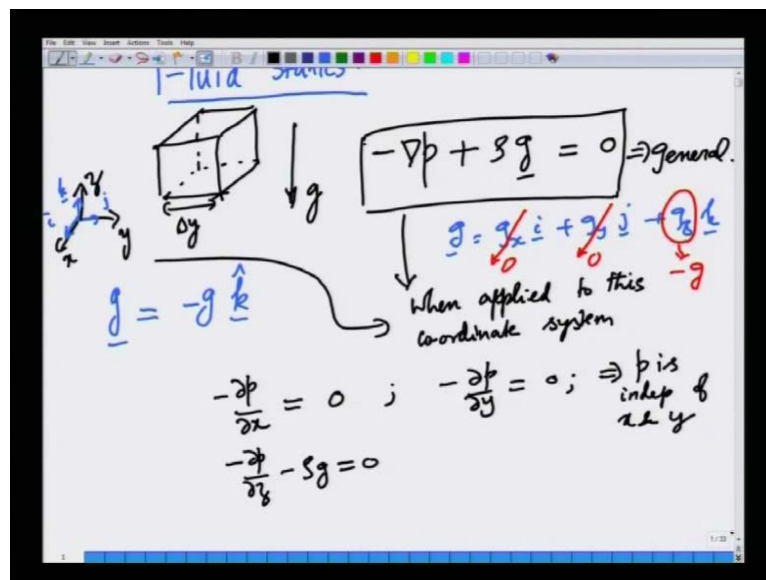


Fluid Mechanics
Indian Institute of Technology, Kanpur
Prof. Viswanathan Shankar
Department of chemical Engineering

Lecture No. # 07

Welcome to this 7 th lecture in this NPTEL course on fluid mechanics for chemical engineering undergraduate students. In the last lecture, we discuss the fundamental equation of fluid statics.

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We were discussing fluids under static conditions, and we derive a fundamental equation for fluids that is, that are present and under the influence of a gravitational field. So we started by taking a volume element, volume element of a fluid under the influence of gravity, and in the limit when, so we took a co-ordinate system x, y, and z. In the limit when, this volume element dimensions of the volume element shrinks, we derive a fundamental equation for fluid statics is equal to 0 minus the gradient of pressure plus density of the fluid times gravity is equal to 0. This is a fundamental equation that is of use in describing several features of fluid statics, fluids under static conditions.

It is common to point the gravity vector along the negative z direction as I shown here. So there are three unit vectors i, j, and k, in the along the x, y, and z direction. So the acceleration due to gravity vector is given by minus g times k, where k is a unit vector in the positive z direction, but g is pointing the negative direction. So, minus g happens, because of that. So, when we substitute this, when we referred this equation to this co-ordinate system, for this particular co-ordinate system, this equation is very general, because it has no reference to any co-ordinate system.

This is general, when applied to the co-ordinate system shown here, we get minus partial p partial x. Now, the vector g can be written as g x times i plus g y times j plus g z times k. It can be dissolved into the three cartesian directions, and in this co-ordinate system g x is 0, and g y is 0, and g z is minus g. So, we can proceed further by saying that d p by minus d p dx is 0, because g x is 0 and minus d p dy is 0, because g y is 0, and minus d p dz minus rho g is 0, in the z direction.

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$$\frac{\partial p}{\partial z} - \rho g = 0$$

$$\Rightarrow -\frac{dp}{dz} = \rho g \Rightarrow \frac{dp}{dz} = -\rho g$$

If ρ is a const; $g = \text{const}$

$$\frac{dp}{dz} = -\rho g \Rightarrow \int dp = \int -\rho g dz$$

$$\int_{p_0}^p dp = -\rho g \int_{z_0}^z dz$$

This implies that p is independent of x and y and it varies only in the z direction. And this implies, since p is independent of x and y, the partial derivative becomes a normal derivative minus d p dz is rho g or d p dz is minus rho g. We can integrate this, if rho is a constant and g is normally a constant under terrestrial conditions, the acceleration due to gravity on the surface of this a constant.

Then $d p / dz$ is minus ρg can be integrated as follows, between any two points, minus $\rho g dz$. Since ρ and g are constants, we can pull them out. So, integral $d p$ between any two points p naught and p is minus ρg integral z is z naught to any $z dz$.

