Fluid Mechanics

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Lecture No. # 31

Welcome to lecture number 31 on this NPTEL course on Fluid Mechanics and the topic that, we are discussing currently is Pipe Flows and Losses. We saw in the last lecture that, the losses in a pipe are characterized by what is called the friction factor?

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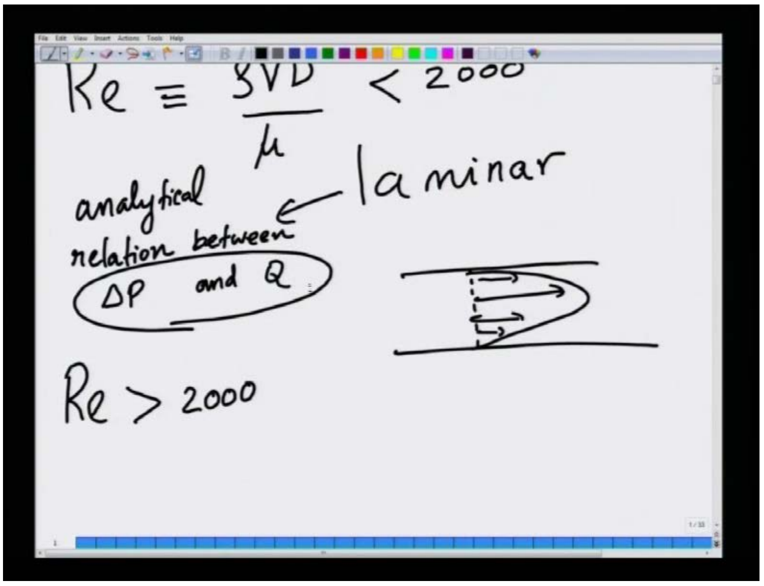

So, just briefly remind you topic is pipe flows and losses. So, when the flow is in there lamina regime in a pipe that is, when Reynolds number, which is defined as the density of fluid times the average velocity of fluid times diameter of pipe divided by the viscosity of the fluid is less then around 2000, then the flow is laminar; then we were able to solve for the velocity profile inside the pipe and we showed that, the velocity profile is actually parabolic.

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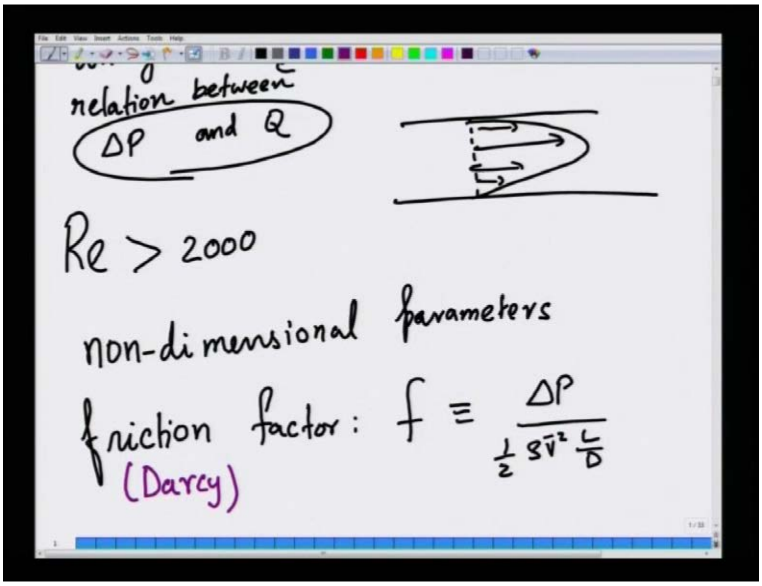

After having solve for the velocity profile, we found the relation between the pressure drop and the volumetric flow rate and that was a very simple relation, but we are also showed that, we also mention that, when the Reynolds number is greater than, so when the flow is laminar, there is an analytical relation that exists between the pressure drop and the volumetric flow rate. That is a very simple expression that, we wrote down in the last couple of lectures.

Now, when the flow is not laminar then, we have to take recourse to experiments that is, the Reynolds number is greater than thousand 2000, the relation between delta P and Q no longer is that was derived by assuming the flow laminar is no longer valid therefore, we have to now take a recourse to experiment.

Now, experimental data we showed in the last two lectures can be best characterized by non-dimensional parameters parameters. So, the non-dimensional pressure drop is called the friction factor. So, the friction factor is nothing but, a non-dimensional pressure drop, friction factor is denoted by the symbol f is defined as, delta P by half rho V bar square L by D and the friction factor thus define is called the Darcy friction factor because, there is another friction factor called a fanning friction factor, which is a half by factor of 8 compare to this or of by a factor of this 4 compare to this. So, you have to be little careful in the definitions, but generally the friction factor are are essentially non-dimensional pressure drops across the pipe of length L and diameter D.

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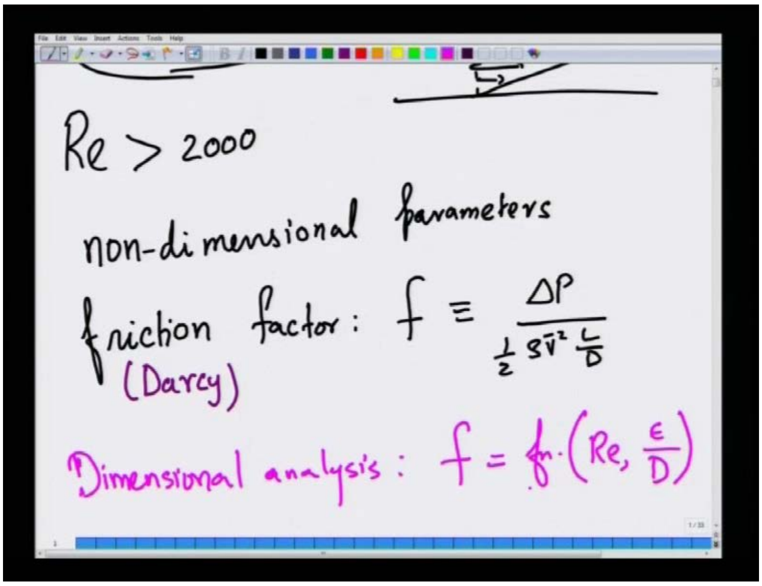

Just by D doing dimensional analysis, so dimensional analysis of the problem of flow in a pipe showed that frication factor is a function of the Reynolds number and the non-dimensional surface roughness of the pipe; epsilon is the measure of the it is a it is a length scale that tells you, how rough the pipe is, for example, it could be the mean square root mean square deviations of the fluctuations on the pipe wall.

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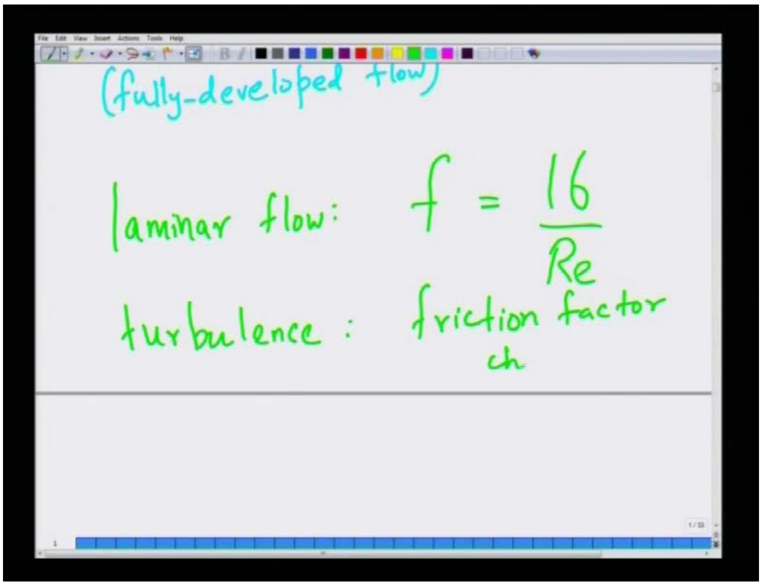

So, this is for a fully developed flow flow in a pipe because only for fully developed flow, we could scale out the dependence on the length of the pipe by saying that delta P by L is the constant at any fully develop region across the pipe, if there are entry and exit regions were, you are interested in measuring the pressure drop and flow rates, then you cannot use this a relation, we have to take into account the explicit dependence on the length of the pipe, so this is only for a fully developed flow.

So, we also showed that, for laminar flow region in the lamina regime, that a friction factor is nothing but, 16 divided by R e, while in the turbulent regime, you have to use the friction factor chart. So, the friction factor chart is merely an experimental observation condensed in the form of a non-dimensional relation.

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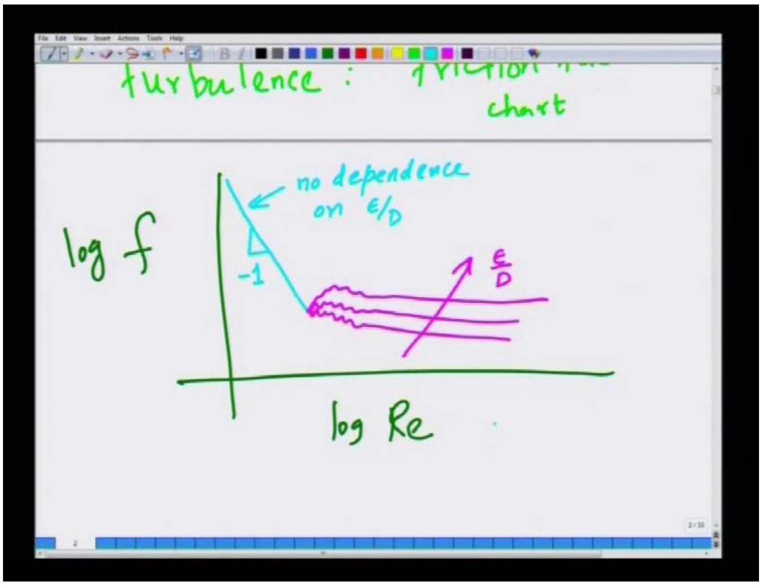

So, you have plotting f verses R e for different values of epsilon by D. So, in the laminar regime, so this is in a sense in a logarithmic plot, so it is log f verses log R e double logarithmic plot; so in the laminar regime f verses R e will have in a double logarithmic plot as slope of minus 1 because, f is inversely proportional to Reynolds number in the turbulent region it is not very clear.

So, there will be lot of fluctuations initially and and this is for different values of epsilon by D in the laminar regime there is in the laminar regime, there is no dependence on epsilon by D experimentally observed dependence on epsilon by D, while in the turbulent regime, it does depend on epsilon by D. So, this is an experimental observation of pressure drop verses flow rate, which is reframe or reformulated in a non-dimensional sense and plotted in terms of friction factor in Reynolds number.

