the z direction. So, after having done, that you land up with an equation 0 is minus partial p partial x plus mu. Now, this is a function only of x; this is a function only of y, because from the y momentum balance, we found that the pressure can be function only of x d p d x can be a function only of x, and from the continuity equation, we concluded that U is the function only of y.

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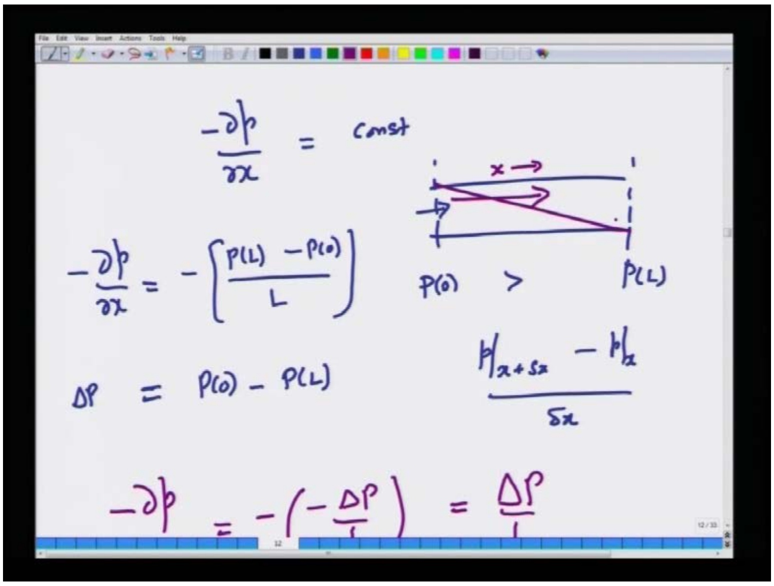

So, each must be equal to a constant, what should that constant be so, minus d p d x minus d p d x is a constant; that means, P is a linear function of x d p d x is a constant then p varies linearly with x. So, in order for the flow to flow fluid to flow in the plus x direction, the pressure at 0 must be larger than the pressure at L. So minus d p d x is nothing, but minus of P at L minus P at 0 divided by L, because a pressure is a linear function of x; so, I can simply take, and the partial derivative of any quantity is defined as p at x plus delta x minus p at x divided by delta x, but since it is a linear variation, I can take it to be Pat L minus P at zero, but I have defined P 0 delta P as, I have defined - already defined the pressure difference is P 0 minus P L, because P 0 is greater than P L.

Therefore, I can write this equation as minus partial p partial x is minus of minus delta P by L or it is simply delta P by l. That is simply put in order for the flow to be in plus x direction. This is the plus x direction; the pressure must decrease in the plus x direction. So, d p d x is a negative quantity. So, minus of d p d x will become a positive quantity.

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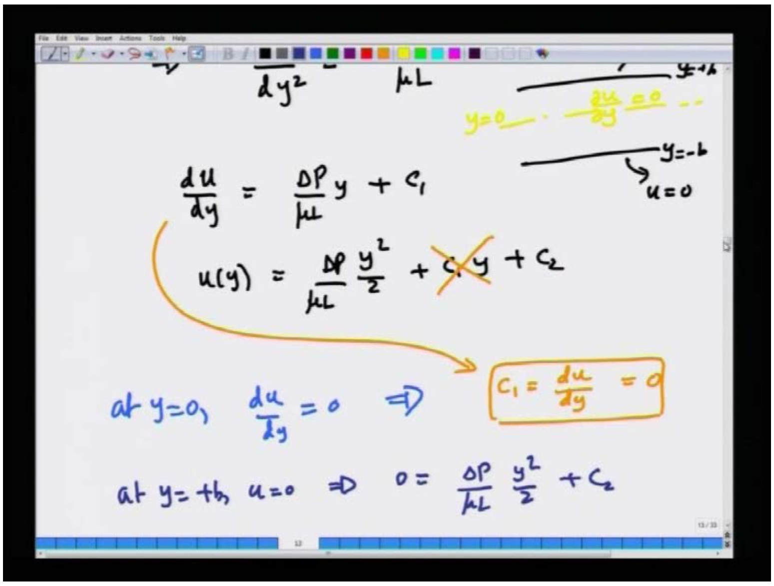

So, the x momentum equation, simply becomes delta P by L plus mu partial square u by partial y square or if I take it to the other side, I can without loss of generality, I can convert the partial derivates to ordinary derivates; I can write the partial derivates as ordinary derivates, because use only a function of y. So, I get delta P by mu L. If I integrate this twice with respect to y, if I integrated once with respect to y, I will get delta P by mu L times y plus c 1, if I integrated twice u of y will become delta P y square by 2 divided by mu L plus c 1 y plus c 2.

Now, the constants of integration must be found from boundary conditions. The boundary conditions, that we know are at y equals minus b; you have no slip u 0, and y equals plus b, you have u zero, but we could also as I point out in the last lecture; use the symmetric plane at y equals 0, and say that d u d y is 0 at the symmetric plane. Because the velocity profile is completely symmetric; the geometry is symmetric, the boundary conditions are symmetric; the driving force is the same. So, the velocity profile must be a mirror image upside, and downside of y equal to 0 plane. So, at y equal to 0, itself u has to be a maximum, because the flow is the plus x direction. So, d u d y is 0. We will use that, because that is simpler to set this constant at y equals 0 d u d y is 0 implies, if I use this equation here, if I put y equal to 0, I get c 1 is d u d y, but at y equal to 0 d u d y is zero. So, c 1 is 0, c 1 - the constant c 1 is 0 in the velocity profile.

So, we get, we can knock this term off. So, at the other constant is found by using the other boundary condition at y equals plus b u is 0, this implies that 0 is delta P b y mu L y square by 2 plus c 2. So, if we bring c 2 to the other side, I will get minus delta P by l. And instead of y, I should put plus b so, b square b squared by 2.

