Fluid Mechanics.
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Lecture No. # 37
Fluid Mechanics

Welcome to this lecture number 37 on this NP-TEL course. And fluid mechanics for undergraduate chemical engineering students the topic we are currently discussing is flow through packed beds.

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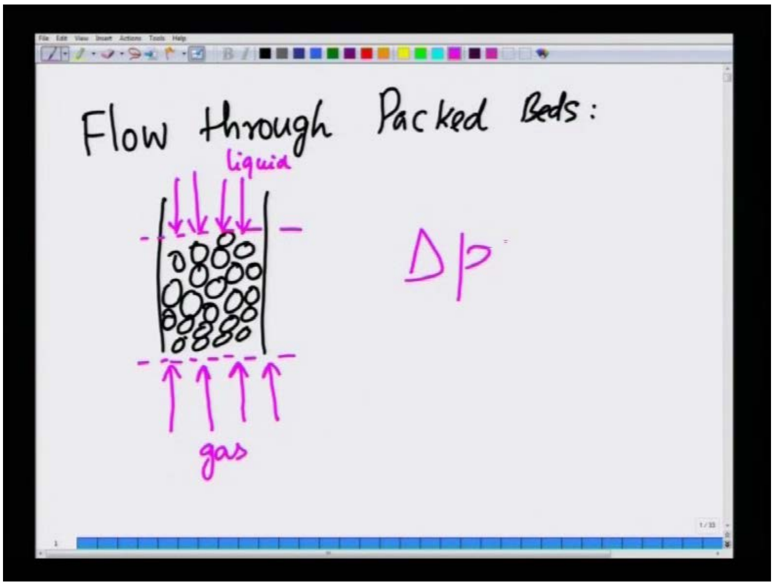

So, we will complete the discussion on that topic flow through packed beds. We started this discussion in the last lecture, lecture number36. So, let me briefly recap what we are doing. So, the pack bed is essentially container with lot of particles packing is of different shapes for simplicity. Let me draw particles of spherical shape in this picture in this cartoon essentially. So, you have a distributor through which fluid is made to flow let say a gas is made to flow.

And the idea of having a pack bed is that the effective surface area for unit volume. For contact between let say the gas and the solid surface or the gas and maybe there is a absorption going on gas liquid absorption going on. So, the surface area per unit volume increases, if we are packing because interfacial constricts area increases dramatically the moment you put in packing.

But, here we are not concern with the mass transfer aspects of the problem. We are rather interested in the fluid mechanics aspects with namely what is the pressure drop that is required to make a fluid flow in such a pack bed.

So, the model we had well before I go to the model let me also make a general comment that this flow process is extremely complex because, in principle if you want to solve it you have to solve for the Navier-strokes equations with mostly boundary conditions in Nemours. Number of particles and each particle is arranged in a very random way. So, it is very very difficult to solve the problem exactly.

It is almost impossible one requires extraordinary competition resources and sophisticated algorithms to solve for the flow through such a bed of particles. But, we will come up with the very simple model which will get the right trend for the pressure drop except for a dimensialized constant.

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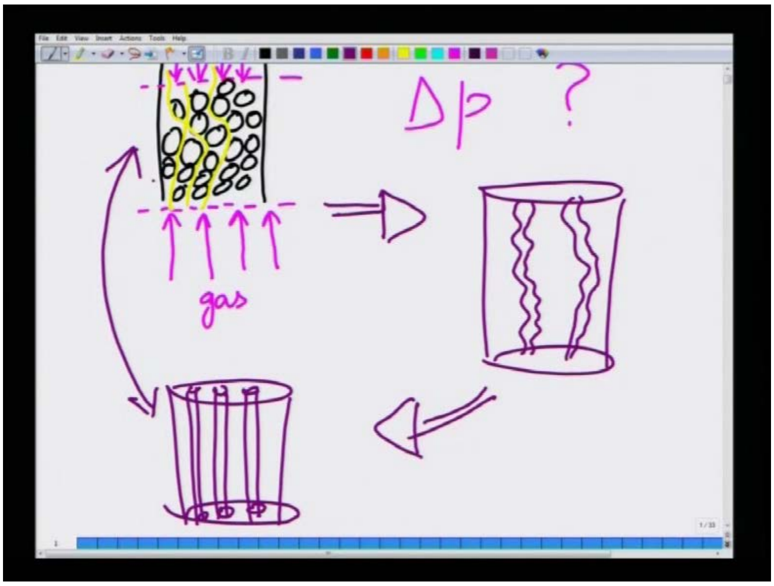

So, the model is the following. If you look at flow through a pack bed there are channels through which fluid is flowing there are empty passages. In the pack bed through which fluid is flowing for example, I am drawing some passages like this. So, essentially the model we are going to have is that the bed of particles can be considered as a collection of bundle of tubes.

So, although the cross section of the tubes are extremely complex because, the path of this fluid in to this bed is extremely complex. But, to simplify this further we are going to assume to be a bundle of straight circular tubes that is the model we going to have for flow through a pack bed.

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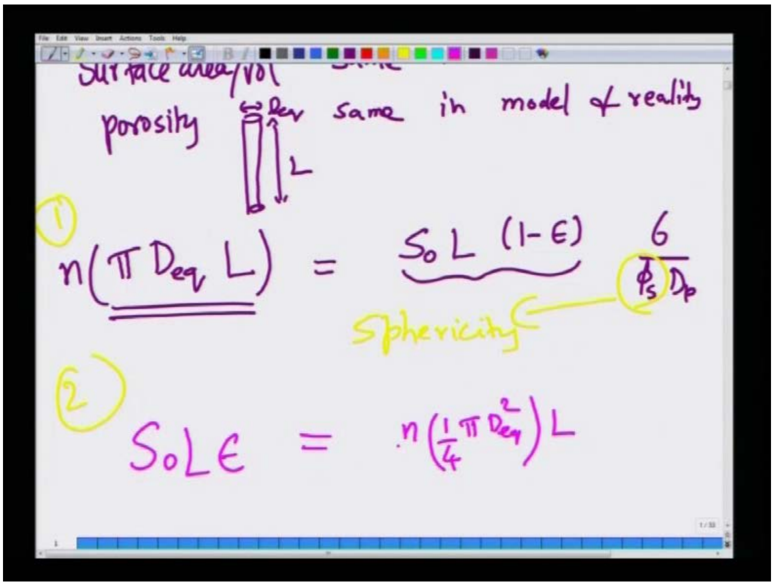

But, the key thing that we are going to match is that from the reality to the model is that the effective surface area the surface area per unit volume is the same, same in model. And reality model is this the bundle of tubes and reality and the porosity or the void volume same is the same in the model and reality with this guiding principles. Let us move ahead.

So, let us assume that first let us equate the surface area. So, let n be the number of such tubes let pi well pi D e q. D equivalent time L, pi D equivalent time L is a surface area of a single tube here, some tube of equivalent diameter and L is the length of a tube here. We are assuming that the length of this tortuous path is the same as the length of the bed although if you can draw if you really measure the length along the contract of the tortuous path.

The actual length will be greater than L. But, for simplicity we are going to assume that it is same as L. And we are going to correct all these with experimentally fitted parameter at the end but, right now let us move along with this assumption. So, this must be equal to the actual surface area that is present in the real in the bed of particles.

Let s not be the cross sectional area of the so, if you have bed of particles this is the cross section area of the bed empty bed. L is the length of the bed 1 minus epsilon is the epsilon is the void volume, void volume fraction or porosity. 1 minus epsilon will be the solid fraction times. So, this is the total volume occupied by particles times 6 by D p for a non spherical particle we corrected that with what is called as sphericity.

It is this converts or it signifies the deviation from a spherical particle, in general it will have phi s is greater than 1. Sphere has the maximum volume per maximum area per unit volume sorry minimum area per unit volume. So, this will be actually sphericity will tell you the deviation from as spherical particle. So, this will be less than one for non spherical particles.

Because, any non spherical particle we have more surface area for the same volume. Now, having done this we also require the second relation this is the first relation. Second relation is that the void volume in the bed which is S not L epsilon is the same as the total volume of the n channels. That is the amount of volume that is available for flow. So, the volume of n channels is pi D equivalent square one forth that is a cross sectional area times are length times n is a number of the such tubes.

So, by equating this by using these two we can get an expression for D equivalent. So, D equivalent is two third is phi s D p epsilon by 1 minus epsilon. That is the equivalent dia diameter of a tube through which fluid is flowing. And the model is that there are n such tubes in a bed of particles, n such tubes that model a bed of particles of porosity epsilon and particle diameter D p and sphericity phi S.