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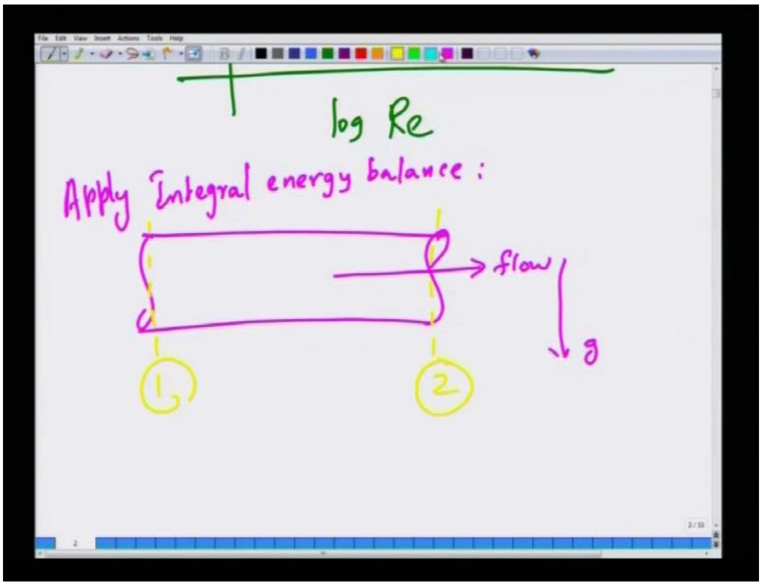

Now, once we know the friction factor then, we can estimate what are the head losses in a pipe, for that we have to take a pipe a straight pipe. So, gravities are acting perpendicular to the direction of flows, so the flow is in this direction and you apply energy balance, macroscopic integral energy balance, when we do that, we take the stations of the control volume 1 to be like this and 2 to be like this.

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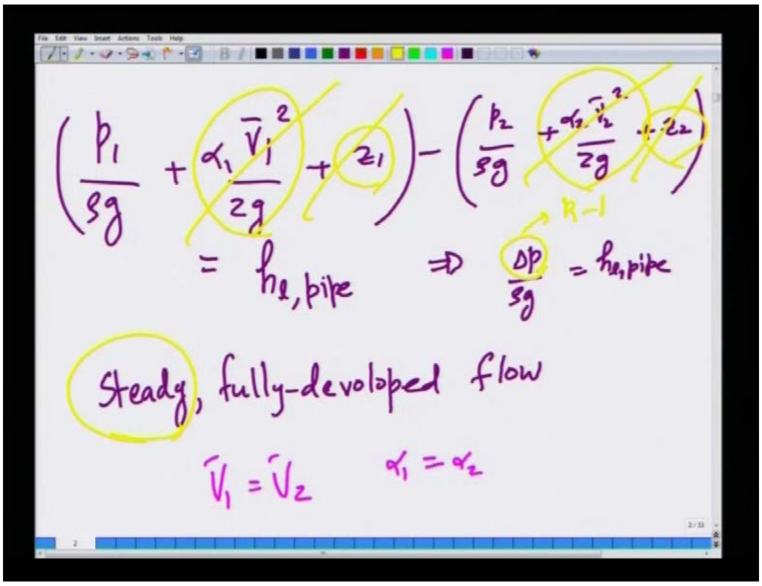

When we do that, we found that p 1 by rho g plus alpha 1 V 1 square by 2 g plus z 1 minus the same quantity evaluated at the exit, p 2 by rho g plus alpha 2 V 2 square by 2 g plus z 2 is equal to the head losses in the pipe because, there is nothing else that is happening, there is no shaft work between the two ends, the control volume 1 and 2. And the only loss is due to the fact that, fluid is flowing and there is viscous dissipation of energy.

So, for a steady fully developed flow, which is what we are going to assume we are not going to assume the flow is laminar or turbulent we want to say that, its steady fully developed flow. I have to make only one remark regarding the meaning of study for a turbulent flow, because it seems like a contradiction in terms, because the turbulent flow as I told you has is inherently unsteady, the flow velocity depends at each and every point in time with time in very random way in (( )) way.

Here, what we mean by steady is that, if you take a substantial time interval and then, average the velocity, so that average time average velocity remains independent of time. So, you can imagine having a pipe flow experiment let us say, the flows in the turbulent regime, because Reynolds number is greater than 2000, then you can let us say measure the velocity at a given point in space for about 5 minutes and then, take an average and then you can measure the velocity at the same point after some time again for 5 minutes. So, that that time average quantity will itself be independent of time. So, what we mean is that, average quantity average quantities are steady.

So, this is called steady in the mean although, there are a fluctuation about the mean flow the mean itself is independent of time. So, that is what we mean by steady flow. So, when we have steady fully developed flow, mass conservation equation will mean that, V 1 is V2, because this is the pipe of straight cross section and alpha 1 is alpha 2, because the conditions are identical in both the cases its either, it is not like its turbulent in one end and laminar in the other end. So, these two terms will cancel each other out.

And since the pipe is oriented horizontally z 1 is also equal to z 2 leaving us with this simple expression delta P by rho g is h l pipe, where delta P is nothing but, p 1 minus p 2 this is our convention the pressure at the entry minus the pressure at the exit, it is the pressure difference across the pipe.

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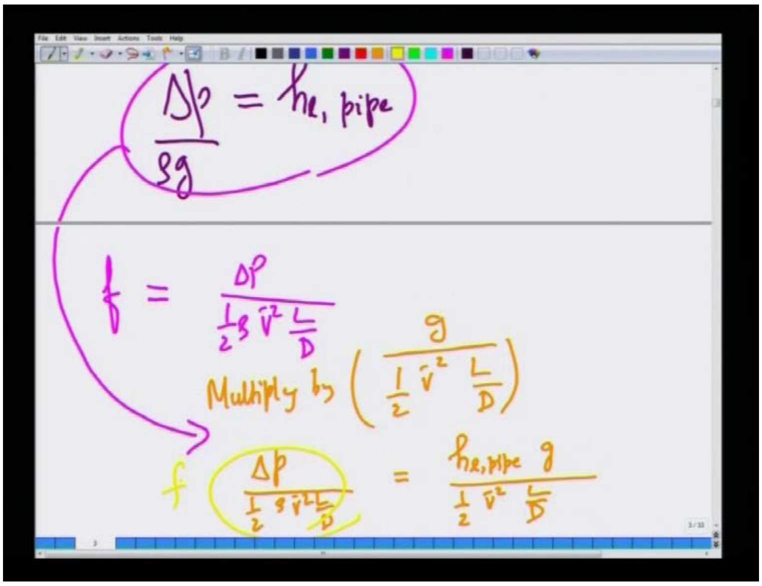

Now, so all we have from the energy balance is delta P by rho g is head loss in a pipe. Now, I want to relate it to the friction factor, if I want to relate it to the friction factor I have to simply include the invoke the definition of friction factor, f is nothing but, delta P divided by half rho V square L by D, if that is the case then, I have to take this equation I have to take this equation, now all I have to do is divide both sides by half V square half V square g L by D because then I will get sorry half V square L by D because g already yeah just half v square L by D and multiplied by g. So, that I get friction factor. So, if I do that I will get delta P by half rho V square L by D because, I am multiplying by g is equal to h l pipe time's g.

So, I am sorry. So, I have to divide by g or multiplied this entire equation let us call it multiply by g divided by half V square L by D. So, that I get friction factor on the left side of this equation, on the right side I will get h l pipe divided by half V square L by D since this is the friction factor, the Darcy friction factor f.

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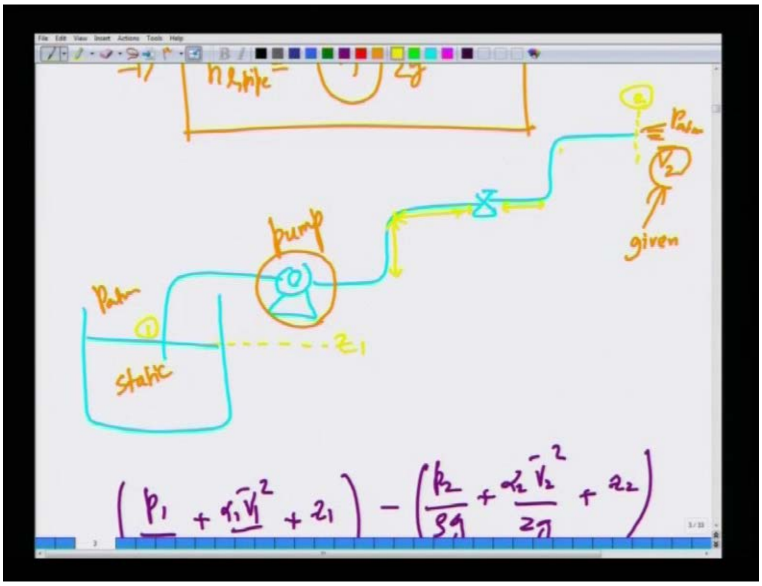

I can write therefore, h l is nothing but, f times half head loss in a pipe half V square L by D divided by g, this is the relation between the friction factor and the head loss in a pipe, this is the relation between the friction factor and head loss in a pipe.

Why this is important this relation important because, when we want to write the energy equation for a complex flow setting for example, in practice you may have problem like this you may have reservoir in which there is a liquid and you want to have a pipe and you want to may be pump the fluid at a given flow rate and there may be valves to some other elevation. So, this is a typical problem, this is elevation z 1 let us say this is point 1 and fluid is coming out to the atmosphere and this is let to say point 2.

If you want to write the energy balance between point 1 and point 2, the general energy balance of course, we know is p 1 by rho g plus alpha 1 V 1 square by 2 g plus z 1 minus p 2 by rho g plus alpha 2 V 2 square by 2 g plus z 2 is equal to head losses in the flow minus any work done by the pump on the fluid. So, there is a pump here sorry which inputs energy constantly to the system.

So, this is a typical setting that one would like to one would like to solve for example, we may want to ask the question, suppose I want this is a static fluid stationary fluid, the flows this no flow here and this is the atmospheric pressure and fluid is exiting to atmospheric pressure at point 2, then the question we may ask is suppose I want the fluid to come up with a specific velocity V 2 this is given to us, now what is the head loss sorry what is the head (( )) generate across the pump, so that we can make the fluid flow at a particular velocity.

What this pump is essentially doing is to overcome the gravitational head between points 1 and 2 and also the viscous losses that occur at a various points, now there are various segments in which the flow is straight, there is a flow the flow is through a straight pipe. Here, we have to estimate the losses using the friction factor chart. So, whenever there are there are segments of pipes, where there are straight segments then, those pieces will the losses in those pieces of the pipe line network will have to be calculated from friction factor chart.

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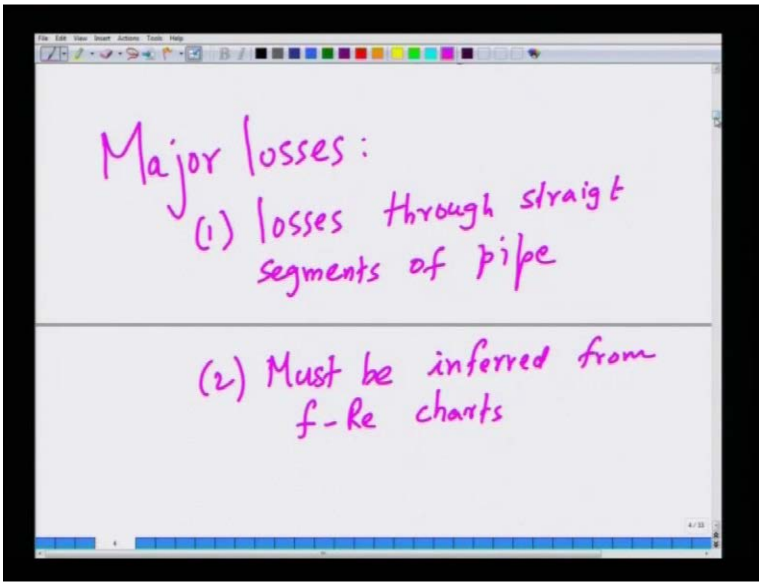

Such losses are called traditionally major losses; major losses because, they contribute numerically the most to the losses present. So, major losses are losses through straight segments of pipes, they must be calculated from friction factor charts must be inferred from f-R e charts, but that is not to say that this is the only source of losses for the flow system because, there are also other pieces such as the flow is static here, but it is entering a pipe. So, these are called entry losses.