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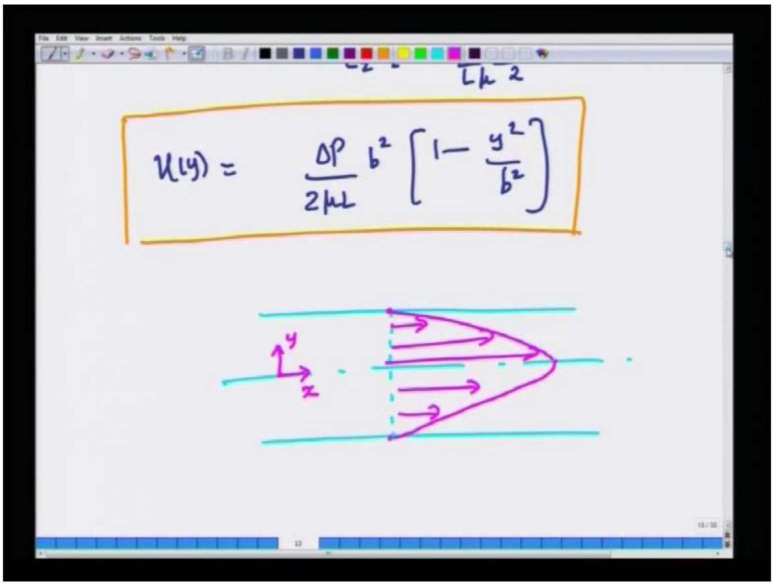

So, the velocity profile u of y becomes delta P by 2 mu L b square times 1 minus y square by b square. This is the velocity profile for (( )) fully developed flow in a rectangular channel, where the width of the channel, and length of the channel is very - very large compared to the gap between the two plates; that is half, that is b is a half width of the half gap of the channel 2, b is the full gap width of the channel.

Now, having derived what else we can do with this expression well; first thing, we can do this, we can plot this expression. And this is the center line, if you want to plot this expression; you will find that the velocity 0 at the walls, and maximum at the center, and it is a parabolic function of the normal coordinate y, the x will also be maximum at the center, and 0 at the two walls. So, that is what this solution will say, but we can also get more information from this solution. Suppose, you want to ask the question, what is the average? What is the volumetric flow rate, that one gets for a given pressure drop inside this rectangular channel.

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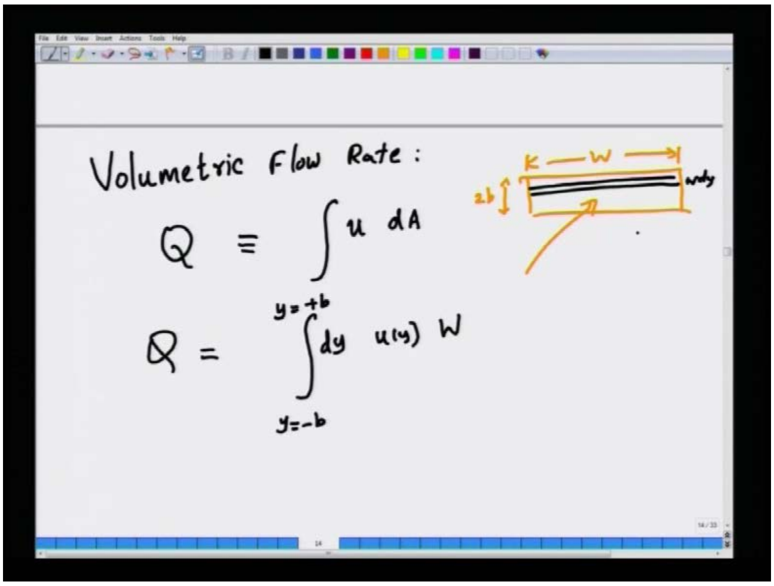

So, what in order to find the volumetric flow rate Volumetric flow rate is the volume - amount of the volume that flows in the channel per unit time; so, it is denoted by symbol Q normally. Q is defined as integral u times d A, where A is the cross sectional, it is important to note here, the area - the cross sectional area through which fluid is flowing. This is w width of the channel, and this is 2 b, and you are integrating over this is the flow area through which fluid is flowing. So, what is it in this geometry Q is integral d y, y equals minus b to plus b u of y, and then the other direction, there is no variation, we can simply type this is w. So, essentially we are taking a strip of thickness w d y, and since velocity is the function of y, we have to do the integration to find the volumetric flow rate.

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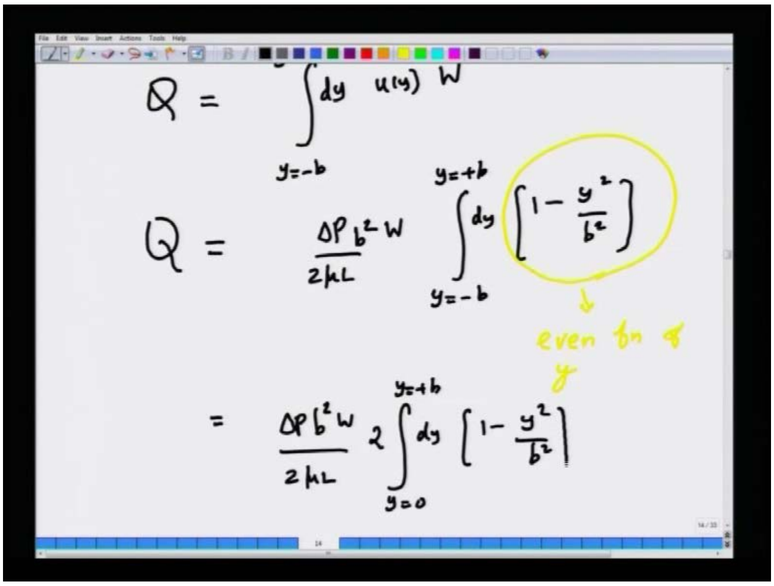

And let us substitute what the form of the velocity is in this, we just found what the form of the velocity is here delta P by b square by 2 mu L. Let us, substitute that here delta P by 2 mu L b square then w - w is a constant, I can pull out. Now, so let me write this minus b plus b dy 1 minus y square divided by b square. Now, the integral is an even function of y of y. So, whether you go to y equals in the positive direction or negative direction, you will get the same value for the velocity. It is the velocity profile is symmetric about y equals zero. So, instead of evaluating the integral from y equals minus b to b, I can write this simply as y equals 0 to b twice, because I am trying to integrate only the half domain. So, I will write twice dy 1 minus y square by b square.

