Fluid Mechanics

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Lecture No. # 38
Fluid Mechanics

Welcome to this lecture number 38 on this NPTEL course on fluid mechanics for undergraduate chemical engineering students. So far in this course, we have discussed various phase of understanding and analyzing fluid flow problems in that occurring chemical engineering applications starting from the basics. When we try to analyze fluid flow problems, we basically spelt out 3 forms of attack.

One is you have the microscopic or integral balances, where in you write the balance of mass momentum or energy through the entire system of equipments such as forms compressors valves and so on. And while we do this approach, we often had to specify, what is that viscous losses that happen in various parts of the flow? Or when you do the momentum balance you have to specify, what are the forces, drag forces that are exerted by the valve on the fluid?

Now these inputs are not available within the domain of microscopic balances so, one often has to resort to experiments. And specifically, when we consider flow in a pipe for example, if you want to understand viscous flows in a pipe, we have to do experiments. And their we found that we can characterize viscous losses in terms of friction factor versus Reynolds number charge.

The other alternative to experiment is to carry out differential balances. And differential balances gave rise to the Navier- stokes equations of for a Newtonian fluid. Which are essentially differential momentum balance for fluid that obeys that obeys Newton constitute relation, that is valid at each and every point in the flow.

Now, the difficulty with the Navier- stoke equation has I have told you is that, it is the highly complicated non-linear couple set of partial differential equations. So, one has to make very drastically simplifying approximation in order to be able to solve the Navier stoke equation exactly. And when we did that, we landed up with very simple flow such as the Eigen possible flow. Which is, which says such the flow velocity profile is parabolic for flow in a pipe?

But, we also mention that, the result that we obtain from this parabolic velocity profile for the variation of well volumetric flow rate with the pressure drop is valid only when the Reynolds number is less than 2000. When the Reynolds number is greater than 2000, the theoretical prediction for assuming this Eigen console velocity profile, parabolic velocity profile fails when the Reynolds number is greater than 2000. And the friction factor be pressure drop volumetric flow rate relation is very very different.

And we attributed this because, of the transition from laminar to turbulent flow that happens in a pipe. And so when ones flow undergoes the transition, the simplifying approximations that we made to arrive at the lamina flow solution is no longer valid. The parabolic flow solution is the lamina flow solution.

That is no longer valid because, the flow becomes un steady and it becomes three dimensional so all set of complications set in once the flow becomes turbulent. So, we have to resort only to experimental data to find out, what is the friction factor for given Reynolds number, in the turbulent regime in the laminar regime of course? We had, we can derive exactly that f is 64 by r e or 16 by r e, depending on what the friction factor is? Whether it is a Darcy friction factor or the fanning friction factor?

We can exactly derive that relation. But, for the turbulent flow regime we only have experimental data. Now we can ask the question, can we make any progress in understanding the turbulent flow regime starting from the fundamental governing equation that is the Navier- stoke equations?

The answer is very dance the question is an extremely difficult question but, an extremely important question also. Because, it is right now believe that the description of turbulent flows is within the domain of Navier- stokes equations. That is, it is believe that all the information about turbulent flow is present in the 3 Navier- stokes, Navier-stokes mass and momentum equations. But, the only difficulty is in obtaining the solution of this Navier- stokes equation because, they are extremely complex.

There are approaches which do try to compute the Navier- stokes equation exactly in the turbulent flow regime. But, again the computation becomes extremely difficult as the Reynolds number becomes larger and larger.

So, can we ask a question, well, the question is can we take a approach where in we sought of do not compute the Navier stokes equations? Solve the Navier- stokes equations computationally using sophisticated computational algorithms. But, can we make some simplifying approximations that are good enough for engineering applications. That is the view point that we are going to take right now.

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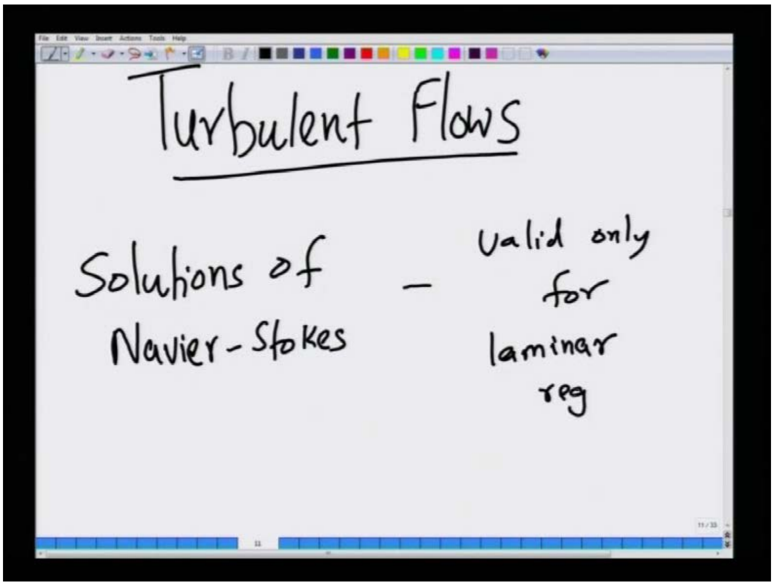

So, essentially, so we are going to discuss turbulent flows in the next in the remaining 2 or 3 lectures that we have in this course. And the key thing is to remember remind our self is that the solutions of the Navier- stokes equations. That are obtained so far are valid only under laminar flow conditions.

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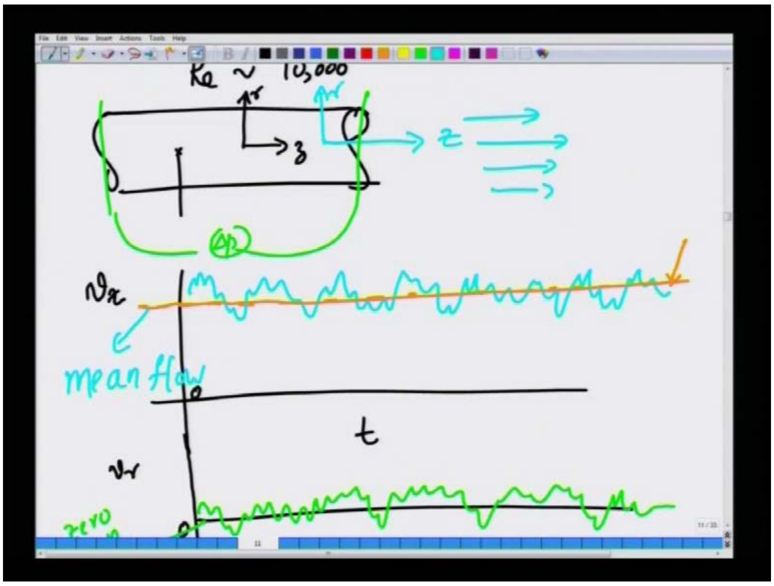

So, in the turbulent flow regime, for example, in a pipe if you were to understand flow in a pipe at a Reynolds number greater than 2000. Let say Reynolds number of the order of 10,000, let say. And if you have to measure the velocity at a given point in the fluid as a function of time, suppose you have this is the flow direction z. Remember z is a flow direction and the normal direction is r the radial coordinate.

Now, if i were to plot v, the z velocity the actual velocity along the z direction, the measure velocity as a function of time it is going to appear like a random signal. Because, in turbulent flow, their velocity fluctuate very widely with time and of course, with space also but, here we are looking at a fixed point and space.

And we are measuring the velocity in the axial direction that is the flow direction this is a z direction as a function of time. And you will find that the velocity fluctuates widely. But, it fluctuates widely about a mean velocity because; there is a mean flow in the z direction.

So, there is a mean flow, if you were to average this velocity over significant time, average this fluctuating velocity over a significant time, you will get this orange line.

That is the mean flow the, mean flow is the time average velocity in a given direction. But, if you were to record the velocity in, let say the r direction, let this is this 0 of the velocity as a function of time. Here, it is going to look like this.

It is again random but, it is random about 0 mean because there is no mean flow in the r direction. The, there is a pressure drop that is exerted in the z direction. So, there is a mean flow in the z direction although there are fluctuation, strong fluctuations about the mean flow. And but, there is none, the less mean flows, if you time average this velocity along a given direction with time.

For a certain time period, you will find that that will be close to this arrangement, so, that is the mean velocity. If we have to do the same thing for the velocity in the normal direction, that is the r direction, remember this is the r direction. In the normal direction, there is no net flow, there is no mean flow. Therefore, (()) velocities in the r direction that are highly fluctuating but, if you were to average this you will get a value of 0.

