Fluid Mechanics

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Module No. # 01

Lecture No. # 35

Welcome to this lecture number 35 on the NPTEL course on Fluid Mechanics for under graduate chemical engineering students. The topic we are discussing is momentum integral equation approach to understanding forces at high Reynolds numbers on solid surfaces, within the boundary layer approximation, so we all are already understood that very close to a solid surface, there is a thin region of a thin region where the fluid velocity varies very rapidly at high Reynolds numbers, and because of this high velocity gradients, viscous shear stresses become dominant very close to solid surfaces even at high Reynolds numbers.

So, in order to predict, what are the forces on solid surfaces, it is necessary to solve in principle, the Navier stokes equation in the boundary layer, but we will not do that in this introductory course rather we will take a simplified approximation.

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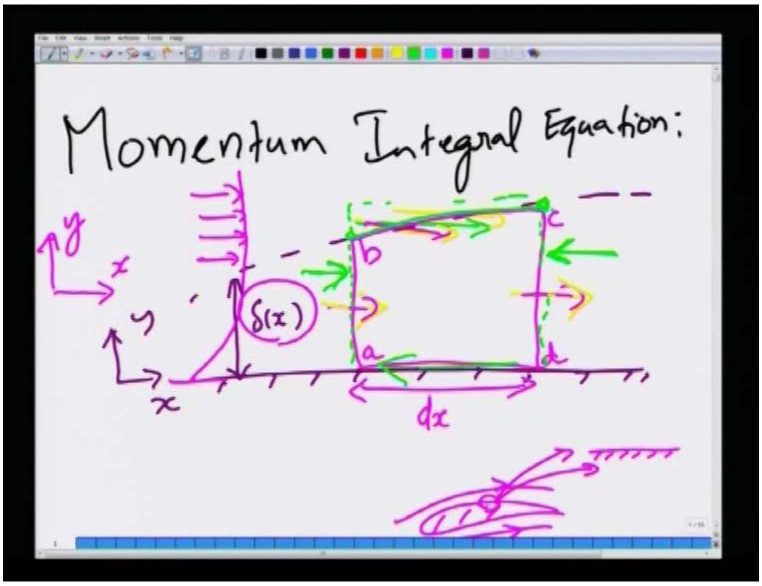

We will take a tiny width d x, tiny distance d x along the length of is plate, for example, we argued in the last lecture that, even if you have a flow past a stream line body like an air foil, locally since the boundary layer thickness is very very small, locally, this surface is going to appear like a flat surface, so that is the reason why we are focusing only on a locally flat surface.

And so essentially we have a fluid flow past a flat rigid plate and then there is a boundary layer, because of the fact that the fluid velocity as to go 0 velocity at the surface of the plate. Now, the boundary layer thickness in general is denoted by the letter delta and we are going to define how the boundary layer thickness is defined shortly, we are going to explain, how the boundary layer thickness is defined shortly. But in principle it could be a function of the distance along the plate, so x is the flow direction and y is the direction perpendicular to the flow, normal to the plate.

Now, we consider a infinitesimal control volume a b c d and then do an integral momentum balance, the integral momentum balance simply says that the momentum flux, net momentum flux due to inflow and out flow through this three faces of the control volume must be equal to the net forces acting on the control volume. The momentum flux can enter the control volume through a, the face a b leave the control volume through the face c d, but it can also since fluid is flowing like this, this is just the edge of the boundary layer it is not a stream line.

So, fluid can enter the boundary boundary layer through the surface b c also, so we had derived mass conservation equation for this control volume in the last lecture and related the various mass fluxes. And so the momentum can enter through all the three faces, along the face a d the bottom surface it is a rigid surface, fluid cannot come in through the face a d because, there is the no penetration boundary condition at the surface a d.

Now, the forces that act on the various faces are firstly, the pressure force along a face a b, the pressure force will be in the plus x direction along the face c d, the pressure force the pressure force will be in the minus x direction, the pressure force will act here in the plus x direction, pressure force will act here in the minus x direction. And then there are also pressure forces that act on this curved surface, which will take as the average of the pressure between the these two stations and then multiplied by the projected area.

This is something that we are familiar from fluid statics on how pressure forces act on curved surfaces. Finally, at the surface a d the viscous shear stresses by the, viscous drag by the plate on the fluid will act in the minus x direction.

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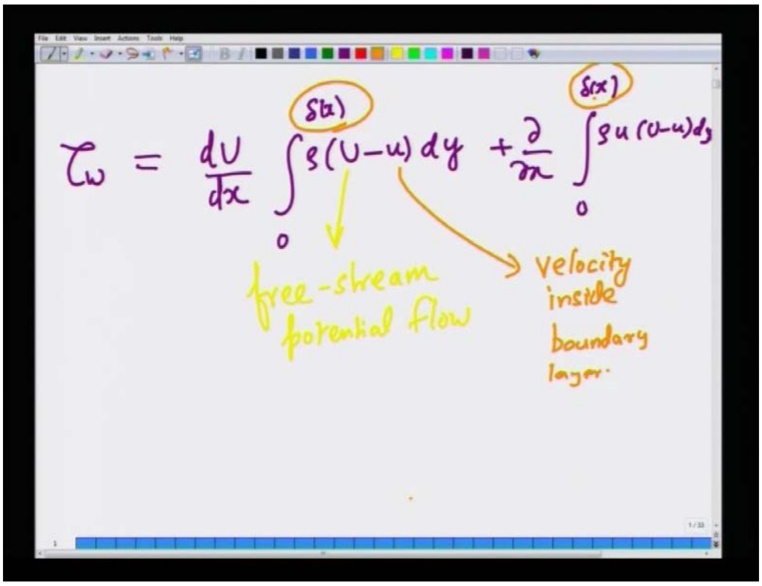

Now, if we account for all these things, we ended up with this final differential equation in the last lecture, which relates the wall shear stress, tau w the force per unit area is equal to d U d x integral 0 to delta rho times capital U minus small u d y plus d d x delta is in general a function of x of 0 to delta of x rho u capital U minus small u d y. Now, just to again familiarize you with the notation, this is the free stream potential flow velocity profile outside the boundary layer.

And the small u is the velocity profile inside the boundary layer velocity profile inside the boundary layer and the same notation goes here also, notice that the boundary layer thickness is in general a function of x therefore, even if you integrate this with respect to y ultimately the boundary layer thickness is also the function of x, so this d d x will also depend on the upper limit of this integral. Now, in order to proceed further, I am going to simplify this even little bit like I did in the last lecture.

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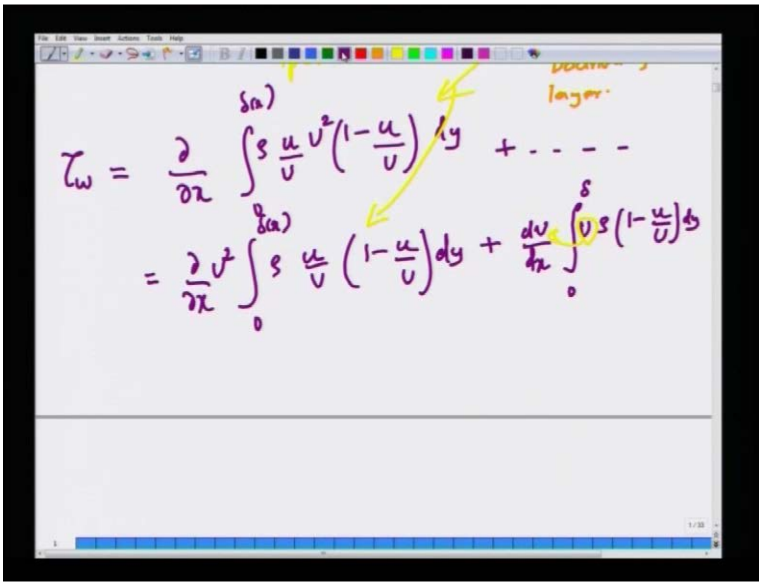

So, I am going to write this as d d x of U square, so if you look at this expression I have U here, so I can write this as d d x of, so let let me do it step by step, so I am going to pull out going to pull out capital U here, so I get let us take the second term first d d x of integral 0 to delta of x rho u capital U times 1 minus u by capital U and that is the first term and d y sorry that is this term.

So, let me focus on the second term first, so let me simplify this further and then plus the other term, which we will come to in a minute, if you simplify this further I can write this as delta x, now I can divide and multiplied by U, which will give you this format and I can pull this U outside the integral, because the integral is only a function of y. So, this becomes rho u by U times 1 minus u by U d y, this is the second term, which further simplifies here.

Now, the first term can be simplified as follows, if you look at the first term, you have u d U d x, so I can again pull out a factor of capital U and write this as d U d x times integral 0 to delta U times 1 minus I am sorry rho times 1 minus u by U d y, so if you look at this term, so I can pull out U out, so I will get a factor of u divided by U and since, this capital U is a function of x only, I can pull this outside the integral and write here outside.

