Fluid Mechanics

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Lecture No. # 40
Fluid Mechanics

Welcome to this lecture number 40, the final lecture in this course on N P T E L course on fluid mechanics for undergraduate chemical engineering students. So, in the last lecture it is good to recapitulate what we have done? And so, far in this course in a very condensed manner and we would also like to see what are the things that we could not do in detail because of lack of time or because of fact that this being the first introductory course in fluid mechanics.

So, this is the plan for this course this lecture that is will first summarize the key concepts that we discussed in this entire course. And then will finally, also discuss topics that we could not finally, mention topics that we could not discuss in detail, but none the less which are very important in chemical engineering fluid mechanics.

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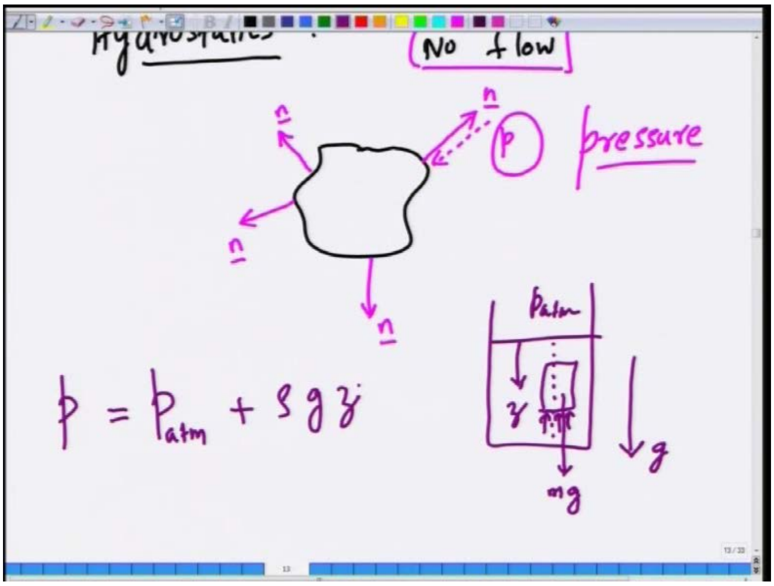

So, the very beginning of the course we started with hydrostatics. Hydrostatics concerns with motion of fluid only under static conditions that is no flow. When there is no flow if you take any fluid element volume element at each and every point in the fluid there cannot be any shear stresses.

So, the force has to be directed suppose you consider unit normal at each and every point on this arbitrary volume element. The force has to be completely in this completely in the direction of the normal. It turns out that the force is compressive. So, it is acting in the direction of minus n, but this if you call n as the unit normal at each and every point the force should be along the normal. There cannot be any shear stresses under no flow conditions because a fluid cannot support non 0 shear stresses under static conditions

The moment you have non 0 shear stress in a fluid it will start flowing. So, by restricting to no flow conditions we can make an important conclusion that the forces acting on any arbitrary fluid element they are purely normal. That normal force is usually compressive in nature that is it acts in the direction of minus n and the magnitude of the normal force is actually the pressure. That is the pressure in the fluid.

So, once we understood that the fact that there is suppose you consider any arbitrary fluid element. The surrounding, the fluid surrounding this arbitrary fluid element exerts a surface force on the surface and that acts in the direction of minus n. That it acts invert and it tends to compress the fluid element.

Now, then by doing the force balance on; suppose, you have a fluid element under the influence of gravity. We took a volume element and we then said that the pressure in a fluid. Now, has to suppose you consider this to be z suppose, this is atmospheric pressure. And if you consider the distance from the free surface downwards as z and let us say gravity is acting like this.

Then we said that the pressure in the fluid at each and every point is no longer a constant in the z direction because the pressure has to increase. So, as to balance the body force that acts on a given partial of fluid. Suppose, you consider a volume element like this. There is a net force acting downwards which is the mass of this element times the gravity.

So, the pressure has to be larger here to support this force in order for the fluid to be in the static conditions. That gives rise to this famous pressure variation and the static conditions p is p atmosphere plus rho g times z where z is the distance from the free surface into the fluid.

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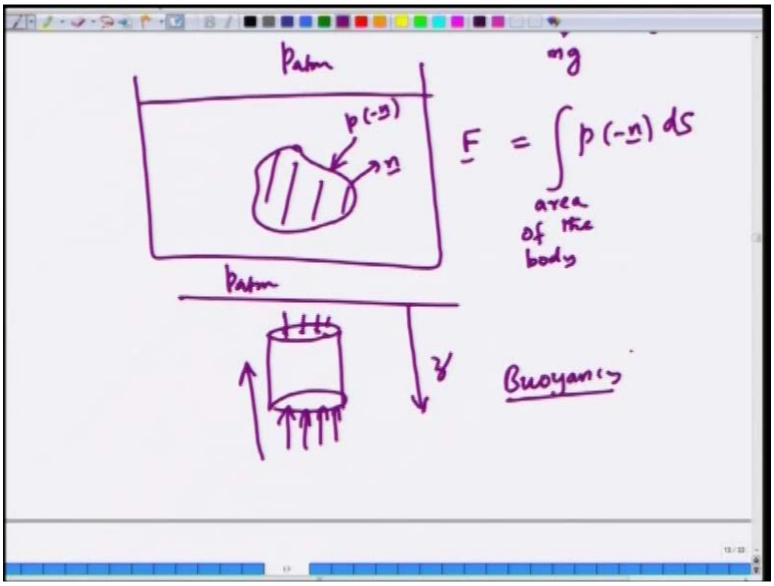

Now, once we know that part. Suppose, you have a free surface this is atmospheric pressure. Now, if you have any object could be a solid object. And then if you want to know what are the forces acting on this solid object. Then all you have to do under static conditions is to integrate. So, if this is the normal the pressure acts in the direction of minus n.

So, the pressure is force per unit area. So, this is p times minus n the vector that is acting in this direction. So, you it simply have to integrate p times minus n over the surface of the object area of the object, area of the body, surface area of the body. Then you will get the net force due to pressure that acts on submerged objects or partially submerged objects and so on.

And this gave rise to the Archimedes principle that once everybody if you consider any body that is present. If this is the atmospheric pressure you consider a body that is immersed in a fluid. Then there is a net force that acts in the upward direction that is called the buoyancy force. That is because of the fact that the pressure varies with respect to the z direction and that means that the pressure the net force. Suppose, you consider a simple geometric object.

Consider a cylinder that is immersed in a in a liquid, solid cylinder let us say. The net pressure force here will be larger than the net pressure force here and the difference will lead to a net force upwards. That is called the buoyancy force and this is the Archimedes principle. And these are the topics that we dealt with in hydrostatics of fluid statics.

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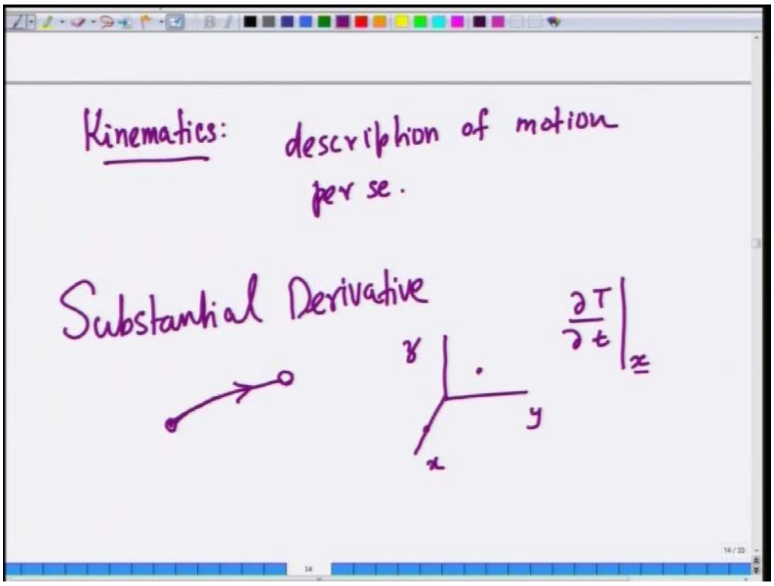

Then we went to kinematics, we told that kinematics is that subject that deals with description of motion without references to the forces that causes the motion. So, the description of motion without reference to any forces that causes the motion. The description of motion as such and in that contest we had come up with an important concept of substantial derivative. (No Audio Time: 06:03 to 06:08)

The substantial derivative plays an important role because often in a fluid if you want to know what the rate of change of a quantity is as you follow a fluid particle. We will choose the so called eulerian coordinate system in fluid mechanics. Where in you place a coordinate system with respect to a lab frame of reference.

So, if you take the partial derivative of any quantity like temperature with respect to position. This means that you are making you are measuring the temperature at a given point in space. Partial derivative of temperature with respect to time at a given point in space. This means that you are keeping the special coordinates of the location of the probe constant, but often we want to address the question as to what is the rate of change of temperature or any other quantity as you follow the motion of a fluid particle. Now, that is given by the substantial derivative.