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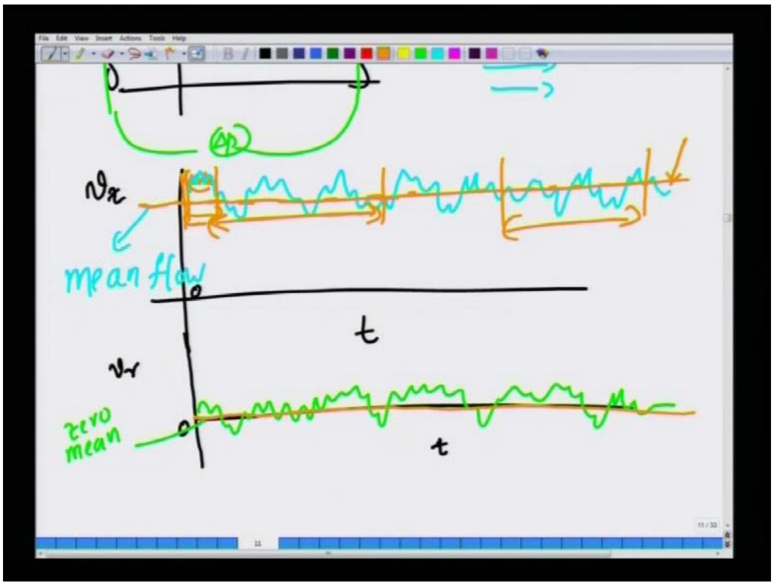

So that is the key thing about turbulent velocity fluctuations. That if you have a studied driving such as a steady pressure drop across a pipe, 2 ends of the pipe. And if the flow is a in the turbulent regime then if the as long as driving pressure drop is steady then the velocity in the axial direction, the flow direction. If you average out over a sufficiently long time, the time average you should, you average should be large compare to the time scale of fluctuations.

Suppose, you average only about this time obviously you would not find the mean to be the same the time average to be same as this orange mean. But, if you were to average over a sufficiently long period, where there a enough, there are equally likely probability of the velocities to be in the both the positive and negative side of the mean. Then first you will have a reasonably well behaved mean which is independent of the time. So, you could take the mean in this time gap or you could take the mean in this time gap. And the mean value will be the same it would not be different for different time.

So, the only criteria is the extent of time over which average should be sufficiently large compare to the time scale of fluctuations. Now, so by looking at this experimental observation, the turbulent velocity fluctuates widely and strongly with both space and time. But, there is a well defined mean flow.

Then we can ask the question whether we can predict these mean flow properties of a turbulent flow? And the reason why this question is of importance in practical applications is because, quantity such as a volumetric flow rate and pressure drop or friction factor, there all are average quantities, their time average quantities.

We are not going to ask the question of a because the pressure drop is over the entire length of the pipe. And so there is a large amount of average in that is going on when you find out, what is that relation between pressure drop versus flow rate in the turbulent flow regime? And so clearly we are not interested in knowing, what is the detail spatial tempo variation of the velocity, for example, v z with respect to r z and time.

So, the question that we are going to restrict ourselves is not to predict the turbulent flow in detail, the fluctuations in detail. But, whether, we can predict the mean aspect of the time average aspect of turbulent flow in some manner. Because, practically important quantities, quantities of practical interest such as friction factor or pressure drop they are actually mean quantities, they do not depend in detail. Well they are average quantities, the fluctuations are average dot when you measure the volumetric flow rate or the pressure drop.

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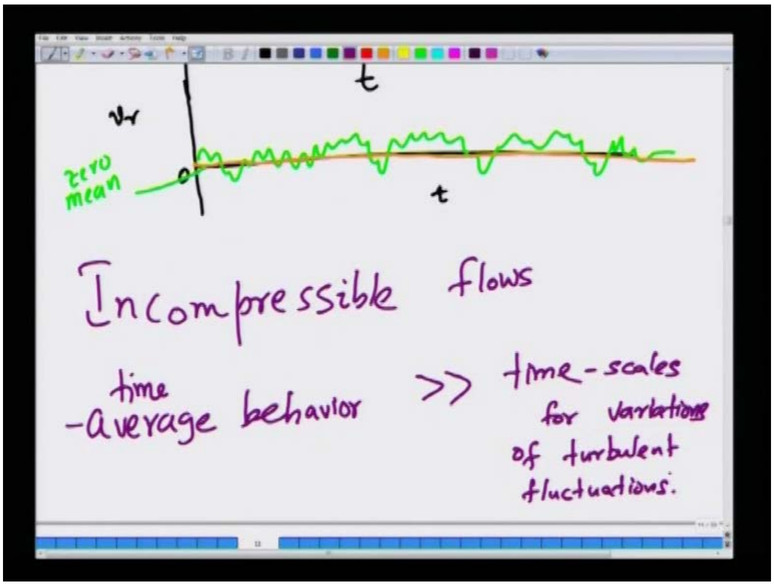

And we are going to restrict ourselves to incompressible flow as before. So we are going to restrict ourselves to incompressible flows. And we are interested only in average behavior large compare to time average behavior, large compare to time scales for variations of turbulent fluctuations.

(No audio form 12:04 to 12:14) So, that is these are the key restrictions that we are going to keep reasonably restricted goal. Not to describe the entire turbulent flow in it is entirety but, to restrict ourselves to average behavior.

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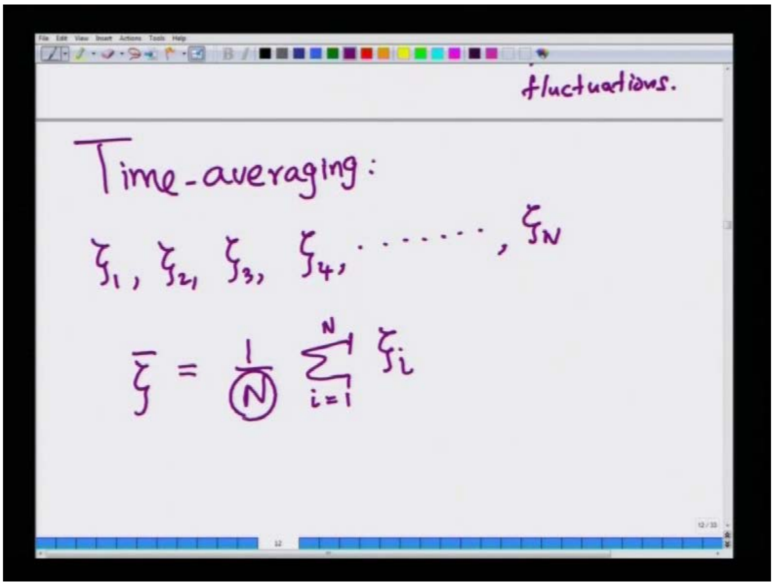

So, we have to say a few words about time averaging. Suppose, you have a variable such as velocity, let us denote the variable as zeta and you have measured at N values at N different times, discrete values N values. Suppose, you want to know what is the average of such N measurements? You will simply say that you will add all the values at various times. That you have recorded using some instrument this could be the velocity for example, and you had everything and divide by the total number of measurements that is N. This is the time average, this is the average ,when you take reading at discrete values of times.

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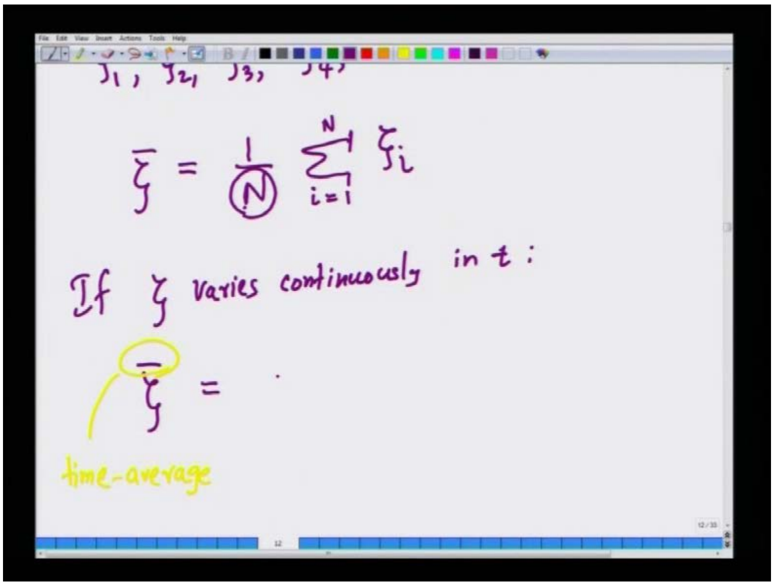

If z varies continuously with time, then you can convert this definition to an integral zeta average. So, time average will be denoted by an over bar. So, over bar denotes time average, from now onwards. So, zeta bar is nothing but, suppose I look at this sketch of what a turbulent velocity fluctuations look like.

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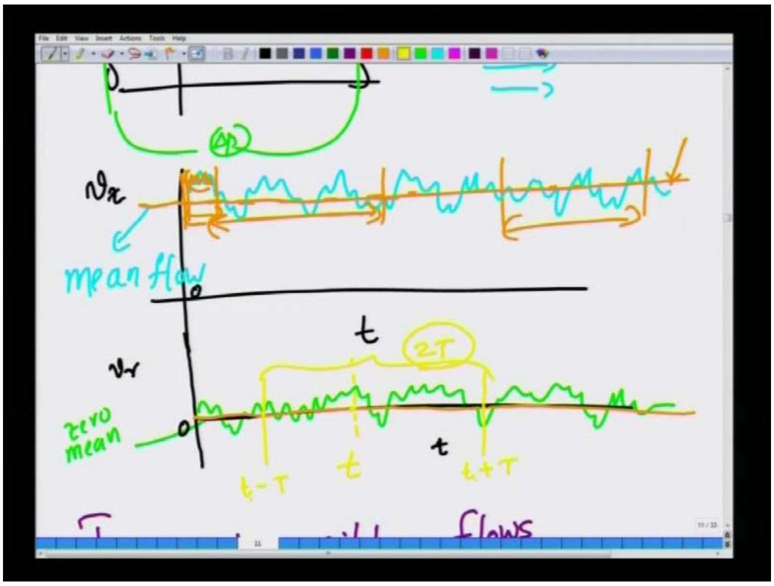

So, I am going to take the value t and I am going to average over, I am going to take a specific value of time, running time. And then I am going to average from t minus T capital T to t plus capital T so, essentially the width or window of time over which I am averaging is 2 t.