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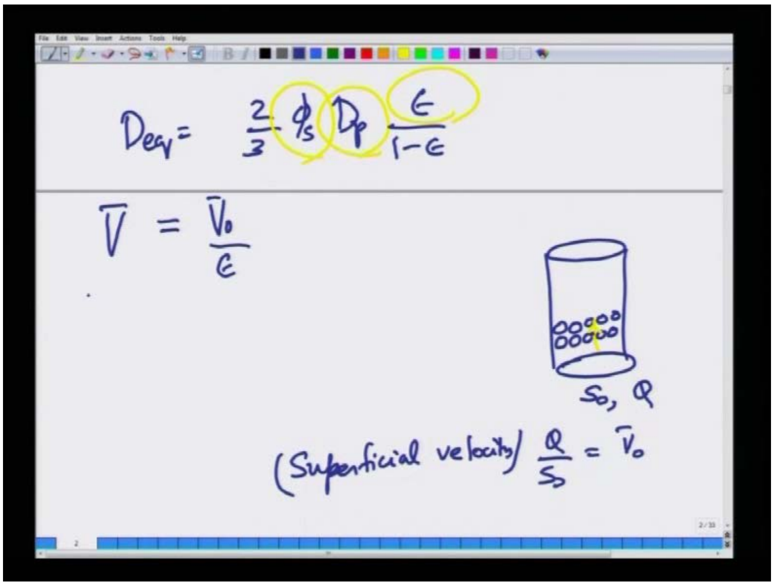

Now, the next thing is what is the pressure drop? The pressure drop will depend on the velocity in the tubes the velocity inside the bed will be different from suppose you have a bed and if the bed is empty if, S not is a surface area and Q is the volumetric fluid. Q by S not is called the superficial velocity. (No audio from 07:41 to 07:47)

If, there were no particles this would be the average velocity of fluid present inside this container. But, we do not have all these space or area available for flow because, only a fraction of it is available for flow. So, you would expect that actual velocity in the interstitial gaps will be greater than V not.

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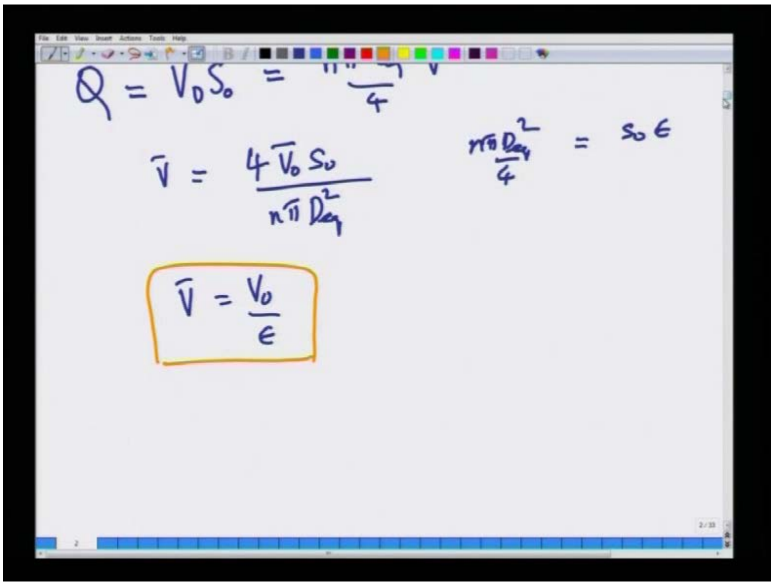

So, that velocity is nothing, but, V not by epsilon if epsilon is porosity then the actual velocity will be greater than superstition velocity. Because, only that much area is available to for the fluid flow. We can derive this in a very simple way Q is nothing but, V not superficial velocity times S not that is also equal to n pi D equivalent Square by 4 times the actual velocity in these tubes.

So, we can relate these two to get what is V in terms of V not this is 4 V not S not divided by n pi D equivalent square. But, we also know what n pi D Equivalent Square is by 4 from this expression from here from this expression. So, we are going to use that. So, n pi D Equivalent Square by 4 is epsilon times epsilon.

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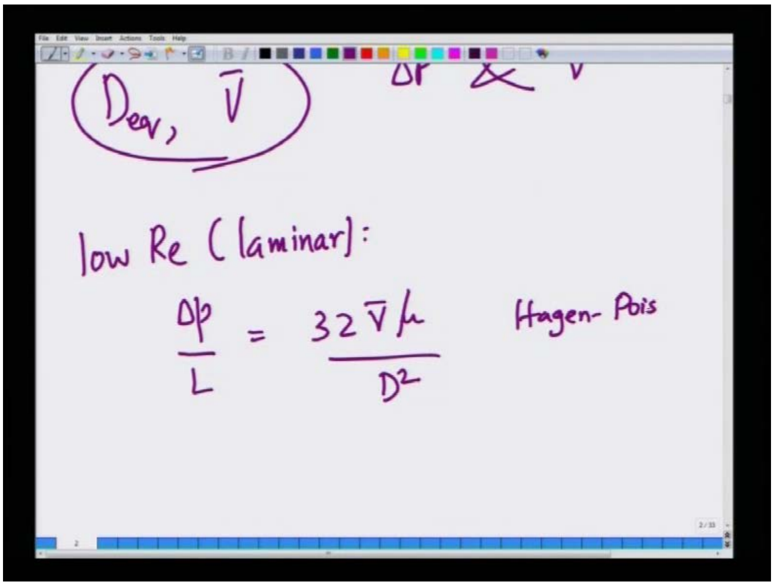

So, I will get V is V not by epsilon as I claim. So, the actual velocity is the superficial velocity divided by the porosity. So, now, we are we have an idea is to what is we know what is D equivalent we know what is the velocity. So, we can know what is the relation between the pressure drop and velocity. If, we know what is the flow regimes in these tubes. So, there are of course, two flow regimes the flow regions could be laminar or turbulent. So, we could take both the (( )).

So, let us assume that the flow is at low R e. So, the flow is laminar. So, we can assume that the delta p by L will be given by the Hagen Poiseulle velocity problem relation for laminar flow in a tube. So, this is the Hagen poiseulle relation for pressure drop, pressures velocity, average velocity. Now, we are going to instead of D we are going to use D e D equivalent and then we are all set to derive the pressure drop relation.

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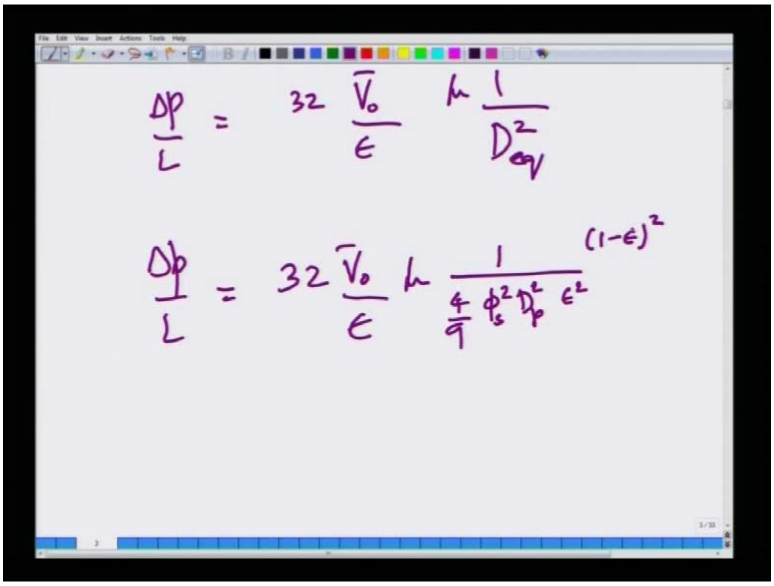

So, delta p by L is 32 instead of V we are going to write V not by epsilon times mu times one by D equivalent square. So, delta p by L is nothing but, 32 V not by epsilon mu times 1 over 4 by 9 phi s square D p square epsilon square times 1 minus epsilon whole square.

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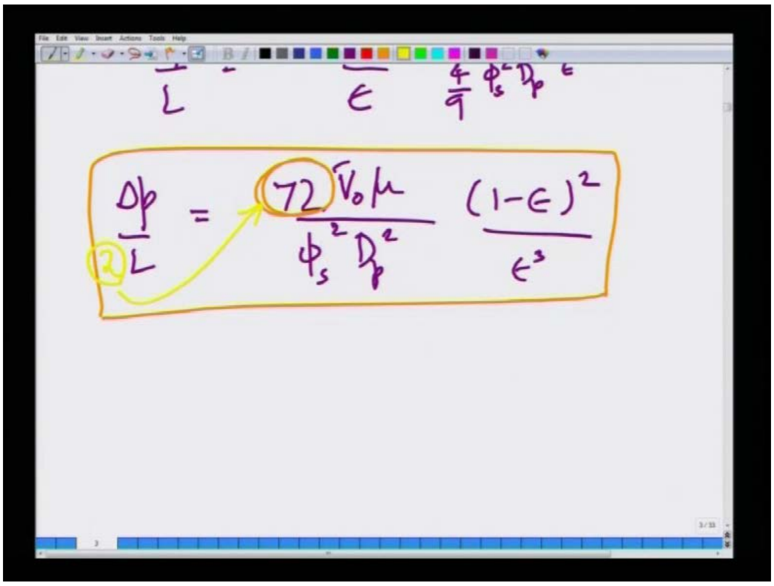

So, delta p by L is if I do the simplification 72 V not mu by phi s square D p square 1 minus epsilon square by epsilon cube. So, this is what we have now, this expression for pressure drop divided by length it has a pre factor of 72. Now, experiments tell us that because of the various assumptions we have made while deriving this expression.

The real problem is extremely complex there are torturous path through which fluid is flowing. But, in reality in this model we assume the channels to be of simple cross section, circular cross section. And I mention that the actual length of the flow path of any fluid particle is not exactly the bed high which is L but, it could be more than L typically it is twice L.