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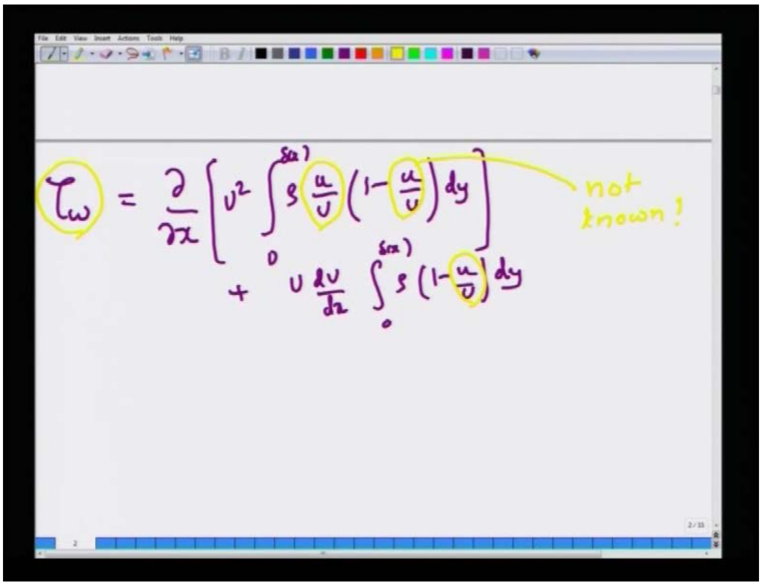

That deals as the final expression for wall shear stress, is d d x of U square integral 0 to delta of x rho u by U times 1 minus u by U d y plus U d U d x. Since, it U is independent of y we can pull it outside the integral 0 to delta of x rho times 1 minus u by U d y, so this is the final expression for wall shear stress. Now, if I tell you what is this velocity profile within the boundary we can do the integration and then we can find an expression for shear stress, but this is not yet known, this is not known, so we have to make some approximations in order to solve this equation.

Before, I do spell out, before I spell out what these approximations are, let me first tell you, what are the definitions of boundary layer thickness, various definitions, so far I have not rigorously defined the boundary layer thickness.

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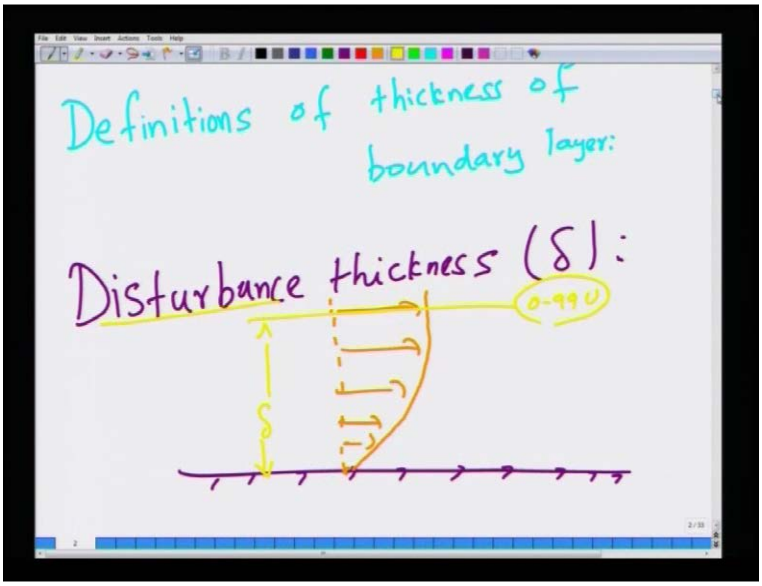

The first definition is called, the disturbance thickness delta normally denoted by the letter delta what it essentially means is the following, suppose I draw the velocity profile in flow past a flat plate. Now, the velocity will be free stream velocity far away from the flat plate and it will go to 0 at the flat plate at the rigid surface itself, but the way in which the velocity approaches the free stream velocity is not abrupt it is only gradual or it is asymptotic.

So, there is no single point where the velocity is exactly equal to the free stream velocity, its slowly approaches the free stream velocity therefore, we can define delta as that suppose you ask the question at what thickness is u the velocity in the boundary layer is pointing 99 percent of the free stream velocity, that is called the the disturbance thickness.

So, one could have different definition somebody could say what, at what distance it is 99.9 percent or simply 90 percent and so on, there could be variations, but there all in a sense there is some arbitrariness in the definition of the disturbance thickness, but once we all agree to this convention that it is 99 percent, then that is the distance over which the velocity varies from 0 on the bottom surface that is a rigid plate to 99 percent of the free stream velocity.

Because, it will take a very very long distance to it since, the approach is asymptotic it will take an in I mean infinitely long distance to approach truly speaking the actual free stream velocity, but you can approach 99 or 99.9 percent or 99.99 percent fairly, quickly and that is a matter of a definition. So, the other definition is what is called the displacement thickness.

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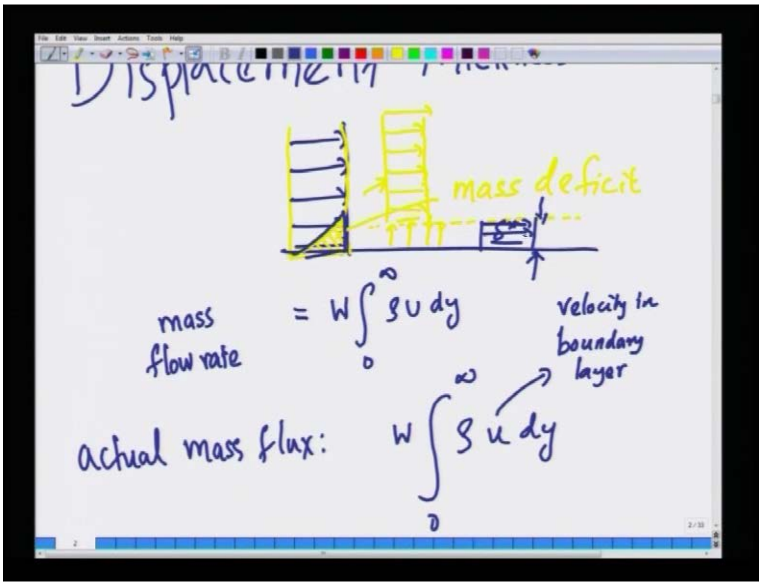

We need all the definitions shortly in our calculation of the shear stress that is the reason why we are going through this discussion, now imagine if you had no boundary layer, the flow velocity is the free stream velocity let say it is a uniform velocity, then the mass flow rate is essentially or mass flow rate per unit area is integral 0 to infinity rho U d y times w is the width, this is the mass flow rate, mass per unit area. Now, the actual mass flux is not the same, because if I were to draw the actual velocity profile it is going to look like this.

So, there is a region of deficit that is there in reality, this shaded region is the mass deficit, because that much is not flowing which would have happened if the velocity profile was uniform right, through so this shaded region is the mass deficit. So, the actual mass flux is actually integral W 0 to infinity rho small u d y, because this the actual velocity profile, the boundary layer velocity profile, boundary layer, so now the way in which, we envisage or we defined the displacement thicknesses.

So, essentially it is almost like what the the net effect of the boundary layer is almost like lifting the plate to some distance, so that after this distance the flow is uniform, and the actual flow rate which is obtained by combining integrating this curve is obtained by simply taking u times, this distance, so essentially that is what we have saying. That the net effect of the boundary layer is as though it is trying to lift the flat surface, so that he mass deficit is accounted for, so how do we do that, we are going to set the losses equal suppose we were, suppose you call this thickness delta star this mass deficit must be equal to this mass deficit, that is the way we are defining delta star.

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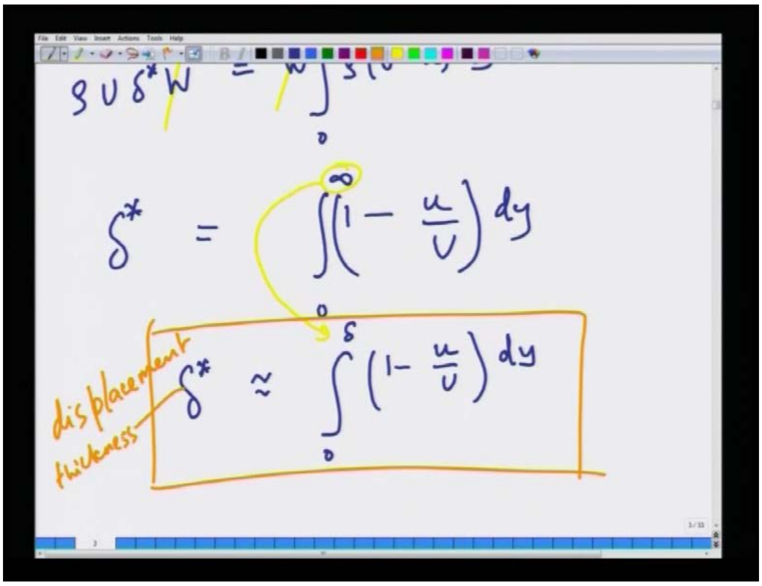

So, the mass deficit due to the lifting of this imaginary surface by an order delta by an amount delta is simply rho U delta star w, but that must be equal to integral rho U rho times capital U minus small u d y, because capital U minus small u is the deficit if you remember, this is the yellow profile is small u here, and the blue profile is capital U, the difference is this deficit which is shaded in this is the deficit region, shaded in yellow.

Now, we have equating this deficit to this deficit there by obtaining a definition of delta star, so delta star and sure enough there is a w here, which will cancel in both sides. So, delta star is nothing but, integral 0 to infinity 1 minus u by capital U d y or delta star is integral since, if you look at these two curves, they are almost the same after a point, after some point of the order delta star itself their almost the same, so we can take that distance as delta which is the disturbance thickness.

So, I can remove infinity in this integral, in the upper limit of the integral to delta x itself, so delta star is approximately 0 to delta 1 minus u by U d y, this is called the, this is the displacement thickness, which is essentially the idea the idea here is that, the net effect of having a boundary layer is that it tends to reduce the mass flow rate by certain amount and therefore, in effect you can imagine that in order to calculate the same mass flow rate from the potential velocity profile, it appears as though you have to lift the solid wall by a certain amount, the amount to which you lift is called delta star, that is the displacement thickness. So, the reason why it is called displacement thickness is because, this essentially means that you are displacing the plate by a certain amount.