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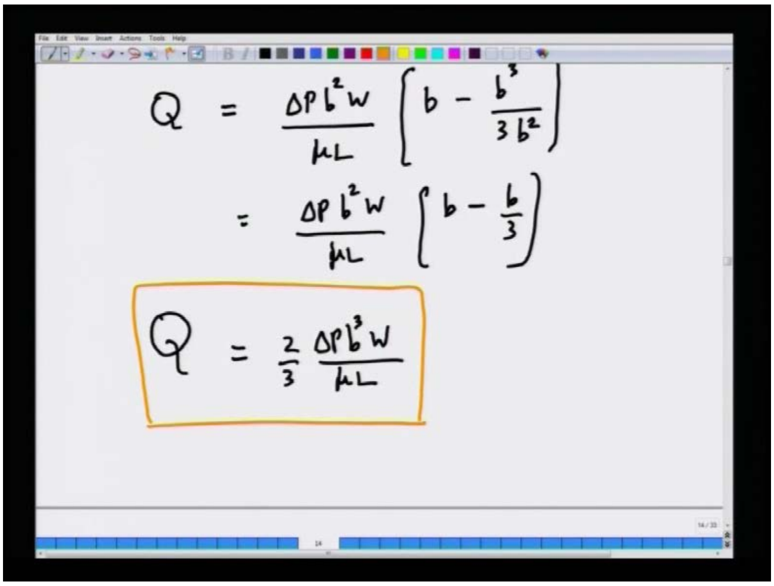

This is just to use symmetry of the problem. So, Q is two will cancel with this two, Q is delta P b square w this is b square by mu L times, if I integrate y, you will get d y just get y integrate y square, I will get y cube by 3 y equals 0 to y equals plus b. So, this is equal to delta P b square w by mu L times, if I evaluate this at y equals plus b, I will get b minus b cube by 3 b square, if I evaluate at 0, I will get 0. So, I do not need to do this. So, this is nothing, but delta P b square w by mu L times b minus b by 3. This is nothing, but Q is delta P b cube w 2 by 3 divided mu l.

So, this is a very - very important result, because it tells you, how the flow rate in a rectangular channel depends on the various parameters such as the pressure drop, the viscosity, the width of the gap thickness of the channel, that is two b, and the other parameters like width, and length of the channel.

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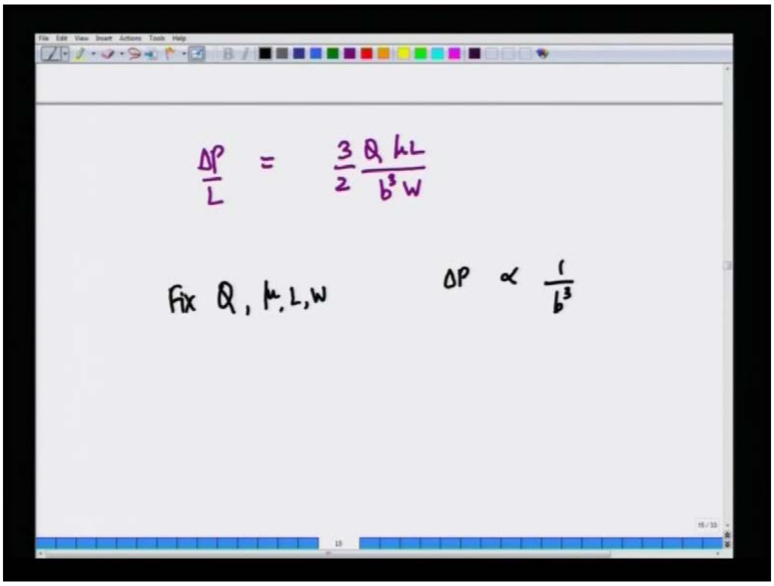

So, as for everything remaining constant, if you so, we can invert this relation to write this as, suppose I want to know, what is the pressure drop, I must maintain in order to achieve a given flow rate. So, I can invert this as 3 by 2 Q mu L by b cube w. So, what this means is that, for if I want to fix Q, and if I fix the viscosity of the fluid, and length and width, then delta P goes as 1 over b cube, that is as the channel becomes the gap becomes smaller and smaller; the pressure drop, that is required to pump a fluid in a channel of thickness, that is gap thickness 2 b, it goes as one over it is inversely proportional to the cube of b.

So, it rapidly decreases; as you decrease the channel dimension, this is a very useful result especially in the context of newer application such as micro fluids, Where the channel with order of let us, hundreds of microns in contrast to conventional applications, where the channel widths are hardly large. So, you get a very quick feel for how large pressure gradients that we must apply, if you want to push fluids by pressure drops inside channels of very small thicknesses. This is a very useful result that one obtains.

One can also obtain from this relation, what is the average velocity. The average velocity v bar is Q divided by area through which fluid is flowing 2 b w cross section area. So, v bar is nothing, but delta P b square by one thirds u l. So, this is another important result It tells you, what is the average velocity of the fluid, we know that the velocity profile is parabolic, which we pointed out repeatedly several times the velocity profile is parabolic. The maximum velocity is in the channel happens at the centers; as I have to simply put delta y equal to 0 delta p b square by 2 mu L the average velocity, which we just found out is delta P b square by 3 mu L.