So, this factor 72 is not really correct. So, this In fact, if you think approximately that the bed should be of 2L. So, if you take this the actual length of the tube is 2L if you take this here ,you would expect something to be around 140 or 144 and in reality that is what happens if you do experiments you will find that this becomes.

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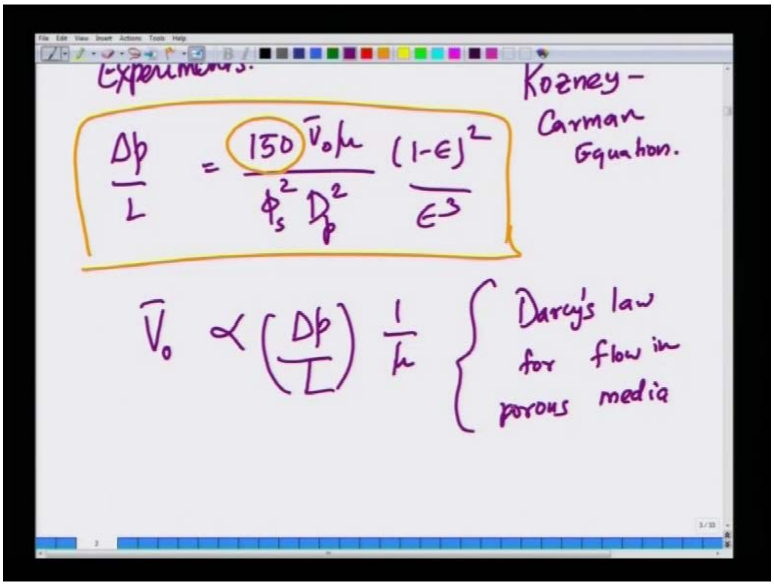

So, experiments tell us that at low Reynolds numbers delta p by L is 150 V not V bar not mu by phi s square D p square 1 minus epsilon square by epsilon cube. So, the 72 is modified to 150 but, what is very interesting. And what is very instructive to understand is that even a simple model is able to predict the function relationship between delta p by L various otherwise such as the superficial velocity, viscosity, porosity diameter of the particles.

And so, on in a very very exact way the only thing that different between the simple model and experiments is the pre factor which instead of being 72 terms of one 150. And that is physically because, of the fact that the length of these tubes through which I am so, to speak the tubes over which are the passages over which a fluid is flowing is not quite the length of the bed it is more than the length of the bed.

That is the reason why you find this pre factor to be different this equation in the literature is called in text books is called the Kozeny Carman equation. (No audio from 13:45 to 13:54) So, this is valid for smaller Reynolds number, if Reynolds number is defined based on the particle diameter and the superficial velocity.

And the fluid properties less than one then you will find that this correlation works very well. And there is one small difference you also include 1 minus epsilon this is the way Reynolds number based on the particle is defined.

Now, so for a given system this appears like you can re write this expression as V not being proportional to delta p by mu sorry delta p by L times one over mu. So, this is called Darcy's law for a porous media for flow in porous media. That the velocity through which fluid is flowing in a porous media is directly proportional to delta p by L and is inversely proportional to the velocity sorry viscosity. So, this is often used in understanding fluid flow through porous materials like the sand and other under ground water transport and so on.

So, whenever you have flow of water in under the ground it is almost like flow through porous bed of mud and sand. So, this model represents this model is useful to describe such flows porous media flows. And now the point is we do not have just flow at low Reynolds numbers we have we could have flow at higher Reynolds numbers also.

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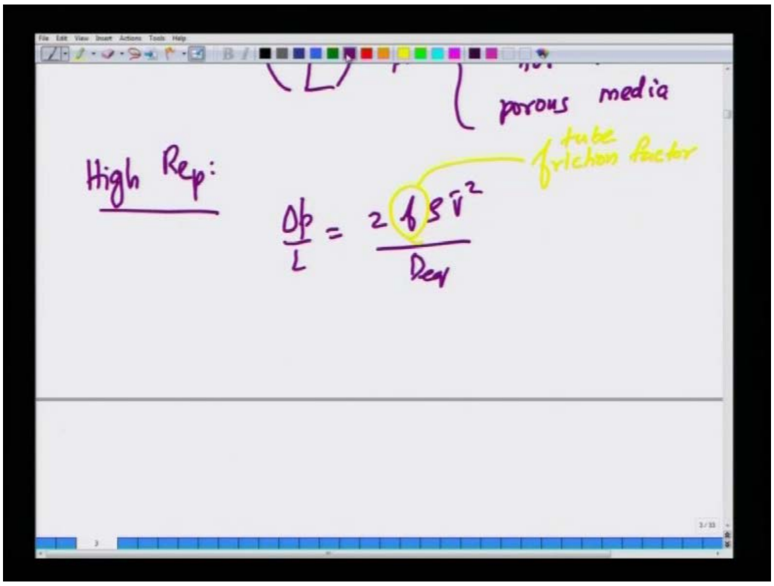

So, for high Reynolds numbers we cannot use the laminar flow, relation flow in a tube. So, remember the delta p by L is two f rho V square by D equivalent. This is the friction factor of a flow in a tube we know that delta p by L is related to friction factor through a definition. And the definition is this is the tube friction factor which we already known from our discussion on flow through tubes.

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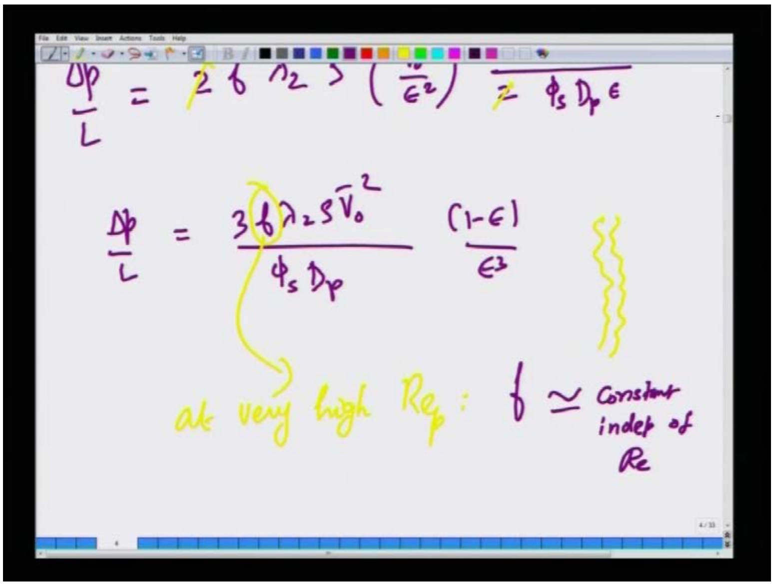

Now, we can now, therefore, write delta p by L is 2 f .Now, we will put a constant lambda 2 and then we will fit the constant based on experiments instead of V. I am going to write V square I am going to write V not square by epsilon square and instead of D equivalent. I am going to write 1 minus epsilon. So, this something that we are already derived for the equivalent diameter is nothing but, 2 by 3 phi s D p epsilon divided by 1 minus epsilon.

So, that is all I am trying to write here, 2 with this 2, 3 f lambda 2 rho V not square by phi s D p 1 minus epsilon by epsilon cube. This is at high Reynolds I mean this is at I am not saying whether which is at high Reynolds number and anything.

But, I am writing the delta p in terms of friction factor. Now, I am going to argue that at high Reynolds numbers at very high Reynolds numbers of the order of 5000 or something.

Now, the flow is turbulent and that the tubes I mean if you look a real, but, the tubes (( )) to speak a very well rough through fluid is flowing. And when the tubes are very very rough friction factor for flow in a rough tube becomes in dependent of Reynolds number in a turbulent flow. So, the friction factor becomes a constant it is approximately a constant independent of Reynolds number R e.

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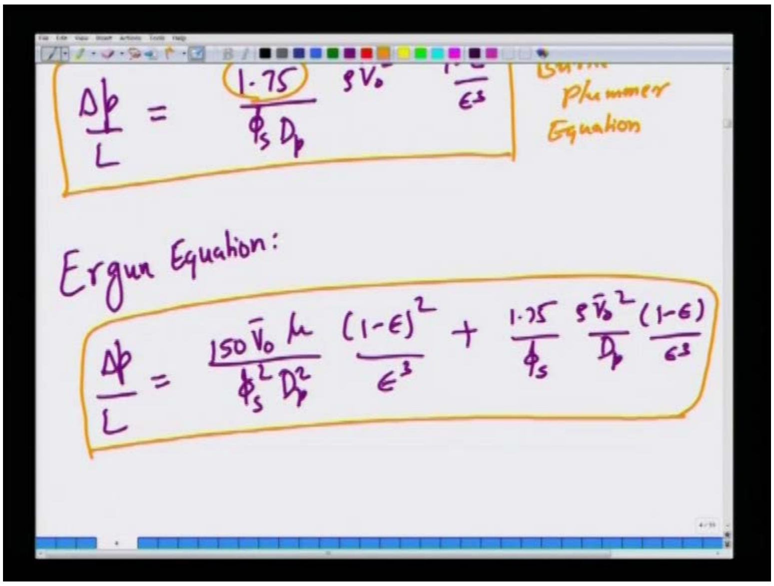

So, I am going to use that input from flow through rough tubes and we will get delta p by 2 sorry delta p by L terms out is 1.75 divided by phi s D p rho V not square 1 minus epsilon by epsilon cube. This is after correlating the data correlating this expression with experimental data people find that this 3 times F becomes roughly 3 times lambda 2 times F becomes a roughly1.75.