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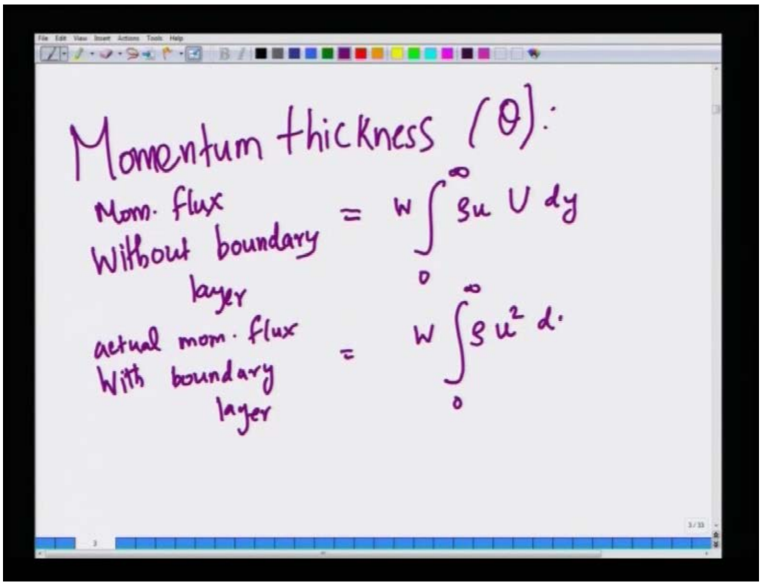

The other definition is momentum thickness, the definition of thickness of boundary layer is momentum thickness, usually denoted by the Greek symbol theta, now instead of worrying about the deficit in mass here, we are not going to talk about the deficit in momentum. So, the question we are asked the the definition is in in that sense very much similar in spirit to the displacement boundary layer thickness definition, where in you worried about the amount of the thickness by which the plate must move in order to so that, the deficit in mass is taken care of, so here we are going to worry about the deficit in momentum.

So, without the boundary layer the momentum flux, momentum flux without the boundary layer is nothing but, integral 0 to infinity momentum flux is simply mass flux times, the actual mass flux which is rho u times capital U d y times W, so with the boundary layer, the actual momentum flux (No audio from 17:19 to 17.35) is W 0 to infinity rho u squared d y.

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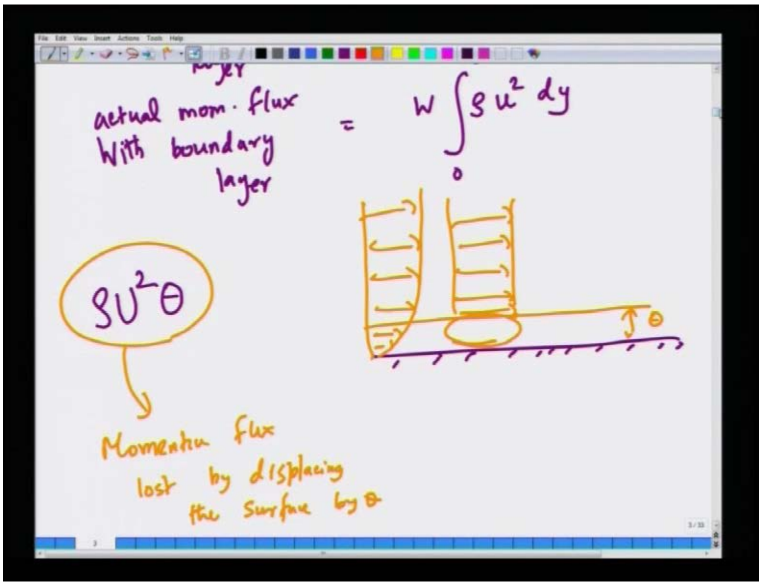

So, the deficit is simply, so imagine that without imagine that you are lifting the boundary layer, this is the actual surface of the plate, now imagine that you are lifting by an amount theta, so that the approximate momentum flux sorry, the momentum flux calculated from potential flow up to this distance is the same as the actual momentum flux carried by the boundary layer, so this is the momentum flux.

That is lost by displacing, that is lost here, because you have displaced the plate by an amount, the surface by theta, that must be equal to the difference between the difference between the momentum flux without the boundary layer and this is the momentum deficit, because this is assumed without the boundary layer, this is assumed with the boundary layer.

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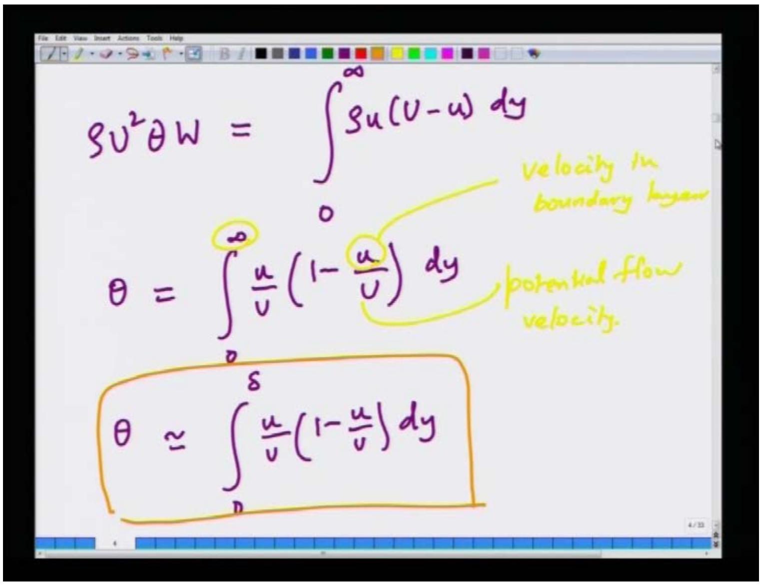

So, by equating the two we will get rho U squared theta w this is the momentum deficit by lifting the plate, displacing the plate or the rigid surface by an amount and this is nothing but, rho times u times capital U minus small u d y now therefore, theta is essentially integral 0 to infinity u by small u times 1 minus u by sorry u by capital U d y.

Again to remind you small u is the velocity profile in the boundary layer, and capital U is the potential flow velocity profile, outside the boundary layer (No audio from 19:36 to 19.44) velocity. So, we have now therefore, three definitions and likewise we just as we did before, we will going to replace the infinity in the integral by the displacement thickness delta, so theta is approximately equal to u by U d y, this is the definition of momentum boundary layer thickness, momentum boundary layer thickness.

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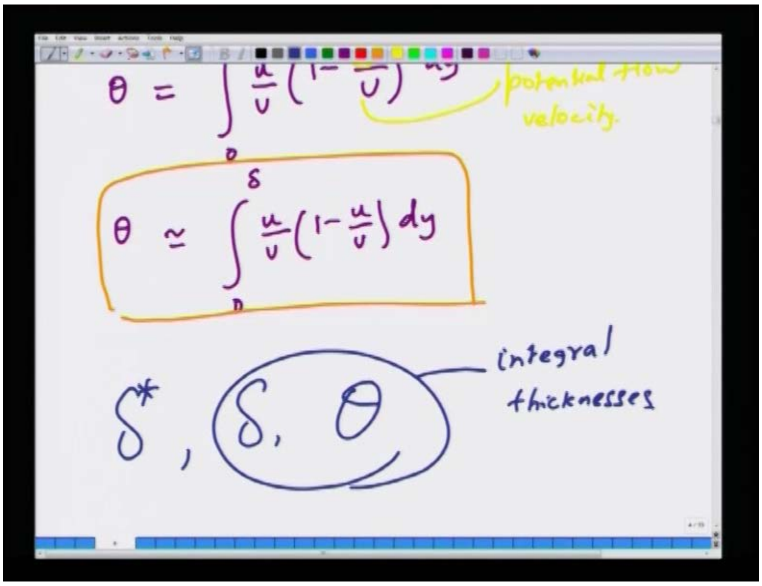

So, you had now three definitions, you have delta star the disturbance thickness, delta the displacement thickness and theta which is the momentum thickness, these two are called integral thicknesses, because they are obtained as an integral of the velocity profile. The mass displacement thickness and the momentum displacement thickness are called the integral thicknesses, while delta star is simply the distance it takes for the velocity profile to reach 99 percent of the free stream velocity, so now therefore, we going to rewrite the momentum equation, momentum integral equation.

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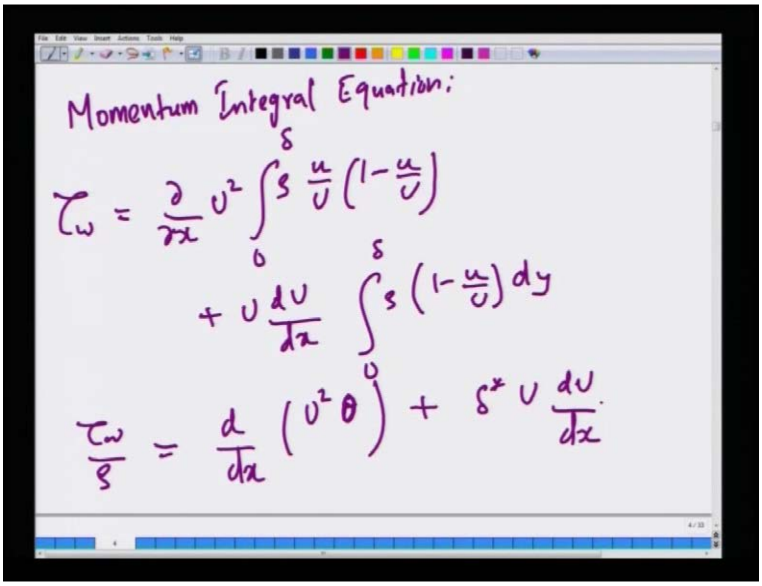

So, let us go back to the momentum integral equation and rewrite it in terms of the newly defined boundary layer thicknesses, the momentum integral equation was simply tau w is d d x of U square integral 0 to delta rho times u by U times 1 minus u by U plus U d U d x times integral 0 to delta rho times 1 minus u by U d y, after using the definitions of the momentum and displacement thickness we get this is simply nothing but, d d x of U square theta plus delta star U d U d x.