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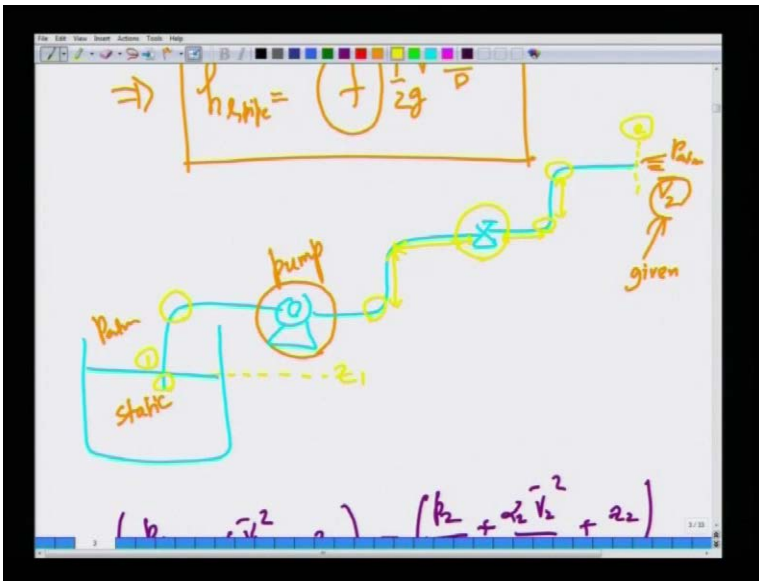

And there is a bend of various types throughout the pipe line these bends will in general have losses, there are valves here which will have losses depending on that nature of the valve, so and there is a exit of fluid is exiting from the pipe to the atmosphere. So, there are losses associated with exit. So, viscous losses are not just their when you have straight sections of pipe as we also saw while doing integral momentum balance whenever you have a sudden expansion or contraction of flow cross section length, there are recirculating eddies in the immediate vicinity of the expansion or contraction, which will contribute to additional losses. So, we incur additional losses due to various reasons those are called minor losses.

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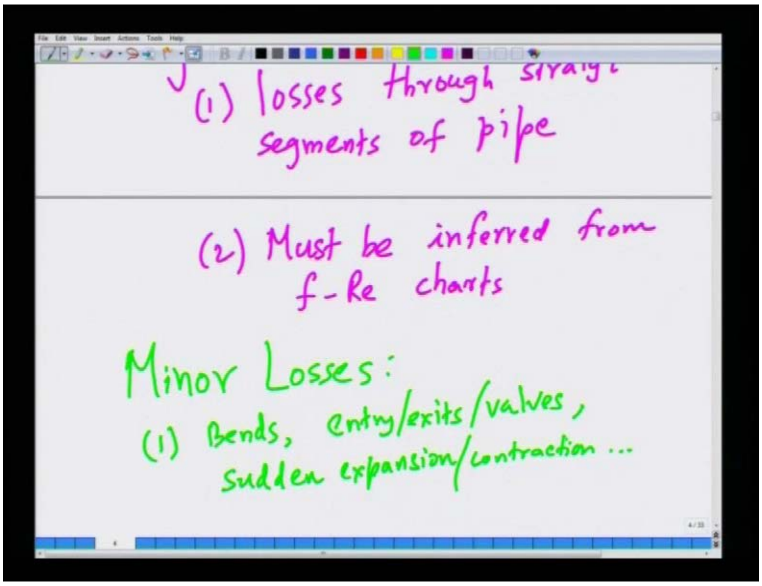

For the simple reason that, they do not numerically contribute much to the losses, they contribute may be 10 percent to the loss, but they are the minor losses typically include bends, entry, exit, valves, sudden expansion, contraction and so on. So, there are various reasons why, there could be minor losses in a problem. So, these minor losses have to be calculated by (( )).

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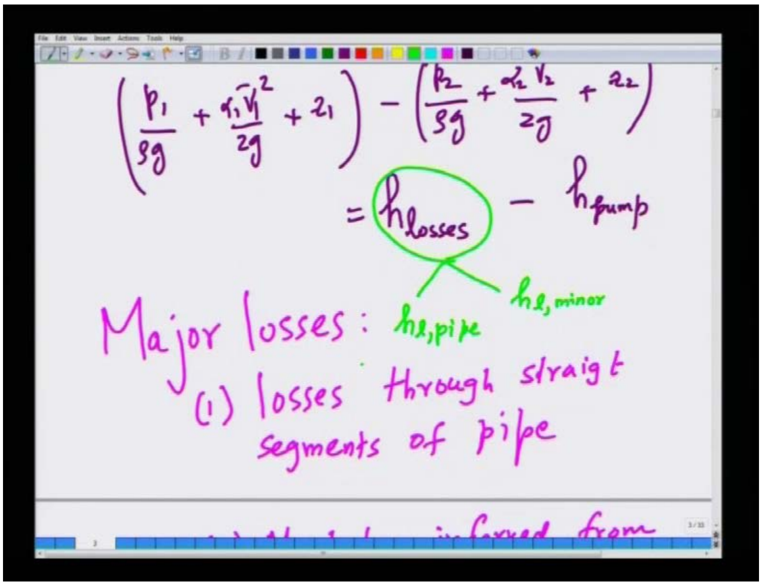

So, let us again rewrite let us split this losses into major losses, which I will denote as h l pipe and minor losses which are just write as h l minor it depends on the nature of the problem and nature of the type of losses that occur in a given flow systems. So, the major losses will have to be calculated from friction factor charts, the minor losses will have to be again derived from experiments, but they have to be you know there are standard values that are written down for minor losses.

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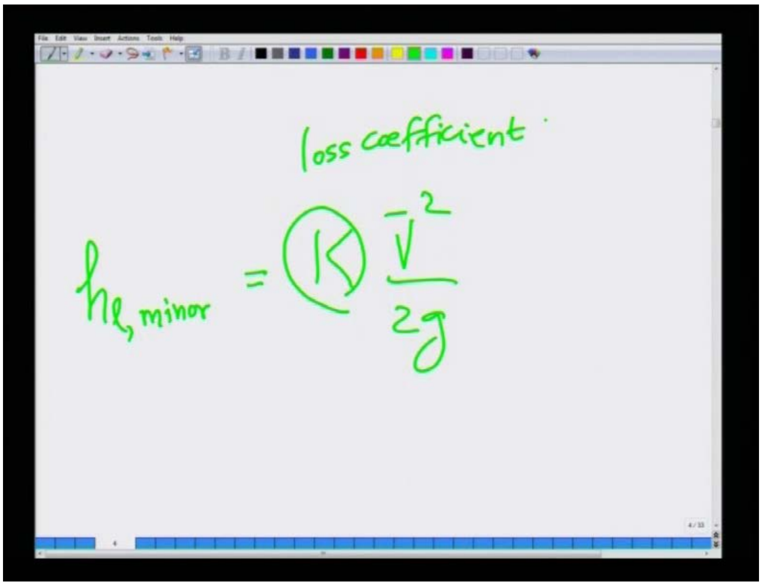

Typically the minor losses are written as are correlated as a loss coefficient time's V square by 2 g, this is called loss coefficient. So, this coefficient will be different for different types of losses for example, for entry this may be 0.8, for a sudden for a valve for a globe valve partially closed valve or fully open valve, it will have different values and so on. So, these are typically given in textbooks and handbooks.

So, these are not these are merely experimental facts, because the actual flow that happens in this bends and through valves this extremely complex for us to be able to calculate this losses exactly using any fundamental means. So, we have to take reports to experiments and in experiments we characterize these losses as the minor loss head loss coefficient which is non-dimensional number and that number varies for varying types of losses various losses.

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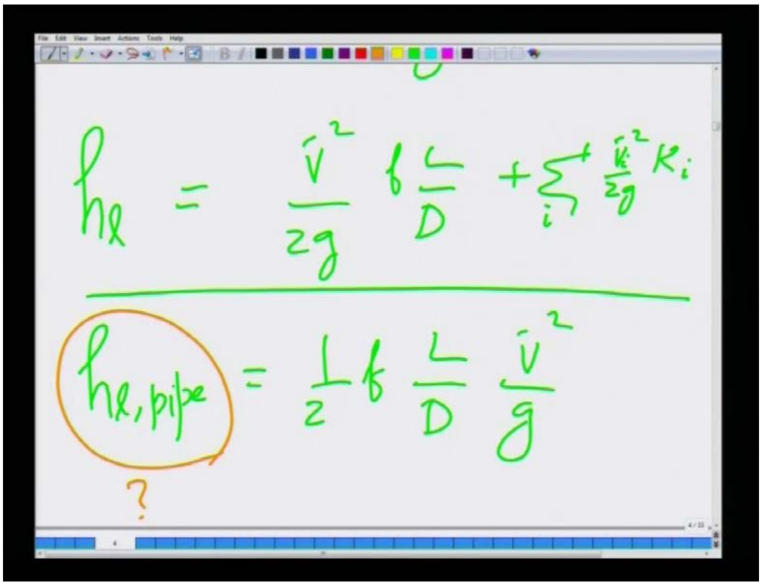

So, the total loss will therefore be, the total losses in the pipe will be V squared by 2 g times f time L by D plus summation over various minor losses V square by 2 g times the loss coefficients varying there could be varying losses, so over sum over all losses. So, this is the typical way in which losses are computed. So, when you want to compute the losses, there is a small detail that one as to understand properly. Suppose we are interested in computing the major loss. So, h l pipe is nothing but, half f L by D V square by g. Now, when can we calculate h L by D h l pipe? So, there are three types of question that can occur.

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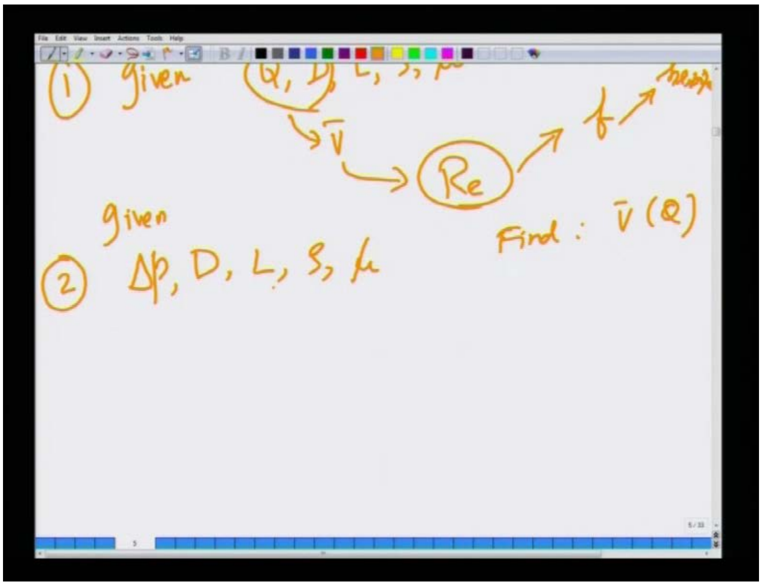

In class 1 question, we are given the volumetric flow rate, that pipe diameter, the length, that density of (( )) viscosity nature of the fluid. So, this is essentially volumetric flow rate and diameter essentially means we are given the average velocity this means that, we can calculate the Reynolds number. So, once we know the Reynolds number we can use friction factor charts to find the friction factor, once you know the friction factor you can find the major loss through this expression.