So, that is the experimental data that tells us that the combination of the numerical constants 3 lambda 2 f not f not is a constant independent of Reynolds number. Now, because we are in the highly turbulent regime in the rough tube so, we get this.

Now, this is called the burke Plummer equation. Burke Plummer equation for high Reynolds number, we are essentially saying that in this bundle of tube models at high Reynolds numbers. If the tube is very rough in the tubes are very rough through which fluid is flowing which is the case, in porous media in packed beds. Then we can actually treat the friction factor to be a constant independent of the Reynolds number. And correlate this experimental data, correlate this expression with experimental data and fix the constant to be whatever that agrees with experimental data terms out to be1.75.

Now, there is another approach which linearly combine which mealy adds these two expressions the cosine Carmenere equation and burkes Plummer equation. And it claims that it is valid for any Reynolds number that is called the Ergun equation. the Ergun equation is merely an addition of the super position of the tube results 150 V not mu by phi s square D p square plus 1.75 divided by phi s rho V not square by D p 1minus epsilon divided by epsilon cube. So, this is the Ergun equation which is merely an addition of the two presences that we have obtain. So, for.

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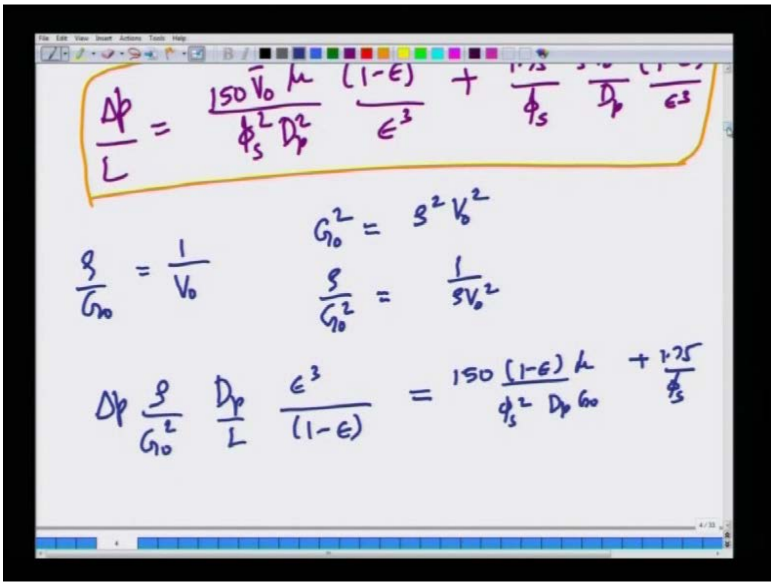

So, people often define rho by G not as one over V not or g not square is rho square V not square. So, rho by g not square therefore, becomes one over rho V not square. So, whenever you have one over rho V not square we convert it into rho not by G not square. So, delta p rho by G not square D p by L this is a non dimensional epsilon cube by 1 minus epsilon this is a dimensional pressure drop this is essentially like a friction factor is 150 times 1 minus epsilon divided by phi s square times D p G not by mu plus 1.75 divided by phi s.

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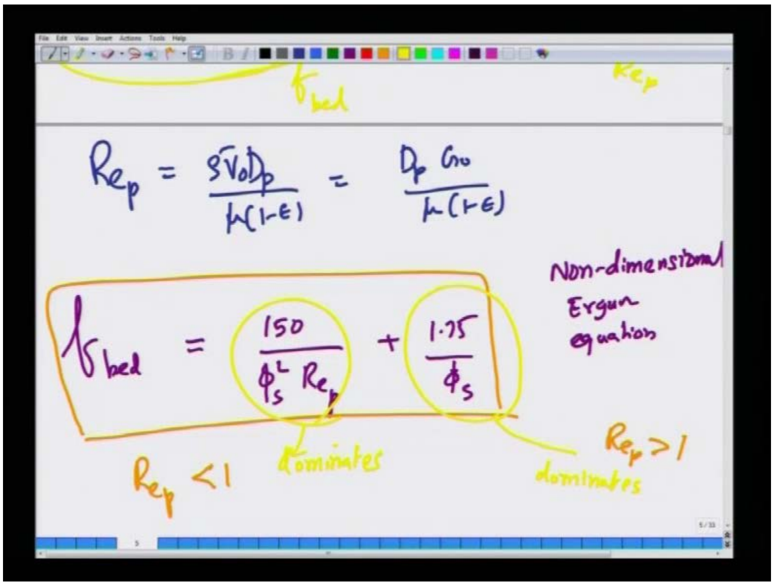

Now, if you define the Reynolds number based on particle as rho V not D p divided by mu 1 minus epsilon this becomes (No audio from 21:39 to 21:45) is becomes D p G not by mu 1 minus epsilon. So, this is essentially one over R e p. So, we can re write this expression if, you define the friction factor of the bed as this is the friction factor of the packed bed then f bed becomes simply 150 times 1 minus epsilon. We can rewrite in terms of the Reynolds number which we defined 150 divided by phi s square Reynolds number based on the particle plus 1.75 divided by phi s.

This is the non dimensional Ergun equation. (No audio from 22:38 to 22:48) So, what this equation is telling you is when R e p is less than 1 the first term dominates, when R e p is that larger than 1 this dominates. So, and when R e p is comparable to one is neither small nor large compare to one then you would have the combination of the two work very well.

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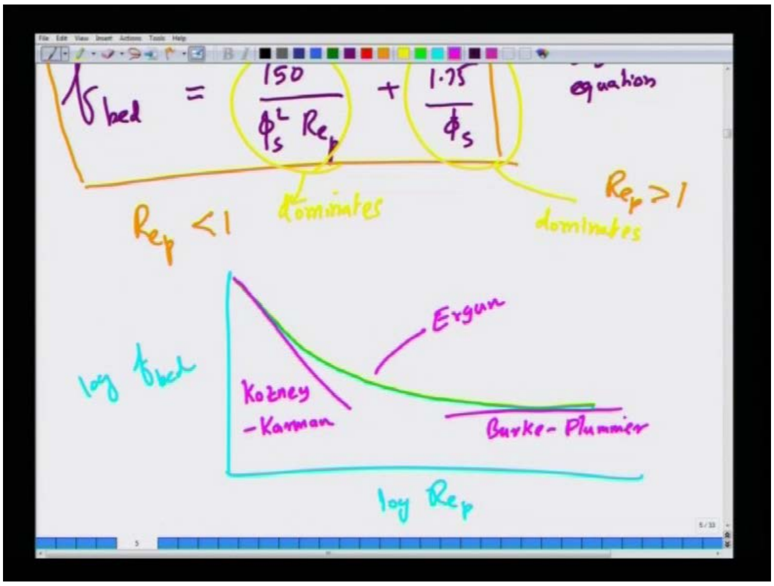

So, if you plot the experimental data in f bed logger f bed verses log Reynolds number based on particle you have the reality to be like this. So, at low Reynolds numbers you have the cosine Carmenere equation high Reynolds numbers. We have the Burke Plummer equation and at intermediate Reynolds number you have the Ergun equation. (No audio from 24:01 to 24:12)

So, this is how one can predicts pressure drop in flow through packed bed. So, these equations are very very useful in the design of unit operations such as absorption towards in involve in packing. And all that we need is what is the sphericity of the particle what is the porosity of the bed which are easily determined experimentally ones. You have that information then we are ready to design the bed. That is at least the fluid mechanics aspects of the bed what is the pressure drop requirement.

What must be the pumping requirement how to design pumps for running a pack bed you need to know first of all what is the flow rate that is desired what is the pressure drop that you required. So, that pressure drop can be calculated using this bed friction factor and ones we have this data we can design a suitable pump for such unit operations.

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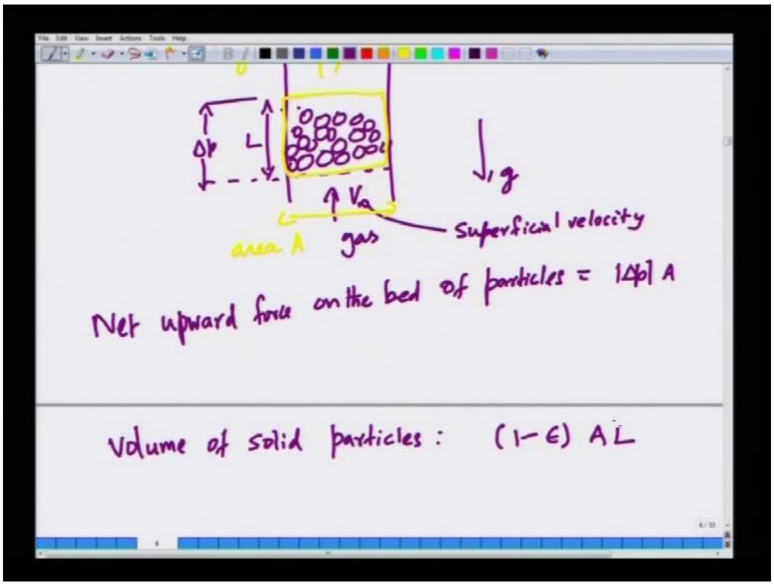

Now, the next topic that we are going to discuss is related to pack bed, but, it is slightly also different is called fluidization. (No audio from 25:14 to 25:21) Now, in many chemical engineering operation, operations you want to increase the contact area between let say if the solid particle or catalyze particles on which reaction is happening you want to increase a surface area. And at the same time you also want to ensure good mixing between various parts of the reactor.