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The whiteboard shows the following handwritten derivation:

$$\frac{dp}{dz} = -\rho g \Rightarrow \int dp = \int -\rho g dz$$

$$\int_{p_0}^p dp = -\rho g \int_{z_0}^z dz$$

$$(p - p_0) = -\rho g (z - z_0)$$

$$p - p_0 = \rho g (z_0 - z)$$

So, p minus p naught is nothing but minus ρg times z minus z naught or p minus p naught is ρg times z naught minus z , after sawing the minus sign.

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The whiteboard shows a diagram and equations for atmospheric pressure. The diagram includes a vertical axis with an upward arrow, a horizontal line representing the atmosphere at height z_0 , and a downward arrow representing height z . The pressure at the atmosphere level is labeled $p = p_{atm}$. The equations are:

$$p(z) - p_0 = \rho g (z_0 - z)$$

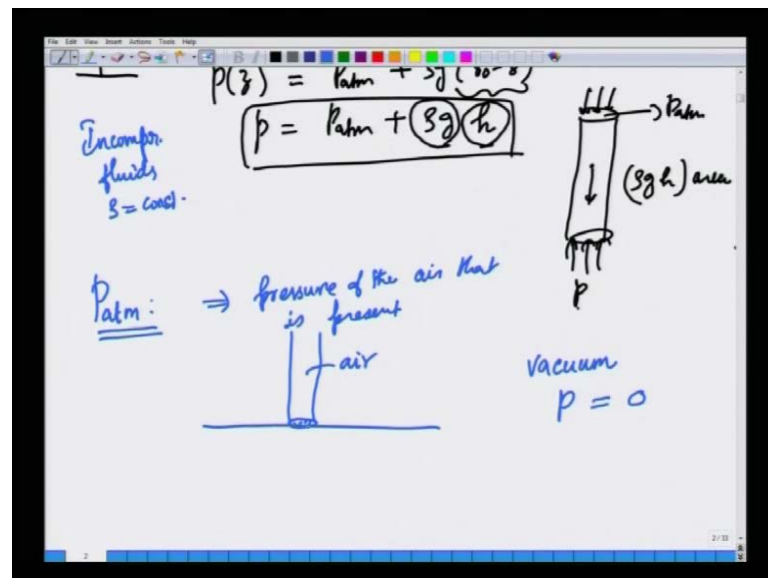
$$p(z) = p_{atm} + \rho g (z_0 - z)$$

$$p = p_{atm} + \rho g h$$

Now this can also be written as p at any z minus p naught is ρg times z naught minus z . It is customary in fluid mechanics, so you have the z co-ordinate going up like this. Suppose you have a fluid, suppose you have a water body which is exposed to atmosphere air. Where the pressure is p atmosphere, the pressure of air in the atmosphere is due to the weight of the air that is present above a given elevation. So, at c level the pressure of air is conventionally called the atmospheric pressure. That is precisely because of the weight of the air that is present above the level, so this is known.

So, if you call this location as z equals z naught which is the free surface, where p is p naught is p atmosphere. Then p at any location z is p naught which is p atmosphere plus ρg times z naught minus z . Z is any location and z naught is this location, so z naught minus z is this depth from the free surface. So this is conventionally denoted by the letter h , so p at any location in the liquid is p atmosphere plus $\rho g h$. This is something that you may be familiar with from your earlier classes in physics. Where the pressure in a column of liquid is an increase with vertical distance in a linear manner and that is precisely because of the fact that the pressure. Suppose, you take a column of liquid and this is the atmospheric pressure, so pressure always acts normally to a surface.

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So if you look at the pressure on this side, this pressure will have to be greater than atmospheric pressure. Because of the fact that under static conditions, the fluid will have to balance the weight of this liquid column, which is precisely given by $\rho g h$ times the area of the element. So that is the precise physical meaning of this equation. And this is valid only for incompressible fluids, where ρ is constant. Now, let me just spend a couple of minutes commenting on the nature of atmospheric pressure. $P_{\text{atmosphere}}$, the atmospheric pressure is precisely the pressure of the air that is present in the atmosphere. And so, if you consider $z=0$ level that is ground level, then if you take a cylindrical column of air, the weight of this air is precisely the pressure that you will feel at the ground level. And at pure vacuum, that is when there is no air, when you go far away from the ground level, far into the atmosphere there is the density of air will decrease significantly the pressure will also decrease. So at pure vacuum, the pressure of air is 0.