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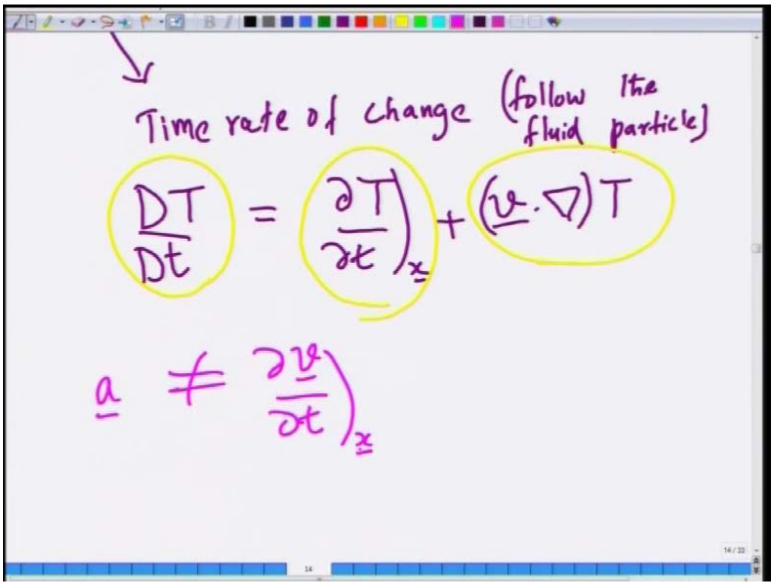

So, this is the time rate of change of any quantity following the fluid as you follow the fluid particle. (No Audio Time: 07:14 to 07:19)

So, essentially this is denoted by this capital D symbol. The substantial derivative of let say temperature is not just equal to the local derivative, but there is also a convicted acceleration or convicted increase of temperature because of the fact that the fluid is moving. And if there are gradients in temperature a special gradients in temperature then if you move from a region of lower temperature to a higher temperature as you follow the fluid particle. Of course, its temperature is going to change.

So, therefore, this is called the local time derivative, partial derivative. This is called the convicted increase in temperature, convicted change in temperature and the addition of this two will give rise to the substantial derivative of any quantity. So, in fluid mechanics we are interested in. For example, acceleration is not equal to simply the rate of change of velocity at a given location in space, but acceleration refers to the rate of change of velocity as a fluid particle is followed.

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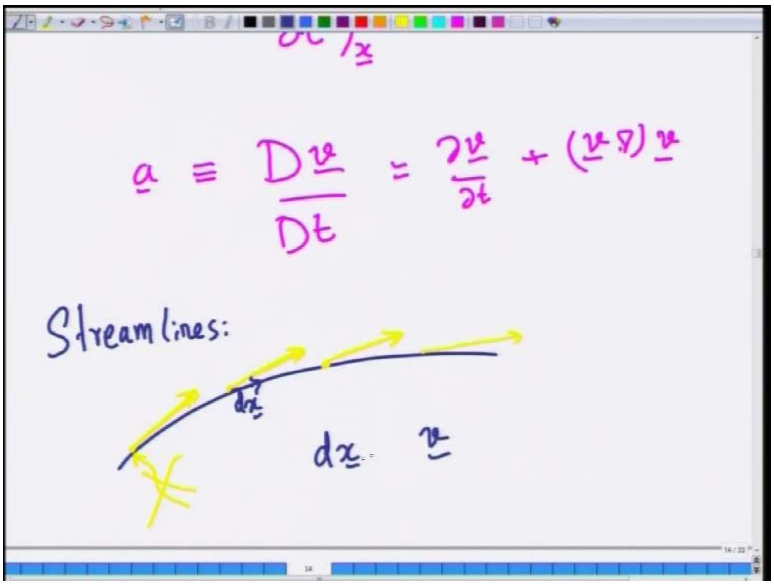

So, it is naturally given by a substantial derivative of velocity and that is simply equal to partial v partial t plus v dot delta. Now, we then discussed the motion of stream lines. A stream line is a line where in which the velocity vector is parallel at each and every point along the line. We consider each and every point the velocity vector will be parallel to that point.

So, by definition there is no normal flow normal to the direction of a stream line because the velocity is parallel to at each and every point along the stream line. So, this means that if you consider a timely displacement d x along the stream line vector and you consider the velocity vector. Since they are parallel the cross product is 0.

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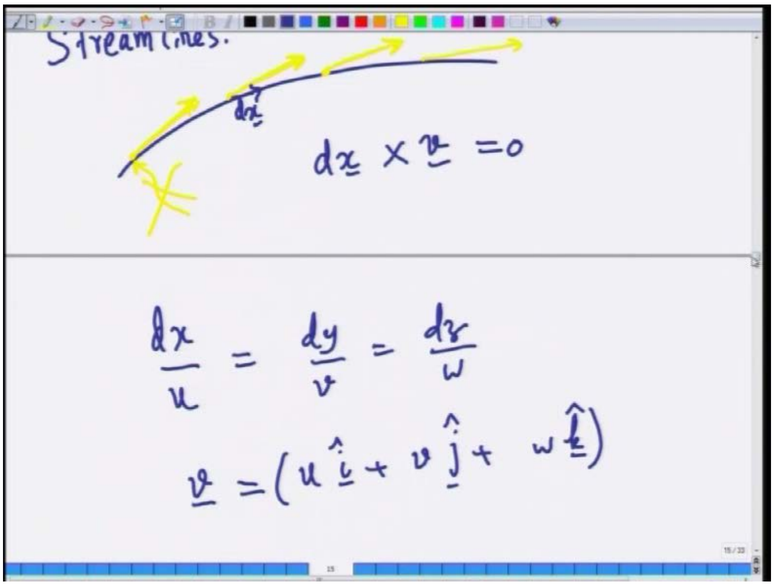

That gives rise to the equation of a governing a stream line which is d x by u is d y by v is d z by w. Where v vector the velocity vector is given by u times i plus v times j plus w times k namely the three components along the three Cartesian directions x y and z directions.

So, these are the u v and w are the three scalar components of velocity. Now, just as substantial derivative told us how to convert time rate of change? How to get the time rate of change of any quantity as you follow the particle?

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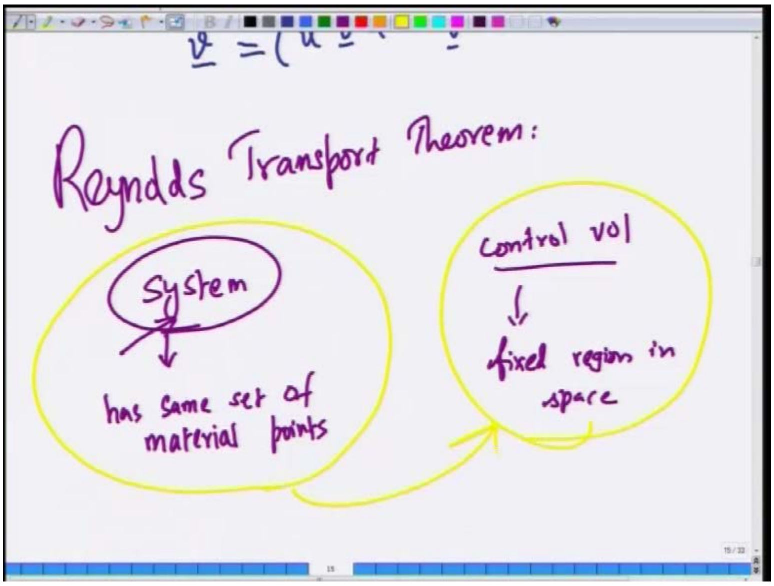

The Reynolds transport theorem pertains to a macroscopic body. Often we make distance between system and control volume in fluid mechanics. The system for example, contains the same set of mass point. Suppose you mark each and every point in the fluid with some dye it is a thought experiment.

Then as the fluid flows if the system will contain the same set of points as the fluid is flowing. Whereas, the control volume is the fixed region in space. It could involve for example, a device like a pump or a pipe line and so on. In space that could contain various unit operations or equipment whereas, a system is actually it has the same set of mass points. (No Audio Time: 10:58 to 11:04) material points.

So, often many fundamental law such as Newton’s second law of motion is applicable only to your system because a system is an identical piece of matter whereas, the control volume is merely a fixed region in space. So, therefore, when we want to apply balance law such as Newton’s second law or first law of thermodynamics. If they are applicable only to a system, but not to a control volume.

But using the control volume approach is very very advantageous in engineering applications because you are really interested in what are the forces acting in a given region in space. Rather than the forces acted upon or forces exerted by the fluid as you follow the fluid element because ultimately you are essentially interested in some devices and some fixed regions in space such as the pipe line network and so on.

So, the control volume approach is very very advantageous in practice. This is what is followed, but the fundamental laws are applicable only to the system approach. So, we need a vehicle or we need a way to sort of transfer the results from the system approach to the control volume approach. That is given by the Reynolds transport theorem.

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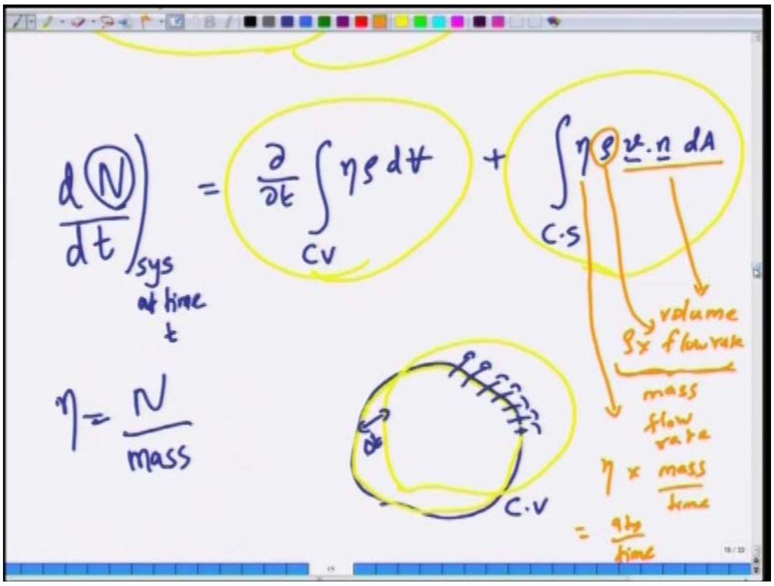

So, what this means is? Suppose, you have a quantity such as mass, momentum or energy which is denoted by this symbol N. So, the rate of change of that quantity of in a system at time t. So, the way we do it is we construct first a C V of interest and then we say that the system and C V coincides at time t equal to 0, the yellow line is the system. At a later time the system would move elsewhere while the C V would remain at the same point at the same place.