So, what is normally done is that you have a bed of particles initially for simplicity I will draw spherical shape particles and let say you pass a gas. So, initially it is a packed bed but, when so, what is happening is initially the weight of the particles act down words. Gravity is acting down words is now, balance by the pressure drop the pressure drop that you exert to make the fluid flow is essentially balance by the weight of particle.

If, we do a macroscopic momentum balance that is what we will show up you take this entire thing as control volume. If you neglect wall friction the main dominating forces are essentially the pressure drop that you apply and the body force that acts on the entire bed which is essentially due to the weight of the particles that are present in the bed.

Now, initially when the drake force around each particle is such that they are not able to lift the entire bed then the pressure drop will go on to just balance the weight of the bed. But, as the velocity is increased further and further there will come a point when the drake force on this particles that present in the bed will overcome the weight of the bed.

That is a pressure drop will be able to overcome the weight of the bed and the bed will start expanding the moment. The bed starts expanding the particles will no longer be static or in contact with each other they will start moving almost like a fluid.

So, such a state of a bed where in the particles are no longer in contact with each other but, they start to move about more easily almost like a fluid is called a fluidized state of the bed or just a fluidized bed. And the onsite of this process is called fluidization of the pack bed.

So, this is the idea. So, we want to in many chemical engineering operations for example, in petroleum refining you have this fluidized catalytic crackers f c c units, which tend to process which are used to in processing crude oil to convert it. To various hydrocarbons. So, we are of course, in this course we are interested only the fluid mechanics aspects of the problem the question we want to ask is based on whatever we have studied whatever you understood. So, for in pack beds can we predict what is the velocity that is required to fluidize given pack bed.

Because, if you want to design a fluidized bed you want to know what is the flow rate at which you must send the gas. So, did the bed remains in a fluidized state other design requirements are such that the fluidization is should not be such that the particles go out of the bed. That should not be entrain of the particles outside the bed because, the gas will come in and go out. But, when the gas is going out of a bed it should not take away particles with the bed. So, we have to worry about those issues as well.

So, we will try to address all these issues based on whatever we have done. So, for now, the idea is very very simple you have a fluidize bed. Let us imagine that you are the length of the bed initially you have a pack bed length of the bed is L and the pressure drop is delta p over a length L and you are initially sending gas with velocity V infinity. The superficial velocity of the gases V infinity and the area is a the cross section area of the bed is a delta p is the pressure drop across the bed.

The net upward force if you do a momentum balance on the bed of particles is nothing but, delta p times a now this will be balanced by this is acceleration due to gravity. The weight of the bed the volume of solid particles present in the bed. Present in the bed is 1 minus epsilon times a times L if a L is the total volume of the bed only 1 minus epsilon fraction of the volume is occupied by particles. Because, epsilon is the porosity. So, only 1 minus epsilon of that fraction of volume is available for the particles to occupy.

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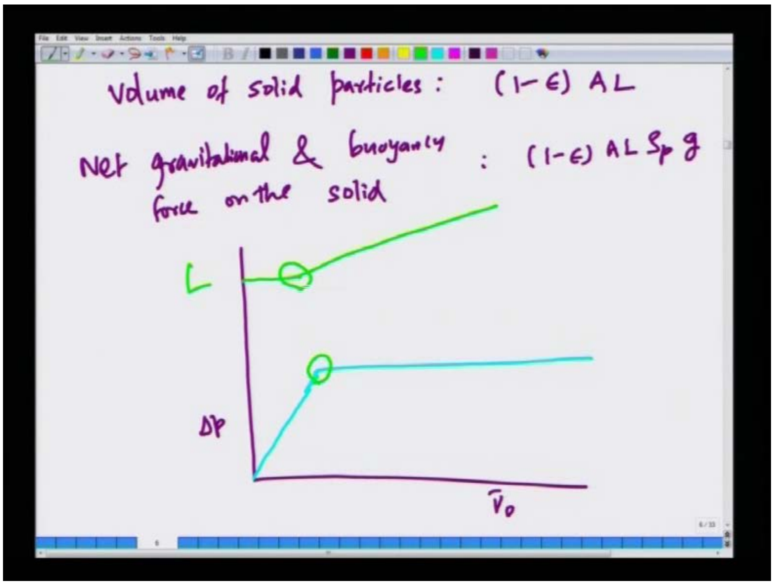

So, the net gravitational and buoyancy force on the solid particles is given by I am going to multiply this solids volume by the solid density the particle density. And then the acceleration due to gravity when these are equal, when these two are equal that is the onsite of fluidization. Because when initially when you if you have to do a thought experiment I am going to plot delta p verses the superficial velocity.

Initially when you increase the superficial velocity you will find that as per the pack bed relation the delta p will be directly proportional to velocity. If the Reynolds number is low and after at the onsite fluidization the delta p will remains constant.

And then the as you increase a superficial velocity but, what is happening instead is that the length of the bed, which was initially constant that will start increasing that is the onsite a fluidization.

So, essentially at the point when the delta p starts stops decreasing become starts to become a constant plato the length of the bed will start expanding. So, we are interested in finding what this onsite point is that is all interested in at this point of moment.

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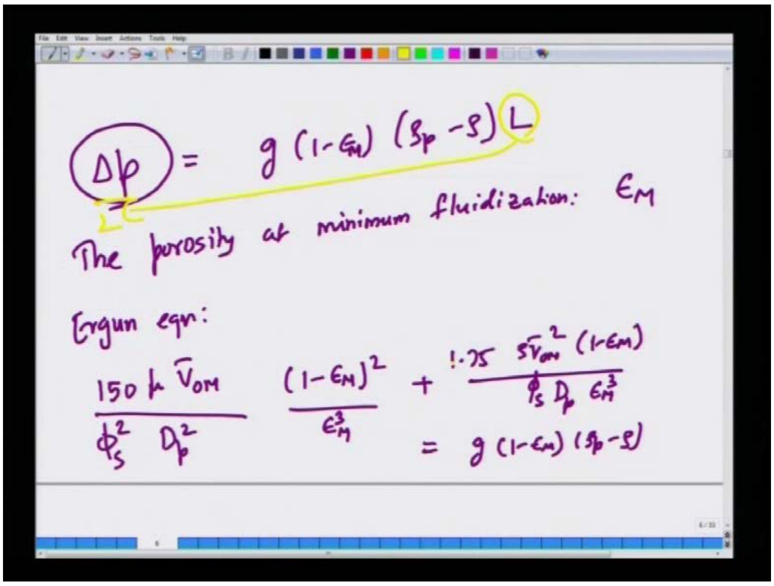

So, essentially we relate delta p times a the force upwards force is equal to the downward force at steady state at the onsite of fluidization. And that is the this will give you the critine for minimum velocity for minimum fluidization.

Now, So, delta p the two factors of A cancel out delta p will be therefore, g times 1 minus epsilon times rho p minus rho times L. So, let us the porosity at the onsite of fluidization will be the minimum compare to what when the bed is fluidized the porosity at minimum fluidization velocity is denoted as because when the length of the bed increases the porosity will also increase. It is the solid content is the same we are not in putting in any solids into the system. Therefore, the porosity has to increase. So, let us call the epsilon I am going to put reach e m that is the minimum porosity at fluidization at the onsite of fluidization.

Now, at the onsite of fluidization delta p will be determined because, the bed just at the until the onsite of fluidization it is a fixed bed, it is a pack bed it is not a fluidize bed. So, if it is a packed bed we can use our earlier correlations to find what is the pressure drop. So, we are going to use the Ergun equation.

So, we will find that 150 mu V o M is the minimum velocity for onsite a fluidization that is the notation in standard text books V m square by E M cube plus 1.75 rho V o M square one minus epsilon m by phi s D p epsilon M cube is equal to g times 1 minus epsilon M times rho p minus rho because, I am writing an expression for delta p divided by L. So, therefore, you will have only the L factor will not be there anymore.

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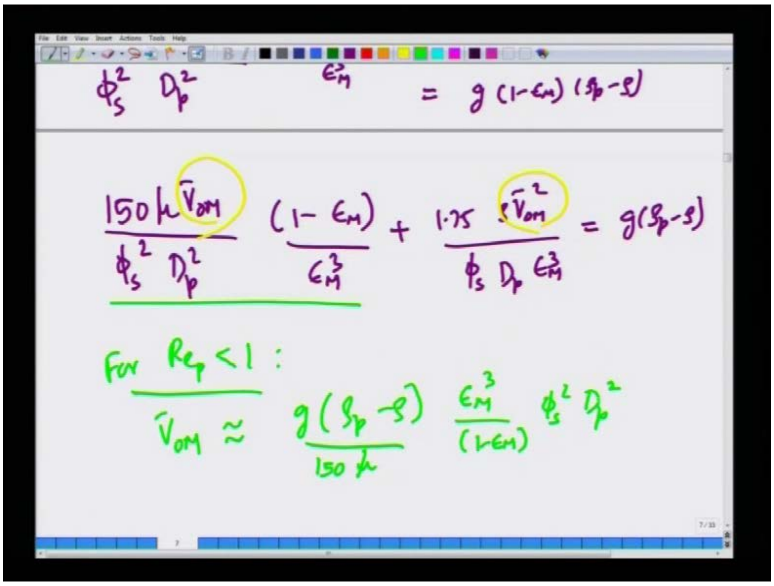

So, I can re write this further after taking into account one minus epsilon m to the other side I will get 150 mu V o M on site of fluidization phi s square D p square times 1 minus epsilon M by epsilon M cube plus 1.75 rho V o M square divided by phi s D p epsilon M cube g times rho p minus rho.