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The image shows a whiteboard with handwritten mathematical derivations. At the top, it states $P_{\text{atm}} \approx 10^5 \text{ Pa} = 10^5 \text{ N/m}^2$. Below this, a horizontal line is drawn. Under the line, it says "If air is treated to be an ideal gas". This is followed by the equation $p = \rho R_g T \Rightarrow \rho = \frac{p}{R_g T}$. Below that, the hydrostatic equation is written as $\frac{dp}{dz} = -\rho g$. Finally, the equation is substituted with the ideal gas law to get $\frac{dp}{dz} = -\frac{p g}{R_g T}$.

So there is no air molecules, there is no pressure and the pressure at the ground level is called the atmospheric pressure. And this atmospheric pressure is roughly 10 to the 5 pascals, is 10 to 5 newton per meter square in S I units. The other thing we discussed was the role of compressibility. So, if air is treated compressible, to be an ideal gas, so we said that ρ is $\frac{p}{R_g T}$, then you had $\frac{dp}{dz} = -\rho g$. Instead of ρ , so this implies ρ is $\frac{p}{R_g T}$, so I can eliminate $\frac{dp}{dz}$ is $-\frac{p g}{R_g T}$ times g , acceleration due to gravity.

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The whiteboard shows the following handwritten equations:

$$\int \frac{dp}{p} = -\frac{g}{R_g T} \int dz \quad \text{---} \rightarrow \text{const. } T_0$$
$$\ln p = -\frac{g}{R_g T} z + C$$
$$p = p_0 \exp\left[-\frac{g}{R_g T} z\right]$$

Arrows indicate that T is constant and p_0 is the pressure at $z=0$.

So I can integrate this in the following way $\frac{dp}{p}$ integral, this is minus g by $R_g T$ integral dz . So, logarithm of p is nothing but minus g by $R_g T$ z plus some constant. Which can be simplified to write as p is p_0 times exponential of minus g by $R_g T$ and T is assumed to be constant in this analysis. The air is a constant temperature, so let us call that constant T_0 times z . This p_0 is the value, pressure at z equals 0. That is the condition, boundary condition we use to fix this constant. This constant is fixed by saying that the pressure at a z equal to 0 is p_0 .

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The whiteboard shows the following handwritten equations:

$$p = p_0 \exp\left[-\frac{g}{R_g T_0} z\right]$$

if $\frac{gz}{R_g T_0} \ll 1$ Taylor expand $e^{-x} \approx 1 - x$ if $x \ll 1$

$$p \approx p_0 \left[1 - \frac{gz}{R_g T_0}\right]$$
$$p = p_0 - \left(\frac{p_0}{R_g T_0}\right) gz \rightarrow \rho_0$$
$$p = p_0 - \rho_0 gz$$

Arrows indicate that T_0 is constant and p_0 is the pressure at $z=0$.

So p is p_0 times exponential of minus g by $R T_0$ times z . This is an equation that is valid, if air is treated compressible. But we also said that, if the value of this exponent $g z$ by $R T_0$ is small compared to 1. This is a dimensionless group, so if this number is small compared to 1, you can Taylor expand and write p is p_0 times 1 minus $g z$ by $R T_0$. So Taylor expand, e^{-x} is approximately $1 - x$, if x is small. So $1 - g z$ by $R T_0$, so p is p_0 minus p_0 by $R T_0$ $g z$. From ideal gas law, this is nothing but ρ_0 , so p is p_0 minus $\rho_0 g z$. So, this is similar to the incompressible equation that we derived by treating density to be constant.

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The image shows a whiteboard with the following handwritten equations:

$$p \approx p_0 \left[1 - \frac{g z}{R T_0} \right]$$

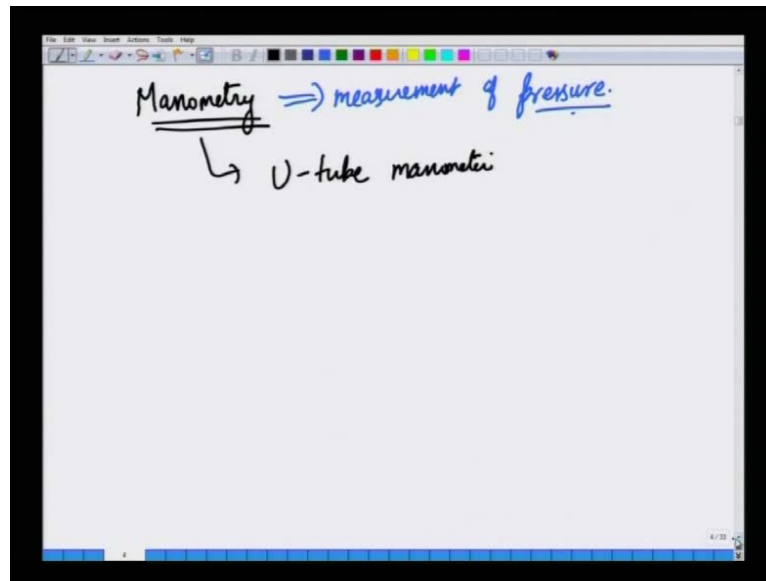
$$p = p_0 - \left(\frac{p_0}{R T_0} \right) g z \rightarrow \rho_0$$