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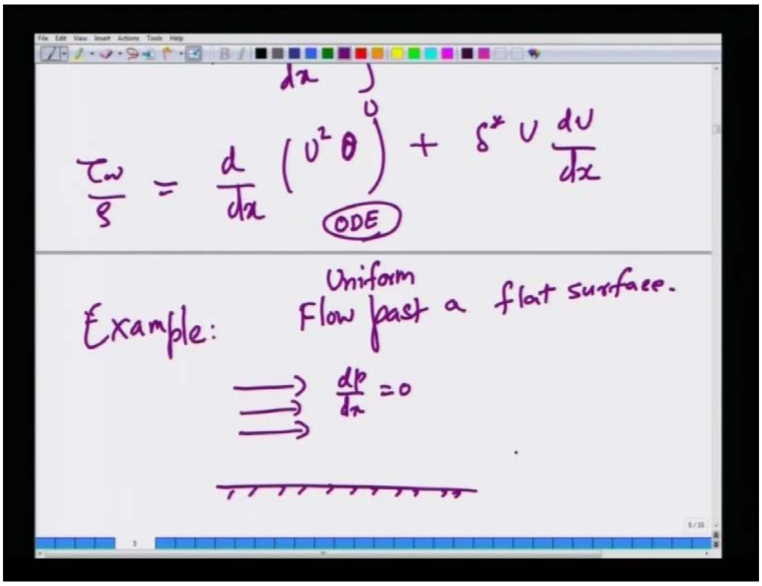

So, this is like an this is an, this is an O D E or it is an ordinary differential equation, in x and it has to be solved to find what is the shear stress. Now, let us look at a special example, let us look at a particular case, this is one of the classic cases in boundary layer theory, you have flow past a uniform flow past a flat plate flat surface, rigid surface. So, imagine that outside the boundary layer, the flow is uniform that is no pressure gradient in the potential flow.

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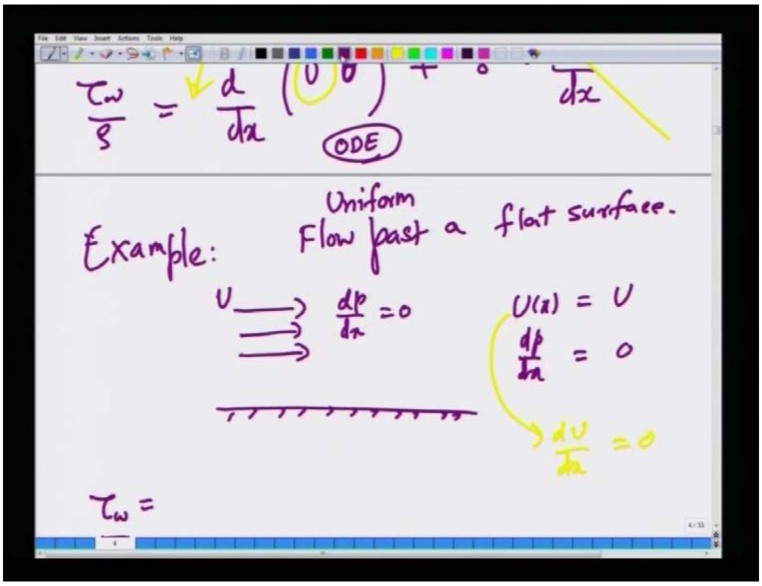

So, this is the uniform flow case, so U of x constant velocity U independent of x and d p d x is 0, these are the two conditions. So tau w essentially becomes since, d u if d u x is U of x capital U then d u d x is 0 it is independent u. So, the wall shear stress expression just written here, this term will go away, because d u d x is 0, so essentially what you will have is tau w by rho is d d x sorry, you will just have this expression since, u is constant you can pull this out also (Refer slide Time: 23:28).

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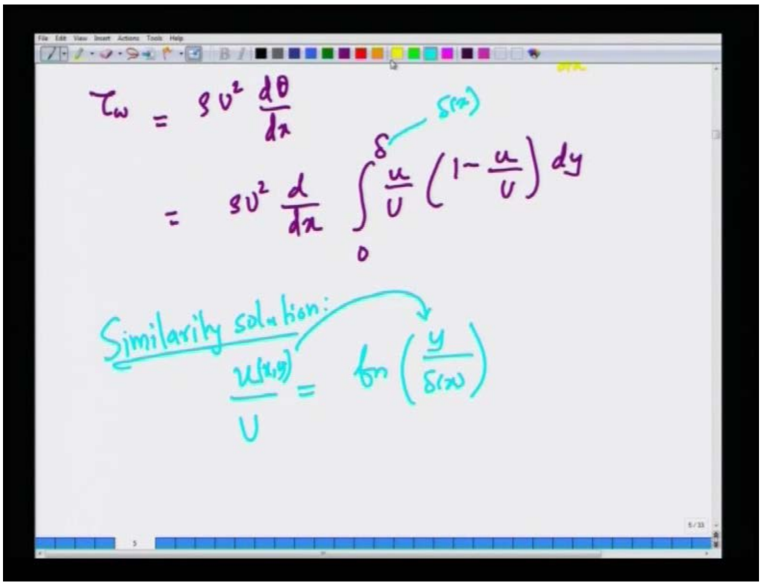

Now therefore, tau w by rho is written as is simply written as, (()) we can write tau w as rho U square d theta d x is nothing but, rho U square d d x of integral 0 to delta u divided by capital U times 1 minus u divided by capital U d y. Now to proceed further, we need to know what is the function small u, what is the velocity profile in the boundary layer, of course, we have not solved for it, and so we have to make an approximation. Usually it turns out that within the boundary layer, there is a similarity solution in the sense that the velocity profile u by U is nothing but, a function only of y divided by delta of x.

So, remember the delta as a function of x here, so velocity profile in the boundary layer is general a function of both x and y but, the similarity solutions puts a constraint on how it is a function of x and y, it is a function of x and y only through this combination y divided by delta of x, so that leads to some simplification.

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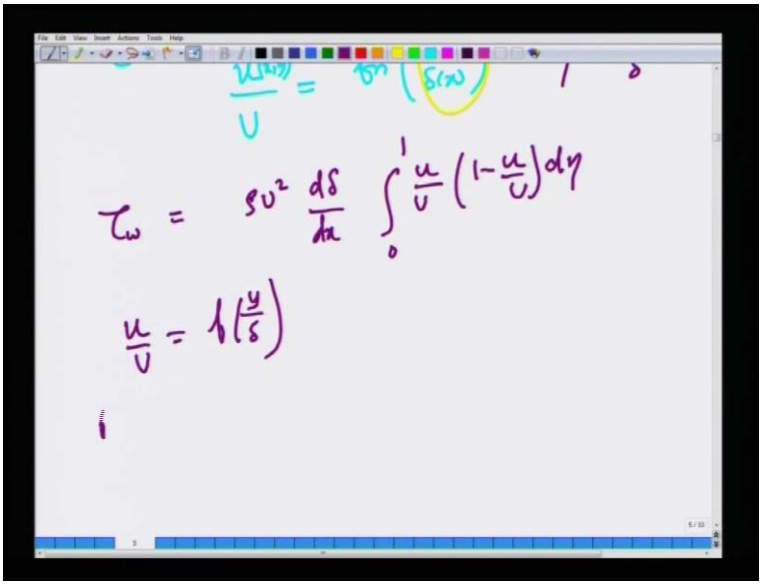

So, you write tau w you, now you define a new variable eta is y by delta and transform this equation as rho U square d delta d x integral 0 to 1 u by U times 1 minus u by U d eta. Now, in order to, so essentially you have to solve this for delta of x in terms of w, tau w, so in order to do that you have to first prescribe a form for u by U, since u by U is a function only of y by delta.

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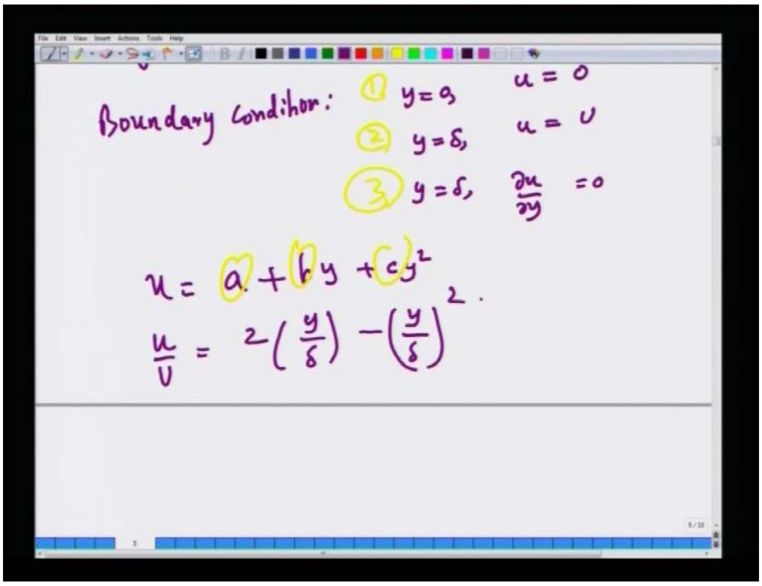

We can stipulate this function as follows using the boundary condition, that at y equal to 0 small u must be 0 no slip condition, y equals delta small u is exactly equal to capital U that is an assumption, because although it is only 0.9 or 0.99 times capital U we are making an error here, but it is it is it is an approximate solution here.