So, this is a very linear progression that is we are given suppose somebody ask the question given the fact that, we have a flow rate how much what is the loss that is incurred in a pipe of given diameter and fluid is known. So, we can readily do this calculation by knowing the flow rate and the diameter of the pipe we can calculate the average velocity by knowing the average velocity we can calculate the Reynolds number, then use the friction factor chart to look up for what is the friction factor corresponding to the Reynolds number, then use this equation for calculating the losses. That is class 1 problem. Now, there can be other class of problems also, suppose you know what is delta P, then diameter, L, rho, mu and we are asked to find the velocity or volumetric flow rate.

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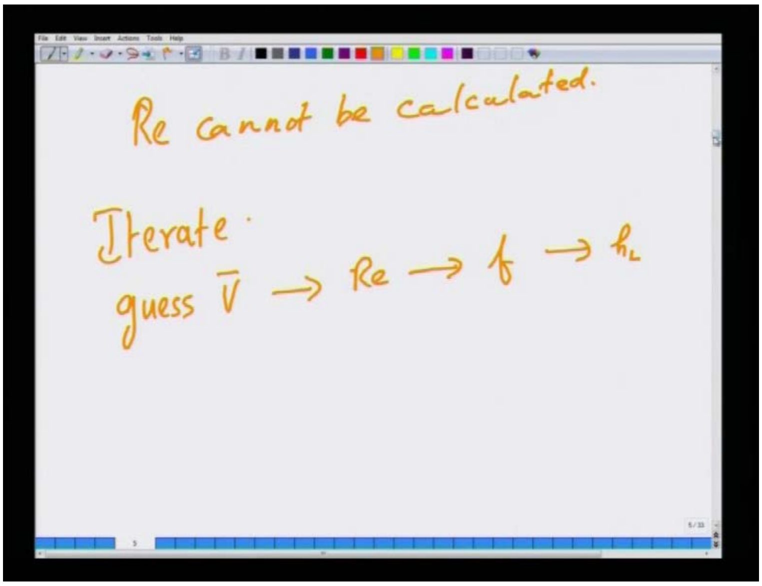

Now, with this set of variables R e cannot be calculated, because we do not know, what is the velocity that is the final goal of our calculation. So, these kinds of problems must be done iteratively that is guessing a velocity, then calculate R e, then calculate f, then calculate head loss.

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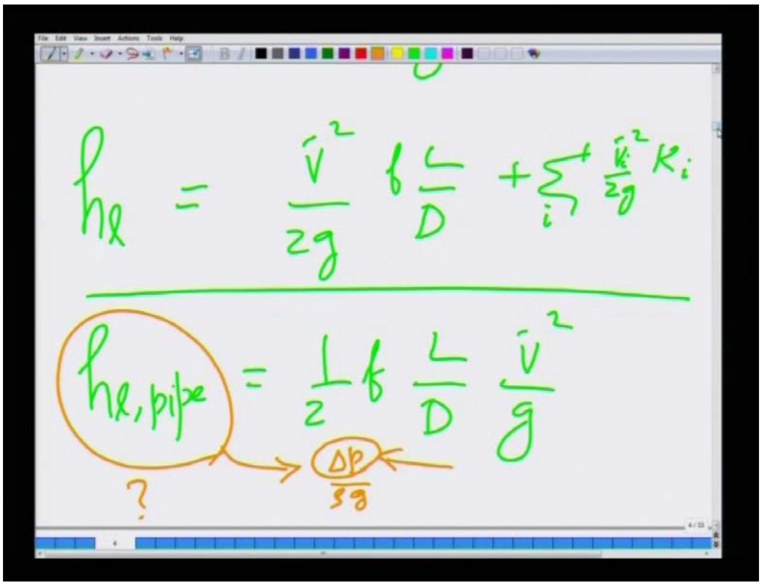

Now, once if you look at the head loss expression, the head loss itself is given by the definition of head loss is nothing but, delta P by rho g. So, given delta P we can calculate what is the head loss, the definition of head loss is delta P by rho g, for a straight section of a pipe if you if you notice this this part, this is delta P by rho g is the head loss in a straight section of a pipe and we are now restricting ourselves to only to straight sections we can add on minor losses little later.

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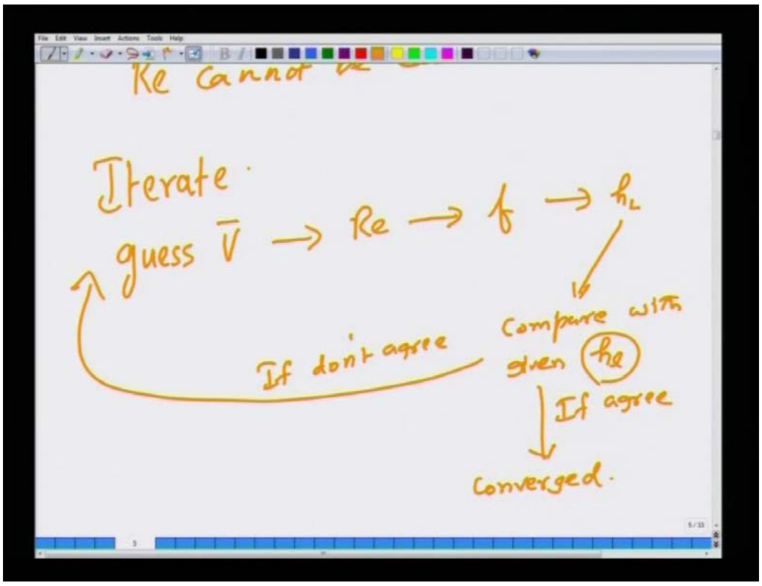

So, you obtain a head loss compare with the given head loss that is, when once you know delta P you can actually calculate head loss. If they do not agree then, keep doing it, if do not agree again refine your guess for V to do this thing if agrees then, you have converged answer as converged. So, if you are given the delta P that is the pressure drop across the pipe and if you are asked to calculate the velocity, it is not a straight forward calculation because, if you are given delta P all we are given is the head loss and if you want to calculate head loss the average velocity from the head loss then, first you have to guess the velocity, then calculate the Reynolds number.

Because, the pipe diameter is given and the fluid nature of the fluid is given to us, properties are given calculate friction factor and then, use the relation between friction factor and head head loss see whether this head loss calculated head loss agrees with the given head loss, if not keep refining and do this iteration until you converge for a suitably satisfactory answer.

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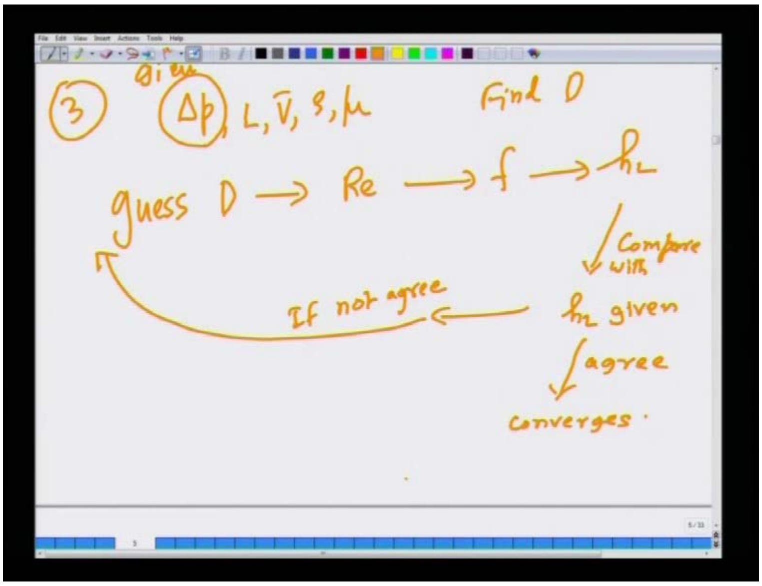

And the third type of problems is given, delta P delta P, length, velocity, rho and mu and you are asked to find the diameter suppose in an application you are saying that well my pressure drop across the two ends is fixed and I want to pump the fluid with the given flow rate and the length of the pipe is fixed, but all I want to do is know, what is the diameter that I have to choose, because in many cases you may we force to work with the given length and you you are pumping requirements are such that, may be delta P is also fixed and you want a particular velocity, so V is fixed. So, the only variable in your hand is actually the diameter of the pipe.

Again, the answer is not a straight forward answer you have to do it iteratively guess D, then everything else is known calculate Reynolds number, calculate friction factor, calculate head loss, now head loss is also given because, delta P is given compare with given head loss with head loss that is given then, if not agree if it does not agree, this again you refined your value of D, if it agrees, then you have a converged answer.

So, these are the three kinds of questions or types of problems that one can often encounter in pipe flows especially, if you have straight sections of pipe, then the it really you cannot use straight forward solution, if you are asked to find for example, the velocity for a given pressure drop or a diameter for a given pressure drop, because those solutions are iterative.

The primary reason being that, the f verses R e chart is a non-linear relation in the turbulent region therefore, you have to use iteration to find the solution. And the data is known only graphically, it is not available as a simple relations, so we have to do iteration in a in a graph in a graph sheet you have to do a graphical iteration to find the solution.

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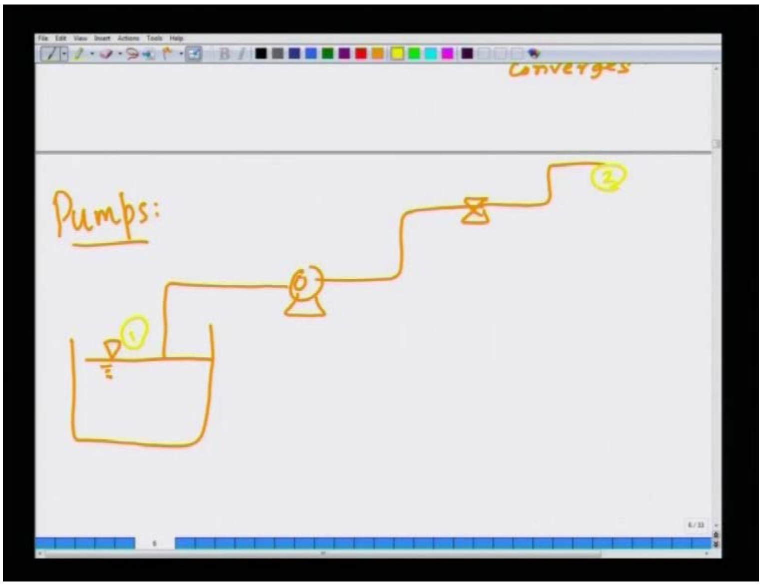

Now, let us go to the case of pumps. So, essentially let us say, you have a system were you have again let us go back to the same example you have a free surface and then, you have a pump and then, you have valves and then, you are writing the balance between point 1 and 2 typically the questions that will happen that will arise in engineering is that, suppose I want to pump the fluid with the given flow rate from station 1 to station 2, what must be the rating of the pump that is what is the horsepower rating of the pump in which which in a sense means, what is the rate at which the work must be done by the pump on the C v.

So, that you can get the required flow rate, which will be essentially the horsepower rating of the pump, if the pump is 100 percent efficient, in principle there may be some mechanical inefficiency is on a pump. So, you will have to account for that little bit that is an experimental fact.