$$\boxed{p = p_0 - \rho_0 g z} \quad \text{if } \frac{g z}{R T_0} \leq 0.1$$

or if $z \leq 800 \text{ m}$

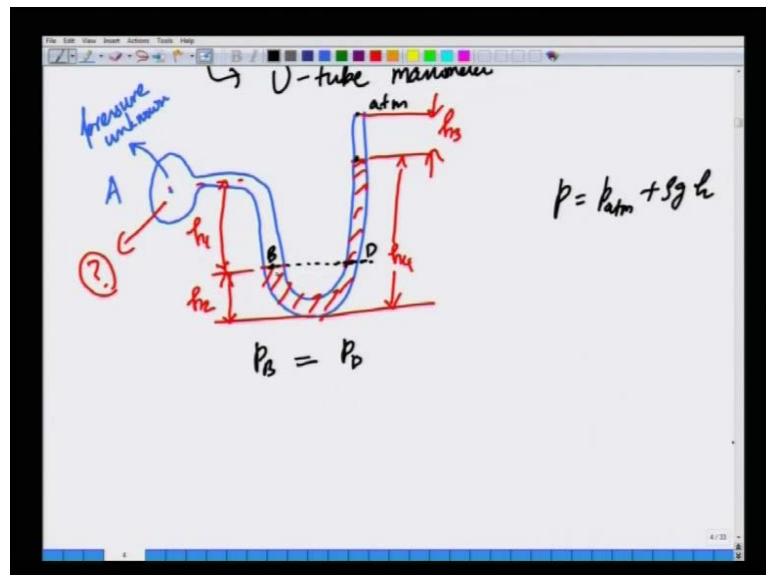
So, this linear variation is a simplification of this exponential variation of pressure and is valid if $g z$ by $R T_0$ is less than 0.1 or if z is less than 800 meter. We can treat the pressure variation even in air which is in general a compressible system to be a linear variation. So, the other thing the next thing we will do is to apply the fundamental equation of hydrostatics to what is called manometry.

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Manometry is that branch of fluid mechanics, which deals with measurement of pressures. And the specific device we are going to use is called a U-tube manometer. So what is the construction? Well, it consists of U shaped tube and one end is exposed to atmosphere.

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The other end is joined to a region, whose pressure we want to know. So here, they let us call this point as A the pressure here is not known, so pressure unknown. The manometer is filled with working liquid is called the manometric liquid. Now this is open to atmosphere. Now the idea is to relate this pressure, unknown pressure to the atmospheric pressure which is known. So, how do we go about doing this? Let us now draw some label some heights here, let us call this height as h_3 , let us call this height as h_4 , let us call this height as h_2 and between point A and this interfaces h_1 . So we want to now apply the principle of the result from fundamental equation of hydrostatics, that p is p atmosphere plus $\rho g h$. Now, if you go from point here to here, the pressure will increase, this is atmospheric pressure, the pressure will increase because of the weight of air, but that is negligible, so we will not worry about this.

From this point, now we are going to between these two points, between these two levels the pressure at this point and the pressure at this point is the same. Because the pressure in this manometric liquid is a function only of the elevation and the elevations are the same to pressure at this point, which is called, let us call it B and the pressure at this point D must be the same. So p_B must be equal to p_D , because the pressure in the manometric liquid is the function only of the elevation. And since these two points are the same elevation, p_B must be equal to p_D . If p_B is, so how do we get p_B in terms of p_A . Well, p_B is nothing but p_A plus $\rho g h_1$.

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The image shows a whiteboard with handwritten mathematical derivations. At the top, there is a small diagram of a manometer tube with points B and D marked. Below it, the following equations are written:

$$p_A + \rho_1 g h_1 = p_{atm} + \rho_m g (h_4 - h_2)$$

$$(p_A - p_{atm}) = \rho_m g (h_4 - h_2) - \rho_1 g h_1$$

Below the second equation, it is noted that $\rho_m \gg \rho_1$. A red arrow points from the term $-\rho_1 g h_1$ to the text "usually negligible".