So, this is this give you a quadratic equation for V o M because, this is linear because this is the quadratic V o M is the unknown. So, we can solve it in principle provided we know all the other properties of the bed such as the porosity and so on. Now, for very small particles for R e p small compare to one only the laminar part will dominate this part will dominate. So, V o M is approximately g times rho p minus rho times epsilon m cubed divided by 1 minus epsilon m divided by 150 mu times phi s square D p square.

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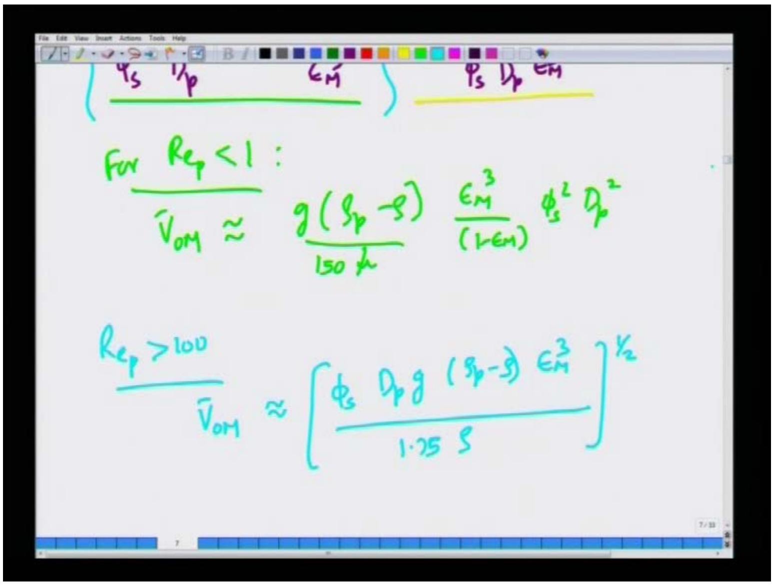

And when the Reynolds number of based on the particle is large 100 let say V o M V neglect the first term and take only the second term. So, this will be the dominant term when Reynolds number is large R e p greater than 100 will use the second term to balance the right side of the equation to get phi s D p g rho g p minus rho epsilon m cubed divided by 1.75 rho this to the power half because, that term has V o M square. So, we will have to take the so, if we neglect this term so if, we neglect this term and take only this term and you have V o M square.

So, let us take the square root to get what is mu r. Now, so, these are the onsite velocities for fluidization at both low and high Reynolds numbers. So, we now know how to estimate what is a minimum fluidization velocity for fluid aspects. And when the particle Reynolds number is small of the order of one and smaller than we can use the relation where in V o M is directly proportional to well.

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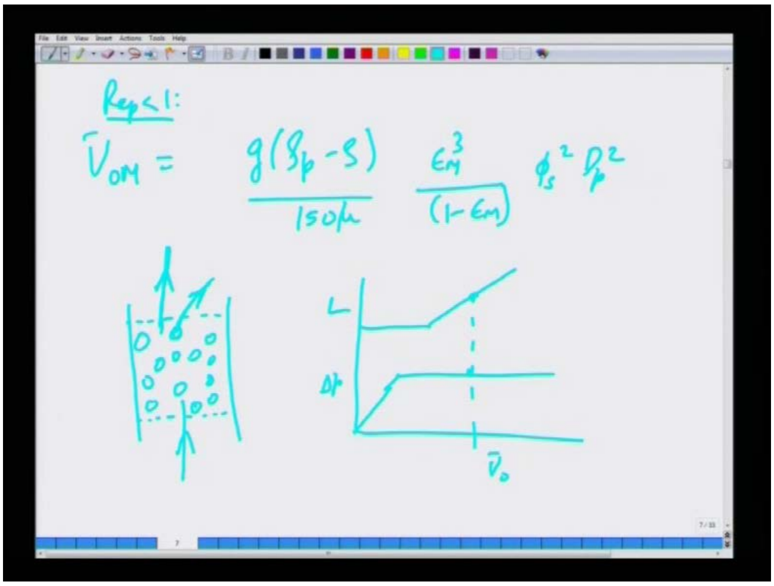

We will just write down this equation again V o M for smaller Reynolds numbers. We need this expression for the following discussion also its approximately g times rho p minus rho divided by 150 mu times epsilon M cube divided by 1 minus epsilon M times phi s square D p square.

And there is a similar expression at high Reynolds numbers now, another question that we can answer with our analysis so, for is that. So, for suppose you have a fluidize bed. The bed is fluidized and you are in this regime that the length of the bed is now expanding and you are at particular this is the length, this is the pressure drop. You are at particular pressure drop where pressure drop is constant this is the superficial velocity and the length is at some expanded value.

The question you want to ask is whether the fluidization velocity in this regime is such that. It can it because during fluidization continuously gases coming and going out while the gas is going out. We should take care enough care that it does not entertain particle with that because, the whole point of our fluidized bed is to keep the particle inside the bed in a animated sense of motion to enhance mixing another transport properties and improve transport properties.

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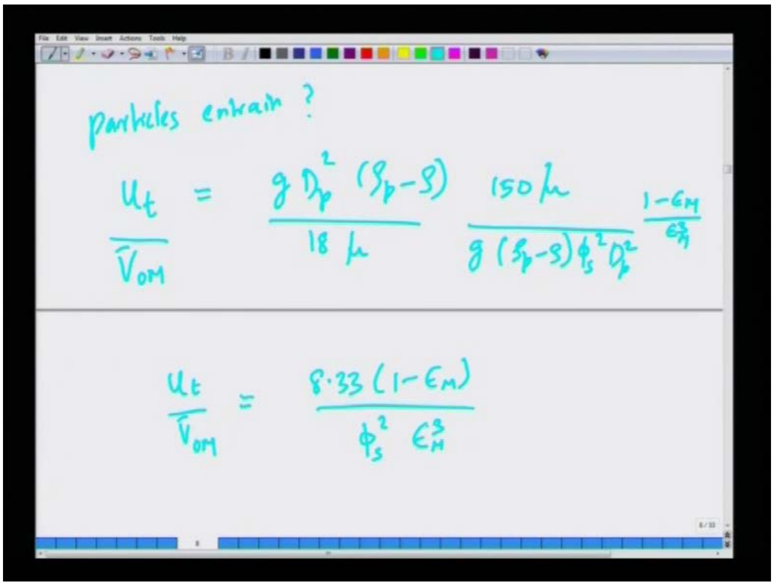

So, how do we check whether the particles will entrain or not. Well, the answer is very simple. So, imagine that the fluid onside the velocity of the particles is slightly about the fluidization velocity. Now, so, the velocity of the fluid so, there will be a force acting on the particles which will tend to drake force acting on the particle, which will tend to push the particle in that direction while there also be if we look at each particle.

Now, let us not assume that the particles are in touch because they are already fluidized they are somewhat independent although they will be colliding ones in a while.

So, we can roughly estimate whether the particle is going to stay inside the bed or its going to go out by just comparing the fluidization velocity with the turbulent velocity of the particle. So, what is the terminal velocity of a particle that we know at low Reynolds number is g. So, we can divide the terminal velocity by the onsite velocity for fluidization g D p square rho p minus rho by 18 mu. This we derived in the last lecture or the lecture before.

And then we divided by the fluidization velocity for low Reynolds numbers and then you will get. (No audio from 40:11 to 40:21) Now, this is nothing but, is 8.33 1 minus epsilon M divided by phi s square epsilon m cube for fewer phi s is 1.

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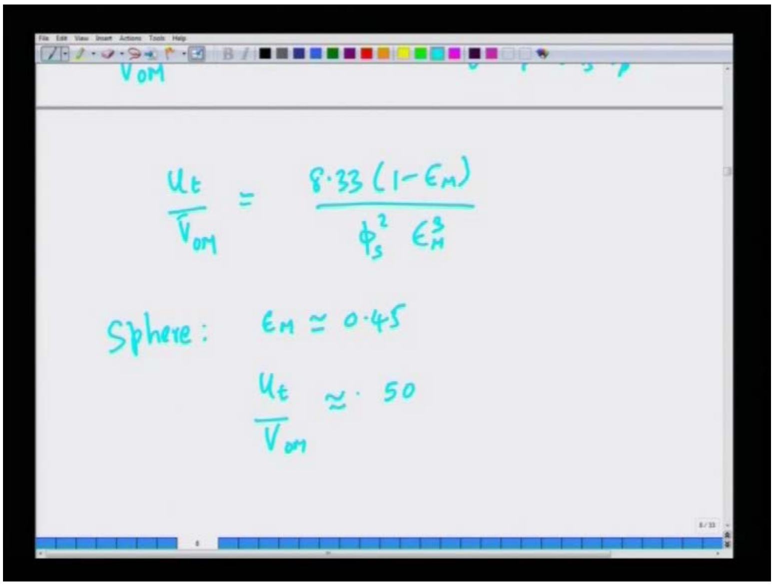

So, spherical particles and if we use epsilon m is roughly 0.45 which is typical value. So, u t by V o M becomes of the order 50. So, the terminal velocity is 50 times the fluidization velocity. Therefore, it is very unlikely for the particles to the entrained by the flow because, there is a very strong component of the terminal velocity which will tend to bring the particle down again. It will collide with other particles and then it will go somewhat up and then it will pushed also by the drag force of the fluid. But, there is a very high chance that it will not go outside the bed it will not be entrained.