So, I am going to take this function, let say we are here. I am going to integrate from t minus T to t plus T over time. And then I am going to divided by the interval over which I am integrating.

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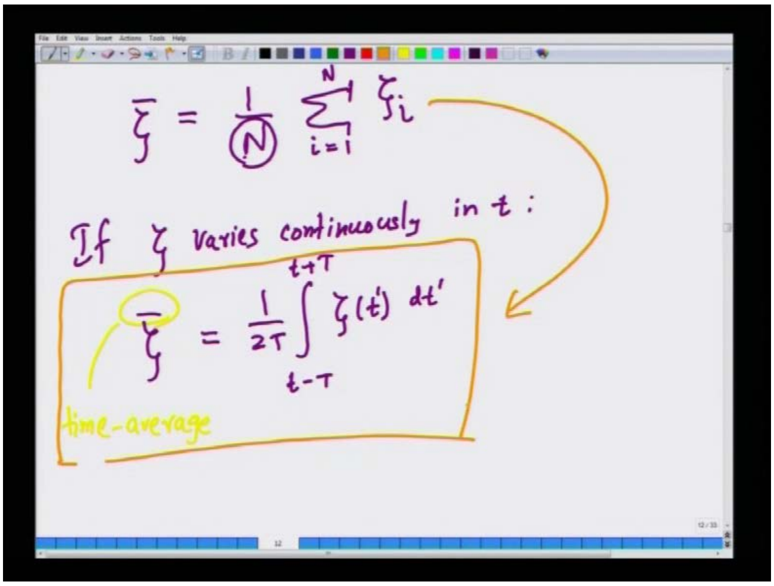

So, this is something like t minus T to t plus T, now we are just writing a generic variable zeta of time. Since we are we can use dummy variable t prime to do the integration. This is the general definition of time averaging, and time averages will be denoted by an over bar. This is essentially a generalization to a continuous time of the discreet time average which we are all familiar with, essentially add all the data that you measure and divide by the number of data points. This is essentially a generalization of that to continuous limit.

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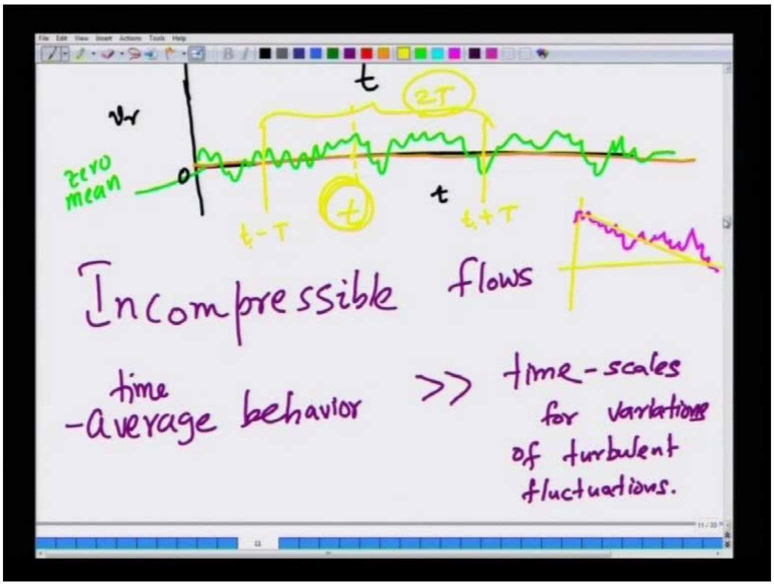

Now, in principle this z this average that I am defining could be a function of time. If for example, my data looks like this, it is fluctuating but, also there is a trend in the fluctuations. So, that could be a mean that decreases in some way but, this is not what we are going to look at s.

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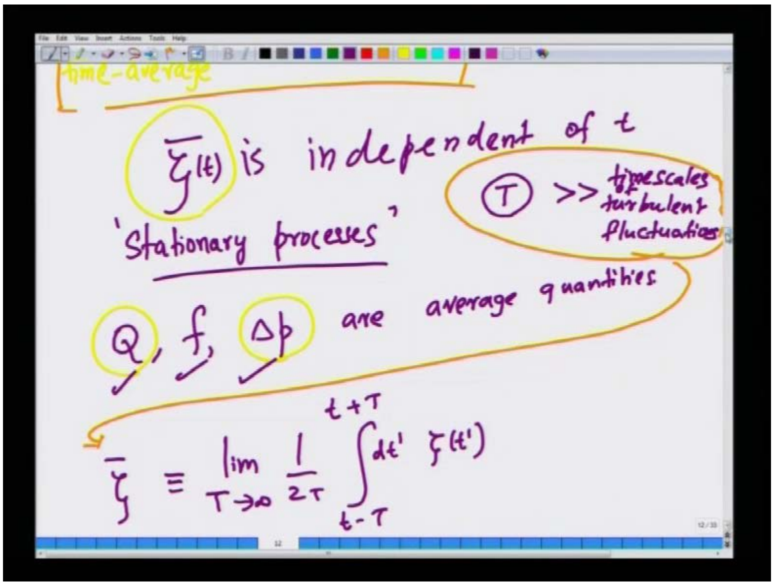

We are going to look at flows that are steady in the mean, that is the time average itself is independent of time. Independent of the running time t, this is in principle function of t because, you could vary you compute this average at different times. And then take the window t small t minus capital T to small t plus capital T. And you can do this integration for several values of time t, that is the running time. But, we are going to restrict ourselves to cases where the c bar of t is independent of time of T small t. Such process are called stationary processes in turbulent flow literature. They are the mean itself the time average itself is independent of time so long as you choose the time window capital T, large compare to any time scales for turbulent fluctuations.

If you have chosen this carefully enough then you would expect that this is independent of t. So long as you are driving force delta p is actually independent of time. Which is what we are going to look at?

So, this is in some sense, what is called steady in the mean. The mean flow is steady of course, there are fluctuations about the mean. And they will do they do play an important role but, in an indirect way.

So, because the reason why we are interested in this such average quantity is that? Quantity such as Q volumetric flow rate friction factor pressure drop are in fact average quantities.

So, they do not change with time for example, if you have turbulent flow in a pipe, you will find that Q delta p and all will be same. Although at each and every point in the fluid in a pipe, the velocities will in fact change, but, these quantities will not change with time. So, that is the reason why we are interested in computing only the or analyzing only the time average quantities.

So, to write this aspect, to describe this aspect that t is large compare to time scale of fluctuations. We are going to express this formally as, limit capital T tending to infinity 1 over 2 t small t minus capital T to small t plus capital T d t prime c of t prime.

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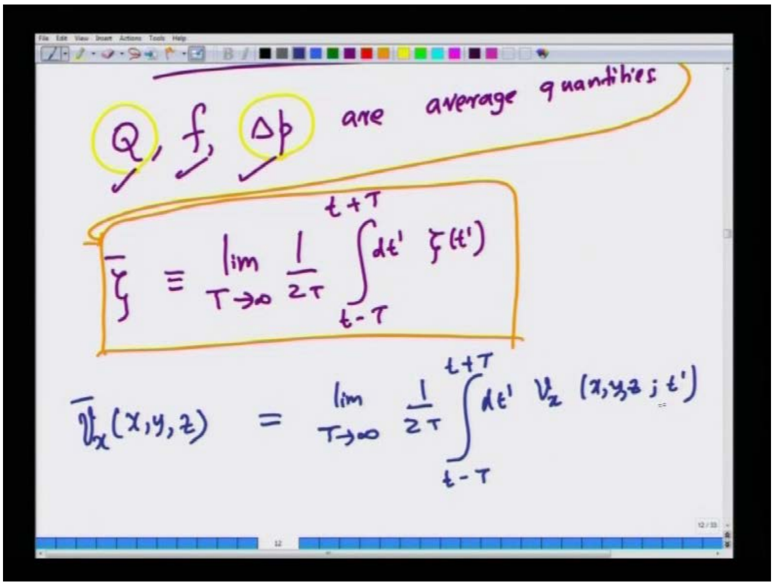

So, what we mean by infinity is not truly infinity but, it essentially the t is large compare to any turbulent time scales, turbulent time scales of fluctuations. So, for example, along this lines we can define the average velocity in the x direction of a flow.