So, the rate of change the infinitesimal rate of change of quantity such as mass, momentum energy present in the system. So, this for example, is an exaggerated version after time delta t system would have moved elsewhere is equal to a local rate of change present in the control volume. So, eta rho d v where eta is N divided by mass. The specific quantity for example, eta is 1 if N is mass itself eta is just velocity. If N is momentum and eta is half v square.

And for example, if N is kinetic energy and so on plus; this is not all this is the local rate of change present in the control volume plus the surface flux term. That is because as the system moves out of the control volume. It also takes material out of it and that surface flux will also take away whatever mass, momentum or energy.

So, the Reynolds transport theorem is in some sense the analog of the substantial derivative. The substantial derivative refers to point wise change as you follow a given mass point. Here it is almost like a collection of mass points macroscopic collection of mass points which is what we call as system. And as you follow the system then how does the rate of change of any quantity? How does mass change as the system moves from a given location to another location? Then how does that relate to quantities are variable pertaining to the C V.

That has two contributions the local rate of change of that quantity in the C V plus the flux the surface flux contribution that because fluid comes in and out of the control volume by virtue of through the control surface. The physical interpretation for this term is like this v dot n d A is the volumetric flow rate through an infinitesimal patch and rho is mass per unit volume. So, if you multiply this. This will give you the mass flow rate. If you multiply this which is quantity per unit mass. So, eta times mass flow rate will give you let us say the quantity per unit time due to surface flux. So, this is the Reynolds transport theorem.

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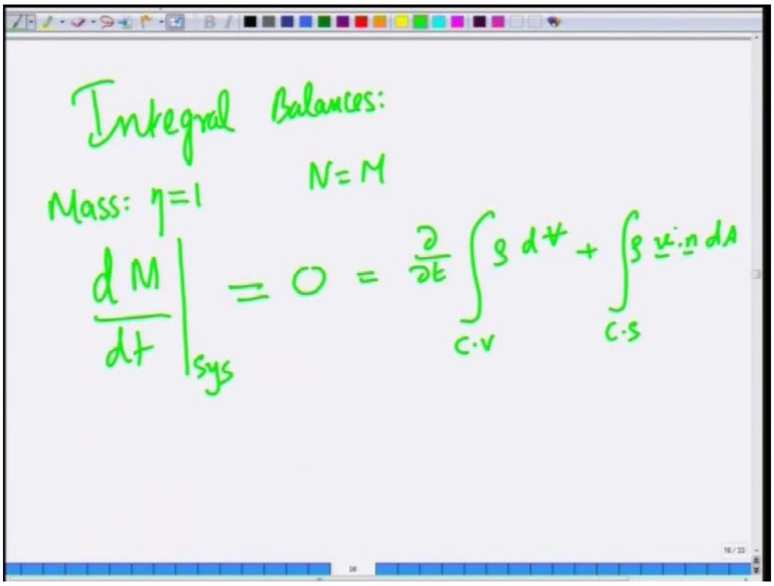

Now, we want to derive the integral balances of mass, momentum and energy using the Reynolds transport theorem. It is very easy because for mass we have to simply set eta is 1 and d N d t d M d t if n is mass. If you follow the same set of mass points that is then rate of change of mass of the system is 0. This is the principle of conservation of mass or the law of conservation of mass.

So, this is equal to the local rate of change rho d v plus the convicted or the surface flux contribution which is rho v dot n d A.

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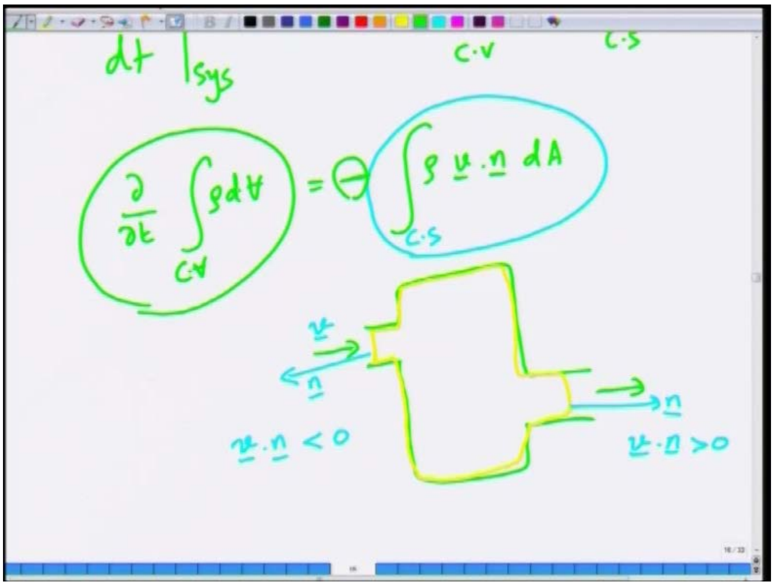

So, this is straight forward application of Reynolds transport theorem. This implies that v d t of integral rho d v over the control volume is equal to minus integral rho v dot n d A. Now, let us try to understand minus sign suppose you have a simple case like this fluid is coming in and going out. And let us say the yellow is over control volume. Now, the key thing that we emphasis many a times in this course is that n is the unit outward normal to the control surface. So, at inlets the velocity and the control and the normal are in the opposite direction is negative v dot n is negative. Therefore, and the outlet v dot n is positive.

So, if this term tells you the net mass flow in or out due to the surface due to the flow in and out of the surface. If rho v dot n is positive; that means, that there is a net a flux of fluid and notice that there is a minus sign here. So, that means that the time rate of change of mass present in the control volume will decrease with time. So, that means physical sense if there is net a flux of mass of course, mass present in the control volume will decrease with time.

But if v dot n is negative, there is a net input of mass then negative of negative is positive. That means time rate of change of mass present in the control volume will increase with time. So, that is the interpretation of the integral balance of consideration of mass.

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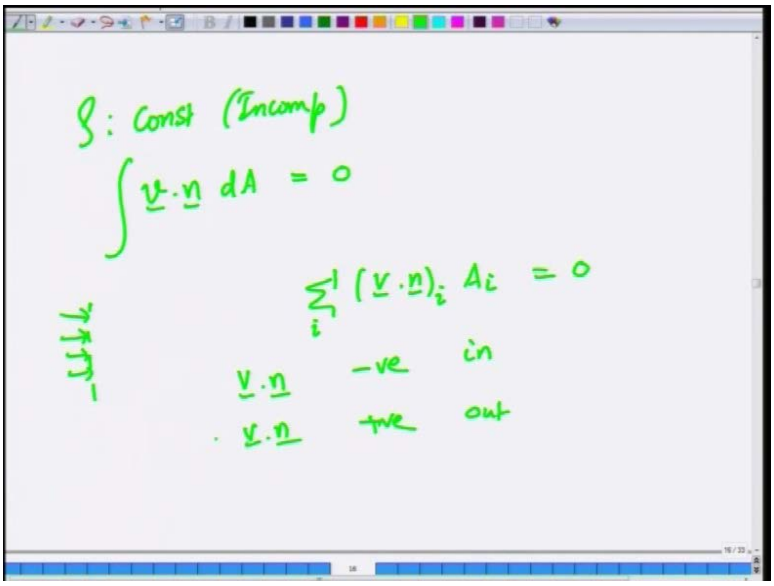

And for an incompressible fluid rho is constant and so, to the mass conservation equation simply becomes v dot n d A is 0. And so, that is the simplest form of mass conservation equation. If you assume, uniform velocity at the various inlets and outlets then you will get summation v dot n times i where i is the i th inlet or outlet. And notice that v dot n is negative for inlets and v dot n is positive for outlets. And this is the simplest form of conservation of mass when the flow is incompressible then the density is independent of time.

So, then that clearly means that the only way, in which there is no way in which the mass or v dot n can be different at inlets and outlets. The other special case is of course, you can say that the flow is steady and the flow is steady then d d t of this is 0. Then you will get this equal to 0, but rho is not a constant there, but for an incompressible fluid rho is a constant.

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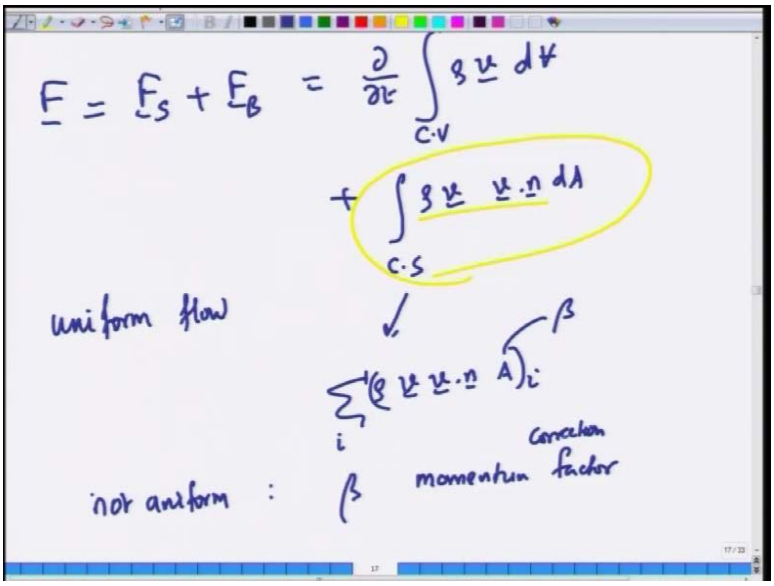

Then we went to the momentum balance, integral balance of momentum which is essentially a restatement of Newton’s second law through a macroscopic chunk of mass points. And then on applying the Reynolds transport theorem the sum of all forces acting on the control volume we said that there are two types of forces. Forces that act only along the surface are called the surface forces and then the force that acts on the entire volume of the body. That is called the body force and that is example is example of body force is gravity.