The final result is boxed:

$$(p_A - p_{atm}) = \rho_m g (\Delta h)$$

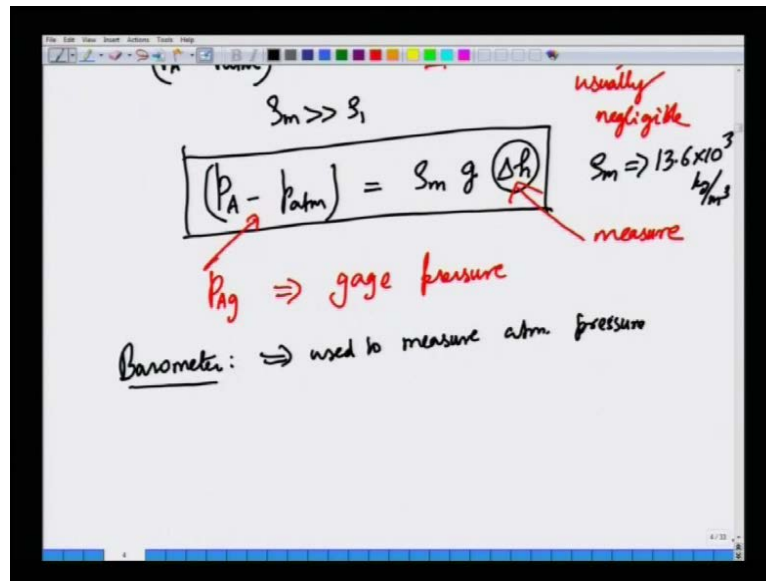
A red arrow points from the boxed equation to the text " $p_{Ag} \Rightarrow$ gage pressure". Another red arrow points from the term (Δh) in the boxed equation to the text "measure".

Let us call this liquid ρ_1 , equate the density ρ_1 , acceleration due to gravity times, this column height h_1 , $\rho_1 g h_1$. This is the pressure at point B. Now p D, the pressure at this point D is given by the pressure at this point which is approximately atmospheric pressure plus ρ_m manometric liquid. Let us call it ρ_m , so density of this liquid the manometric liquid is $\rho_m g$. This total height is h_4 , this is h_4 and so this total height is h_4 , this is h_2 , so that this weight of this, the height of this column is h_4 minus h_2 .

So, we can write p_A minus $p_{\text{atmosphere}}$. The difference in the value of the pressure at the point A minus atmospheric pressure is nothing but $\rho_m g (h_4 - h_2) - \rho_1 g h_1$. So, by measuring these two heights, by measuring the height difference $h_4 - h_2$, $h_4 - h_2$ is basically this height. I am measuring this height that is this and typically the density of the manometric liquid is very large compared to density of the working fluid. So this is usually negligible, so we can get the difference in the value of pressure at point A from the atmospheric pressure to be the density of the manometric liquid times, the acceleration due to gravity times, and the height difference between the two lengths of the manometer.

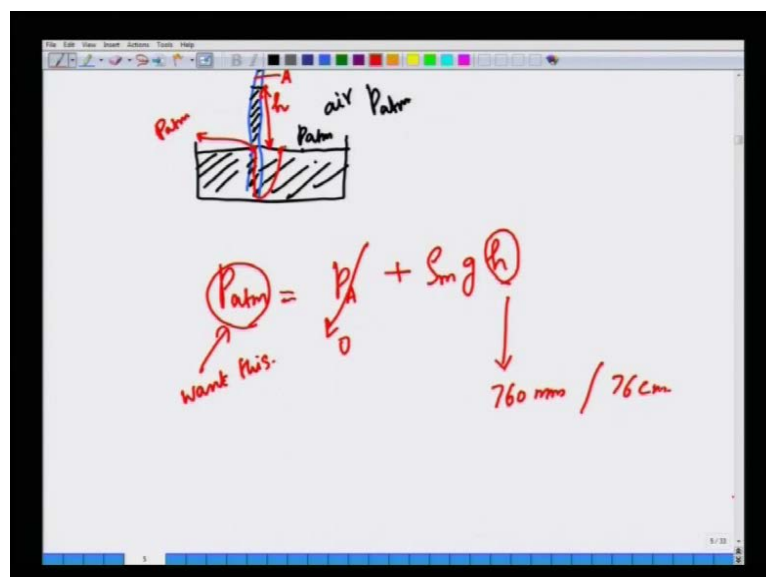
So this is a fundamental equation of manometry. And by just simply measuring the height, we measure this to obtain the unknown pressure. Now, this difference is called the gage pressure, as I mentioned in the last lecture. The difference between the values of a pressure at a point from the atmospheric pressure is called the gage pressure, and because that is what is measured by a pressure measuring devices, such as the manometers.