So, once you do the time average it becomes independent of time because now we are considering quantities only that are stationary process. The turbulent flows are stationary, that is the average quantities themselves do not depend on the time, running time small t. So, the average quantity is limit time average velocity in the x direction for example, is small t minus capital T to small t plus capital T, d t prime v x the actual velocity which is of course, the function of time. So, while the actual velocity is function of time the average velocity we are assuming is time independent, the process is stationary or steady in the mean.

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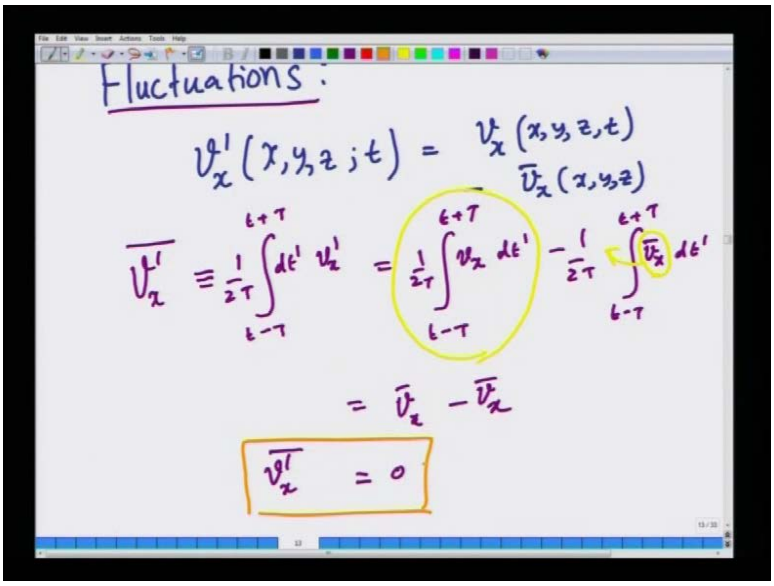

Now, we can define fluctuations as deviations from the average. So, we can define velocity fluctuation v x prime as the actual velocity. Now the fluctuations themselves are of course, functions of time because, you are subtracting from the actual velocity which as fluctuations. The mean velocity which as no time dependence so clearly that, this velocity fluctuations, will have time dependence.

By definition, if I take the time average of a velocity, of the fluctuating velocity I am sorry, if I take the time average of a fluctuating velocity this is 1 over 2 t, t minus T 2 t plus T d t prime, v x prime, this is integral t minus T to t plus T, v x, d t prime, minus 1 over 2 t, integral t minus T, t plus T v bar x d t prime. But, v bar x is independent of a time because, it is a constant we have assume that so, we can pull it outside the integral.

So, the first thing will exactly give you v bar x. And the second thing will also give v bar x because, once you pull it outside the integral this becomes simply 2 t divided by 2 t, so, v bar x, so, this is 0. So, the time average of a fluctuating quantity is generally 0 of a single fluctuating quantity.

Because, there is if you give the time interval is you take the time interval to be sufficiently large compare to the time scale of fluctuations. That is an equal likely hood of the fluctuations to be both on the positive side as well as the negative side of the mean. Therefore, if you just average the fluctuations over a sufficiently long time, they have to go to 0 if the fluctuations are truly random. And in turbulent flows, the fluctuations are in fact apparently random, so, we do have this to be satisfied.

So, this is an important result that will keep using. When we go further in deriving the time average Navier stokes equations. Now in our discussion will also have opportunities to work with quantities like this.

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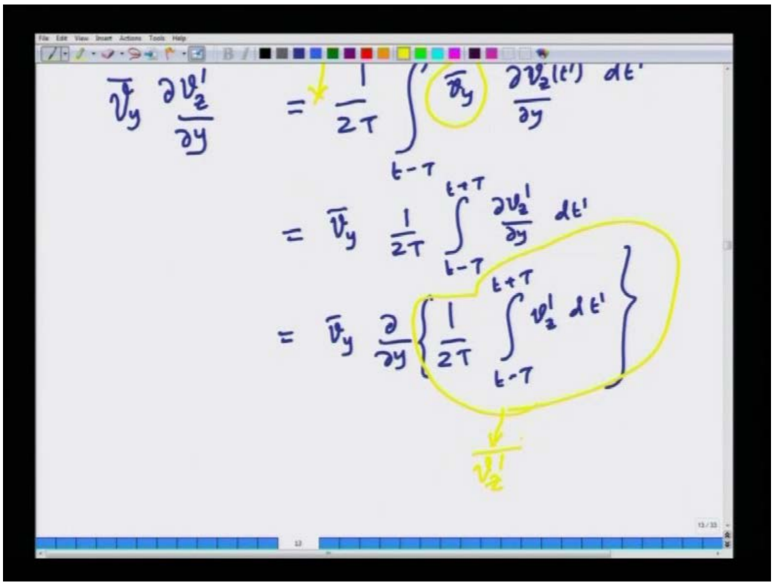

Suppose you have a product of a mean and a fluctuating quantity. We may have to do, a time average of this entire quantity. Now, this can be written in the following way, this can be understood in the following way. 1 over 2 t integral t minus T to t plus T v y d v z by d y of t prime d t prime. This is what the meaning of the time average is.

Now, since v y is constant, v bar y is constant, it is a time average. And we are assuming that the time averages are independent of the running time t small t. So, we can pull it outside the integral. So, this becomes v bar y times 1 over 2 t t minus capital T to t plus capital T d v z prime by d y d t prime.

Now, the integral is merely a summation over various values. So, the summation and differential sine can be interchanged to give d d y of 1 over 2 t integral t minus T to 2 plus T v z prime d t prime. This entire quantity inside the curly braces is nothing but, v z prime average time average.

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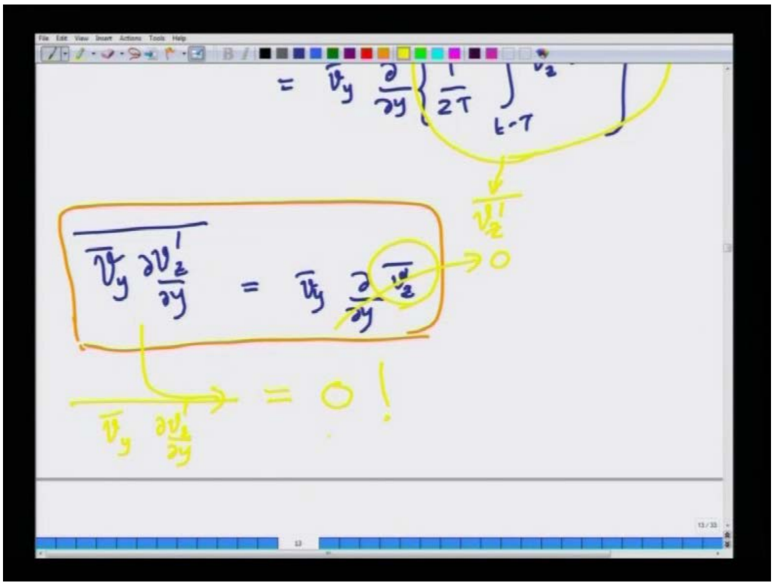

So, we have v y bar times d v z bar prime d y whole average is nothing but, v y bar times d d y of v z prime average. So, this is also, something that we will use, when we derive the time average in Navier stokes equations.

So, what we are trying to say is that generally, if you have a time average of quantity that is. Now, sorry, this is one step that we can also proceed one step ahead and say that since this is time average of fluctuating quantity, this has to be 0. So, whenever you have a product of an average times of fluctuating quantity. For example, like this and if you take the time average that will always be 0 is a very useful relation. Because, we will see that, we will have several terms that linear in the fluctuating quantities, when we derive the time average differential Navier strokes equations. And in such circumstances one can through away these terms because, they are identically 0.

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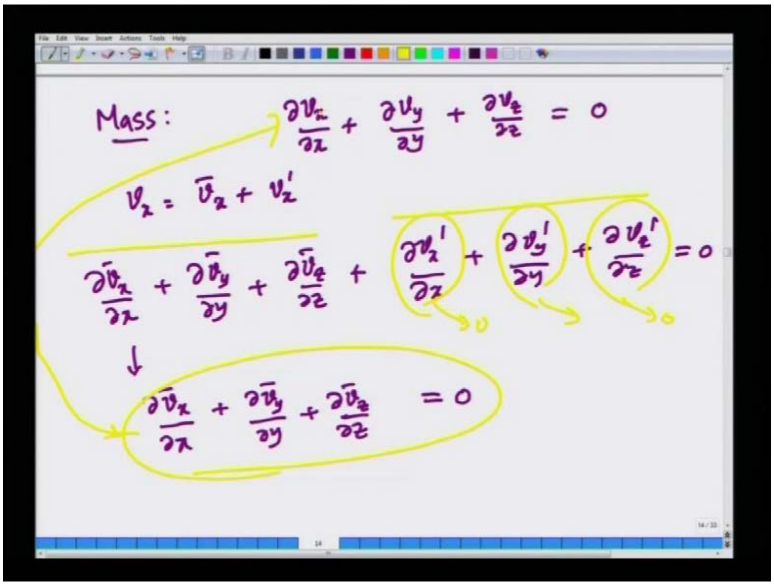

Now, I am going to take the mass conservation equation and do a time averaging. Mass conservation equation in Cartesian co ordinates will stick to Cartesian co ordinates for simplicity, is simply this. Now I am going to write v x as v x bar plus v x prime spitted in to 2 and then write this expression as z plus v x prime by partial x.