So, this is equal to d d t C V rho v d v. This is the time rate of change of momentum present in the control volume plus the momentum in flux and a flux due to flow. Rho v through the control surface. This is the momentum balance and for uniform flow the key thing in momentum balance is this evaluation of this term the flux term.

So, for uniform flow you can imagine that all these terms are constants. So, you can pull this out of the integral and integral of d A is simply A. So, this becomes simply this term simplifies to summation of rho v v dot n times A various inlets and outlets of this system. And of course, we understand that v dot n is positive for an outlet and negative for an inlet. Suppose if the flow is not uniform then we came up with this momentum correction factor beta. Which will enable us to write this equation in this form, but with a beta term in it and beta is identically equal to 1 for uniform flow.

But for other flow such as laminar flow in a pipe or turbulent flow in a pipe beta is not exactly equal to 1. So, we can actually momentum correction factor. So, we can take into account the effect of non uniformity in the velocity profile in a simple way by using this correction factors and we have derived what these values are for laminar flow of a fluid in a pipe and so on.

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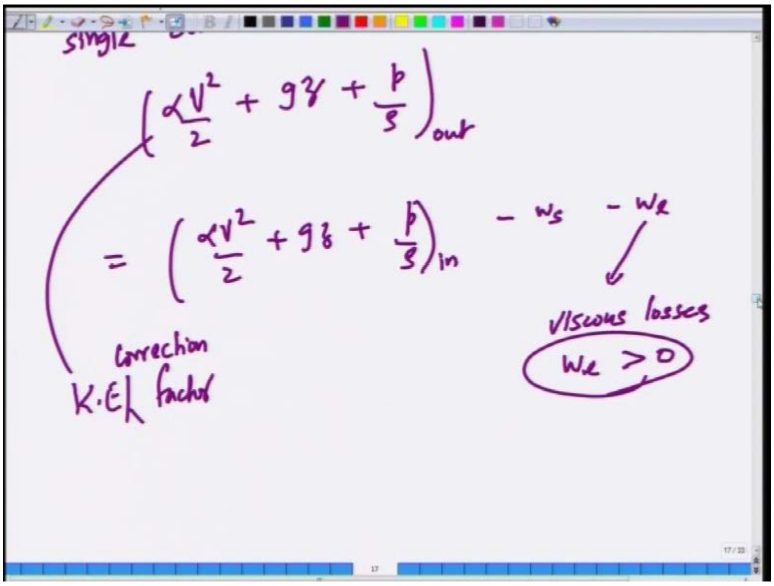

Then we discussed the integral balance of energy. Now, again it is a restatement of first law of thermodynamics through a flowing fluid. And when we have a single inlet and single outlet we will say. Then energy balance simplifies to summation over all outlets sorry summation all outlets. Let us keep it to single inlet single outlet for simplicity alpha V square by 2. Alpha is the kinetic energy correction factor that tells you the corrects for the deviation of the velocity profile from uniform velocity out is equal to (No Audio Time: 22:13 to 22:20) in minus the amount of shaft work done by the fluid on the surroundings.

So, we took the convention that work done by the C V on the surroundings is positive work done by the surroundings on the C V is negative. So, if this W s is positive; that means work has been exerted out of the C V W s is negative. Then work is been put in to the C V minus the viscous losses. The viscous losses are always positive W l is always greater than 0. That tells you that whenever you have a fluid flow that is a unidirectional conversion of macroscopic energy to molecular degrees of freedom essentially heat. So, that is the laws term and alpha is the kinetic energy correction factor. That corrects for the non uniformity of the velocity profile.

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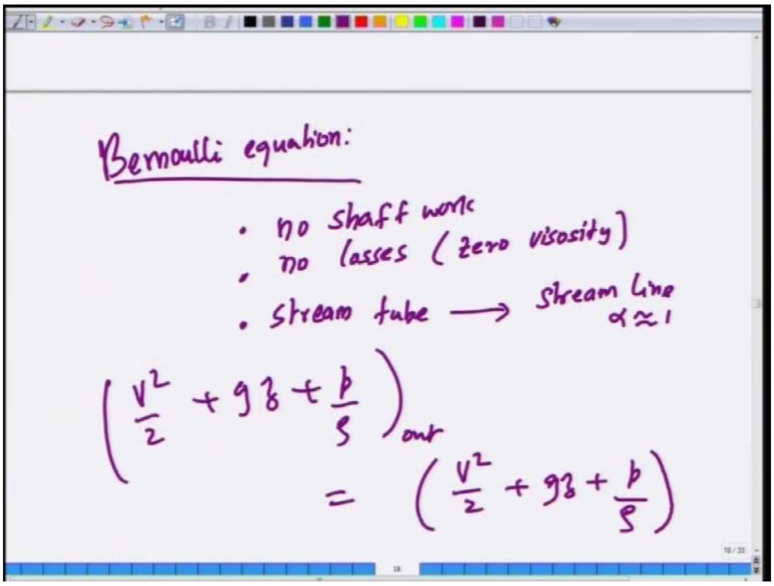

Now, we also said that this equation has some connection to the classical Bernoulli equation, but one has to be careful because there are similarities at times are superficial. Because the Bernoulli equation is applicable when there is no shaft work, no losses that is the fluid is 0 viscosity.

So, it works only for an in visit fluid or hypothetical fluid with 0 viscosity and we restricted we tried to apply that energy balance to a stream tube and then shrink it to a stream line. So, the Bernoulli equation is valid to essentially to a stream line where in for an in visit fluid. So, in that case. So, when you have a stream line essentially alpha is approximately 1.

So, we said that v square by 2 plus g z plus p by rho at the outlet must be identically equal to v square by 2 plus g z plus p by rho at the inlet. So, there is a superficial similarity between the energy equation and Bernoulli equation, but one has to be very careful. Because the energy equation can be applied to across different equipment and construct the C V as big as possible including all such of equipments and valves and compresses and so on, piping and so on. Whereas, the Bernoulli equation is special equation that is valid only for an in visit flow along the stream line.

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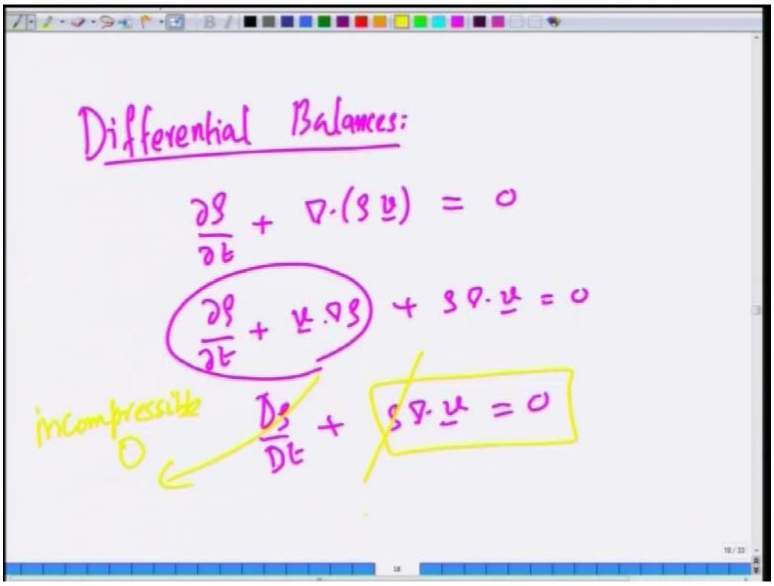

Then we came to differential balances or microscopic balances of mass and momentum. We did not do energy, but one could do energy as well and that could form part of the course of heat transfer. Because heat transfer is essentially understands the transfer of energy due to flow as well as modes of heat transfer such as conduction and radiation. So, we did not do the energy balance in the differential sense that typically forms the part of the courses in heat transfer.

So, the mass conservation equation becomes d rho d t plus del dot rho v is 0 or you can write d rho d t plus v dot del rho plus rho del dot v is 0 and this is nothing, but the substantial derivative of density plus rho del dot v is 0. For an incompressible fluid the substantial derivative of density is 0 because if we follow a fluid particle it is density will not change. So, for an incompressible fluid this term is 0 leaving us with simply this as the continuity equation. Since, rho is a constant that cannot be 0.

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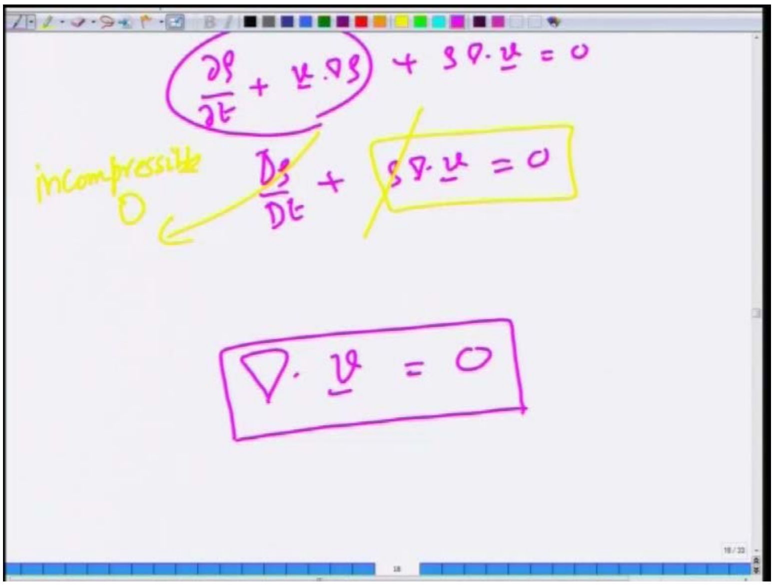

So, the continuity equation or the mass conservation equation boils down to simply saying that the velocity field. The velocity vector which is function of all three special directions and time always satisfies the divergence of the velocity vector is 0. That is the divergence free for an incompressible fluid.