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Now how do, how does one measure atmospheric pressure itself? In order to that, we have, what is called a barometer. Usually the manometric liquid is mercury ρ_m is the typically mercury, which is 13.6 times 10 to the 3 kg per meter cube, so very large density liquid. Now, a barometer is used to measure atmospheric pressure. So, this is the device that is used to obtain what to estimate the atmospheric pressure. How is it done? Well, the construction of the barometer is very simple. You take a trough of mercury and then in this trough, we invert a tube which already has mercury in it. Now the mercury in this tube will raise.

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So this is mercury, will this raise to in this tube will rise to a particular height, this is air atmospheric pressure, p atmosphere. Now in this part, it is largely vacuum, it has some molecules of mercury vapor. But it is since a vapor pressure of mercury is very very small, this is essentially a vacuum, there is no pressure here. So the pressure at this point is atmospheric pressure, from the fact that this liquid is exposed to air. Now, the pressure at this point must be the same as the pressure at this point, because this is connected by the same liquid is connecting these two points.

And since they are at the same elevation, there cannot be any pressure difference. So the pressure at this point is also atmospheric. So the pressure at this point, which is atmospheric pressure is the pressure at point A, which is let us call it p_A which is 0, because it is a vacuum plus ρ mercury g times h . Where h is the **column** height of the column of mercury that is present in the tube. So, p atmosphere which is what we want to calculate want this. Can be obtained by this measuring what is the height of mercury. So, typically this height is 760 mm at normal conditions of temperature and altitudes or 76 centimeters.

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The image shows a handwritten derivation on a whiteboard. At the top, it says $(1 \text{ atm}) =$ with a note "Want this." and a diagram of a U-tube manometer. The left side of the tube is labeled 'A' and '0', and the right side is labeled '760 mm / 76 cm'. The density of mercury is given as 13.6×10^3 . The calculation is as follows:

$$p_{\text{atm}} = 13.6 \times 10^3 \times 9.8 \frac{\text{m}}{\text{s}^2} \times 0.76 \text{ m}$$

$$p_{\text{atm}} = 1.03 \times 10^5 \frac{\text{N}}{\text{m}^2} = 1.03 \times 10^5 \text{ Pa}$$

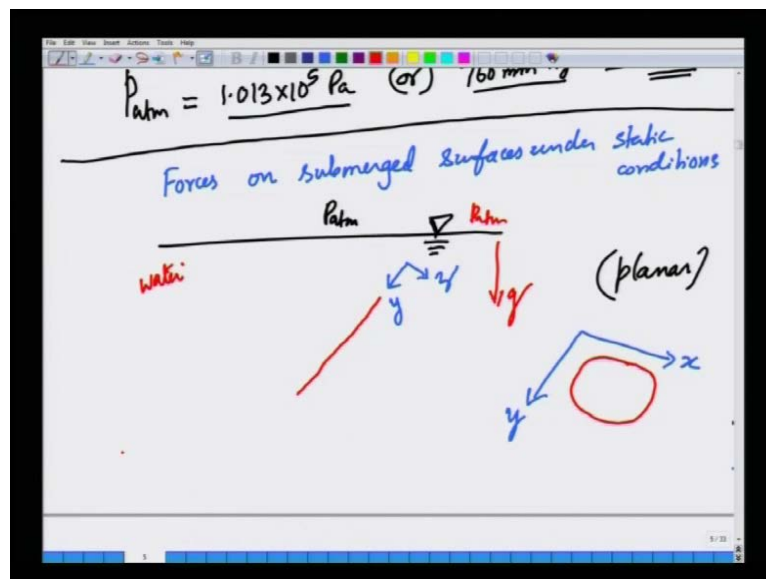
$$p_{\text{atm}} = \underline{1.013 \times 10^5 \text{ Pa}} \quad (\text{or}) \quad \underline{760 \text{ mm Hg}} \quad \equiv \underline{1 \text{ atm}}$$

So, the atmospheric pressure is nothing but density of mercury, this is 13.6 times 10 to the 3 times 9.8 meter per second square times 0.76 meters, this is height. When we do all these, we get atmospheric pressure to be 1.03 times 10 to the 5 newton per meter square or 1.03 times 10 to the 5 pascals. So the barometer is the simple construct that is used to

calculate the atmospheric pressure. So, the atmospheric pressure in several text books or even hand books is denoted in terms of S I units as 1.013 times 10 to the 5 pascals or it is written as 76 mm Hg. Because that is the height of the mercury column that raises to counter balance atmospheric pressure. So, it is sometimes refer in terms of mm h g and this is also trivially called as one atmosphere.