So, the average velocity is two thirds the maximum velocity in a rectangular channel through which fluid is flowing. The average velocity is this is another important result, which we already sort of hinted at when we did momentum correction factor as well as kinetic energy correction factor. We pointed out to the fact that the velocity variations within a channel or a pipe for that matter is not the velocity flow, profile is not uniform, it varies across - across section; and in that context, we had to evaluate the average velocity, and we found that we use these relations without proving; now we have actually prove this relation that the average velocity for flow in a rectangular channel is two thirds the maximum velocity. The maximum velocity happens at the center of the channel by symmetry, and the average velocity is two third two thirds the maximum velocity.

So, this is a very important result again, which we already used in the context of kinetic energy correction factors, and momentum correction factors, when we did integral balances, but at that time it was merely as statement without proof. Now, we have actually proved it. So, this is another important result that we just derived. So, few comments that, I want to make regarding the logic of the solution for the navier stokes equations.

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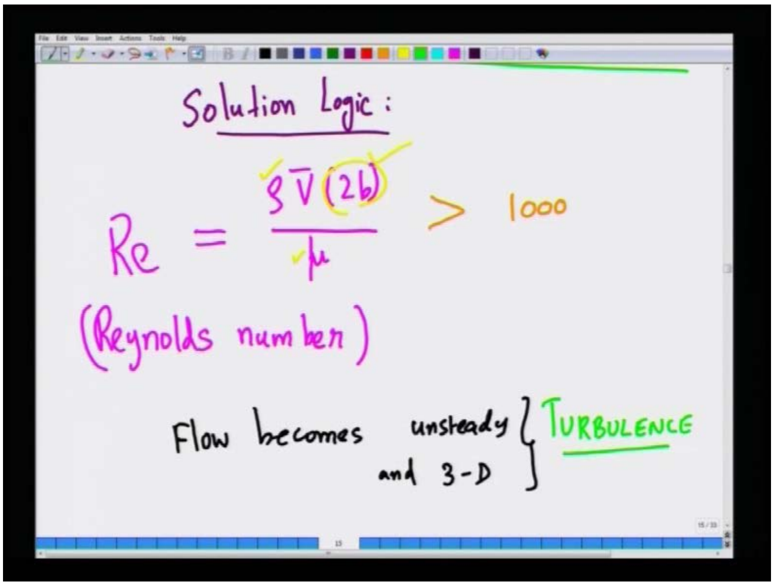

Solution logic, first we make some assumptions about the nature of the velocity field. We said that the velocity field, we said velocity is only in the x direction; there is no component of the velocity z direction, because the there is no driving force in the z direction in the first place; and there is no variation of the x velocity in the z direction, because the width of the channel is very - very large compared to the gap thickness b. So, we said that the flow velocity u is independent of z.

It could be a function of y, because of course the no slip condition is valid at two walls. So, this is one assumption we made, and then the other assumption we made was that of steady flow which is reasonable, because when the applied pressure gradients are steady, it is not unreasonable to expect that the flow should also be steady. The third assumption, we made is that of fully developed flow, that is the flow velocity is independent of the flow direction, and that is true, when you are far away from the entrance and exits, because when you are far away from the entrance, and exits. There is no reason for the flow velocity to vary in the flow direction with these assumptions. We proceeded to solve the momentum equations, and we throw away several terms, and ultimately we landed up with a solution for the velocity profile in the x direction, and it depended only on the y coordinate in terms geometrical coordinates; of course, it depend on viscosity, and the and the and the thickness of the channel and so on, gap the thickness and so on, but in terms of coordinate variation it is a function only of y.

Once you did that, we got a consistent solution in the sense, that the solution that we arrived at does not contradict any of the assumptions, we made in more complex flow situations; it could happen, that you could make an assumption; and the continuity equation will say that the assumption cannot be made. So, it is an internally inconsistent assumption. So, the physical assumptions that we make so, should also confirm to the fundamental differential balances; that is namely continuity equation, and momentum equation. For example the momentum equation for this problem in the y direction, told us that the pressure gradient is function only of x; it is a very - very useful piece of information; and we use that in the x momentum equation to find out that d P d x, and d square d u d y square d u each must be a constant.

So, the equations sort of tell us additional informations, and they also confirm to our initial assumptions, if the equations do not corroborate your initial assumptions; that means, that those assumptions are physically invalid. So, we make assumption based on some intuitive ideas, but those assumptions should also agree with the fundamental equations of fluid mechanics, that is the differential mass balance, and differential momentum balance. So, at least in this example, we had no contradicting assumptions; we had no contradicting results in our answers, when compared to assumptions.

Now, another important point, that we have we have to make at this juncture is that the fact, that we obtained one physically consistent solution, does not necessarily mean that, this will be the solution; that is obtained or seen in experiments. The reason for this is, that the navier stokes equations are a non-linear set of equations, and whenever we have non-linear set of equations, you cannot prove that a given solution is unique. If you have linear system of equations differential equations, then uniqueness is proved always a given solution - any given solution that satisfies the differential equation, and boundary condition is the unique solution; there is no other solution. But such uniqueness proofs do not exist for non-linear differential equations therefore, we are left to wonder, whether the solution that we derive is the only solution, because in principle there could be other solutions. So, how can we settle the issue? The issue can be settled only by carrying out experiments in the lab by carrying out experiments in the lab; you have to measure the flow rate for a given applied pressure drop, and see whether the theoretical relation that we derived is valid, because you can compare the theoretical predictions with the experimental observation to see whether the flow rate varies with pressure drop in the same way as we have predicted.