So, now I will do the time average of the entire equation, this plus this, I am going to time average is entire equation. Now, if a time average and is already existing average this will simply give the same value. Because, it is already average over time these are constants. So, if the average that I will simply get the same value, same expression but, if a time average, this each term will give you 0, because, these are linear in the fluctuating quantities.

So, if a time average this entire thing is, they are individually 0, if time average then. So, I will get one relation that the mean velocities also satisfy the same continuity equation. Partial v x bar by partial x by partial v y bar by partial y by partial v z bar by partial z is 0. Now I subtract this from here, then I will get after using v x is v x bar plus v x prime.

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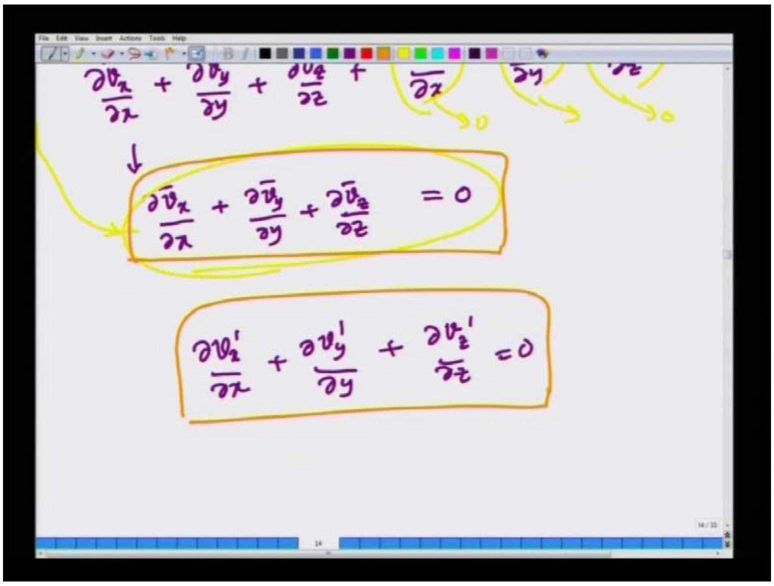

Then I will get that the fluctuating quantities obey this equation which is also the same as the continuity equation, the mass conservation equation. So, we are found that the mean velocity satisfies the mass conservation equation. The actual mass conservation equation del dot p is 0 and we have shown that the fluctuating velocities also satisfy in mass conservation equation. So, this is the result from doing time averaging of mass conservation equation.

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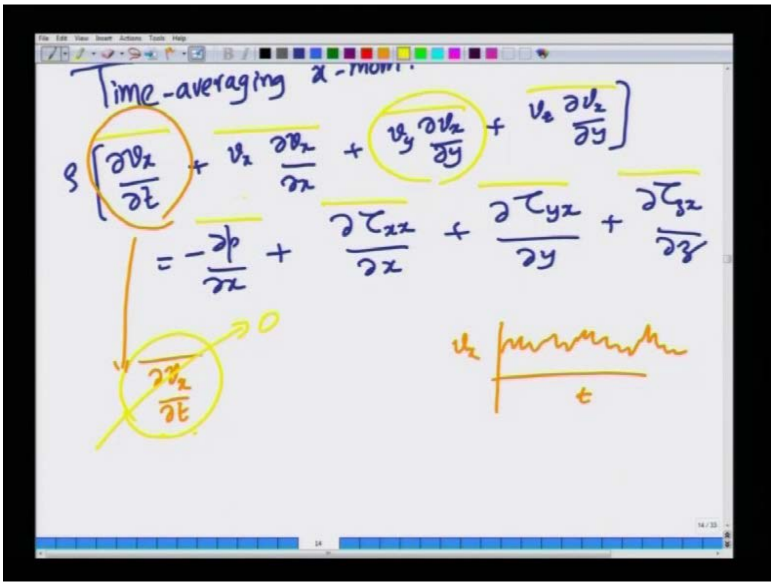

Now we are going to do time averaging of the momentum balance. I am going to specifically take the x momentum balance, first write the entire x momentum balance in Cartesian direction, sorry, in Cartesian co ordinates in the x direction.

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Plus you have. Now I am going to write my expressions in terms of only the stresses and if the reason for this will become clear little later. I am not going to use a Newtonian constitute law. I am going to write every, this is the Navier strokes equation before substituting the constitute relation for the stresses. This is what we call the Cauchy momentum equation.

Now, I am going to substitute, I am going to average this entire equation. So, I am going to put a bar on each term, and then we will have to simplify each term. Now, if you look at the first term, you look at the first term, this is the time average of the rate of change of velocity. Now, if you look at the velocity, the time rate of change of velocity it is the velocity itself is a very randomly fluctuating variable time. If you look at, if you average the rate of change of velocity, since a fluctuations are completely random. You should time average it, it has to be 0 for a truly random fluid so, we said that to 0.

Now, let us look at any of these terms, for example, let us look at this terms, they are all similar. They involve product of two terms, this is the convictive non-linearity, this is in the Navier strokes equation.

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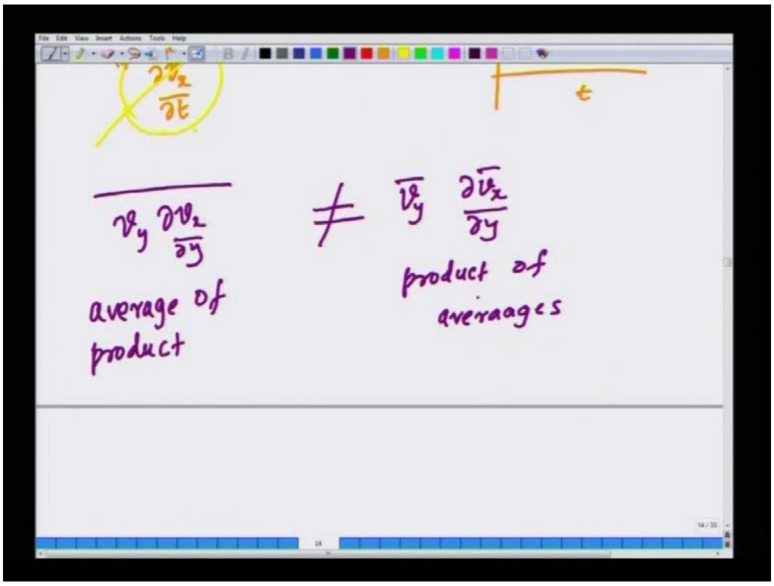

So, let us look at v y partial v x by partial y. Now, this is the average of two products but, the average of two products is not equal to the product of the averages in general. You might so, this is the average of product of two quantities, this is the product of the averages of the same two quantities. But, for fluctuating variables, these are not generally is the same and we will we will show why it is. So, because when you substitute.

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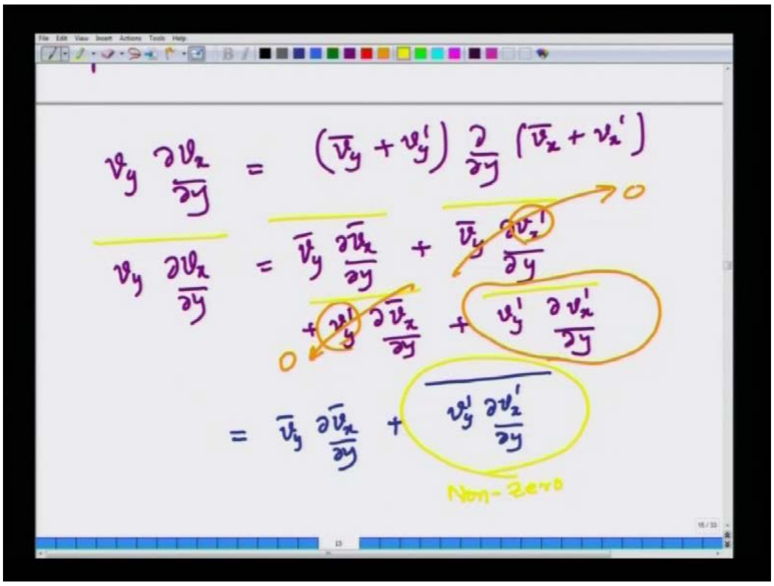

For example, this is v y bar plus v y prime times v x bar plus v x prime. So, if i expand out I will get 4 terms. So, we will get v y so v y partial v x partial y this v y bar partial v x bar partial y plus v y bar partial v x prime by partial y plus v y prime partial v x bar partial y plus v y prime partial v x bar by partial y.

So, if I average this entire term you will have all the terms, all the 4 terms present. So, clearly the average of product of two quantities is not the product of the averages. Because, that is only the first term but, the other terms to worry about, now, let us look at each term in detail. The first term if you do the averaging, we will give you the same result because, you are averaging an already averaging quantity. So, it is simply going to give you x by partial y.