So, y equals delta, we are saying that d u d y is 0, that is velocity profile as you approach the edge of the boundary layer it becomes independent of the y direction, it becomes a flat profile. So, once you assume a function for y, u I am sorry that satisfies all this, so let us assume u is a polynomial of the form a plus b y plus c y square, you have three constants unknown constants a b c and you have 1, 2, 3 equations once you solve for this, we will get a functional form for u by U u by u becomes 2 times y divided by delta minus y divided by delta square or u by u is nothing but, 2 eta minus eta square minus eta square.

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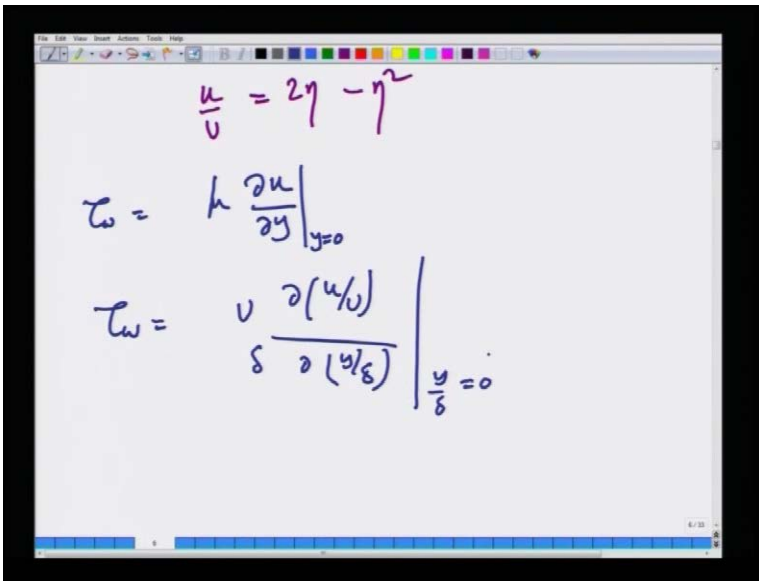

So, once you substitute this back in here, we can now integrate this expression and get an expression for tau w, now tau w is nothing but, mu times partial u by partial y evaluated at y equals 0. Now, since we have everything in terms eta, we write it in terms of and capital U by small u by capital U, we write it in terms of y by delta times delta evaluated at y by delta is 0, this is nothing but, tau w this is mu u by delta times d of u by u divided by d delta evaluated at eta e equal to 0.

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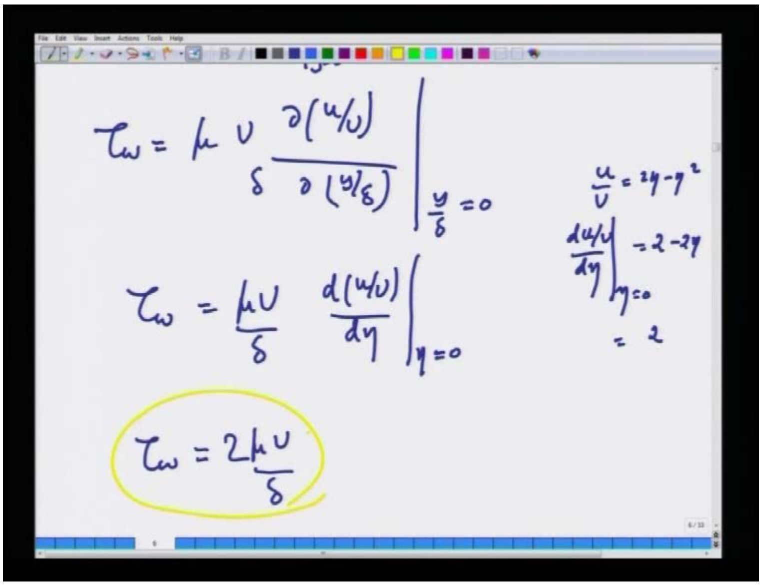

So, tau w is nothing but, mu u by delta times you have to set the derivative of 2 eta minus eta square, so u by U is 2 eta minus eta square d of u by U divided by d eta is nothing but, 2 minus 2 eta, if you evaluate it at eta equal to 0 this becomes simply 2, so we will get the answer 2 mu u by delta, that is the shear stress. Now, you have to substitute this back in the integral momentum equation out here instead of tau w, we are going to write 2 mu U divided by delta, and we are going to substitute for u by U the profile 2 eta minus eta square, everywhere here and then we will get a close form expression for delta (Refer Slide Time: 28:36).

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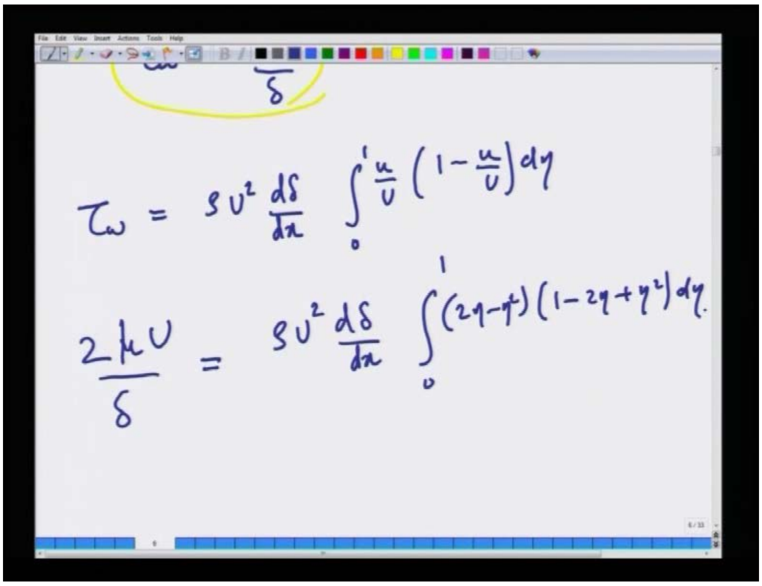

So, we will just do the algebra, we get we already had tau w is rho U square d delta d x times integral 0 to 1 u by U times 1 minus u by U d delta, so 2 mu U by delta is nothing but, rho U square d delta d x times integral 0 to 1 2 eta minus eta square times 1 minus 2 eta plus eta square d eta.

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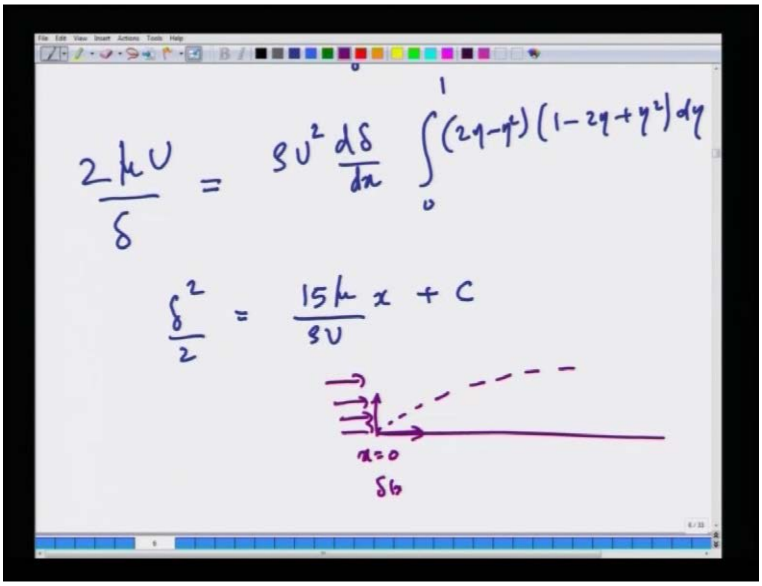

Once you solve this expression, you will get delta square by 2 is nothing but, 15 mu by rho U x plus C, so now let us assume that at the leading edge of the boundary layer, at the leading edge at x equals to 0 the boundary layer thickness, the fluid is just hitting the flat plate at x equal to 0. So, at x x equal to 0 we are going to assume that, delta x is 0 that is the boundary layer thickness just starts from 0 value.

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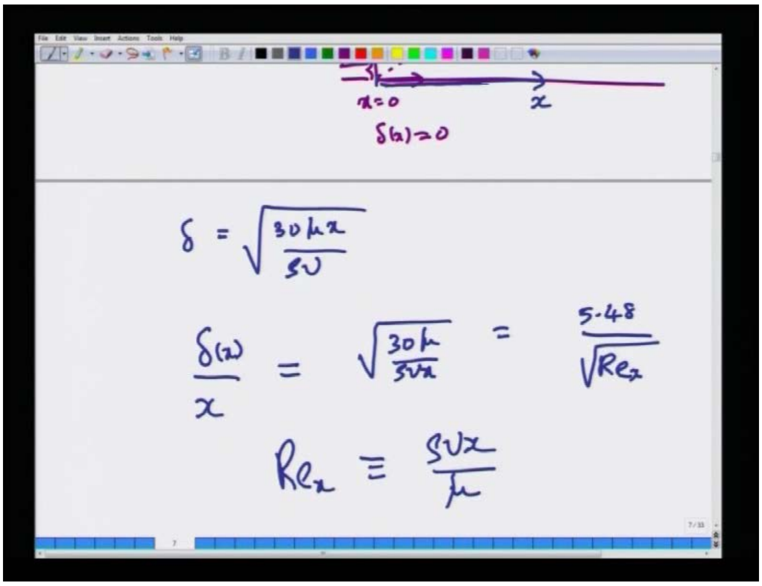

So, the constant is fixed to be 0, so you get delta therefore, is square root of 30 mu x by rho U or if you define, if you divide delta divided by x the local distance from the, x is actually the distance from the leading edge of the boundary layer. So, that becomes square root of 30 mu by rho U x this is nothing but, 5.48 divided by the local sorry local Reynolds number, which is defined as rho U x divided by mu where x is actually the distance from the leading edge of the boundary layer, from the from the distance of the origin of the plate, so this is the Reynolds number.