Now, it turns out that in reality; that the flow rate relation, the flow rate was a pressure drop relation for a channel - rectangular channel; that is - this relation is valid in a certain regime for a lower velocities, while when the velocities are very high, then this relation breaks down. Now, we are going to return to the topic a little later also, but I will just point out by saying that, there is a there is a non dimensional ratio called the reynolds number; reynolds number which we will come to shortly after we are finished with differential balances. The Reynolds number is a non dimensional group; that means it is a quantity with no physical dimensions, it is just a number.

It is defined as rho times; the average velocity times; the width of the channel divided by the viscosity. We will see later, how this number naturally emerges, and so on, but right now, I am just telling you a fact that for a given fluid, what is fixed rho is, fixed mu is fixed for a given channel width 2 b is fixed. So, reynolds number is essentially for a given channel, and given fluid, it is a non dimensional velocity in some sense. So, as you increase the velocity; you are increasing the reynolds number, when the reynolds number is greater than about thousand for this problem for flow in a channel; then this assumptions break down the flow becomes unsteady, and 3 dimensional; that is not it we first claim that only one component of the velocity is non 0; that is the x component of the velocity, when the flow becomes when the reynolds number, which is a non dimensional velocity at this point of view - at this point of time, we can just think of it as a non dimensional velocity, when the reynolds number is greater than 2000 sorry 1000 for this problem; the flow in the laboratory becomes unsteady in 3 dimensional, and you also your complicated state called turbulent flow, at which point our all the assumptions break down. So, this pressure drop versus flow rate relation will also break down.

So, the relation that we have derived by making simply assumption to navier stokes equations namely that the flow is steady and then there is only one component of velocity, and fully developed, and so on. They are not valid, but when the reynolds number is greater than 1000 or so, but until that these are valid, and they provide an accurate description of what is happening inside the two plates in of a rectangular channel. But when reynolds number is greater than 1000 n 1 of these predictions are valid. And we have to have more a sophisticated theory to understand, and describe, how fluid flows inside a channel in the turbulent regime; that is, when the reynolds number is greater than 1000. So, that is something, we must always keep in mind, when we derive simplified flow solutions for from the navies stokes equations, because these are not always it, there is no guarantee that, if you obtain a solution to the navier stokes equations by making physically motivated simplications.

It is not a priory evident that the solution, that you get will be - actually be observed in the experiment, because the reason is the there is no uniqueness proof. The solution that you obtain is not necessarily unique, because navier stokes equations are a set of a non liner partial differentiation equations, and there is no reason for us to expect that a given solution, that we obtain is the unique solution. So, having done that example on flow through a rectangular channel, and having pointed out the key features of a solution. We now next move to another important example, flow through a pipe.

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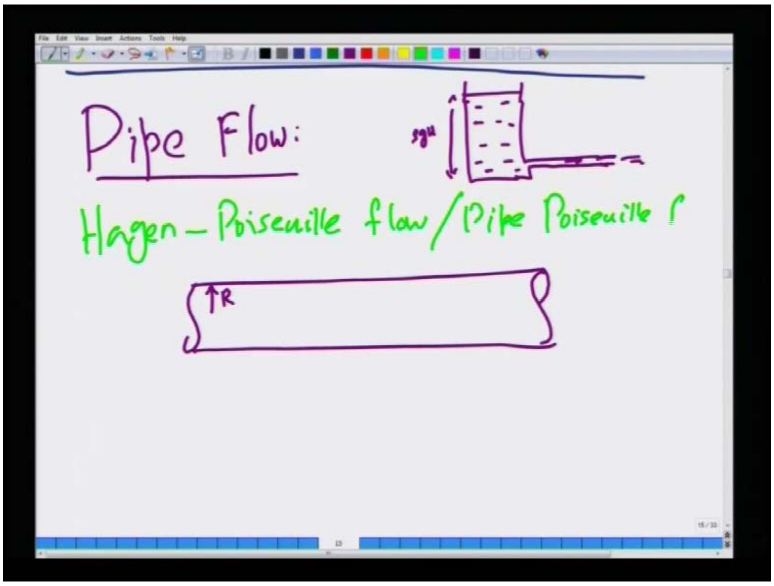

So, imagine having a long pipe, a rigid pipe, a circular pipe or radius r radius of the pipe is R, and this ends of the two pipes are let us, you have one end of a pipe is connected to a huge reservoir, and the other part end of the pipe could be open to a atmosphere. So, because of this virtue of this pressure head present in this. Let us say liquid is present fluid is present here, and because of by the virtue of this gravitational pressure head rho g H.

The fluid is going to flow in this direction, and question again that we want to ask is, what is the relation between pressure drop, and flow rate? And the reason, why I am going to do this; there are two reasons, why I am going to do this one is that - this is one of the most practically encountered flow geometry in chemical processing industries that you have plants pipe flows in pipes in various dimensions diameters in any chemical plant. In the second is that from a fundamental perspective; this will also tell us, how things are different when we go to a different coordinate system for example, in the previous two cases, we have worked out two examples of the solutions of navier stokes equations. One is plane couette flow, other is plane poiseuille flow. But both these cases, where amenable to solution in cartesian coordinate systems, because the geometry is such that you can nicely use a cartesian coordinate system to do define the flow to describe the flow, but not so, in such the present place so, before I proceed with the problem about the names; this is variously called as hagen-poiseuille flow or pipe poiseuille flow or simply pipe flow. So, the first thing to note is that is that so, since this problem is very, very similar what we just discussed namely flows in a channel. Let us, focus more on the differences between this problem, and the previous problem. The first thing is, what is the coordinate system, that we have to use here there is a natural symmetry associated with the circular nature of the pipe. So, you have a pipe like this. So, it is better not to use the cartesian coordinate system, and it is better to use a cylindrical coordinate system.