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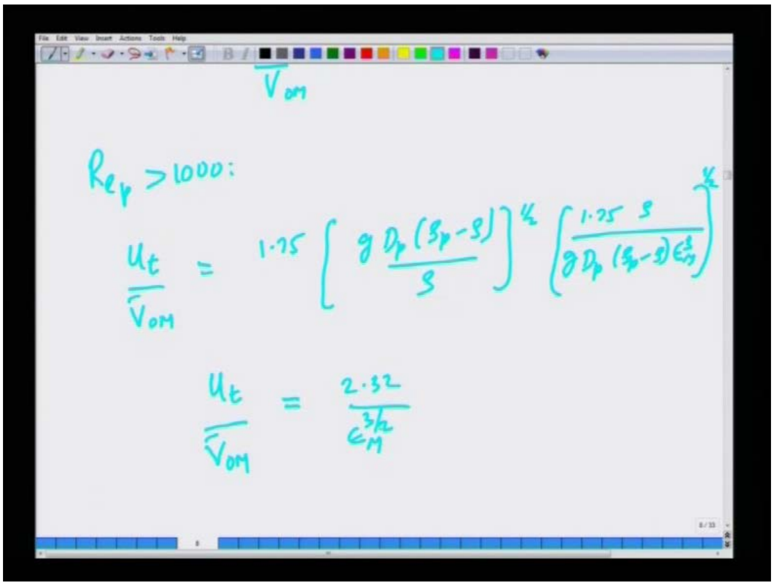

But let us look at the same case at high Reynolds numbers let say R e p greater than 1000. Then will use the turbulent flow correlation for the onsite of fluidization velocity for onsite of fluidization. Then you will get its 1.75 times g times rho p minus rho by rho rho (No audio from 41:52 to 42:01) half. So, u t by V o M is equal to 2.32 divided by epsilon M to the power 3 half’s.

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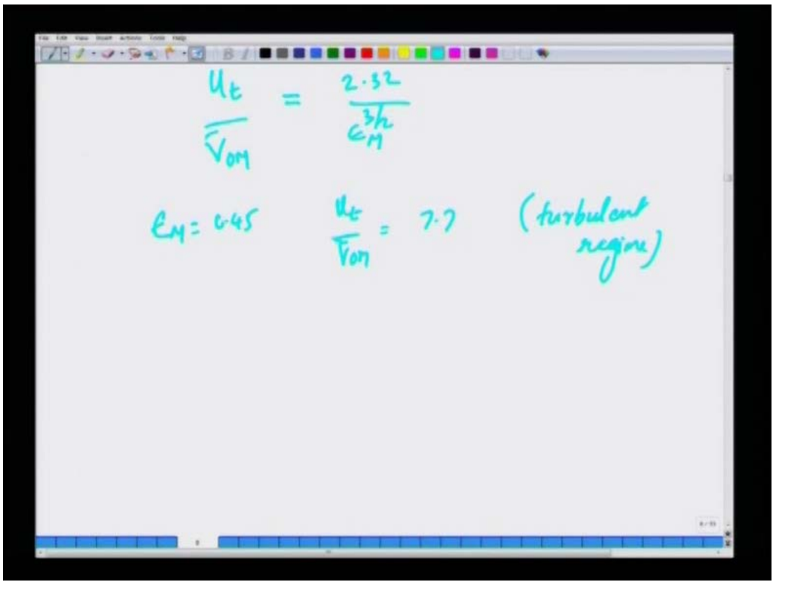

So, if you put epsilon m is 0.45 you will get by u t by V o M for the turbulent regime just 7.7. Now, this implies that there is a higher chance of particles being entrained when the flow is in the turbulent regime when you compare it with ratio for u t laminar regime where u t by V o m was about 50. So, in the turbulent region there is a chance at the velocity the particles will be entrained by the flow. Because, the ratio of the terminal velocity to onsite of fluidization velocity is very very it is not that high as in the case of laminar flow.

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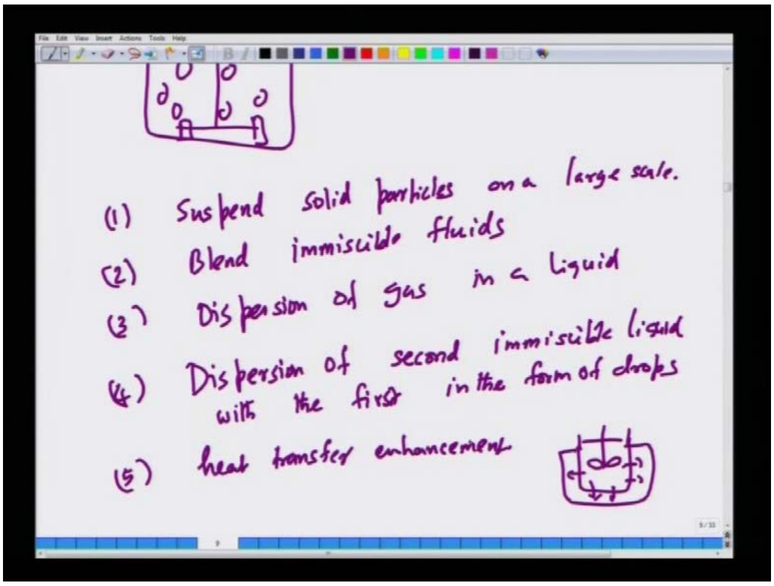

Now, these are I want to talk about fluidization. The next application I want to talk about is agitation and mixing of liquids. (No audio from 43:07 to 43:18) So, agitation and mixing of liquids is a very important unit operation in chemical industries because, we have many reactors and separators in which we have we have the necessity to mix two streams for example, you could have a gas stream and liquid stream.

And you want to suppose you have liquid and then you are bubbling gas and you want to able to mix this two things well typically. We use what is called an impeller which is a rotating element to agitate the whole system. So, that these two things are coming to the these two phases are well mixed with in the reactor or a separator.

So, purpose of agitation the reasons why we agitate is to suspend solid particles on a large scale. We want to blend immiscible fluids (No audio from 44:27 to 44:34) and thirdly we may want to disperse a gas through liquids like I just discussed dispersion of a gas in a liquid and or dispersion of a second immiscible liquid in a in another liquid with the first in the form of drops (No audio from 45:04 to 45:15) and of course, we want to enhance heat transfer. Let say you want to remove heat effectively from the suppose you have container you have jacket at coolant or a heating fluid then in order to remove heat very well you have to mix this thing very well heat transfer enhancement and so on.

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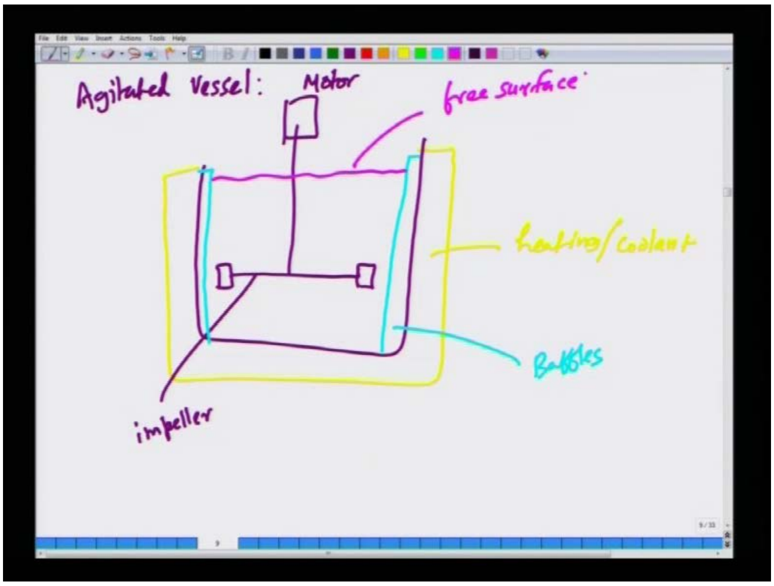

There are many reasons why agitation is used in chemical industry and a typical agitator will look like this you will have the typical agitator vessel will look like this you have a jacketed vessel. So, let me draw the jacket where in you use heating fluid or a coolant (No audio from 46:08 to 46:15) or a coolant and then you have an impeller and the impeller is driven by a motor and this is an impeller and then you have on the sides. You have graphics to reduce the tangential motion of the liquid. So, this is and typically the liquid that is present will have a free surface this is the free surface of a liquid.