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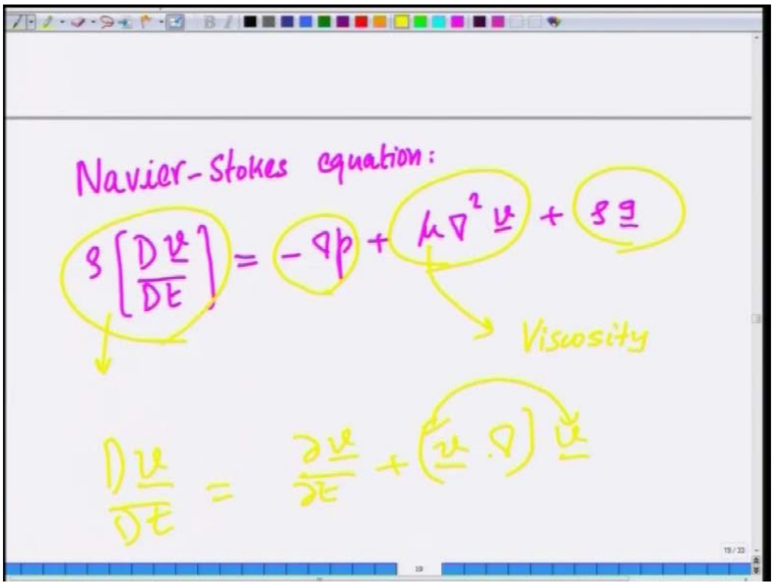

Then we went on derive the navier stokes equation, the momentum equation (No Audio Time: 26:42 to 26:50) rho times substantial derivative of velocity is minus del p plus mu del square v plus rho g. This is a very simple statement of force balance. This is the rate of change of momentum per unit volume. This is the mass per unit volume times the acceleration. Remember that the acceleration of a material of acceleration of fluid particle is the substantial derivative of velocity not just a normal derivative of velocity not the partial derivative of velocity.

So, this is the mass times acceleration per unit volume. So, this is equal to the sum of all forces. So, this is the pressure forces acting on per unit volume on a fluid. These are the viscous force acting on per unit volume of the fluid and these are the body or gravitational forces acting per unit volume of the fluid. And mu is called the viscosity of the fluid tells you how difficult it is to make a fluid flow?

Because as I have told you if the fundamental distinction between a fluid and solid is that as fluid resist the rate of deformation. It simply does not where as the solid resist deformation. That is a solid catches how much you extend it or deform it? Whereas, the fluid continues to deform. So, we cannot say that the stress is proportional to or related to deformation itself. It has to be related to the rate at which deformation is happening. So, different fluids will deform at different rates rather than through the extent of deformation itself is not a good quantity to think about in a fluid.

So, the navier stokes equation are complex because of the fact that the substantial derivative is a non-linear term in the velocity. So, D v d t is partial v partial t plus v dot del v. So, here the unknown velocity comes as a product and these are all partial differential equations because you have to write this vector equation in three component forms in the individual component forms. For example, you have the x y z components in the Cartesian coordinate system.

So, each component is coupled to the other component by virtue of this non-linear term the convective term. So, when navier stokes equation are in general very very difficult to solve, but we had made several simplifications while solving the navier stokes equations.

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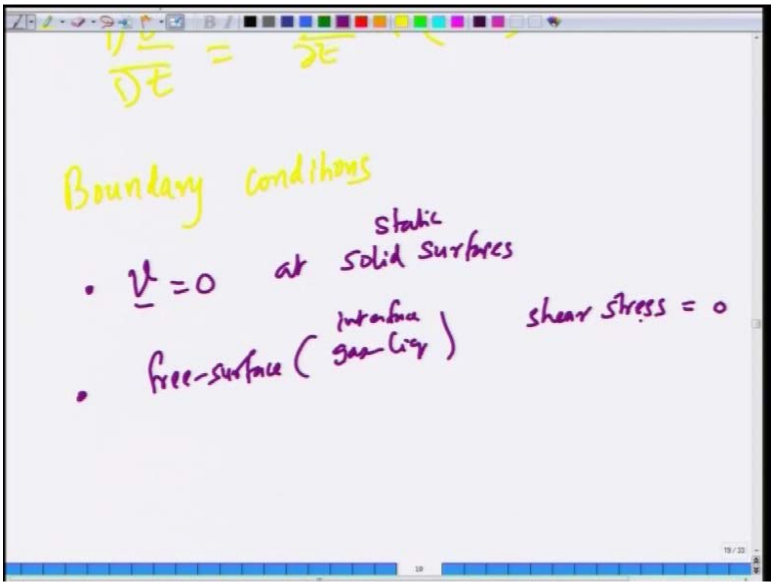

But before solving the navier stokes equations we need boundary conditions. Which are that velocity is 0 at solid surfaces at stationary solid surfaces or if the solid is moving the velocity of a fluid will take the velocity of the solid. That is called the no slip condition and that is one type of condition and at a free surface gas liquid interface the shear stress is 0 is another type of condition because the gases typically have viscosity which are of third or of 100 times smaller than the liquid viscosity. So, the shear stress exerted by the gas on the liquid at the interface has to be very very small. So, we can neglect it.

So, the other thing that has to be kept in mind while following the navier stokes equation is that only in Cartesian coordinate this various terms will become very simple. Then you have this various operates involving del and so on del square. I have told you many times that the form of these operates the gradients,, the divergent or laplacian. They are different in different coordinate systems.

So, when you want to solve problems. It is better to take a look at the various forms of the detailed form of navier stokes equations and various coordinate system that are available in various hand books and text books. Instead of blindly generalizing the Cartesian form of navier stokes equations to other coordinate systems.

So, then we proceeded to solve some canonical problems in simple laminar flows of Newtonian fluids. Where in we solved the plane quirt flow problem driven by the relative motion between two solid surfaces; the plane puzzle flow problem. That this flow in a rectangular channel driven by pressure difference. Then we discussed the pipe possible flow problem. That is flow in a pipe laminar flow in a pipe and all this enabled us to derive the pressure drop versus the relationship between analytically the relationship between the pressure drop required to make the fluid flow at a given volumetric flow rate.

So, that was solved and then we also did once special problem on coating of a wire. And then we asked the question as to what is the diameter of the coated wire in terms of various parameters such as the dyed diameter, wire diameter and so on. So, we solved all this problems under very using very various types of simplifying approximations such as steady flow, fully developed flow, flow only in one direction and so on. And we also pointed out that these assumptions are highly restrictive in the sense that even though you find an the exact solution to the navier stokes equation.

It is not guaranteed that the solutions will be observed in reality in experiments because the navier stokes equations are a non-linear set of deferential equations. And whenever you have a non-linear set of deferential equations there could be multiple solutions. And the solution you derive is not the unique is not necessarily the unique solution. So, it turns out that for example, flow in a pipe the laminar flow velocity profile or the result for flow rate versus pressure drop obtained using the laminar flow assumption works for Reynolds number less than a 2000.

But when the Reynolds term is greater than 2000 then the flow becomes unsteady 3 dimensional and turbulent. So, one has to be very careful while using this simplified solutions of navier stokes equations. For newer settings because it is not necessary for these to be actual solutions to navier stokes equations, but they must also be observed in reality. So, that that matter can be settled only by doing experiments to check whether the theoretical predictions are valid.

So, the 3 major approaches to solve fluid problems in engineering, in chemical engineering. Especially is the macroscopic approach using the entacle balances, the microscopic approach using the differential balances and when both these approaches are not you know possible. Then one has to do experimentation because especially in chemical engineering the unit operations involving fluid flow and flow equipments are extremely complex. And that does not allow us to use the fundamental principles of fluid mechanics in using the navier stokes equation to solve problems exactly. So, one has to do use the integral balances and couple it with experimental data to find the losses and so on.

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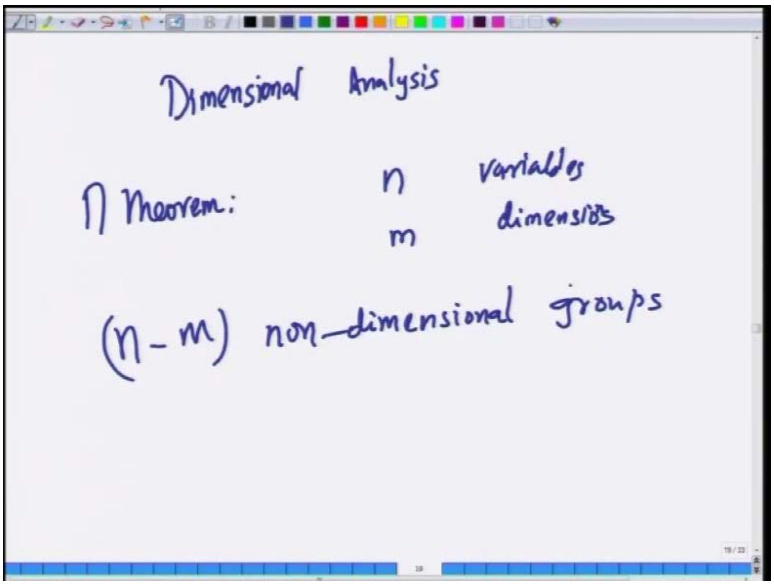

In that context we introduce the notion of dimensional analysis. Dimensional analysis is exception extremely useful in analyzing experimental data in presenting experimental data and in deciding how many experiments to do and so on. Essentially, we did the pie theorem which says that bucking hams pie theorem which states that if there are n variables and m fundamental dimensions. Then n dimensional physical variables and m fundamental dimensions then there are n minus m non dimensional groups. There are only n minus m non dimensional groups.