Now, the second term the fluctuation is linear we just showed that such a term is 0. And same goes in third term 0 but, key thing is to understand. What is the type of the forth term? Is it 0, because it appears like, these are also fluctuations but, remember carefully that these are products of 2 fluctuations. So, this term in general is not 0, because, this is not just quantity, that is linear in the fluctuation that is the quantity, that is the product of, that is actually an average of product of 2 fluctuations. And in general such quantity cannot be 0, because, trivially.

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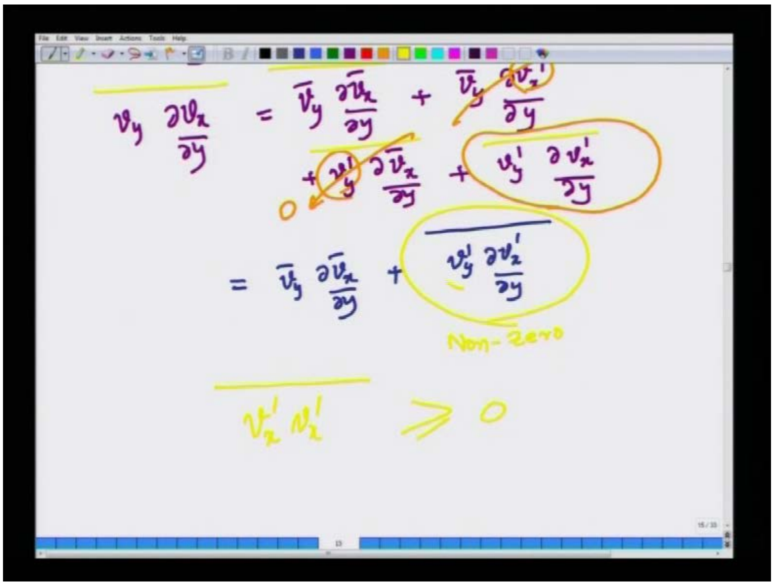

If you look at quantities like v x prime times v x prime, and if you average v x prime times v x prime is either positive or negative. So, if you square it becomes always a positive quantity this is always greater than or equal to 0.

So, obviously in such cases, where if you have a quantities like v x prime multiplying by itself, we trivially see, that the time average of that quantity, even though it involves fluctuations is not 0. Likewise, if you have v y prime times partial v x prime by partial y. If these two fluctuations are co related over certain time then you cannot set this to 0. Because, they, if they are co related, then if you multiply them and take a time average, they will in general give you a non 0 value.

So, we cannot through away this term, specifically. Generally, we cannot through this term and they will able to contribute in a major way to turbulent flows.

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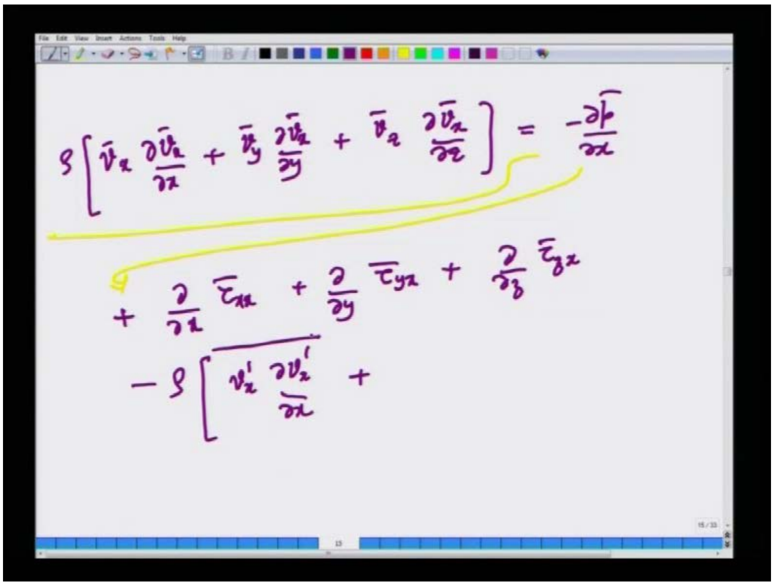

Now, let us going to, let us look at the Navier strokes equation. We said that the time derivative is 0, if you average it because, there is equal likelyhood of time derivative to the positive or negative, the average over a significantly longer time. So, we said that 0. So, the various convective terms, I am going to first write the mean term. (No audio from 33:07 to 33:17) And there are also fluctuating terms, which are I am going to write in the other side of the equation. So, the pressure term plus so I am going to continue from here, plus, now you have this various stresses partial by partial x of tau x x.

So, these are simply normal averages because, they are linear terms. If you average a quantity you just get it is average, because, the integral sign and differential sign can interchange. (No audio from 33:52 to 33:58) Now, there are other terms that we could not neglect.

So, we have to write them here, they come with a minus sign because, they come to the other side of the equation (No audio from 34:09 to 34:15) bar average plus v y prime partial v x prime partial y average plus v z prime partial v x prime by partial z average this is the entire relation. Now, I am going to simplify this further.

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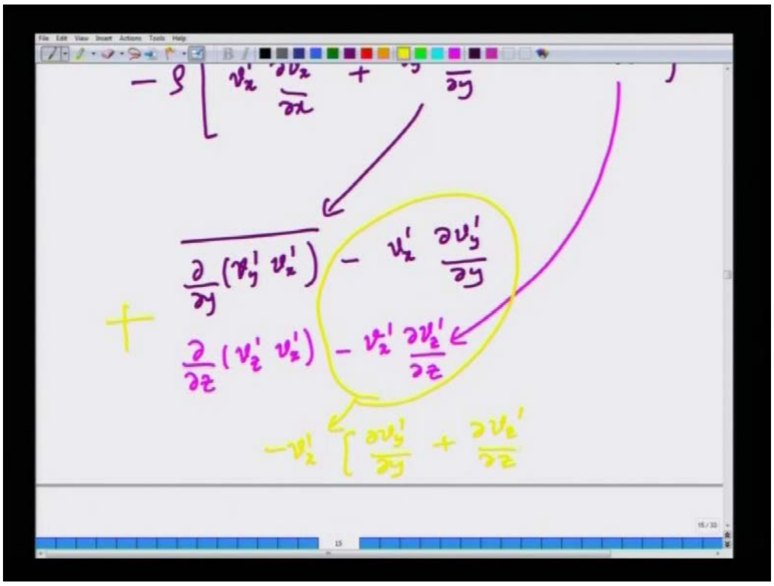

So, let us take this term first, we can write this as d d y of v x prime v y prime whole average minus v x prime partial v y prime by partial v y. Likewise, this term will become d d y or d d z I am sorry, of v x prime v z prime, let us write v z prime x prime for clarity. Likewise we are going to write v y prime times v x prime. Although, it makes no difference but, just for the sake of clarity because, it will help us when we write down the most general time average equation. So, let us keep it like this minus v x prime times partial v z prime by partial y, partial z I am sorry.

Now, if I combine these two, now these must be added. If you remember all these terms, these two terms added, we have to add these two terms. Now, if I combine these 2, they will be like minus v x prime times partial v y prime by partial y plus partial v z prime by partial z.

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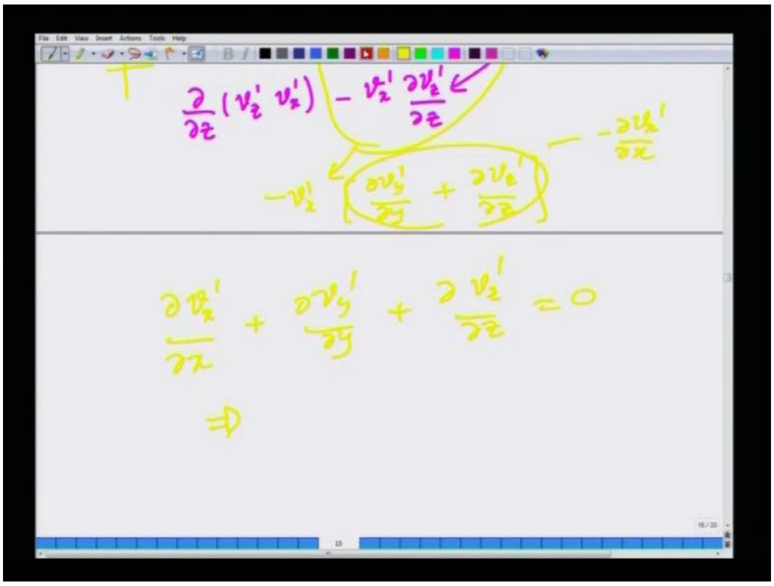

Now, we can use the continuity equation, which we derived from the mass conservative equation, which said that partial v x prime by partial x plus partial v prime by partial y plus partial v z prime by partial z is 0.

So, instead of partial v y prime plus partial y by partial y plus partial v z prime by partial z, I can write that as. So, this is essentially minus of partial v x prime by partial x. So, if I multiply these 2 minus signs, it will becomes a plus. So, if I go back to this equation instead of these two terms essentially. What I am getting is?