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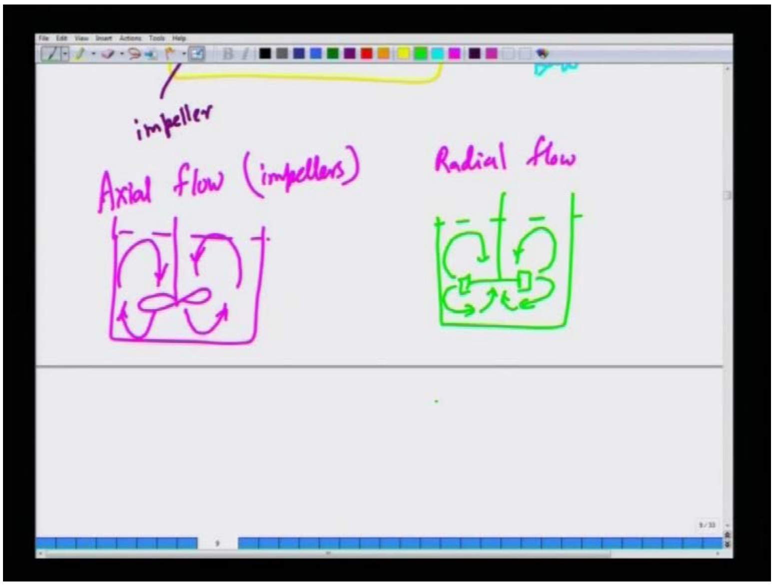

So, there are two types of impellers one is axial flow impeller impellers and radial flow. The major difference is that the axial flow impeller which looks like this it is like a propeller this is the free surface of the liquid. It tends to push the fluid like this and by continuity the fluid has to come like this backend to the impeller.

In contrast radial flow impellers will look like this is a free surface the fluid will go out like this and then coming like this. So, this push pushes the liquid radial and tangentially there is no vertical motion at the impeller here at the impeller there is vertical motion. And that is the key difference between these two impellers.

Now, the flow patterns that happen in an impeller they are very very complex and. So, if you want to design an impeller what we would like is from a fluid mechanical point of view suppose you want to make the impeller rotate at a particular velocity angular velocity in the question. We typically ask is what is the power requirement of the motor that is from a purely from a fluid mechanical point of view.

There are other important questions related to mixing an agitation for example, how does a flow pattern change the efficiency of mixing and so on. Those are more microscopic aspects of fluid flow and transport phenomena which we cannot deal in detail in this course, but here we will restrict ourselves to purely macroscopic fluid mechanical aspects such as what is a power requirement of the impeller. Although the other questions are also very important but, those are beyond the domain of this course beyond the confess of this course.

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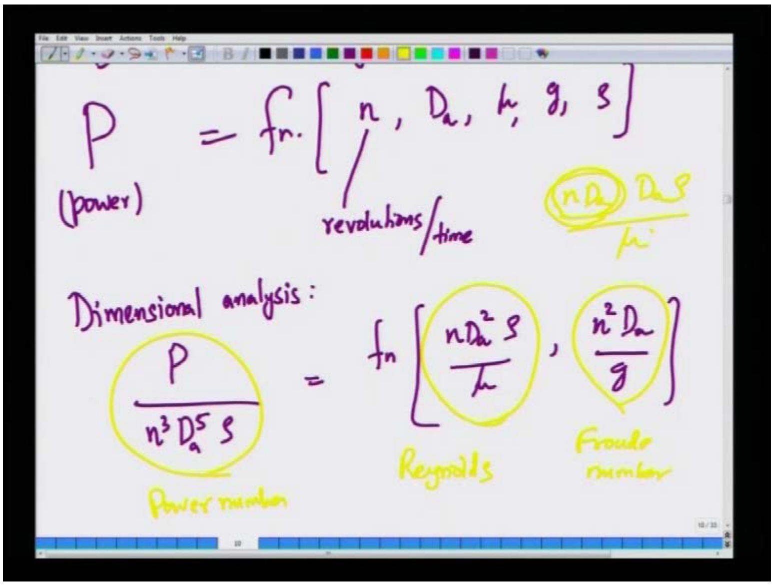

So, essentially we look at we use dimensional analysis whenever we have such a very complex problem in engineering applications a useful tool in design. Is as I mention I have been mentioning several times in this course is experimentation guided by dimensional analysis. So, what are the various variables present in the problem one is the power rate at which work is done by the impeller on the flow. That is the power requirement of the pump it must be a function of the angular velocity of the impeller.

That is denoted by small n in this notation. So, this is the revolutions per second, so for time. Now, the diameter of the impeller viscosity of the fluid gravity density gravity is important because there is a free surface so, the formation of vertices and so on, will be affected by gravity.

So, you have so many dimensional variables will use the pi theorem. I would not go in to the details. So, if you do dimensional analysis by using the standard recipe that I have mentioned before in this class. We will get pi rho is function of n D s square rho by mu times n square D a by g this is the Reynolds number this is called the power number.

These are all non dimensional groups power number this is the fluid number n times D a is the linear velocity of the tip of the impeller times D a rho by mu is a Reynolds number Reynolds number is a velocity scales times the length scale times density by mu velocity scale is given by n times D a in problems involving rotary motion of the impellers.

So, this is essentially a Reynolds number n D square rho by mu power number is a non dimensional power of the system or requirement of the system. This is analogous to friction factor in a pipe in a friction factor we worry about the non dimensional pressure drop here we worry about the non dimensional power requirement of the system.

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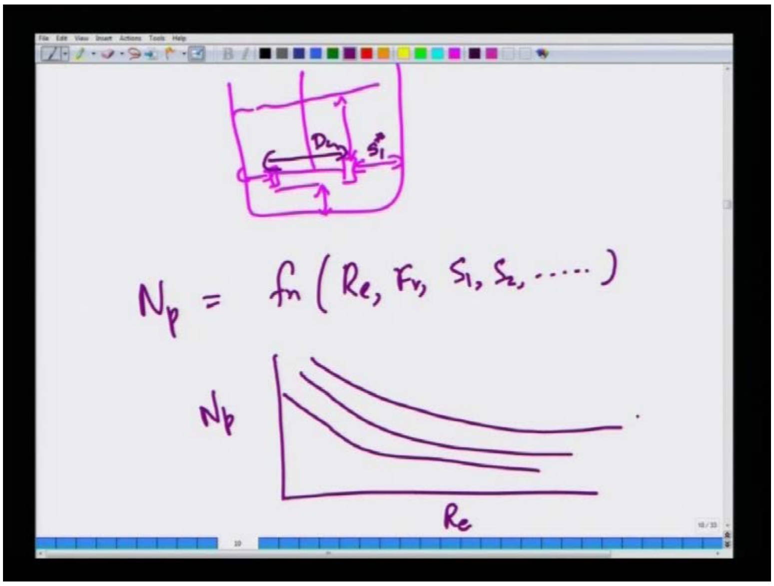

So, by doing dimensional analysis we found this but, in reality there could be other shape geometrical non dimensional ratios for example, what is the distance between this is not all because you have to worry about all the other geometrical similarities the distances is here. And the distance between the free surfaces on the impeller all these non dimensional ratios must be the same if, you want to be able to scale up across sizes and across from lab scale to the prototype.

So, there are other shape factors or geometrical factors which must be included in the correlation. So, we will just include them like this comma S 1 S 2 S 3 these are various shape factors which are essentially non dimensional ratios of let say this length S 1 star divided by the diameter of impeller is S 1 and so on.

There are various other lengths ratios that must be equal in order of our as to be able to in order for us to be able to ensure geometric similarity across scales. We cannot merely require, we cannot say that only Reynolds (( )) number are same. You will have similarities because, there are other shape factors also which are important ones we do that now the power number is a function of the Reynolds number fluid number and shape factors.

Now, then you can use experiments to correlate the power verses speed data and then you will get various curves. So, these are the curves that are obtained experimentally and for different impellers and so on for different. So, these are the curves that are obtained and ones these are curves are obtained.

One can use these information ones these curves are there in the form of handbook one can use this information to scale up impellers and mixing agitation vessels from lap scale to prototype in. So, for as the power requirement of course, as I mentioned there are other issues involved in the design of agitators and mixers.

Because, we have we are now, we have so for not worried about the actual purpose the purpose is not to just design an impeller with power requirements are ultimate purpose is to worry about mixing efficiencies and so on. But, though those are beyond the scope of the present introductory course and those will form parts of advanced courses in transport phenomena.

So, right now all we can convey at this point of understanding is that in order to design mixing agitation vessels. We have to use the concept of power number to scale up as a function of Reynolds number to scale up from lap scale to industrial la prototype scale in order to find what the power requirement of mixing an agitation vessels is.

Now, in the remaining few classes lectures that left with it we are going to discuss in detail what are turbulent flows we have discussed few applications in chemical engineering process. Industries namely flow thorough pack beds flow through fluidized beds then we had discussed mixing an agitation. And we have discussed a little bit on flow of particles and settling velocities and settling chambers and how they can be understood using fundamental principles of fluid mechanics with suitable approximations.

And the next segment list last remaining 3 or 4 lectures. I want to focus on understanding turbulent flows in a little bit more detail than what we have seen. We have so for only describe what is the turbulent flow will try to understand them in little bit more detail.

We will stop here at this point in this lecture, will continue in the next lecture on turbulent flows.