Suppose you have 5 variables and 3 fundamental dimensions mass length and time. Then essentially you have only 2 dimension groups and this was the example we gave for the drag coefficient. When you consider drag forces is a function of parameters such as the diameter of the sphere, velocity of the sphere, density and viscosity. And finally, so you have 5 variables. Then when you reduce it using dimensional analysis it becomes a relationship between the drag coefficient and the Reynolds number.

So, it leads to lot of simplification because you are now able to a multi dimensional function relationship in terms of just 2 dimensional function that is a one variable function. So, that is a very very important simplification. Quite apart from that, by expressing the experimental results in terms of non dimensional groups. One is able to now scale up or scale down data experimental data to different conditions provided the actual condition and the model laboratory condition or geometrically similar.

So, once you have similarity in geometric scales and then we can ensure that results such as the drag coefficient. They will be the same provided the Reynolds number is the same. So, keeping the Reynolds number being the same is actually called dynamical similarity. So, keeping the geometrical scales to be the ratios of the geometrical scales to be same is geometric similarity.

So, once you ensure geometric and dynamic similarity then the results when presented in terms of non dimensional groups will be the same for the model as well as the proto type condition. So, this is the very very extremely important result because this is the most this is the often used in many many engineering applications to scale up or scale down experimental data from lab to bigger or smaller scales respectively.

So, once we discussed dimensional analysis then we moved on. We also while discussing dimensional analysis we also non dimensional navier stokes equation and then we saw that parameters such as Reynolds number popped out naturally by non dimensional using navier stokes equation. And this also tells us the physical meaning of various non dimensional groups and we found that the Reynolds number is essential ratio of inertial forces present in the fluid to viscous forces present in the fluid and we had other numbers such as fluid number and so on.

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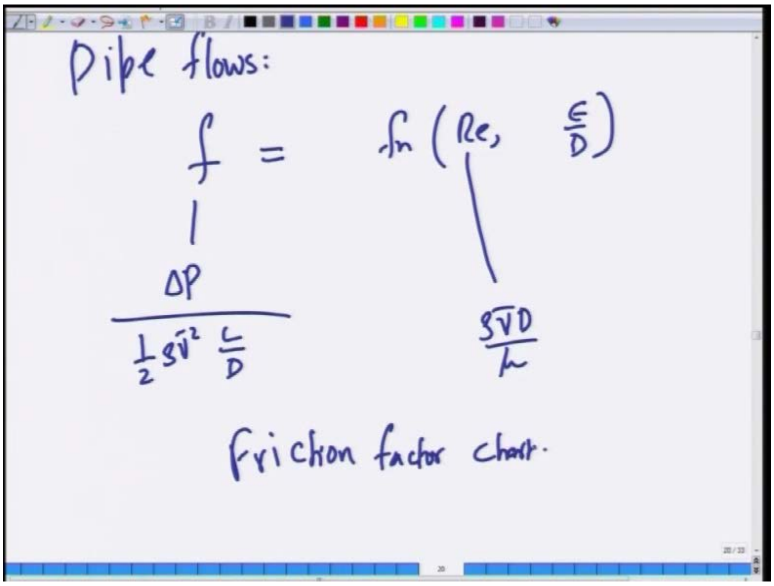

Then we moved to pipe flows. So, we discussed that when we did simple laminar solutions of flow in a pipe. The relationship between pressure drop and flow rate that we got was not valid for all Reynolds number. It is valid only when Reynolds number is less than 2000 when the flow is laminar. After that the flow under goes a transition from laminar to turbulence and in that context if you want to solve problems in engineering fluid mechanics. It is not sufficient to know only the pressure of flow rate in the relation under laminar conditions.

So, we have to go to turbulent conditions as well. So, it is useful to do use experimental data and there we again use dimensional analysis to say that the non dimensional pressure drop is called the friction factor. Which is essentially delta P by half rho V square L by D is a function only of Reynolds number and pipe roughness. Epsilon is the roughness, D is the diameter of the pipe, Reynolds number is rho V bar D by mu. So, this is the data this relation when plotted graphically is called the friction factor chart. (No Audio Time: 38:02 to 38:07)

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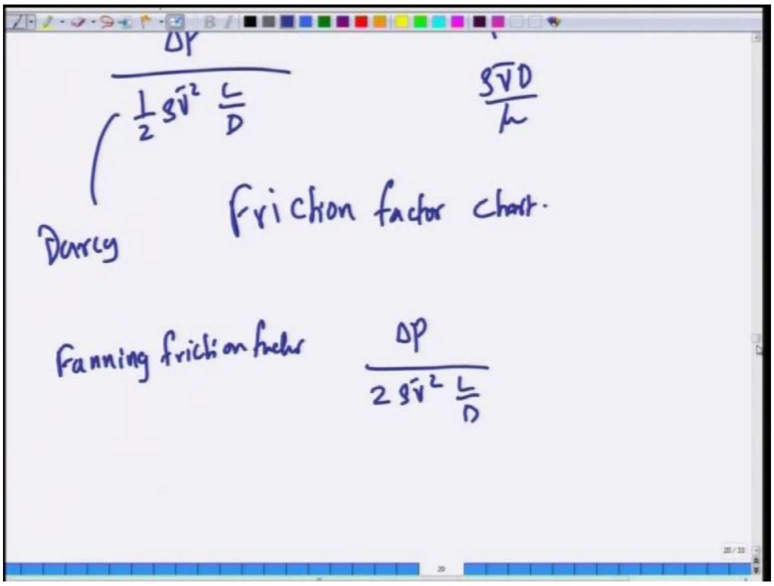

Also please remember that there are two types of friction factors used. This is called the Darcy friction factor; the other friction factor is called the fanning friction factor which will go something like delta P by 2 rho V square L by D. So, there is a factor of 4 that that differs between this friction factor and the other friction factor.

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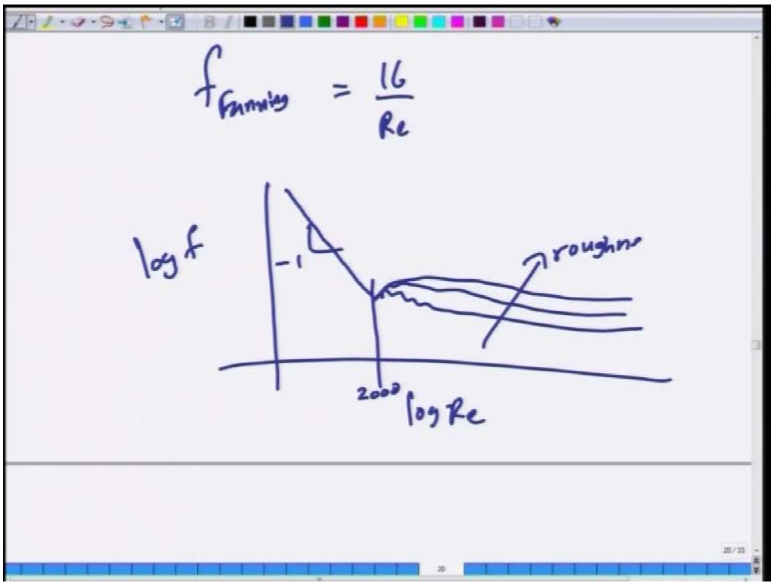

So, different text books can use different friction factors, but one has to be aware. When using darcy friction factor then the laminar flow relation between the pressure drop and flow rate reduces to 64 by Reynolds number. While when we use the fanning friction factor it becomes 16 by Reynolds number. Both are equivalent so, only that the definitions for the friction factor are different, but this is only under a laminar conditions.

Suppose, you plot the friction factor versus Reynolds number in a log log plot. So, when the flow is laminar this relation will be a straight line with slope minus 1, but after some point the flow becomes turbulent. After the Reynolds number of 2000 the flow becomes turbulent. And when the flow is laminar the result between the friction factor and Reynolds number is independent of pipe roughness, but when the flow is turbulent it does depend on pipe roughness. That is an important and this graphical information is used in designing many pipe line networks in asking the question what is the rating on a pump power rating on a pump? In order for us to make fluid flow over a particular distance and so on.

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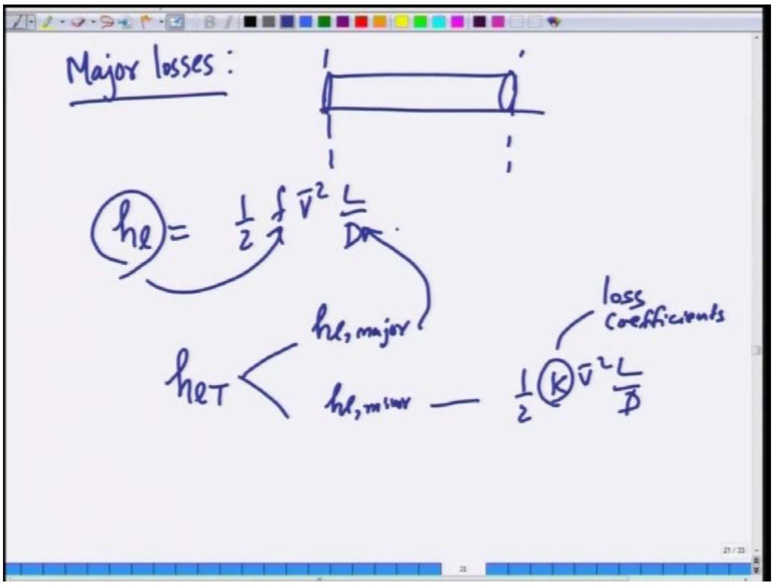

And then we discussed major and minor losses in a pipe. So, these are major losses are because of flow in straight sections of the pipe and minor losses are because of bends or valves sudden expansions or sudden contractions and so on. And once we so, the major losses can when you apply the integral energy balance to a straight section of the pipe and use the friction factor relation. Then we can relate the friction factor to the head loss because the friction factor is defined in this manner. This is the friction factor. Now, the head loss occurs in the integral energy balance in the form of delta p by rho g.