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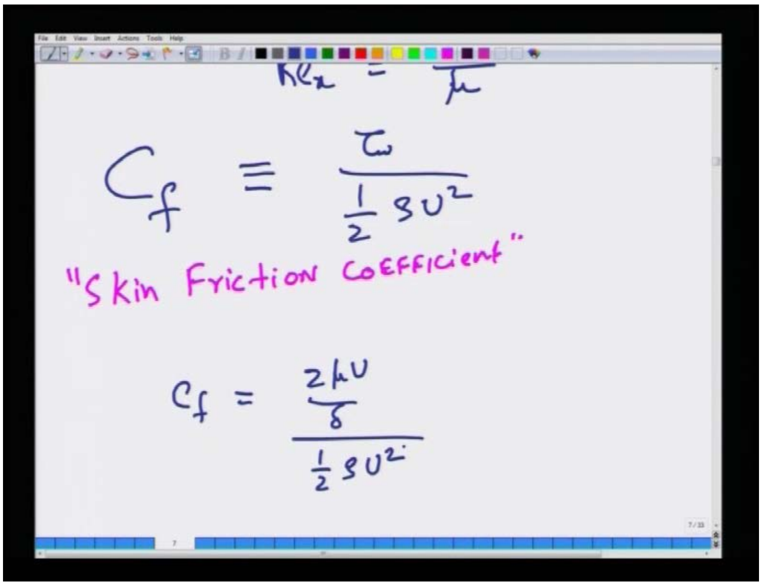

Now, the wall share stress coefficient is defined as tau w by half rho U square, this is called the skin friction co efficient, so this is nothing but, now I know what is taw w, which is 2 mu U by delta divided by half rho U square.

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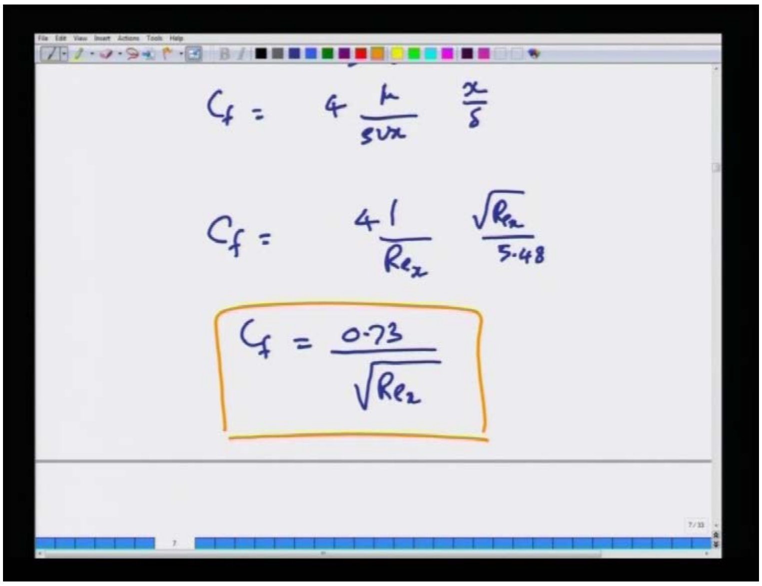

So, C f is nothing but, now after substituting what is delta I will get 4 mu by rho U delta C f is nothing but, 4 mu by rho U x times x by delta, so C f is nothing but, 4 times 1 over R e x, the local Reynolds number times x by delta is nothing but, square root of R e x by 5.48, so C f is nothing but, 0.73 divided by square root of R e x. So, this is the expression for skin friction coefficient which is the non-dimensional shear stress exerted by the fluid on the wall.

So, this is an example of how one can get from reasonably simplifying assumptions, that the velocity profile is simply a polynomial of the form, the constant plus something times y plus some other constant times y square, I am fixing those constants by physically motivated boundary conditions. Then we can actually solve for the momentum integral equation and get and an expression for the force exerted on a flat plate per unit area. Now, if you want to know, what is and that its self is a function of the distance from the origin of the plate therefore, if you want to have force exerted on an entire plate, you have to integrate the stress over the distance of along the plate from 0 to l, since is the function of x we have to integrate this from 0 to l and that will give you the force.

So, this tells you very, in a very simple way how boundary layer ideas can we use to find forces on plates, now the same thing can be carried over two more complex objects, now here we took simplest possible case, namely force on a flat plate, the same thing is valid.

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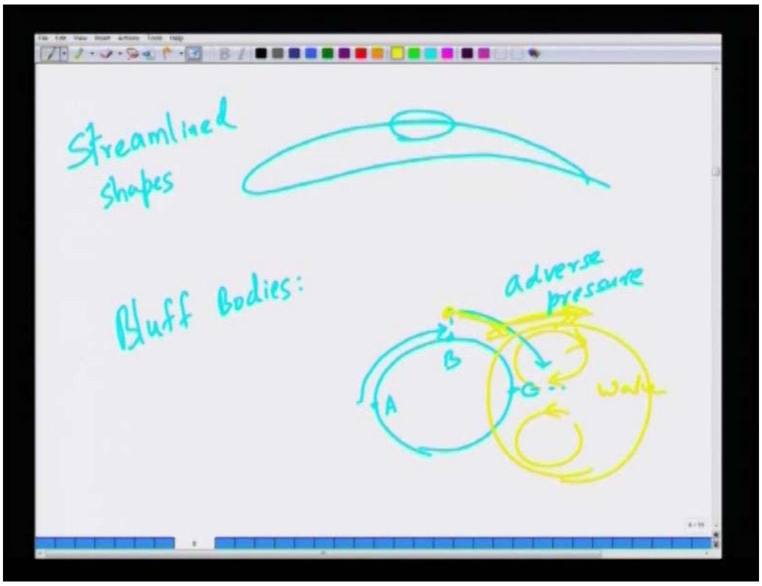

If you consider even an airfoil, because locally the boundary layer is actually flat, so you can integrate over the contour of the surface to get the drag forces, so for stream lined objects such as, remember that stream line object or stream line bodies shapes or stream lined shapes have no separation. So, no market separation on the rear side of the flow, so of the object, so you can calculate the entire drag force to a good approximation you sing only the skin drag, the form drag is usually negligible, but if you consider bluff bodies then like a sphere or a cylinder, then it is not easy to calculate the drag force on a sphere on the cylinder, only using the boundary layer theory.

The reason is because, as I have been mentioning, you know that as the fluid flow from point A to point B, the pressure goes from a maximum to a minimum, while the velocity goes from 0 to maximum, from B to C the pressure goes from minimum to maximum, so B to C is a region of adverse pressure gradient, adverse pressure gradient. So, as a fluid partial which moves from B to C, it moves by virtues of its kinetic energy and that is being converted to pressure head according to the Bernoulli equation.

But Bernoulli equation does not viscous effects in reality close to the solid surface there is a boundary layer, and there will viscous effect which will tend to retard the fluid particle, at some point the partial will come to static, stagnant case case and then, it will be washed away by the surrounding flow.

So, this region of recirculating fluid flow is called the wake is called, the wake and the bound layer theory fails here, because no longer the viscous effects are confined only to a small region close to the solid surface. It is actually a very macroscopic region, comparable to the size of the sphere or cylinder itself therefore, the the simple boundary layer analysis that is design to work for steam line objects without separation, does not work for block bodies.

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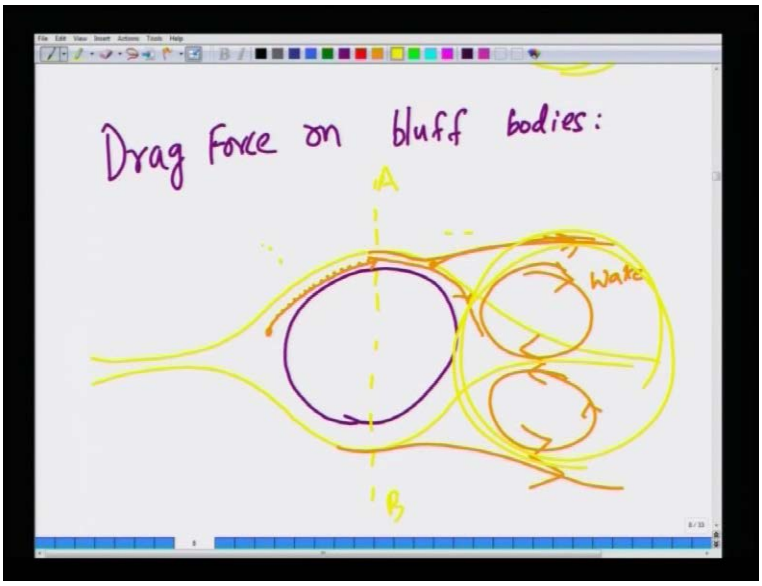

So, when you consider flow past bluff bodies, like cylinders or spheres bluntly shaped objects, and then the bound layer theory is not enough to compute the drag force from first principles, because of this phenomenon called separation, suppose you consider flow past a cylinder.

Let us say, and this is the potential flow velocity profile, this is completely symmetric about the about the fore and aft of the cylinder, this is the potential flow velocity profile but, what happens in reality is that, when you consider a fluid partial which goes from here to here it is favoring, it is facing favorable pressure guidance. The pressure is decreasing as it goes but, when it goes from here to here it is facing adverse pressure gradients, so the particle comes to a stance still at some point, because of the viscous effects close to the boundary layer and it gets washed away the flow profile, actual flow profile will look like this.