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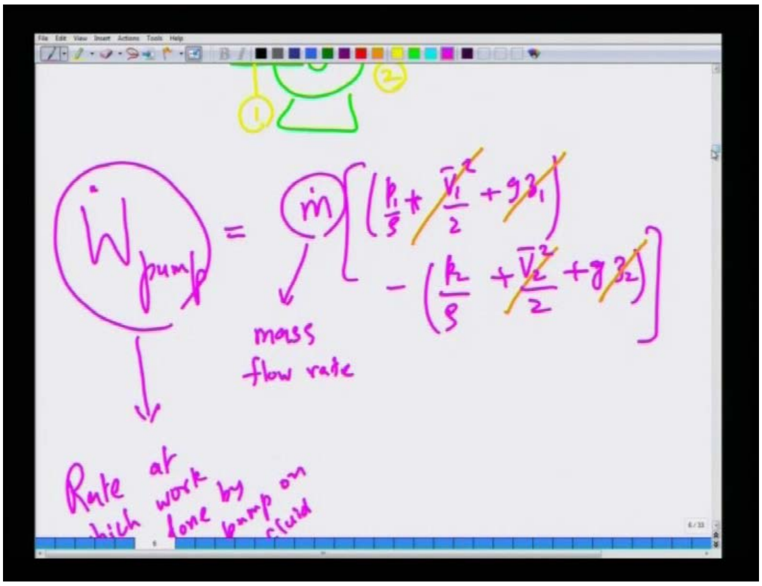

But, theoretically speaking what is the rating of a pump for a given application in order to do this you have to just write the energy balance across the inlets and exists of a pump. So, the rate at which pump does work W dot pump is m dot times p 1 by rho plus V 1 squared by 2 plus g z 1 minus p 2 by rho plus V 2 square by 2 plus g z 2, this is the mass flow rate.

Now and this is the rate at which work is done by the pump, work is done by the pump on the fluid. So, this is the rate at which work is done by the pump on the fluid. And so let us assume let us simplify this balance. So, p 1 minus p 2 is not negligible, because the whole point of having a pump you want increase the pressure across the pump and if the pipes that connect the inlet and outlet of the pump are of the same dimensions, then you can neglect V 1 square and V 2 square is approximately same and the elevation difference between point 1 and 2 is also typically negligible.

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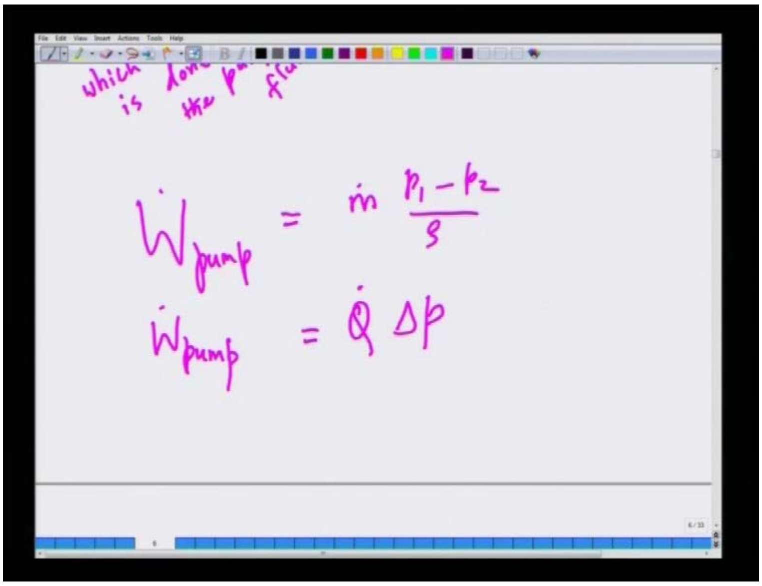

So, W dot pump is nothing but, rate at which work is done by the pump on the fluid is nothing but, m dot times delta P, p 1 minus p 2 divided by rho, now m dot is mass per unit volume rho sorry m dot is mass per unit time, rho is mass per unit volume. So, m dot by rho will essentially give you Q dot times delta P, this is a volumetric flow rate of the pump times the pressure difference across the pump will give you the rate at which work is done by the pump work must be done by the pump in order for the fluid to in order to generate pressure difference of delta P.

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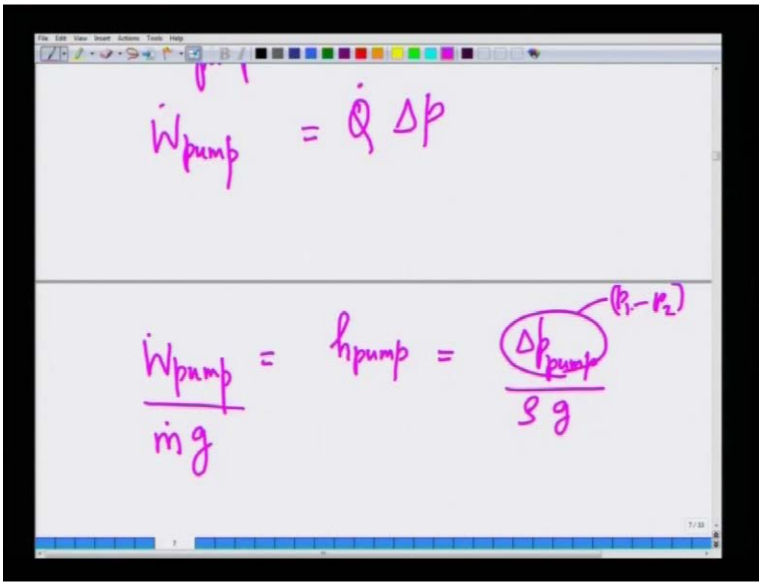

So, W dot pump to go back is nothing but, so if we have this notation of heads which is nothing but, W dot pump divided by m dot times g. So, this is nothing but, delta P pump by rho g. So, this h pump is nothing but, delta P pump by rho g, where delta P pump is the pressure difference p 1 minus p 2 across the pump. So, p now sorry this one sorry this is at station 2 minus stations 1 I made a mistake (Refer Slide Time: 32:37). So, this is minus of W pump.

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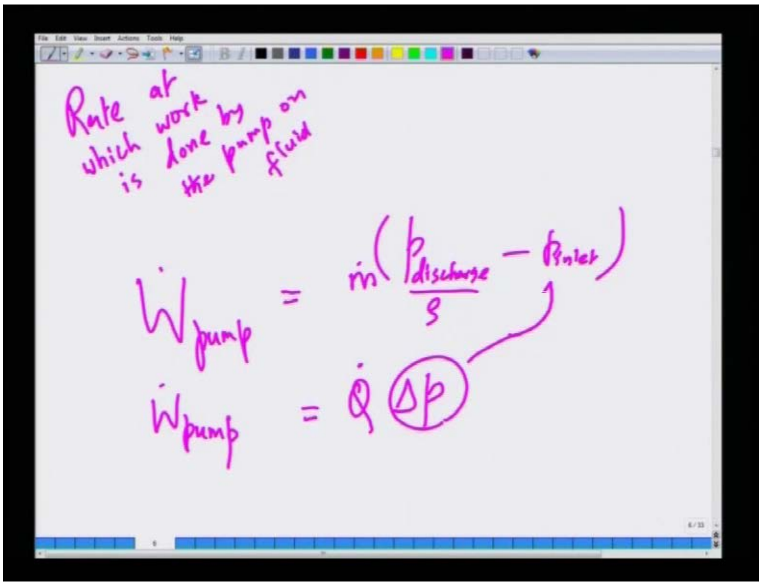

So, this is p 2 minus p 1, because p 2 is greater than p 1 in a pump, the pressure at the exit is greater than, so delta P in a pump is defined as lets instead of calling it p 2 minus p 1 let us call it p discharge minus p inlet and this is delta P, it is a positive quantity, the discharge pressure is greater than the inlet pressure.

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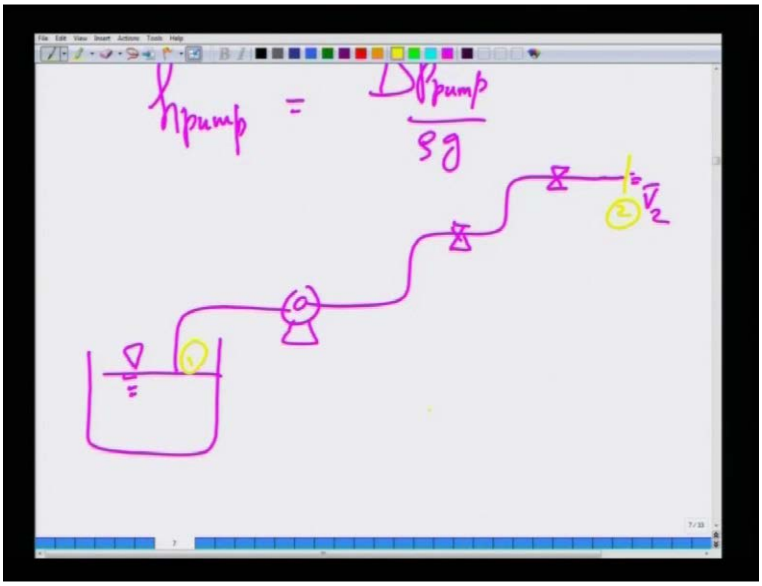

So, W dot h pump which is what we require in our calculations is nothing but, delta P pump by rho g. So, suppose you have this typical problem in which we have a reservoir and it is atmospheric pressure here and you have a pump that has to pump it through series of bends and say various valves to finally, again to atmosphere with another velocity V 2. So, let us call this station 1 and this station 2, suppose we are given and these stations are at different elevations, z 1 and z 2, the two stations are at different elevation.

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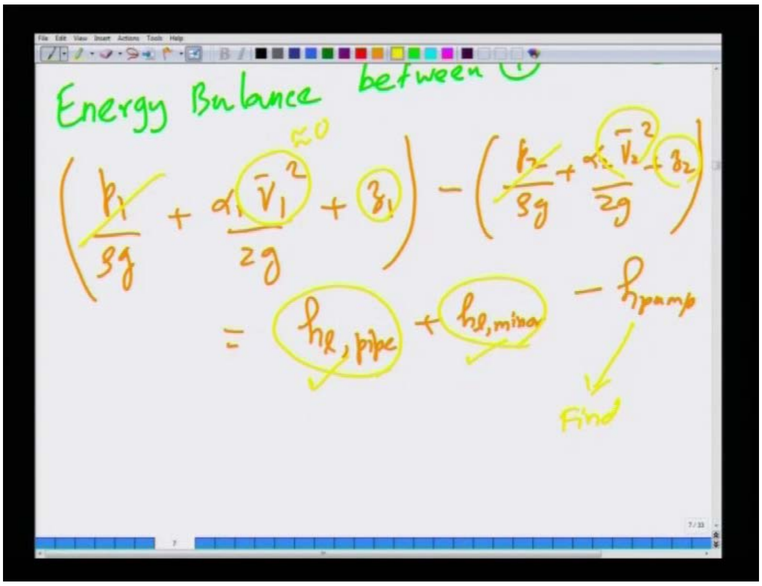

So, if you write the energy balance between point 1 and 2 all you will get is p 1 by rho g plus alpha 1 V 1 square by 2 g plus alpha sorry plus z 1 minus p 2 by rho g plus alpha 2 V 2 square by 2 g plus z 2 is equal to the total losses, which is the major loss due to pipe plus the minor loss due to bends and valves and so on minus h 1, where this is the head this is the in some sense the work done by the pump on the fluid.

So, that is why it is coming with the negative sign remember that, if you have if you have work done by the fluid on the surroundings then it will this would have come with the positive sign, since in our case we have use the convention that work done on the system is negative this comes with the negative sign. So, if you have this kind of a problem. So, we have p 1 is p 2 is atmosphere. So, we neglect this, now V 1 is stationary fluid is stationary there static. So, V 1 is approximately 0, but you have z 1 minus z 2 which is non-zero and V 2 is non-zero we know what the value of V 2 is using what is V 2 the velocity in the pipes we can calculate the major loss as well as the minor loss from which you can find h pump.