Because it is a normal pressure that is encountered in atmosphere, it is of the order of 10 to the 5 pascals. So, one atmosphere is essentially 1.013 times 10 to the 5 pascals is also 760 mm of Hg, so mercury column. So, all these are used interchangeably while reporting the values of pressure. Now, that we have done all this, now we are going to worry about the next topic, which is forces under static as forces on solid surfaces or forces on sub merged, surfaces under static conditions.

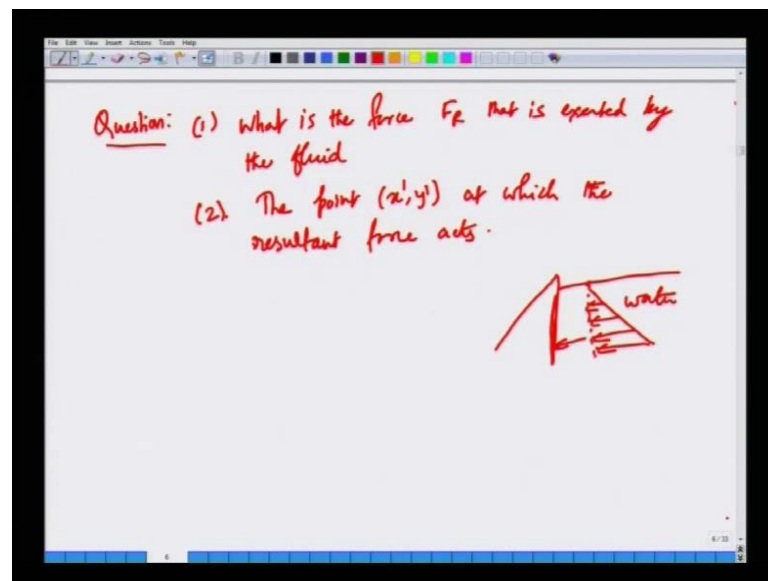
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So the issue that we are going to understand is the following. Suppose, you have a liquid surface, where it is exposed to atmosphere, p atmosphere, and within the liquid, there is a surface, a planer surface, so we will look at planer surface for simplicity, so we look at a planer surface. So you have a plane and this plane extends in the third direction. So let me put co-ordinate system, this is the z co-ordinate, this is the y co-ordinate, and the x co-ordinate runs in the direction perpendicular to the board.

So, if you look at the $x-y$, this surface may look like this. Now, since and in the $y-z$ plane, this will look like a line, because this is a planer surface, this is like a plate with some arbitrary shape. Now, we want to know and this is let us say liquid like water. We want to know and gravity is acting in this direction and this is the liquid surface, where the pressure is atmospheric. We want to know, what is the force that is exerted by the fluid on one side of this solid surface, on this planer surface.

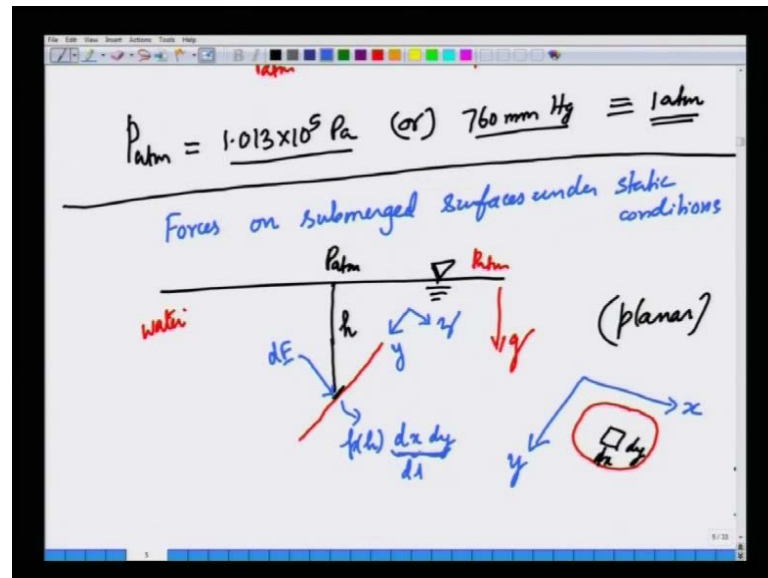
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The question that we are asking, the question that we want to answer is what is the force? F_R that is exerted by the fluid. Now, we also want to know the point at which the point x' , y' , at which the resultant force acts. So we want to calculate these two things. Now, why is this thing important? Well, this thing is important in several applications, where suppose you are interested in construction of a dam. So, a dam is something that stores or obstructs water, this is water. And this surface has to be constructed in such a manner, that it withstands the force due to the water. And the reason why hydrostatics is different is, because the pressure varies with depth. So, the pressure at this point is merely atmospheric, but as you go down the pressure will increase linearly.