And this region of increase circulation is called the wake, this is called the wake, now what is the inserting about the wake is that, it completely alters the picture that the boundary layer where viscous effect are supposed to dominate is only a thin object because, the entire region of the wake viscous effects are important. So, this actually for block bodies therefore, the picture that the boundary layer is is is region of very small thickness compare to the object, objects dimension itself is valid only before separation and the movement of flow separates from the body due to adverse pressure gradient.

Therefore, the bound layer picture breaks down drastically and what is important is that, now the the force on the object is dominated not just by the skin friction which is due to the shear stresses on the boundary layer. But, this is also dominated by the difference between the pressure on the fore and aft side and that causes that imbalance and pressure causes the huge contribution to the drag force that is called the form drag.

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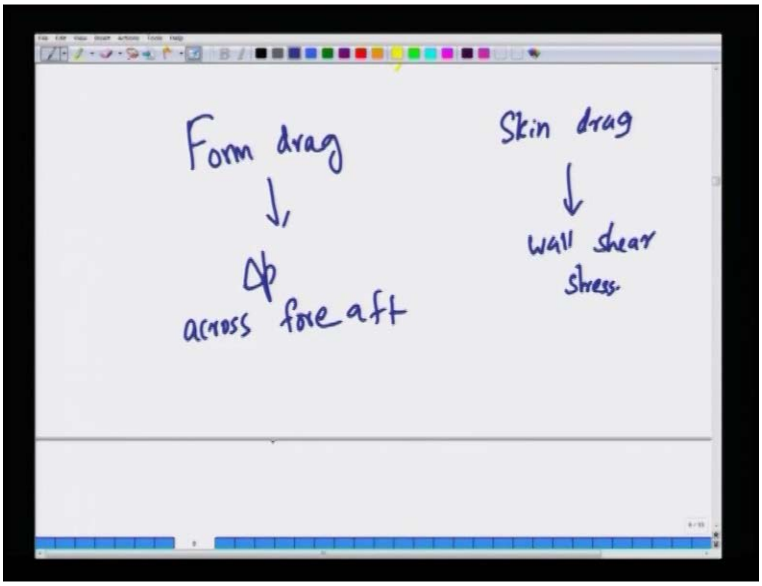

I suppose to skin drag which is due to wall shear stress, the form drag is due to the pressure differences across the fore and aft of the of the body skin drag is due to wall shear stress. Now, therefore, in order to calculate, what is the drag on object such as, fear of cylinders it is not just possible to do theory or analysis alone, you have to do experiments.

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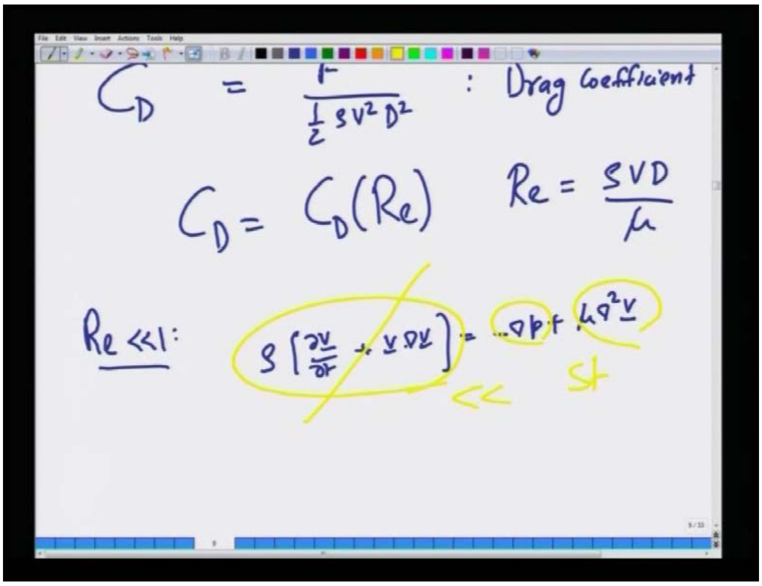

And experiments, if we remember we did dimensional analysis tell the drag force in terms of a non-dimensional drag coefficient which is nothing but, the drag force F divided by half rho v square D square, where if D is a diameter of sphere, then that is the drag coefficients, called the drag coefficient. And we found from dimensional analysis a few lectures back, that C D is a function only of the Reynolds number, where Reynolds number is rho V D by mu based on the diameter of the sphere and the free steam velocity for away from the sphere, now the Reynolds number we now is, a measure of inertial to viscous stresses in the fluid.

It is a ratio instill viscous stresses in the fluid therefore, C D is a function of Reynolds number when Reynolds number is very very small, you would expect that the inertial effects are much much small compare to the viscous effect right. And therefore, we can make an approximation, if you remember the Navier stokes equation, we have rho partial V partial t plus V dot del V is minus del p plus mu del square V.

Let us not worry about gravitational body force affects now, now when the Reynolds numbers small the inertial terms are very small compare to the viscous terms, and the pressure forces, so this can be neglected. And this equation these are called the stokes equation, that is you just set the balance between inertial and pressure forces in the flow.

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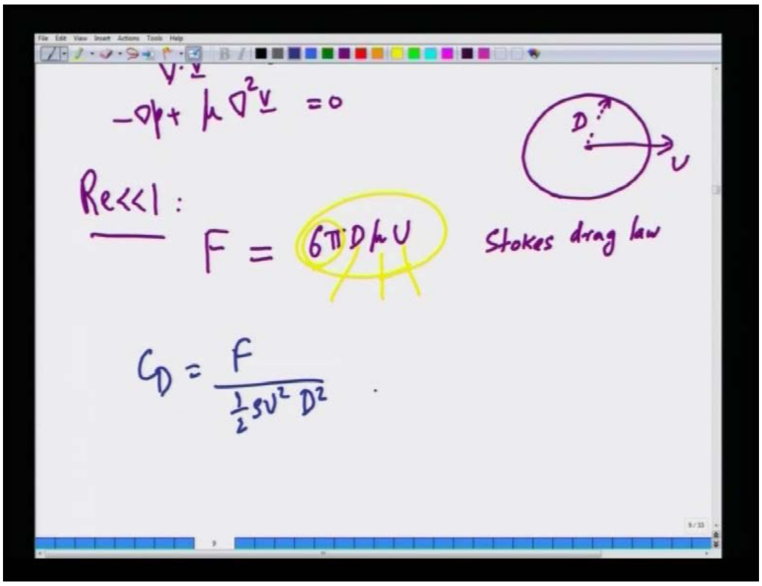

So, you get mu is 0 and subject of course, the mass continuity, mass conservation equation, point wise mass conservation equation. So, when the Reynolds number is very very small, these equations can be solved to find what is the force on a sphere or a. Now, that results in the famous stokes drag law the force, drag force experienced by a sphere moving with a constant velocity U sphere of diameter D in a fluid of viscosity mu and density rho is the celebrated stokes drag law F is equal to 6 pi a eta U, all if I use mu for viscosity, so mu is the viscosity, so this is the stokes drag law, valid at very very small Reynolds number.

Importantly it is the density of the fluid drops off from the, drops out of the equation and that is, because the fact that we are thrown away inertial terms in the fluid therefore, that means that density will not play an important role anymore therefore, the force is the function only of the instead of a l you, we should put D diameter viscosity and the free stream velocity and 6 pi is a numerical constant.

Therefore, if you want to calculate what is the drag coefficient, we have to simply divide the force by half C D is half rho U square D squared, so I mean let me let me redefine my times the area of the sphere pi D square by 4, dimensional analysis will will tell you that is simply half rho V squared D squared but, conventionally it is defined like this, so pi D squared by 4.

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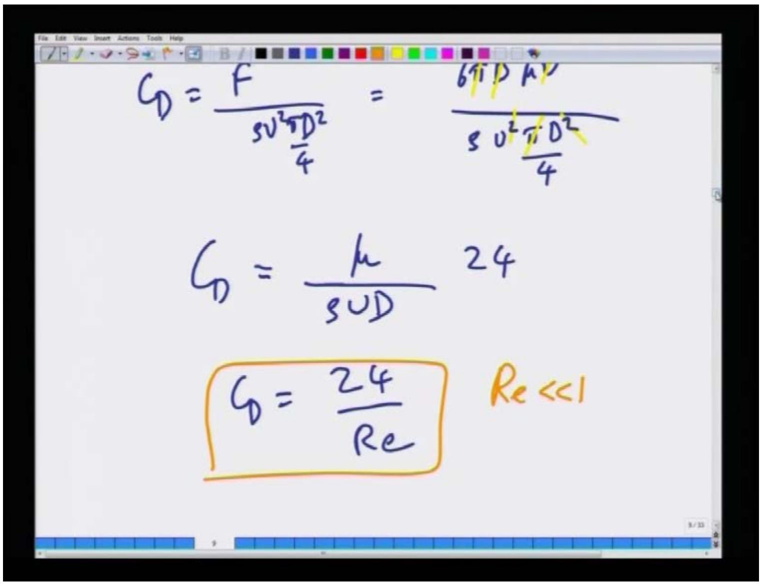

If we do that, if we substitute 6 pi d mu v by half rho U square by 4 then the pi goes off one factor of U will go with one factor of U, one factor of D will go with one factor of D, to give C D it is nothing but, an so you will also find that this is nothing but, let us do the algebra mu by rho U D times, so this half should not be there in the definition. Its a rho U squared D squared times pi D squared by 4, so you get this 4 goes above to give times 24, so C D becomes 24 by Reynolds numbers; so the drag coefficient is defined without the half rho v squared times the area, so that becomes 24 by Reynolds number. So, this is the drag coefficient when the Reynolds number is small compare to 1 at the, when the inertial force is in the fluid are negligible.