So, we can apply the energy balance between a straight section of a pipe and then relate the head loss to the friction factor then you will get this. So, essentially whenever you have head loss you can write it in terms of friction factor in the governing equation. And given the Reynolds number range you can find what the friction factor is and then substitute that to find what is the viscous loss due to flow in a straight section of a pipe. These are called major losses, minor losses are characterized. So, when you want to write the energy balance you will write as total losses h l major which is given by this and h l minor which is correlated like half K V square L by D. These are the loss coefficients for various losses minor losses these are the loss coefficients.

So, these loss coefficients are documented for various losses in text books and hand books.

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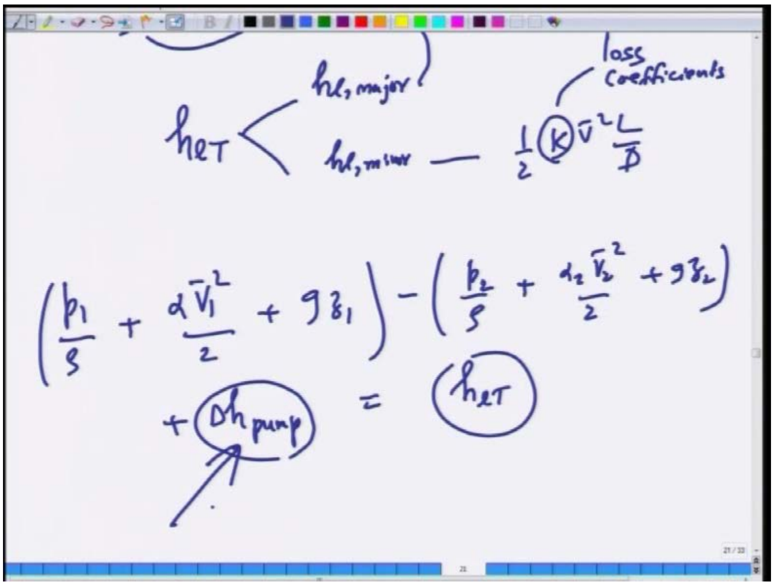

So, one can use this the major loss is given by this expression. Now, having done that we wrote the integral energy balance plus alpha V 1 square by 2 plus g z 1 is minus p by rho between two points in a control volume, by 2 plus g z 2 plus delta h pump is h l T. So, this has both major and minor parts. This is the work; this is the rating on the pump if there is pump that has to be used in order to make the fluid flow from point 1 to point 2. It has to work against the gravitational force, it has to work against the kinetic energy, and it has to work against the kinetic energy losses, it has to work against the total losses and so on.

So, this can be used to find the power rating on a pump by using this expression by calculating the losses. Then we can back out what is the power rating on a pump.

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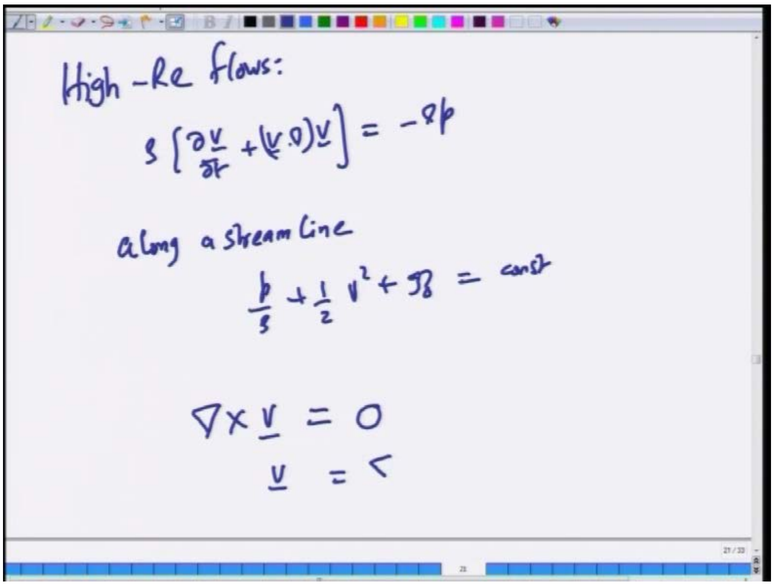

So, we discussed that and then moved to high Reynolds number flows. Wherein we used the Euler equation without the viscosity term in the navier stokes equation. Now, using the Euler equation we derive the Bernoulli equation along a stream line. So, the Bernoulli equation is essentially restatement of the Euler equation, but it is valid only along the stream line p by rho plus half V square plus g z is constant along a stream line. This is the Bernoulli equation and we also said that whenever you have a fluid flow at high Reynolds number except flows to solid surfaces the flow is irrigational. That is del cross v is 0. There is no water city, there is no rotation. Then v can be written as gradient of a scalar functions called the velocity potential.

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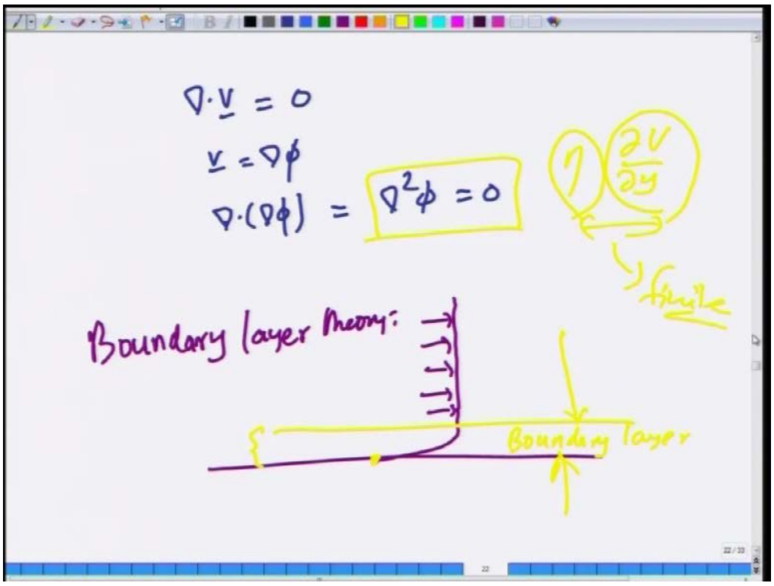

So, when you have del dot v is 0 and v is del phi. Then you have del dot del phi is 0 which means del square phi is 0. So, the velocity potential satisfies the Laplace equation. So, it is much simpler to solve and then we showed that and then the Bernoulli equation serves to compute what the pressure is? And we solved we said that instead of solving for the Laplace equation we will use the super position idea. Wherein we have some simple flow patterns such as uniform flow, flow due to a source or a sink, flow due to a line vertex flow due to a dipole and so on.

And by superposing various simple flows we were able to generate more complex flows and we said that we could simulate or mimic flow pass potential flow pass through a circular cylinder by simply superposing a uniform flow with a dipole at the origin. And by computing the velocity profile we tried to calculate the force due to the pressure forces on force on the cylinder due to pressure force. So, there are no viscous forces in the din visit fluid. By doing that we found that there is no force in potential force past a cylinder when there is a flow is steady.

So, this is a very paradoxical result because in reality no matter how high the Reynolds number is there is always a resisting force that is placed by a solid object due to viscous friction. And since we have neglected viscous friction that force is not there and this aspect was corrected by the boundary layer theory (No Audio Time: 45:14 to 45:21). Wherein we said that closed to a solid surface when you have flow past high Reynolds number past any solid surface. The flow is uniform far away from the solid surface, but very close to the solid surface the no slip condition has to be satisfied.

And the region over the which the velocity varies rapidly from 0 at the wall to the free steam velocity is called the boundary layer. And since the velocity varies rapidly that is the gradient of velocity become larger as Reynolds number becomes large. Therefore, no matter how high the Reynolds number is? The viscous stresses will become important because viscous stresses are essentially eta times d v by d y.

So, it is not just that the viscosity is small in order to keep the Reynolds number high, but the gradients become large. So, the product of these two becomes finite. So, that was the basic idea behind bound layer theory. And then we solve the bound layer problem for a flow past of packed beds using a simplifying approach. Namely the integral momentum approach because rigorous solution of the bound layer using the navier stokes equation is a very difficult task. One has to use some sophisticated approximations of navier stokes equations to get to the solution. It is not impossible, but it is not within the scope of the present study.