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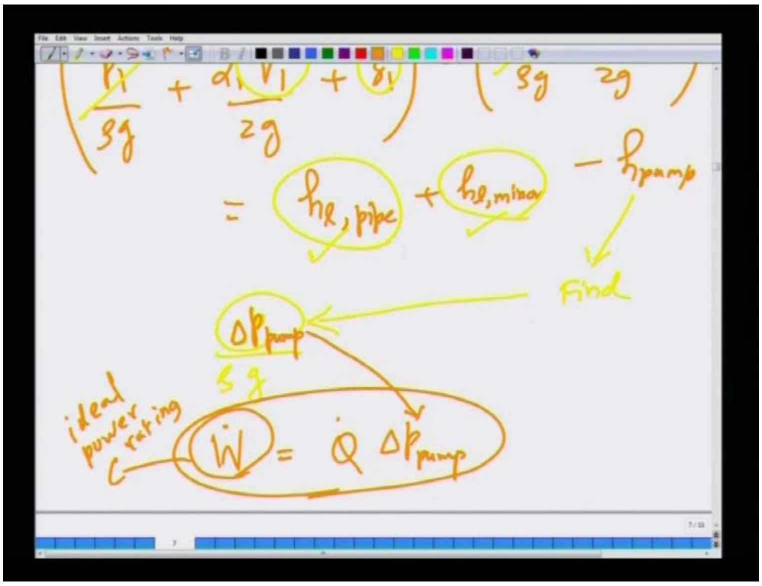

h pump is nothing but, delta P by rho g, now once you know what is delta P once you know h pump you can find delta P, once you know delta P then, the rate at which work must be done by the pump on the fluid is nothing but, Q dot times delta P, this is delta P across the pump and if you take this delta P you can find the horsepower rating of the pump.

This is the power requirement of the pump provided the pump is 100 percent efficient of course, if there are mechanical inefficiencies then, one has to correct them by increasing the power actual power rating of the pump, because in principle the pump will be characterized a mechanical efficiency. So, the rate at I mean the there is a convention factor from the electrical power restricted supply and the mechanical power that is got as an output from the pump.

So, this will be the ideal horsepower rating of a pump ideal rating ideal power rating of a pump. So, this is a very important application of the notion of losses in a pipe major losses and minor losses in a pipe line network and typically this they are used to calculate quantity such as, what is a rating of a pump or alternatively suppose you have installed a pump of a given rating then, you know what is h pump, then you can ask the question what is the velocity in such a case the answer is not straight forward, because you are we have to find out the velocity through an iterative procedure because, we do not know what the Reynolds number is in order to find what are the losses.

So, both kinds of problems are equally possible in practical engineering applications in chemical process industries that is, if you want to design a pipe line network no matter how complex it is the ideas are essentially what we have been discussing typically you may have a pump and then, you may want to find what is the velocity in a pipe and that is a iterative problem, but if you are asked to design a pump that is if you want a given flow rate and then, if you are asked to design a pump that is what is the horsepower rating of the pump, then you have to follow the procedure that we just outlined.

This essentially completes my discussion on pipe flows and losses and the topic that we are going to discuss next is fluid flows at higher Reynolds numbers. In many industrial applications when you calculate the Reynolds number at which fluid flow is occurring typically happen at very high Reynolds numbers and Reynolds numbers by higher means Reynolds numbers of the order of 1000 and higher may be 5000, 10000 things like that.

So, it is good to have some fundamental understanding of what is happening at high Reynolds number flows. Now, the problem with as I told you the problem with Navier-Stokes equations; the Navier-Stokes equations are written down about 200 or 300 years back early.

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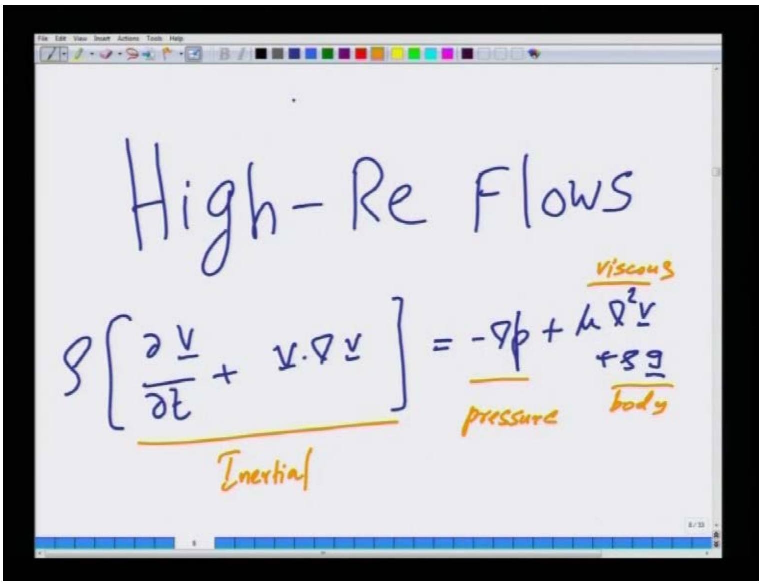

And but, they are extremely complex to solve the Navier-Stokes equations for an incompressible fluid, they are non-linear and partial differential equations. So, let us write it. So, it is extremely difficult to solve the Navier-Stokes equation in a general setting without making suitable assumptions because, they are extremely complex to solve.

So, often to gain some insight into a problem it is useful to simplify the Navier-Stokes equations by neglecting some terms, which are at the outset some what small compare to other important terms. The Navier-Stokes equations are is essentially a force balance equations. So, there are various forces on the various sides of the Navier-Stokes equations, the left side you have the inertial forces and on the right side you have the pressure forces the viscous forces and the gravitational body forces. So, these are the various forces that act on a fluid element by looking at particular flow regimes.

For example, when the Reynolds number is very small we know from interpretation of Reynolds number that Reynolds number is ratio of inertial forces in the system to viscous forces in the system, when the Reynolds number in very small we can presumably neglect inertial forces to begin with and hope to get a reasonable approximation to the actual flow by just worrying about a balance of viscous forces pressure forces and body forces likewise when the Reynolds number is very high very large that is of the order of 1000, then we can hope that since the Reynolds number is very large we can hope that, the inertial forces in the fluid must be dominant compare to viscous forces in the fluid and therefore, we must be able to get a suitable approximation by neglecting viscous forces all together.

So, it seems like a reasonable proportion and that is what we will do to begin with also this approximation which seems like a very reasonable one to make to begin with has some serious difficulties and we will point that out little later when we come to boundary layer theory and we will keep pointing out the deficiency of the approximation as we go along, but right now we are going to non-dimensional write down the non-dimensional version of Navier-Stokes equation.

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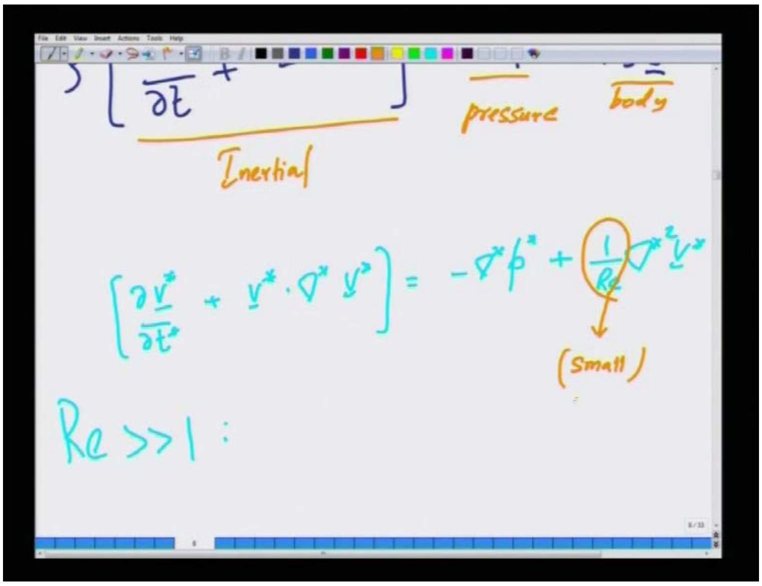

It is nothing but, R e times del V del t these are all star variables because, there are non-dimensional plus it is nothing but, minus del star p star plus 1 over R e I am going to neglect body forces for the moment, because it do not play an important role right now. So, we can say if there is no body forces then this is the I am sorry I have taken Reynolds number to this side. So, when Reynolds number is very large, large compared to 1 that is it could be of order of 1000, then 1 over R e is small.

So, the viscous forces the non-dimensional viscous forces that occur in the Navier-Stokes equation are multiplied by a small number. So, we can hope that we can neglect viscous forces all together to begin with.

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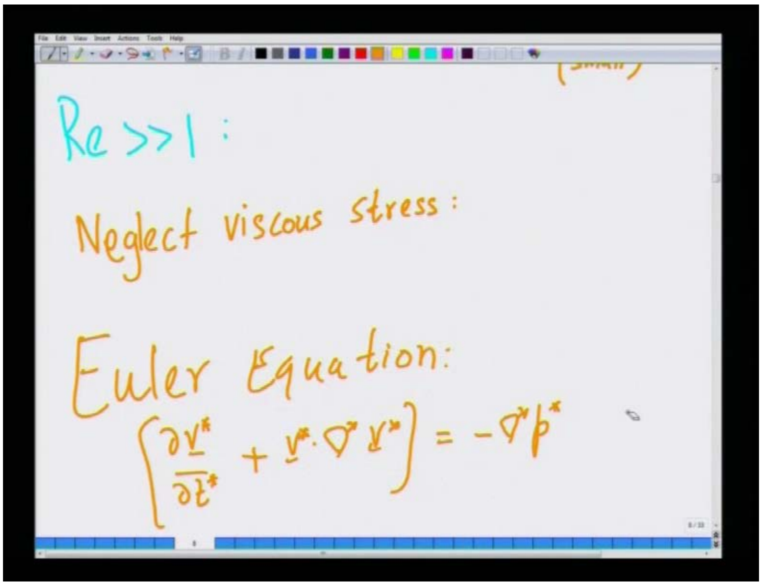

And if we neglect viscous stresses then, we end up with what is called the Euler equation. So, the non-dimensional Euler equation is simply partial V star partial t star plus plus minus del star p star and if there is gravity we can write gravity, but there is non we can just leave it like that. So, this is the non-dimensional Euler equation.

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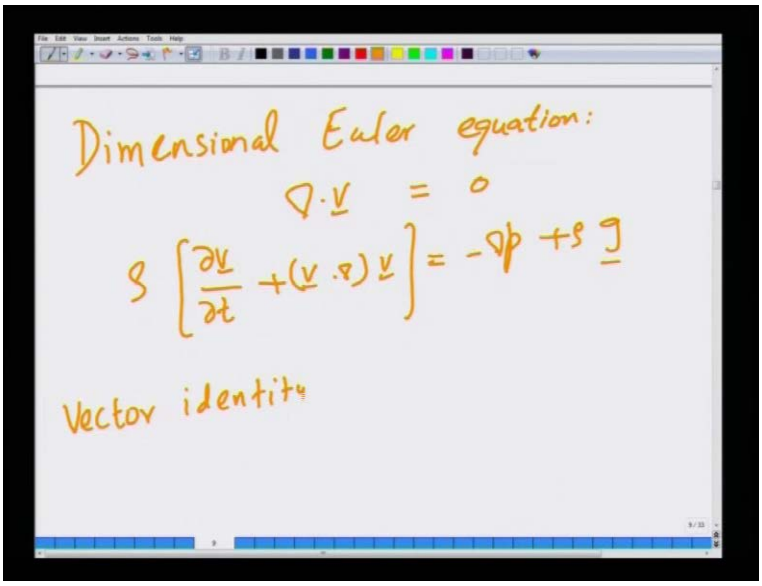

But, we will get the dimensional we want to work with the dimensional Euler equation to begin with. So, we will write down the dimensional Euler equation which is nothing but, rho V is minus del p and if you have body forces plus rho g, this is rho now I want to further simplify this equation or rewrite this equation for which and then of course, we have the continuity equation or the mass conservation equation for incompressible fluid, which is the divergence of velocity vector is 0.