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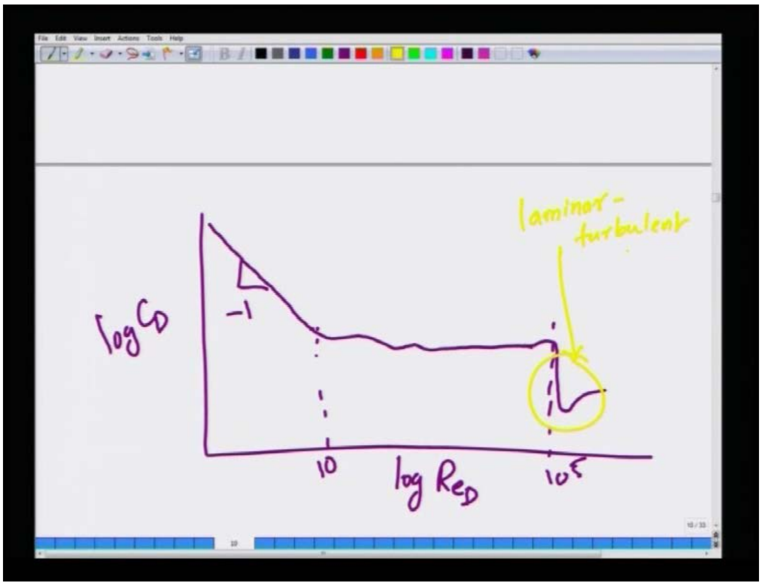

So, when we do experiments therefore, if you plot log C D, log of the drag coefficient versus logarithm of the Reynolds number, initially when the Reynolds number is less than 1 or 10, it will be a straight line with slope minus 1, because C D is inversely proportional to Reynolds number. Now, after that the C D will appear like this there will be a zone of constant drag and then suddenly the drag decreases at a Reynolds number of 10 to the 5 is about 10, now here there is a sudden decreases, because the fact that there is a transition from laminar flow to turbulence.

The the flow boundary layer pass the sphere it as become turbulent, as in any case, as in any flow when the Reynolds number increases for example, many flows are susceptible to the transition from going, under going from laminar to turbulent flow, so this also does that, therefore, you do find. But, what is interesting is you find a decrease in the drag coefficient, a sudden decrease in the drag coefficient that marks the transition from laminar to turbulent flow and that is, because of the fact that, the separation in a turbulent flow is delayed compare to laminar flow.

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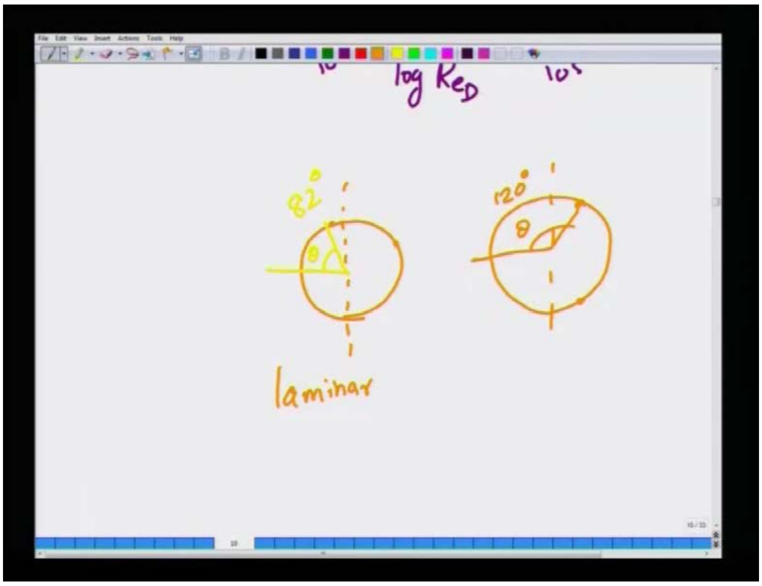

So, remember that the drag drag forces on blunt bodies like spheres and cylinders are determined by the flow separation, and if the flow separation happens early on the surface of the sphere compare to late, then the drag will be more, it turns out that in turbulent flow the drag separation is delayed and therefore, you find lesser drag in turbulent flow compare to laminar flow. So, that is some peculiar feature of laminar turbulent transition, peculiar feature of drag forces on bluff bodies such as flow past a sphere, that there is a sudden decrease in drag when the flow under goes a transition from laminar to turbulent, turbulence, because of the fact that the separation is delayed on a turbulent flow compare to laminar flow.

For example, in a laminar flow if you measure the angle from the upstream, so the separation occurs at the angle of 82 degrees while in a turbulent flow it happens at an angle of 120 degrees, the separation. So, when the separation is delayed to the rear of the sphere, then the extent of pressure asymmetry the form drag is reduced when you turbulent flow past a sphere, thereby leading to lesser drag in a turbulent flow compare to laminar flow.

Which is at a first sight it appears counterintuitive in fact it runs counter to our normal intuition the turbulent flow should have more drag, but here the reason why the drag is reduced is not, because of reduction in skin friction but, it is in fact due to the reduction in the form drag, because of the fact that the separation is delayed on a turbulent flow past sphere; so that is the reason why we have, lower drag in a turbulent flow compare to laminar flow.

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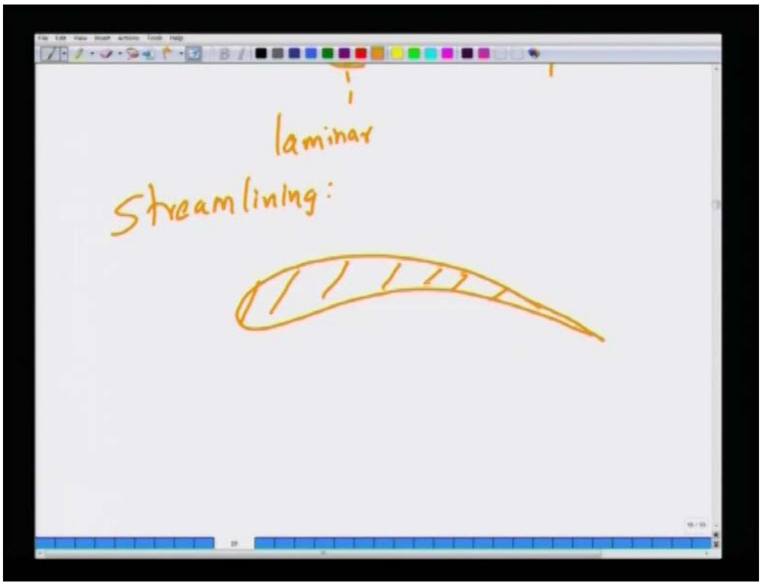

And so, when we when we when you want to prevent separation, you have to do, what is called stream lining, when you want to prevent this increased drag due to separation that is, because of this bluntness or the bluff nature of the body, the flow separates and that leads to a huge form drag, but if you want to prevent separation we end up an what is called stream lining, where in you do not allow the flow to separate and therefore, the form drag contribution to the total drag becomes negligible.

All though now since, we are increasing the surface area, because you are now elongating the object the skin drag contribution will be more, so there as to be a balance between this two opposing effect that is while on the one hand form drag is reduced, on the other hand you are trying to increase the skin drag.

But, you can always strike balance and then find that at some values, intermediate values of the shape of the body stream lining body you do find a total decrease in drag, because you are, we have you know effectively removed the form drag, at the cost of some small increase in skin drag, so this is called stream lining. And this is used in many application such as, the design of air plane wings wherein you do not want increase drag on the air plane wing, which helps you to lift the air plane but, you do not at the same time you do not want the drag to be more, so you stream line the shape of the air plane wing, so that the drag is less.

And same goes in the shaping of various automobiles such as, cars and so on, as long as you have stream lined body the drag force the resistance force experienced by the moving car or moving automobile is less compare to when the case when there is no stream lining. So, this is the fundamental reason behind shaping of objects when they move in flow especially at high Reynolds numbers, because you are trying to prevent the form drag loss in such objects by making the by making the object, shape in such a manner that they are stream line, and there are no separation that occurs.

Now, so far in this course we have been discussing very many fundamental issues in fluid mechanics, that is we have discussed, first of all integral balances of mass momentum and energy, and then we proceeded to discuss differential balances of mass momentum, and leading to the Navier stokes equations and we worked out various solutions of the Navier stokes equations in simplifying approximations, using simplifying approximations

Then we moved to dimensional analysis, as the third way of solving problems in engineering fluid mechanics that is when, we cannot solve all problems using differential balances, you have to use, you have to take recourse to experiments wherein, we have amply demonstrated that dimensional analysis play a very very important role.

Now, then we moved on to this analysis of pipe flows and fittings and losses in pipe flows using the friction factor charts and finally, we also discussed various topics in high Reynolds number flows such as potential flows, bernoulli equation, solutions for potential flows using super portion principle. And finally, we moved to boundary layer theory, which led to the notion of, which which led to the calculation of the drag force on objects such as, stream lined objects such as, you know air foils or cross sections of air plane wings and then we also discussed the notion of separation in bluff body flows.

Now, the next remaining lectures of this course, we are going to focus on applications that are closer to chemical engineering, and the applications of fluid mechanics in many chemical engineering operations. Fundamentals of fluid mechanics are general, and they are as applicable to, chemical engineering applications as to any other applications, of course, we did focus a lot on applications even before, when we discussed pipe flows and losses when we discussed flow measurements using various orifice venture meters or piton tube, they are all applications that are related to chemical engineering.

But, now we are going to look at some context of fluid mechanics which are peculiar only to chemical engineering, and then we will see how, we can make useful approximations, suitable approximations and tackle those problems; we will continue that in the next lecture.