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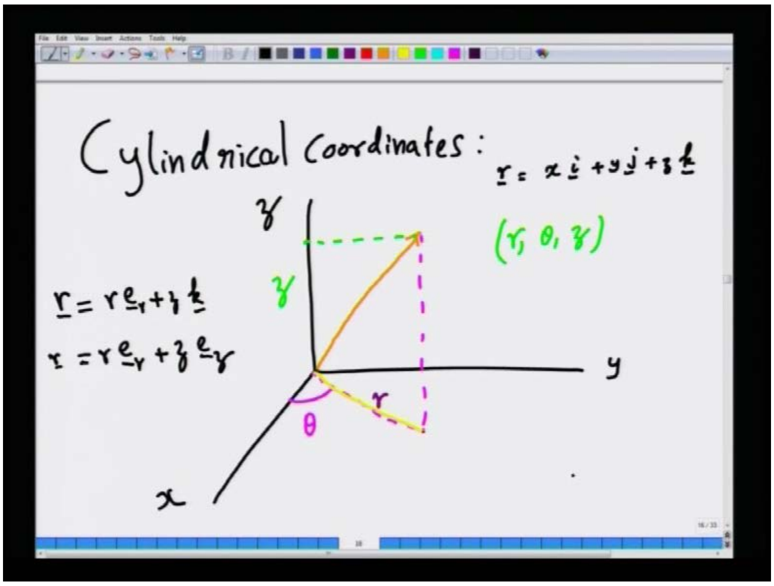

So, we quickly explain, what is the cylindrical coordinate system, you have a 3 cartesian coordinate vectors x, y, and z, any point in space is from the origin; you connected how is it described in cartesian coordinates, we decompose it into so, let us try in cartesian coordinates any position vector is x i times i plus y times j plus z times k.

In cylindrical polar coordinates, we do not use x y z instead we take the projection of this vector on the x y plane, and the angle this projection makes with the x axis is theta, and the length from the origin of this projection - the length of the projection from the origin is r; r is the radial distance from the from the origin to the projection, and essentially the length of the projection on the x y plane theta is the angle; the projection makes with the x axis, and z is normally z. So, the other coordinate is z. So, the three coordinates are r, theta, and z, where r is the length; r is the project; r is length of the projection; the position vector makes with the x y plane, and theta is the angle between the projection as well as the x axis, and z is the normal z coordinate that you have in cartesian coordinate system So, any quotient vector is now presented as r e r plus z times k.

Now, since we are now moving away from the i j k notation we write this as z times e z. So, that is way in which any position vector is described Now, what is convenient about the spherical sorry cylindrical polar coordinates is that, if you have a pipe.

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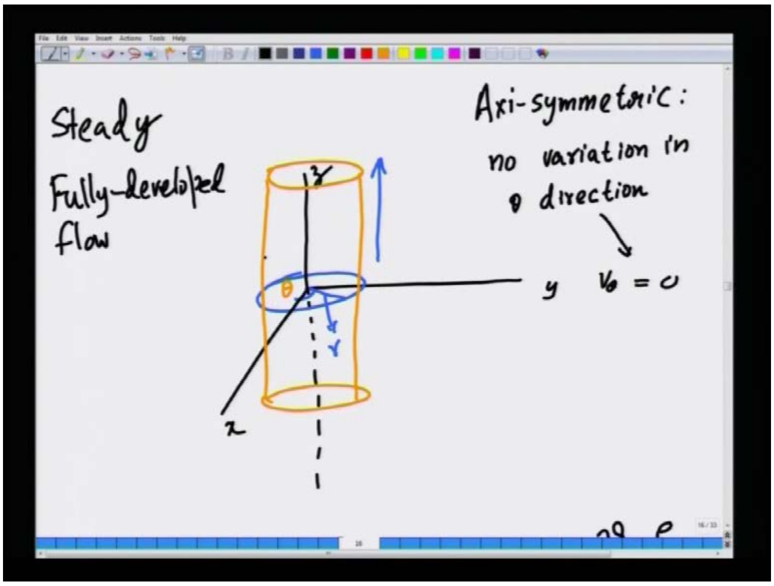

Let us, first draw the coordinate system; this is x y and z, if you have a pipe, you can nicely place the center of the pipe; you can align the center of the pipe with the z axis, with the z axis. So, you can nicely align the center of the axis of the pipe with the z axis. So, you are exploiting the symmetry of the pipe by just nicely placing the axis of the pipe along with the z axis of the cylindrical coordinate system.

Now, so you have in any cross sectional plane you have r. So, you have distance from the center as r, and theta is the theta is the angle made by the projection of any point on the x y plane with the x axis. So, that is r is this distance from the center that is the theta.

So, when fluid is flowing, we can align the flow to be in the plus z direction. So, there is pressure gradient in the plus z direction.

Now, what are the components of the velocity? The velocity vector is written as v r e r plus v theta e theta plus v z e z. Now, we are going to make some assumptions again like before again, as I mentioned just few minutes back the validity of the assumptions can be tested only by checking the predictions of our analysis with experiments; there is no other way, we can justify these assumptions right away. so what are the assumptions that we are going to make we are going to make the assumptions that since the pressure drop is across the z direction, the flow must be symmetric around the theta direction such flows are called axis symmetric flows. By axis symmetric we mean no variation in theta direction.

So, we make the assumptions, the flows axis are symmetric the assumptions; we are going to make are as before steady flow velocity is independent of time, and fully developed flow; the flow velocity in the z direction is independent of the z direction itself. So, now axis symmetry also means that, there is no theta velocity, because if there is a theta velocity; that is going to break the symmetry in theta direction. So, not all points in theta direction equivalent, if there is a theta velocity. So, theta velocity is 0; that is what axis symmetry means, and we are also going to use the fully developed and steady flow assumptions.