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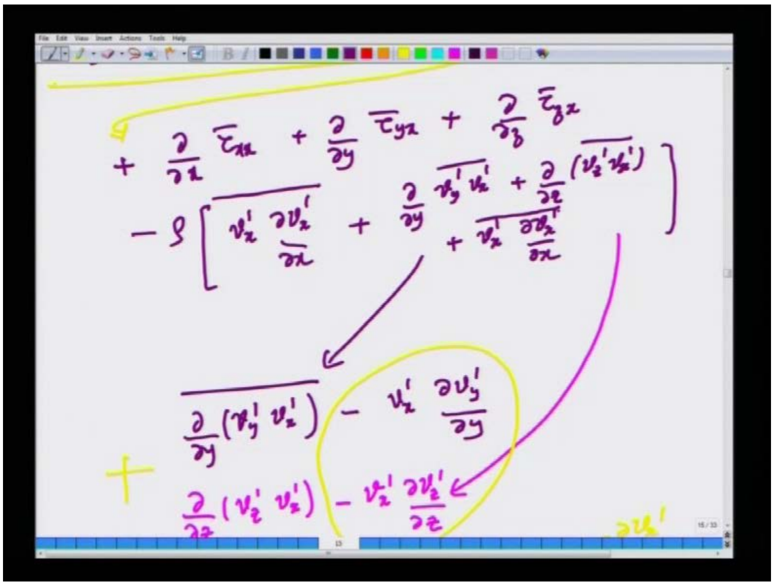

I am getting that term like d d x of v y prime v x prime plus d d sorry, d d y and d d z of v z prime v x prime plus another v x prime partial v x prime by partial x whole average. So, when we combine all these together after doing this simplifications. Now, we realize that there are two such terms, so I am going to write all I am going to put everything together now.

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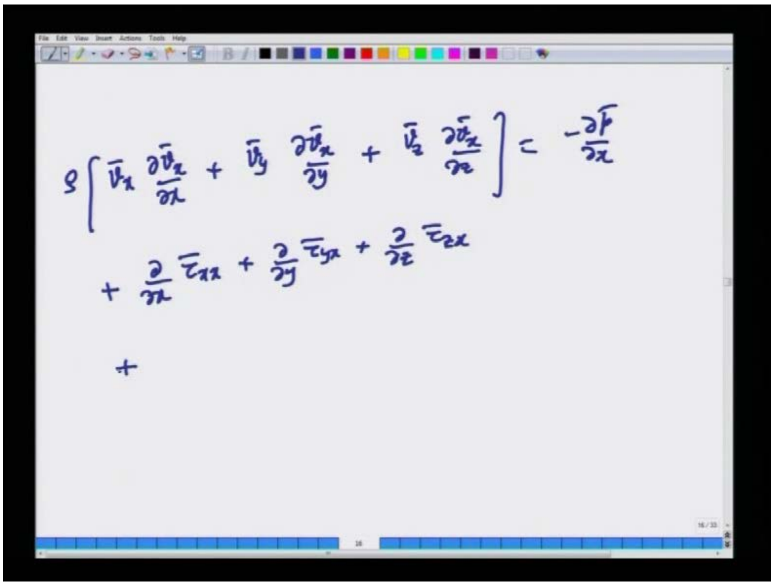

So, the left side will become of the Navier strokes, the time average Navier strokes equation will become rho times v x partial v x by partial x plus v y bar partial v x by partial y plus v z bar partial v x by partial z is minus partial p bar by partial x. And I am continuing this here partial by partial x of tau bar x x plus partial by partial y tau bar y x is partial by partial z tau bar z x plus.

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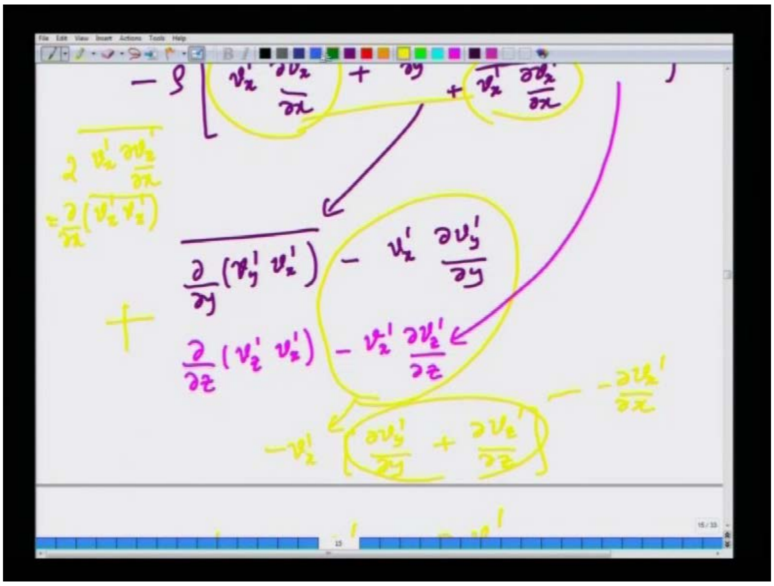

Now, if you look at this I have so let me simplify these two terms I have twice v x prime partial v x bar by partialx whole average. This I am, I can simply write as, partial by partial x of v x prime times v x prime whole average.

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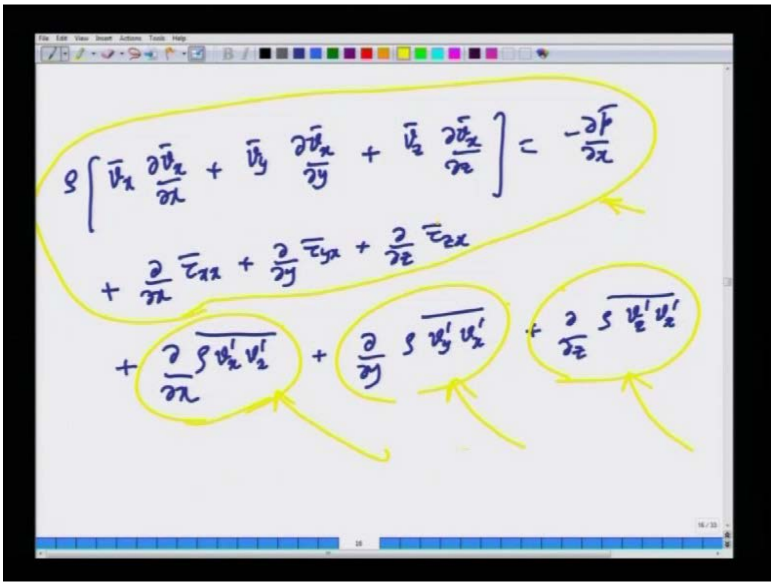

So, ones I realize that, we write this as partial by partial x of rho v x bar v x bar because, rho is also there, rho is a constant for an in comfortable fluid plus partial by partial y of rho v y bar sorry v y prime v x prime plus partial by partial z rho v z prime v x prime. So, this is the time average Navier strokes equations that we get and we have told that these are in general not 0.

These quantities are in general not 0, because, there could be correlation among fluctuations. So, if we look at this time average Navier strokes equation without theses yellow terms, which have circled here. This is exactly the same as the usual Navier strokes equation except that all the quantities are replaced by their average values, time average values. But, the time average values in a turbulent flow, is not exactly the same because, as laminar, so, this looks like normal Navier strokes equation. But the time average values are affected by the fluctuations through these 3 terms.

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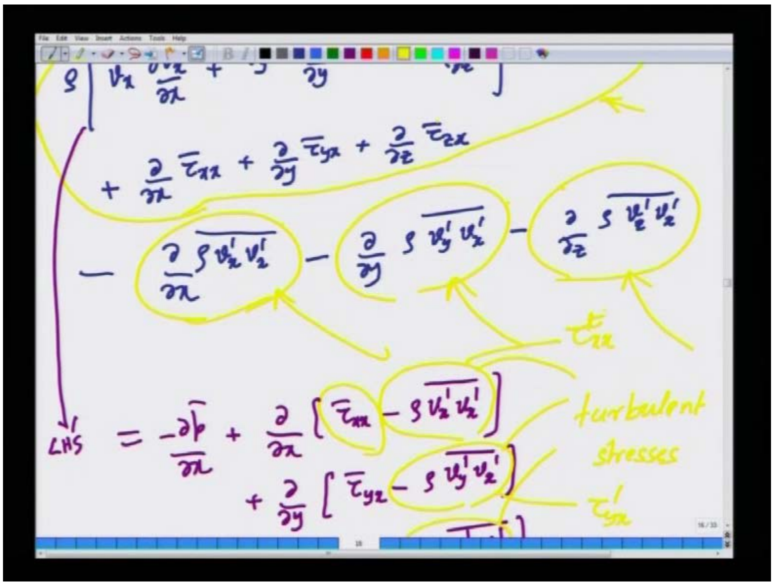

And in fact, we can write this right hand side as, sorry, the right hand side remains the same, the left hand side, sorry, the left hand side remains the same. The right hand side can be written as minus partial p bar partial x the same LHS is equal to plus partial by partial x of tau x x minus, I am sorry, there is a minus sign here and all these quantities. (No audio from 40:30 to 40:39) So, let us put the minus here, in all these quantities there is a minus sign.

Therefore, this becomes, rho v x prime v x prime plus partial by partial y of tau y y minus rho v y prime v x prime plus partial by partial z of tau z z, sorry, this is not tau y y this is tau y x and here this is tau z x minus rho v z prime v x prime average.