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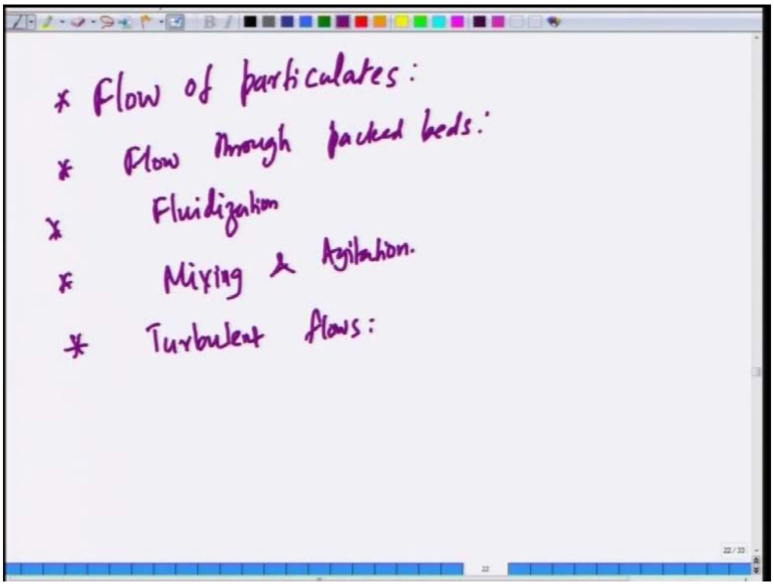

Now, after we finish the bound layer theory. Then we move to a flow of particulates and fluids and we move to applications in close to chemical engineering, closer to chemical engineering flow of particulates. We discuss settling of particles and how that is used in finding out the minimum diameter that can be captured in a settling chamber. That is one application we did. We also found that we can derive the turbulent settling velocity of a sphere be it in gravitational field or in a centrifugal field.

So, that is one thing that we discussed. And then we discussed flow through packed beds where you have a bed of particles through which fluid is made to flow. And the question we are asking is a purely fluid mechanical type question. What is the pressure drop required to make a fluid flow at a given flow rate through a bed of particles? And we found that by modeling this bed of particles as a bunch of tubes present a bundle of tubes with the same porosity and the same surface area per unit volume provided. That is the same between the real situation and the simple model.

Then we were able to derive simplified relations for pressure drop versus flow rate and what is interesting is that despite this drastic simplification in the model. We were able to get the functional form up to a dimensional, non dimensional constant which was fixed by experiment and this gave rise to the Kozeny–Carman equation. And we also found that when the flow is at low Reynolds number. At high Reynolds number we could use that the fact that the flow is through very rough tulles and therefore, the friction factor is constant. So, we got the Burke plummer equation.

By combining these two over all Reynolds number ranges. We got the ergun equation for flow through packed beds. Then we discussed flow through a fluidization. That is when you have bed of particles we keep on increasing the flow rate. Initiating the pressure drop will increase, but after a point when the drag force on the particles over comes the weight of the bed. Then the whole bed will be suspended and these particles will start moving. They will not be stationary anymore and that state is called fluidia state is often used in many chemical engineering reactors fluidias bed catalyst reactors and so on; Fluidias catalytic crackers and so on.

And we were able to use the same ideas such as what is the pressure drop of a flow in a packed bed and use that idea to find the incipient velocity required for onset of fluidization? Which we were able to do? Then we briefly discussed a simple dimensionless or non dimensional approach to understanding mixing and agitation. Where we came up with the notion of power number as a function of Reynolds number and other geometric parameters. Finally, in the last two and half lectures we discussed turbulent flows by using what is called by decomposing the velocity field in the fluid to a mean time average mean plus fluctuations.

And we found that the fluctuations are do not go away even if you want to describe only the mean quantities. That is because the fluctuation has got momentum and they come as stresses and these are called the Reynolds stresses or turbulent stresses. And in order to solve the problem we have to be able to write down an expression for a Reynolds stress.

And in that context we wrote down this the simple eddy viscosity model, where we wrote the turbulent stresses as equal to eddy viscosity times the velocity gradient. And the eddy viscosity was modeled using the prandtls mixing length hypothesis in analogy with the kinetic theory of gases and by using simplified forms physically motivated forms for the mixing line.

We were able to get some fairly robust expressions for example, velocity profile very close to the solid surface you had this viscous sub layer where the velocity is linear in position from the wall and a little away somewhere in between from the center and the wall you had this logarithmic velocity profile. And these are all universal velocity profiles because they do not depend on the nature of the geometry of the flow. That is it does not matter whether it is flow through a pipe or flow through rectangular channel or flow past a flat plate.

These are very robust universal features on shear flows past rigid surfaces. Once we were able to get the velocity profile we were also further able to integrate the velocity profile to find the friction factor. And we did find a very reasonable fit with very reasonable agreement with experiment.

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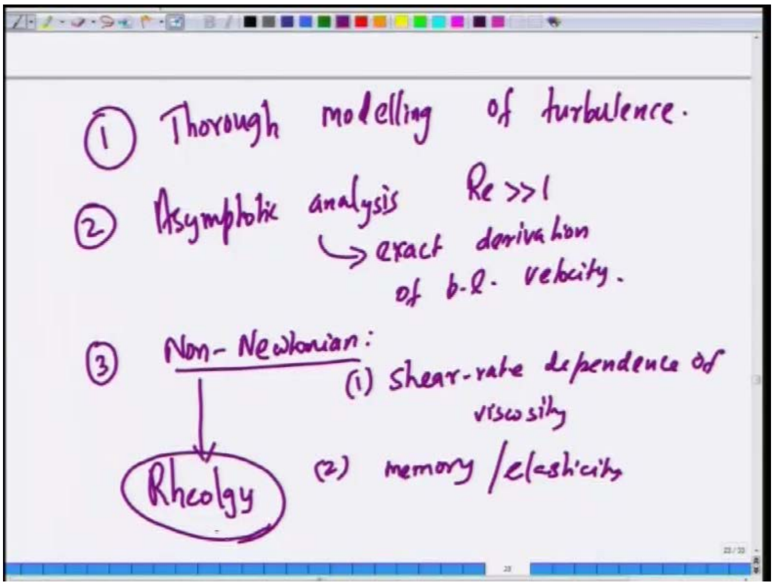

So, this for this is what we covered in this course, but it also is important to understand things that we could not cover and which are nonetheless very very important. One aspect is of course, more thorough understanding of turbulent flows is very very important which we did not do. So, which could form of a more advanced course in a fluid mechanics and we did not do the bound layer analysis in a rigorous manner. We did this simplified integral momentum approach, but it is possible to use what is called asymptotic analysis.

By exploiting the fact that the Reynolds number is a large parameter. When a inbound layer flow and then we can derive the bound layer velocity profile exactly. In the limit of high Reynolds numbers by throwing a victims that are systematically small at high Reynolds numbers. It is possible to derive the velocity profile in the bound layer. Thirdly the flows that we discussed in this course or Newtonian flows, but many chemical engineering applications involve Non-Newtonian flows. The key difference between Newtonian flows and Non-Newtonian flows are two flow.

One is that we assume that the viscosity is a constant independent of the velocity gradient, but in many Non-Newtonian flows viscosity is a function of the velocity gradient itself. So, that such a behavior is called shear thinning, shear thickening. Depending on the whether the viscosity decreases with the velocity gradient or increases with the velocity gradient.

So, Non-Newtonian flows are shear dependence or shear rate dependence of viscosity and (No Audio Time: 53:05 to 53:11). Secondly, they also show memory effects they also show elastic effects. So, they are not purely Newtonian fluid in the sense. They are not purely viscous liquid they have bought viscosity and elasticity. That is why they are called viscous elastic fluids. And many many many chemical engineering applications such as involving polymeric liquids such as molten polymers or polymer solutions are in fact, Non-Newtonian.

And it is very very important to in many chemical engineering processing to understand of fluid mechanics of Non-Newtonian fluids. That branch of mechanics that deals with deformation of more complex materials is called Rheology.

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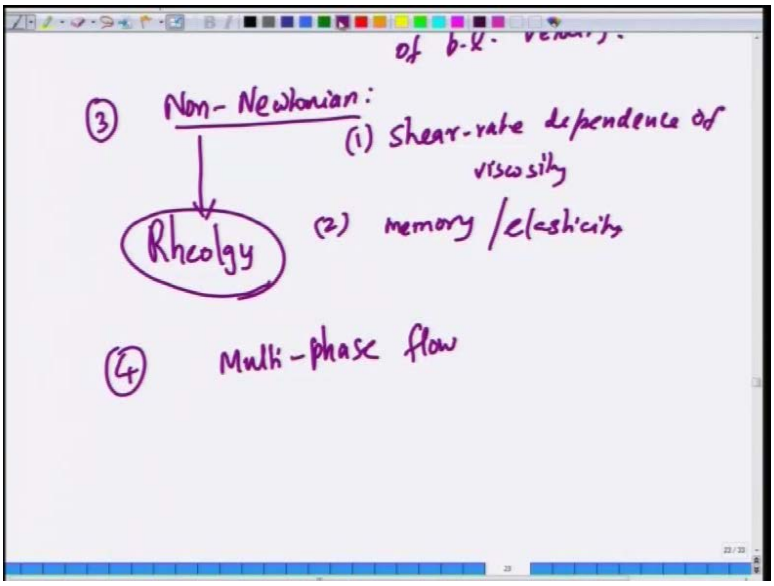

So, there is lot of focus on understanding the rheological nature of behavior of complex fluids because it has an important bearing on the processing of such fluids. And another related topic is Multi-phase flow. In many chemical engineering applications you would not have flow of a single Newtonian fluid. You will have may be a suspension of solids and liquids or suspension of bubbles of gases in a liquid or suspension of one liquid in another immense liquid and so on.

Such flows are much more complex then single phase flows. So, these are called two phase or multi phase flows you could have three phase flow also. So, the clear I mean understanding of multi phase flow is very very critical in a many chemical engineering applications such as reactors, bubble column reactors and so on. Where you do have flow of bubbles in a liquid? Suppose, you want to design these bubble column reactors you have to understand at some level the complex flow patterns that are present in multi phase flows and so on. So, these are very very important and in the context of a practical applications.

In chemical engineering which we did not have which we did not have the time nor the scope in this course. So, clearly these are very very important topics and it is useful for the students of chemical engineering to get a brief exposure at least at an undergraduate level to all these different topics. So, with that we will end this course and thank you for your kind attention.