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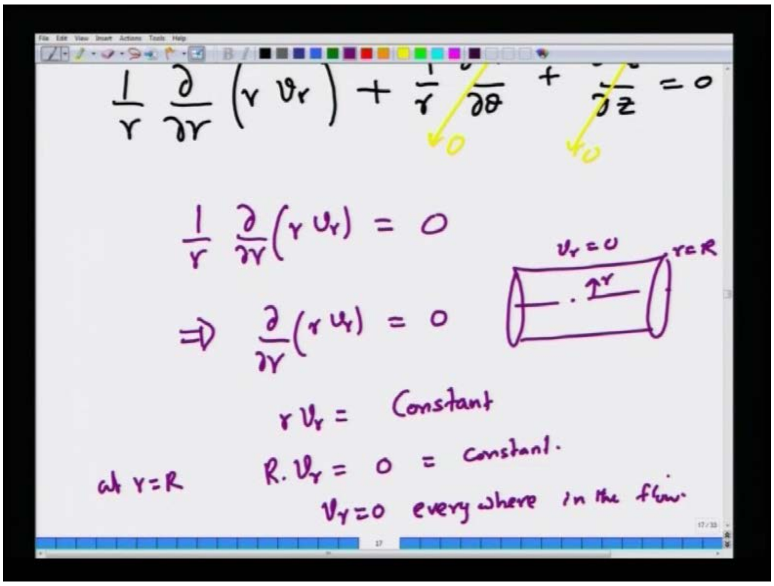

Now, the next task the next task in our hand is to look at the continuity equation or the differential mass conservation equation. Again as I have repeated several times, we cannot write the continuity equation in a steady in a in a the vector form delta; the continuity equation or the mass conservation equation becomes del dot v zero, but you have to look up tables, and text books in order to write that in cylindrical coordinate systems for reasons, that are mentioned several times before in this course. So, the continuity equation in cylindrical coordinate system is as follows.

Now, let us use the assumptions that there is no theta velocity, a flow is fully developed. So, v z is independent of z. So, all we get is one over r from the mass conservation equation r v r is 0. Now, this implies that d d r of r v r is 0 or r v r is a constant. What should that constant be? Now, if we look at the pipe, this is the r coordinate at r equals; the radius of the pipe, when you reach the radius of the pipe, there is no normal velocity v r is 0. So, r time's v r is 0 at equals r. So, that constant must be since it is a constant; it must be 0. If that constant is 0, then v r is 0 everywhere in the flow in the flow. There is no normal velocity. The assumptions of axis symmetry, and fully developed flow conditions, automatically ensure that there is no normal velocity; that is the r component of the velocity is 0. So, that the information that one can gain from the continuity equation for this problem.

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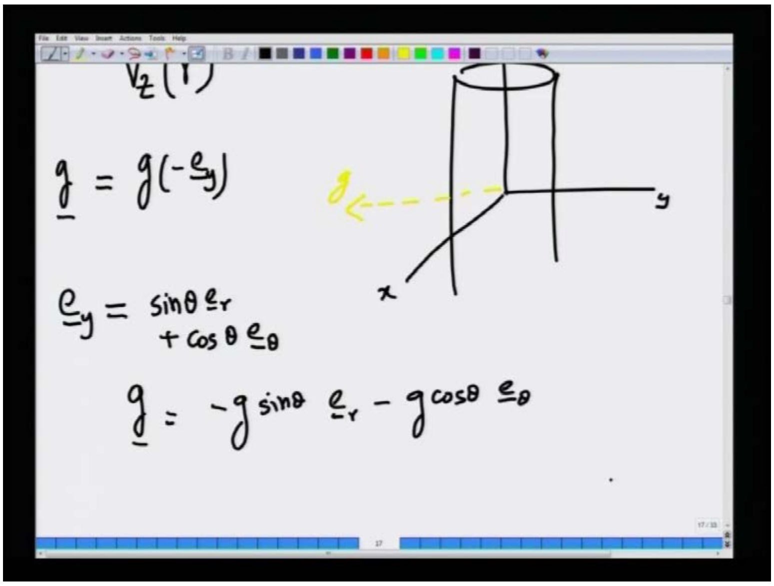

So, what we have ended up this with the fact; that you have a velocity field steady, velocity field v z, it is a function only of r direction. It is not a function of theta direction axis symmetry; it is not a function of z direction, because of fully developed flow conditions. Now, and only in that case, you get the velocity in the z direction to be a function only of r. Now, let us look at the navier stokes equation in the all the three directions. So, let us this is our pipe; this is a z direction; this is a x y plane.

Now, let us assume that the flow is perpendicular to the direction of gravity the direction of gravity. Let us, say is in the minus the acceleration due to gravity vector is in the minus y direction; and the flow is directly perpendicular to the gravity vector, that is the pipe is horizontal in some sense. So, if we do that then the gravity vector the acceleration due to gravity vector, because the gravity vector is pointing in the minus j direction, this is in the cartesian coordinate direction. We have to write in the terms of the unit vectors in the cylindrical coordinates, which are e r, and e theta. So, g is g times minus e y; that is the unit vector in the y direction.

Now, from geometry we can write e y is sine theta e r plus cos theta e theta. So, g is minus g sine theta e r minus g cos theta e theta. So, that is the acceleration due to gravity vector, and we have resolved it in the two directions; namely r, and theta direction, there is no acceleration due to gravity vector in the z direction, because it is perpendicular.