Now, so the same, now this appears like again the same Cauchy momentum equation except that you do not have just the mean stresses coming from the Newtons constitutive relation. But, we also have this external contribution to the, these can be interpreted as stresses. Because, they come along this have the dimensions of stress and they come in the same way as the divergence of the stress. So, this have the same meaning as stress we can be interpreter stress. So, these are called these 3 terms are called turbulent stresses.

These are the stresses that are transported by due to the that are caused by the turbulent flow. In other words you can think of stresses as momentum flux vectors, that is, there is momentum transfer from regions of higher velocity to lower velocity. And stress is a momentum flux vector so if you think of a stress as a momentum flux vector, the first term will tell you the momentum flux due to the normal viscous effect.

The second term tells you, the momentum flux due to turbulent flow because, the turbulent flow have fluctuations. And, the fluctuations are able to transport momentum over and above, what is normally possible due to molecular effects such as viscosity.

So, these are called the turbulent stresses and they are denoted by the same indicial notations, this is tau x x prime, this is tau y x prime, this why I am writing y x here. And this is tau z x prime and they are symmetric as the normal stress. You can verify that because tau x y prime is simply rho v x prime v a prime but, the order of the products is not important because, they just multiplying to quantity so it is not important.

So, these I am sorry, these are not called prime, these are called t turbulent stresses, just avoid confusion with fluctuations, these are the turbulent stresses.

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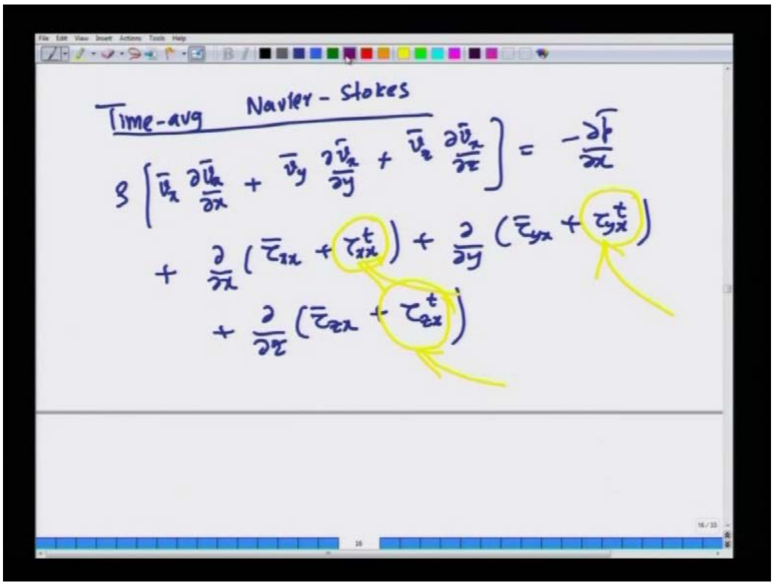

So, we can rewrite the Navier strokes, time average, Navier strokes equation as (No audio from 43:39 to 43:44) as rho by partial x this is the convective side on the left side of the Navier strokes equation (No audio from 43:55 to 44:02) is minus partial by partial x. And I am continuing here plus partial by partial x of tau x x bar plus tau x x turbulent plus partial by partial y tau y x bar plus tau y x turbulent plus partial by partial z of tau z x bar plus tau z x turbulent.

So, this is the time average Navier strokes equation. It for the mean or time average quantities with bar but, this is not the same as the time averaging of the Navier strokes equation. Because, ones you time average there are these additional contributions which are in general non 0. So, you cannot simply put bars in the entire Navier strokes equation and say is time average. Because, the nonlinearities in the Navier strokes equation, in the convective acceleration term has non; if you substitute for the velocities as pro as some of an addition of mean flow plus fluctuations.

You have products of fluctuations and time averaging that in general will not give you 0. Because, if the fluctuations are correlated over a period of time then, that will give you non 0. So, this is a very very important input to turbulent flow. So, even though you are trying to average the fluctuations, average of the fluctuations. And write equations only for the mean flow quantities, that is the time average quantities. The fluctuations do comment indirectly through the time averages of the correlations. This is specifically because, of the fact that the Navier strokes equations are non-linear.

If there are no non-linear t, then you are not have these extra stresses these turbulent stresses. So, this additional transfer of momentum comes only because of the convictive nonlinearities in the Navier strokes equation.

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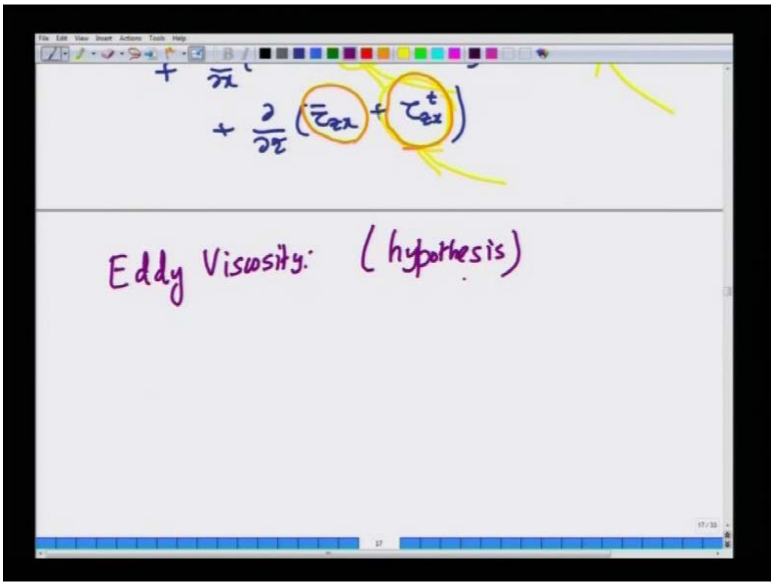

Now, so one, how do we, now how are we now going to tackle this? One more model is called the Eddy viscosity model. So, because we know, what is the Newtonian stress tenser and that will give you what this is, we can find what tau z x is by averaging the constrictive relation Newtonian constrictive relation. But, for this we have to specify some additional inputs. Otherwise, we will not be able to solve the problem. So, the Eddy viscosity model hypothesis, it is a hypothesis, essentially hypothesis which has to be checked with experiments.

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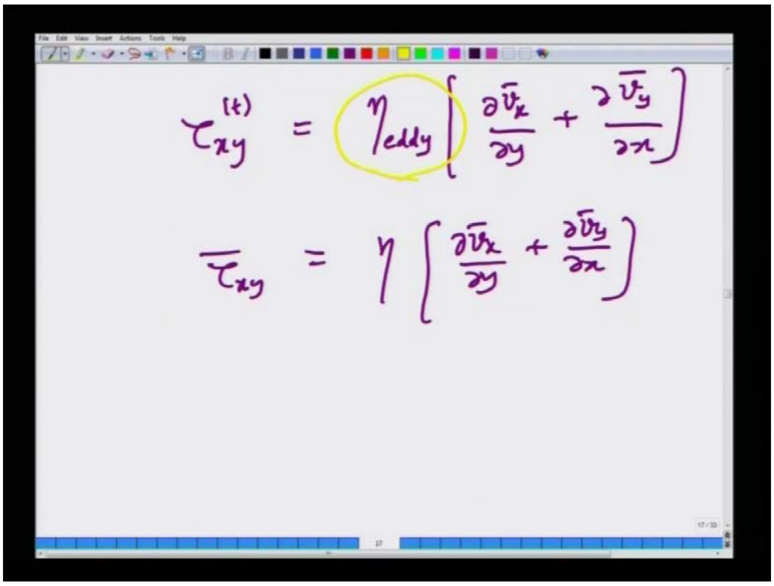

Eddy viscosity hypothesis says that, tau x y turbulent is nothing but, an Eddy viscosity times, we are trying to relate the turbulent stresses to the mean velocity gradients. Just as you related the viscous stresses tau x y bar is the normal viscosity, times partial v x bar by partial y plus partial v y bar by partial x.

So, we are saying that, now, we are going to do a similar model for the Eddy turbulent stresses. But, if Eddy viscosity is a constant then adding these two will give you a new viscosity, which means it almost it is like different fluid with a slightly different viscosity. So, clearly the Eddy viscosity cannot be a constant, it has to be a function of position, in order to make, in order to correctly describe the physics of fluid turbulent.

So, essentially we cannot say that the Eddy viscosity is a constant because, otherwise it becomes a trivial problem. So, we have to bring in spatial dependence of the Eddy viscosity on the various spatial coordinates. So, we are going to and we are saying that all the turbulent all of turbulent can be described by just one function. Essentially, it is a function of spatial positions but, that it is also very questionable, that is certainly questionable unless it is a backed up by experiment.

So, although at this point of time, we are merely hypothesizing the partialating the Eddy viscosity idea. It turns out that it has some moderate success, in describing turbulent flow that are encountered in engineering applications such as pipe flow turbulent. So, we are going to continue with this line of a tag. And we are going to specialize this Eddy viscosity approach for turbulent flow in a pipe, in the next lecture.