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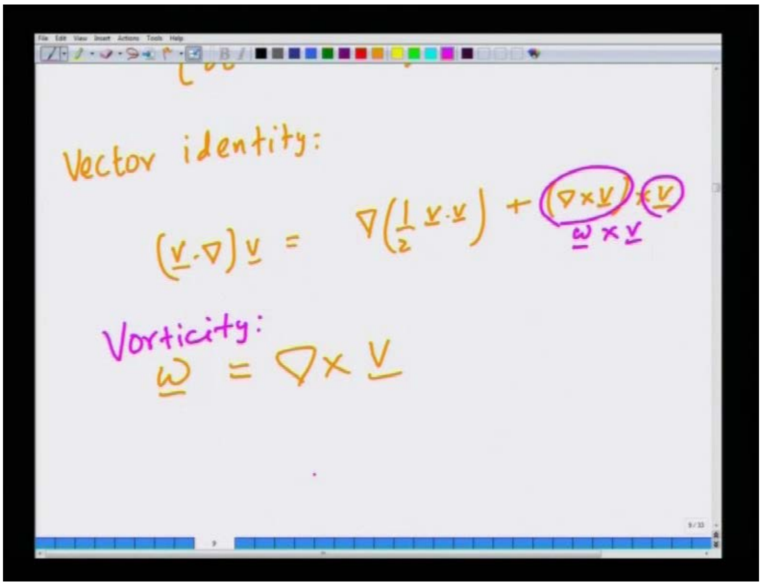

We will use a vector identity to rewrite this equation. So, there is a standard vector identity that V dot gradient of V can be written as, del of half V dot V plus del cross V cross V, now del cross V is the curl of the velocity vector is called the vorticity vector in fluid mechanics, it is the curl of the velocity, velocity is a vector if you take the curl of velocity you will again get a vector that is called the vorticity vector. So, this is essentially what nothing but omega cross V.

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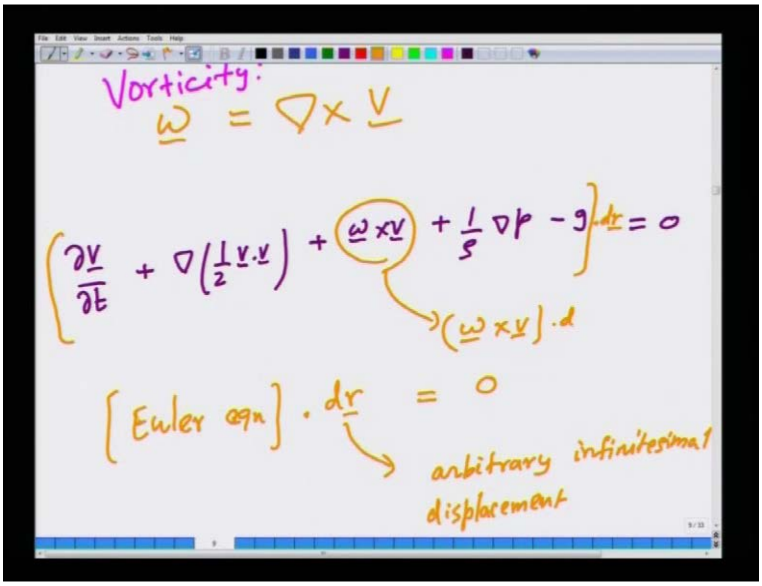

So, I am going to write the Euler equation rewrite the Euler equation as follows, plus del of half V dot V plus vorticity cross V plus 1 by rho grad p minus g, now I am dividing the entire equation by rho. So, that I get rho here. So, I am dividing this entire equation by rho and divide by dividing by rho to get just a rho and del p by rho and this rho and this rho will cancel out minus g is 0, this is the Euler equation rewritten using the vector identity.

Now, I am going to take the Euler equation the above equation and dotted with a small vector displacement vector d r along the fluid, this is an arbitrary displacement vector between any two it is an infinitesimal vector arbitrary infinitesimal displacement vector between any two points in the fluid, it is a vector.

Now, in order to simplify, so we will write this above equation therefore, as this entire equation dotted with d r is 0. So, I am taking the entire equation and dotting out to 0 in order to simplify this equation I should I should somehow get this term omega cross V dot del r to 0.

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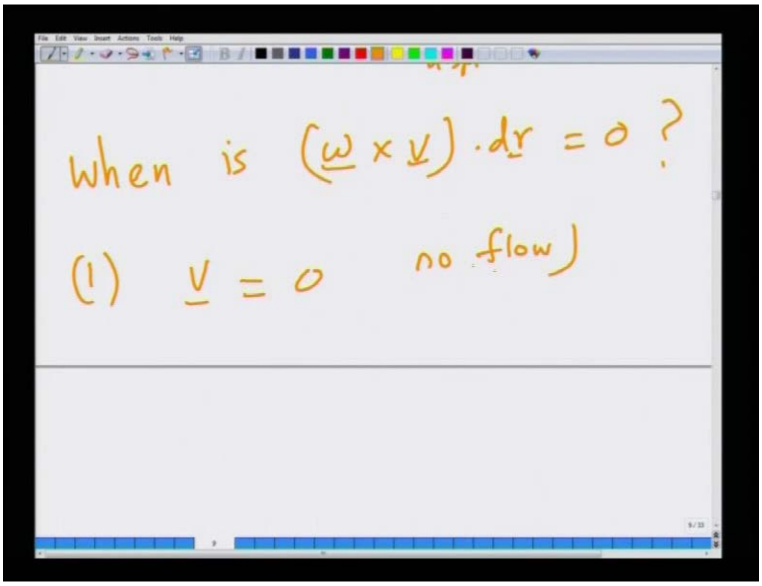

Let us discuss the circumstances, when is omega cross V dot del r is 0, the first thing you will say well, when V is 0 and there is no flow that is a trivial case. So, we will not discuss this.

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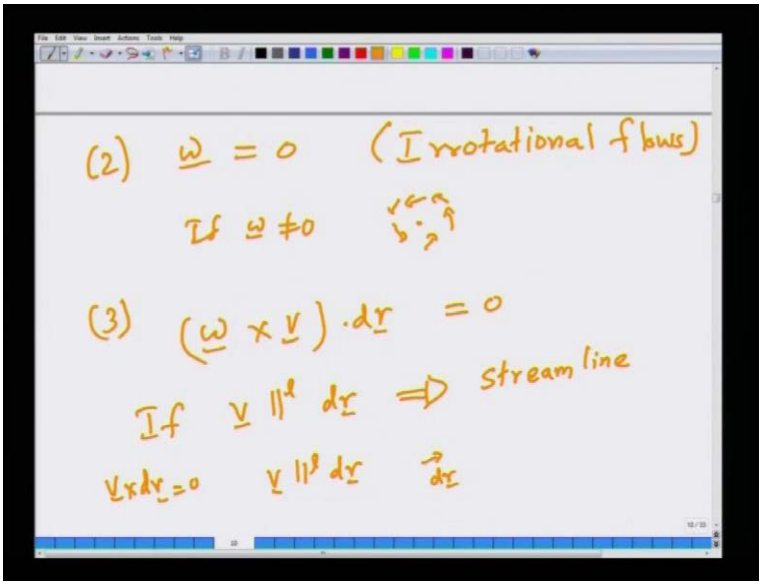

Secondly, when omega is 0 such flows when the curl of velocity is 0 they are called irrotational flows, because omega it turns out tells you how how much circulation is there in a velocity in a velocity fill in a flow fill that is how much there is fluid rotation that is there about a given point, it does not tell you about the rigid rotation of the entire body of fluid, but it tells you if omega omega is non-zero; that means, that about the given point in fluid, if omega is non-zero; that means, about a given point to end fluid that neighboring points undergo a rotation motion, if omega is 0 no such things exists. So, such flows are called irrotational flows.

So, that can happen and omega cross V dot d r can be 0, if V is parallel to d r that is because, if V and d r are parallel then, omega cross V will point in a direction perpendicular to V, because the cross product of two vectors points in a plane perpendicular to the two vectors, if V and D are in the same direction, then omega cross V will be orthogonal to d r. So, the dot product of the two vectors is 0.

When V is parallel to d r what we get is a stream line because, if you take if you take any two points d r the definition of stream function is such that, V cross d r is 0 that is V is parallel to d r and V cross d r is 0. So, this can also happen that for along a stream line omega cross V dot d r is again 0.

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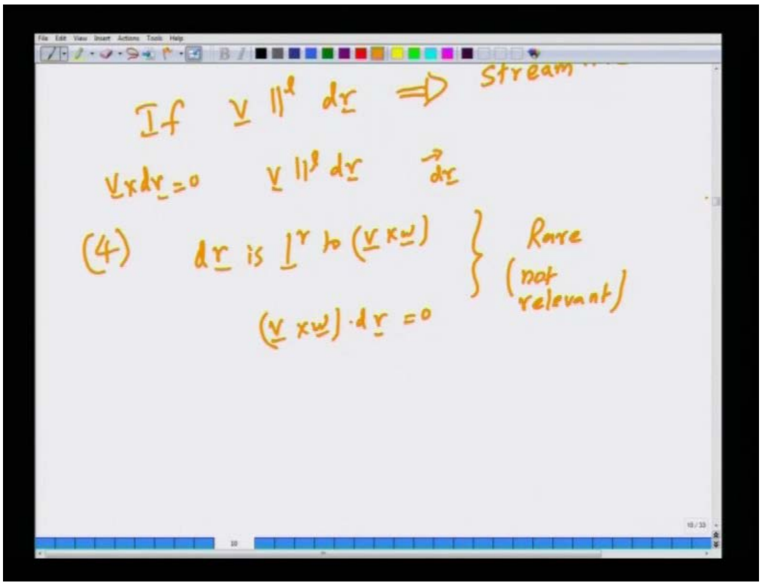

And finally, there is a very obscure or very very rare case that is d r is perpendicular to V and omega because then, V cross omega dot d r sorry yeah will be 0, because if d r is perpendicular to V cross sorry not V and omega it is V cross omega, d r is perpendicular to V cross omega; obviously, this term is 0, but that is a rare case, un important, it is not important.

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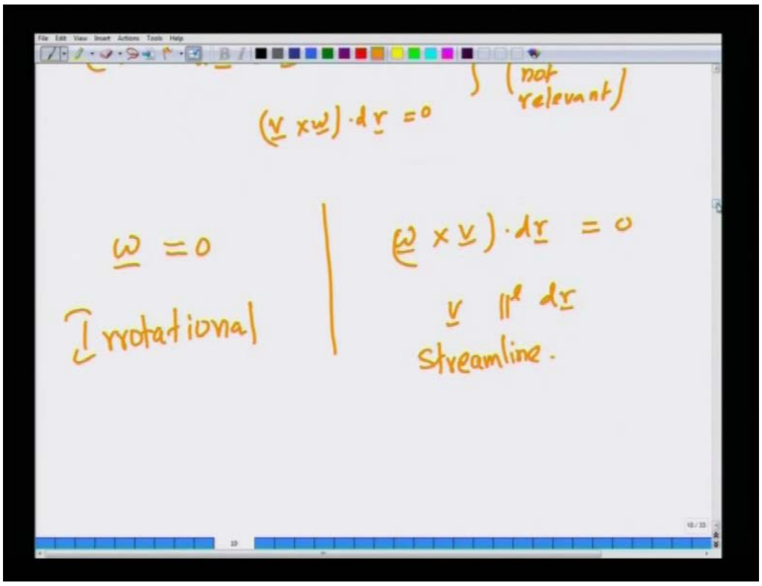

So, what we are interested in are in two cases that is omega itself is 0 that is irrotational flows, which is much more stronger and then another case, where omega cross V dot d r is 0, not because of omega is 0 because, but because V is parallel to d r that is a stream line.