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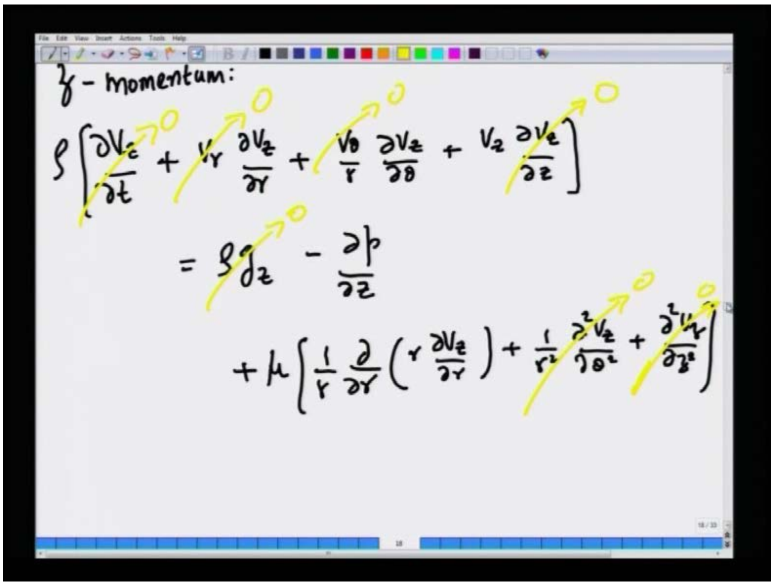

So, all the if you look at the r momentum equation r component of the momentum equation, and theta component of momentum equation. All they will say is that the pressure variation is hydrostatic in the y direction. So that is all, they are going to say so, there is no need to worry about the other two directions.

Now, we are going to write the z competent of the momentum - differential momentum balance, the navier stokes equations again to remind the navier stokes equations have different forms, and different coordinate systems, because the gradient operators as well as diversion operators; they are different in different coordinate systems. So, we have to look up a table, I am merely writing it for the sake of our convenience here, but you can look it up in any tables in many text books is equal to rho g z minus partial p partial z. Since now, it is slightly bigger; let me, write is here plus mu one over r partial partial r of r partial v z by partial r plus one over r square plus square plus partial square.

Now, let us use the assumptions to throw away terms flow steady, and there is no v r velocity, there is no v theta velocity. Flow is fully developed, there is no gravity component in the z direction flow is axis symmetric flow variation in v z, and theta direction flow is fully developed.

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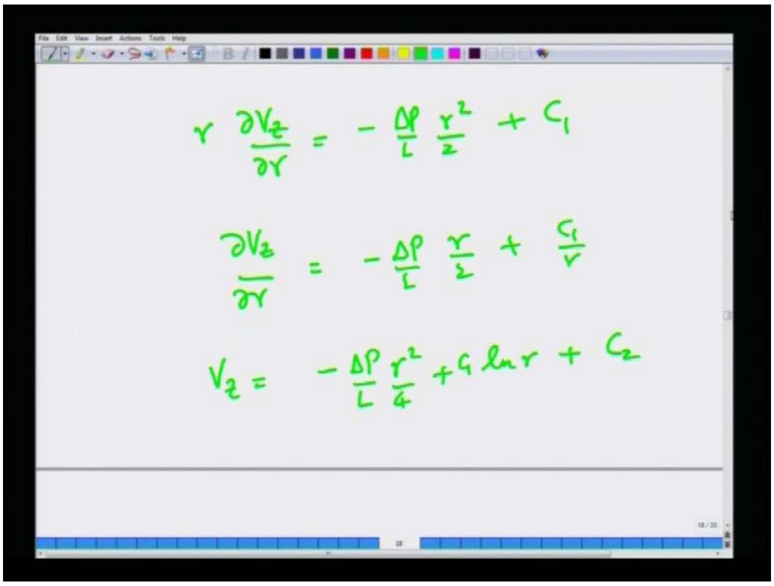

All these terms are 0, leaving us with only just as in the previous case, you have mu times is equal d P d z, and what is d P d z well just as before just to save time; I am going to invoke the previous discussion on channel flow, which we just did. We can say that this is equal to minus P at 0 minus P at L divided by L is equal to minus delta P by l just as in the previous case. Now, so, this is equal to one over r partial r of r. Now, we can integrate this in the following manner d d r of r d v z d r is minus delta p r by l r, if we integrate this once r d v z d r becomes minus delta P L r square by 2 c 1, if I divided by r, I will get r by 2 plus c 1 by r. If we integrate this once more, I will get minus delta

P by L r squared by 4 plus c 1 L n r, because if you integrate d r by r, you get L n r natural log of r plus c 2.

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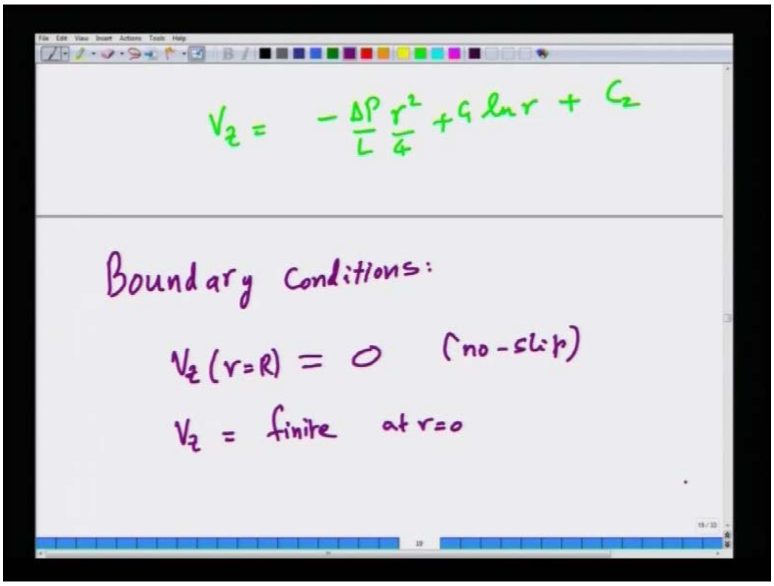

Now, you have to evaluate this, you have to evaluate the two constants with the boundary conditions. (No audio from 54:50 to 55:05) What are the boundary conditions? The boundary conditions are V z at r equals R is equal to 0, this is no slip condition at the boundary, and V z must be finite at r equals 0. You cannot have V z to be tending to very large values at r equals 0. We will stop here at this point, and we will continue in the next lecture.