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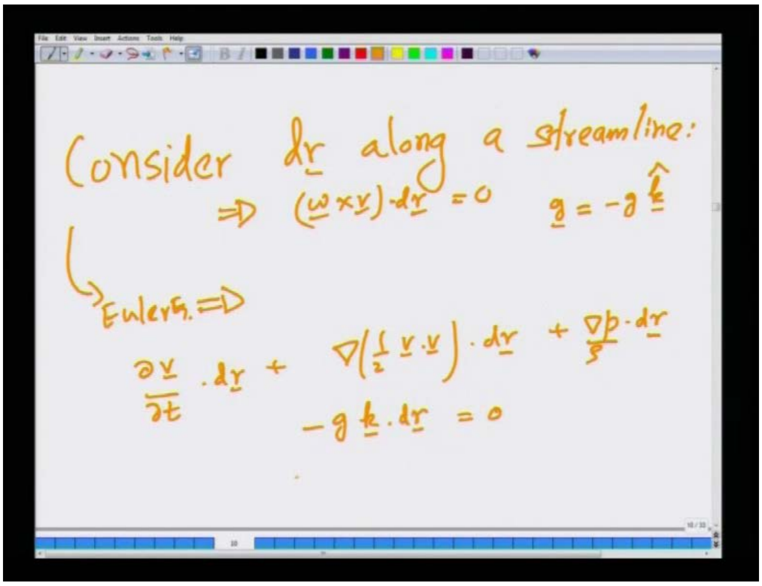

So, we will consider this case consider along a stream line, consider d r along a stream line. So, that you get this immediately says that, omega cross V dot d r is 0 and the Euler equation for such a case becomes Euler equation becomes del V by del t dot d r plus D, so you have gradient of half V dot V dot d r then, you have plus del p dot d r plus g del p by rho dot plus g. Now, let us take the acceleration due to gravity vector in the direction of minus z. So, it is let us written this g k and so unit vector in the k direction. So, you have the acceleration due to gravity.

So, you have let us see you have the minus g minus g vector I am sorry this is a vector this is a vector here I am sorry for that. So, you have minus minus g k dot d r is 0 now that is the relation.

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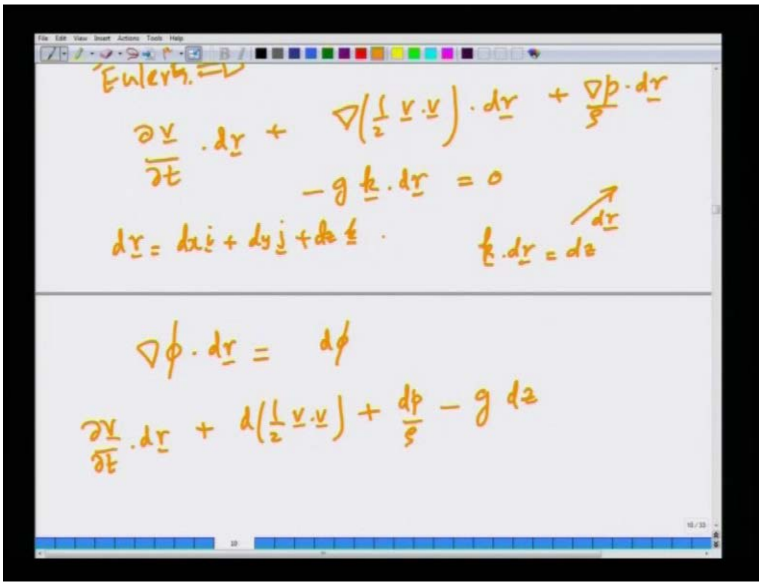

Now, let us try to simplify this further, now gradient of any scalar quantity dotted with d r will give you the differential change in quantity along the two end points of the vector. So, this equation therefore becomes partial V partial t dot d r plus the change in half V dot V plus the change in the pressure across the two points minus k dot d r is nothing but, the difference in suppose you have a unit vector sorry you have a differential vector in some direction this is that projection of that vector along the k direction.

So, this will merely tell you what is z 2 minus z 1 suppose you are let us just write it as minus g d z. So, k dot d r is d z because, d r is d x times i plus d y times j plus d z times k. Now, so this is equal to 0 now I am going to integrate between any two points, this is a differential relation that is valid only for a very small displacement d r vector.

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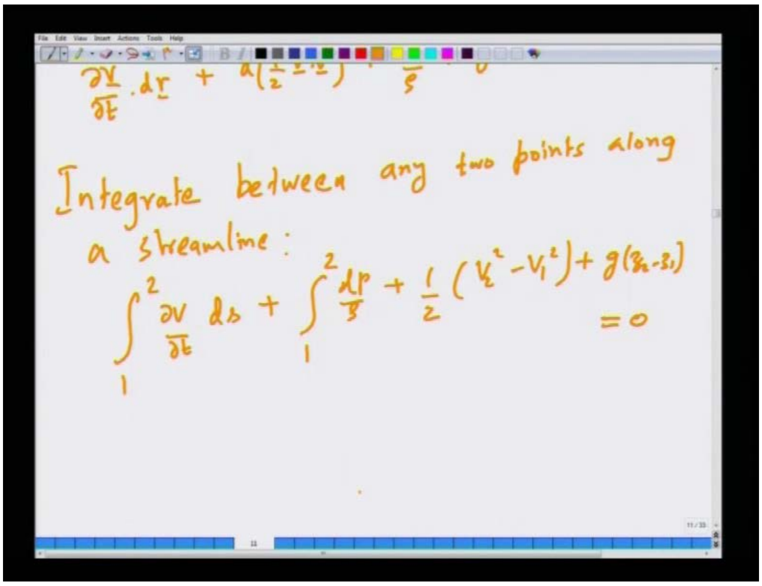

But, if I integrate between any two points along a stream line which I am free to do between any two points along a stream line I get integral 1 to 2 partial V partial t, this is the arc length along stream line. So, you will get this becomes as scalar plus integral d p by rho plus half V 2 square minus V 1 square plus g times z. So, I am sorry there is since g is in the plus k direction, g is actually in the minus k there is already minus g here, so it will become a plus g. So, there is a plus g here plus g times z 2 minus z 1 is 0, there is a minus g because, if you look at the original equation it happens with the minus g and since g was aligned in the minus k direction, the two minuses will cancel to give a plus sign this is equal to 0.

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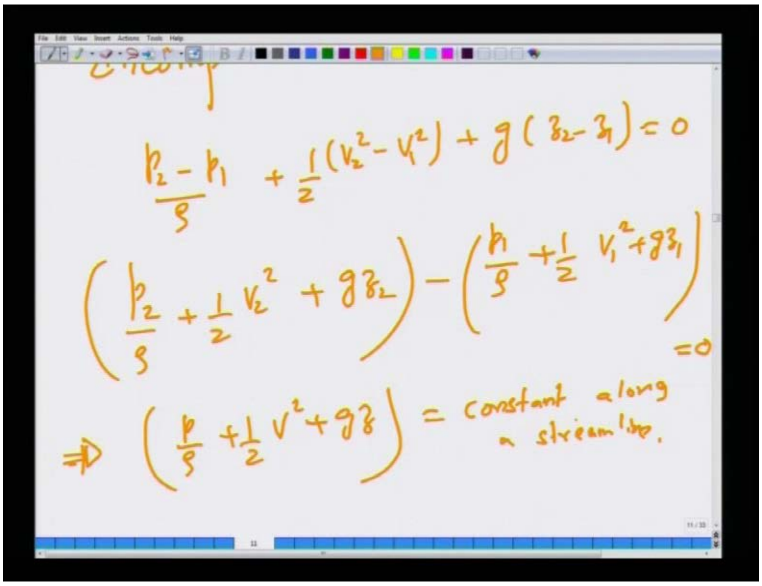

Now, let us consider an incompressible and steady flow, steady flow then, we will have so steady means this is 0. So, we will have p 1 p 2 minus p 1 by rho, rho is constant so we can integrate this. So, it becomes integral of d p, which is p 2 minus p 1 by rho plus half V 2 square minus V 1 square plus g z 2 minus z 1 is 0 or I can rewrite this as p 2 by rho plus half V 2 square plus g z 2 minus p 1 by rho plus half V 1 square plus g z 1 is 0.

Since, points 1 and 2 are arbitrary all this means is that, p by rho plus half V square plus g z is a constant along the stream line.

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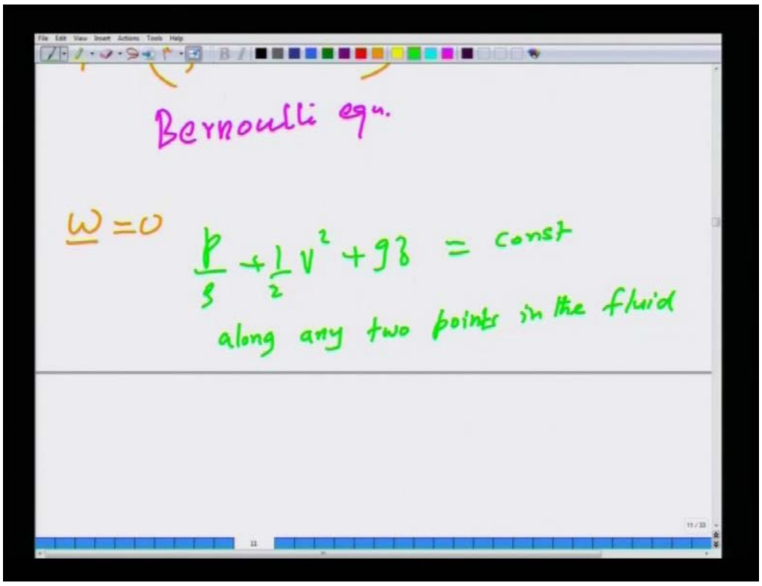

And this is the famous Bernoulli equation, which we derived it as a consequence of internal energy balance, but we had to make assumptions that, the control volume is essentially a stream tube and we shrunk the stream tube to the limit of a stream line and assume that, there is no viscous shear stresses by the surrounding fluids of course, that assumption is there here also that, we are assuming the fluid to bean inviscid fluid; but once you assume the fluid as inviscid fluid the Bernoulli equation directly comes out as a consequence of the momentum balance, the Euler equation and by manipulating the momentum balance we were able to derive the Bernoulli equation which says that, p by rho plus half V square plus g z is a constant along a stream line, so that is the Bernoulli equation.

Now, we can also say what will happen if omega is 0, that was along the stream line, if omega itself was identically 0 then, it means that p by rho plus half V square plus g z is a constant, not just along the stream length stream length, but along any two points in the fluid.

So, these are two different assumptions, in the first case, we are not saying omega is 0 we are merely saying that, the flows is inviscid that is a viscosity is 0 then, the Bernoulli's equation is valid only along the stream line at along any two points along the stream line, but if you are saying that the flow is a irrotational, then the Bernoulli's equation is valid along any two points across the fluid even across the stream lines. So, these are two different types of assumptions that are used and one has to be careful in interpreting the two assumptions we will stop here at this point and we will continue with this theme in the next lecture also, thank you.