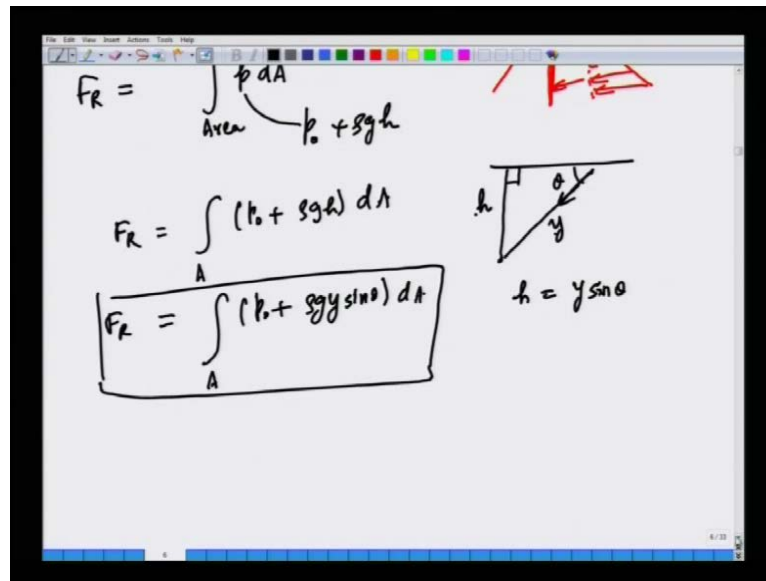
So the force will not be a, the force cannot be obtained by simply multiplying the pressure by the area. It as to be obtained by integrating the pressure with respect to the vertical co-ordinate. So this what we want to do. So in order to do this, the way we are going to proceed is by taking a tiny strip in the, you take a tiny area element of length dx dy.

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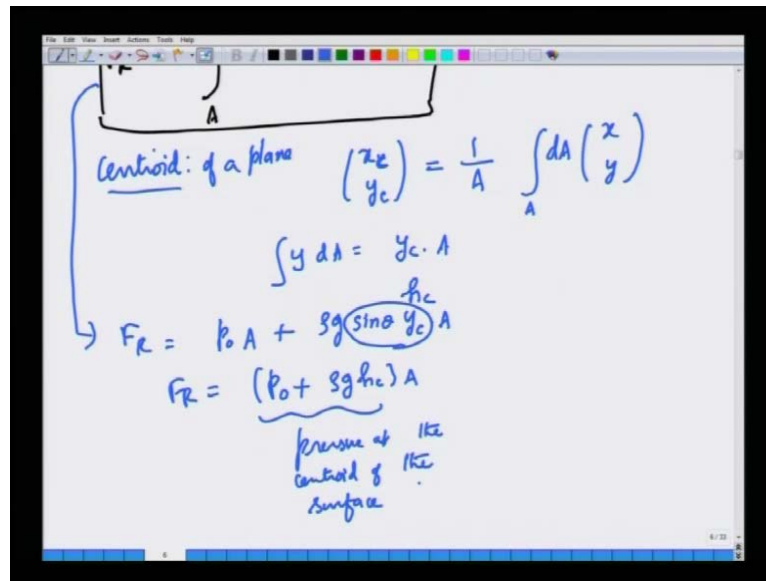
So it will appear like a strip here, this basically a tiny area element. And this tiny area element is at a distance vertical distance h . And on this area element; the pressure force will be acting purely normally, because the fluid is static. So the differential force acting on this area element is purely normal. And what is this differential force? This is the pressure at this vertical location h from the free surface times the area $dx dy$. This is the essential idea of doing the whole thing. So, $dx dy$ is dA this all we will do, in order to calculate the effective force.

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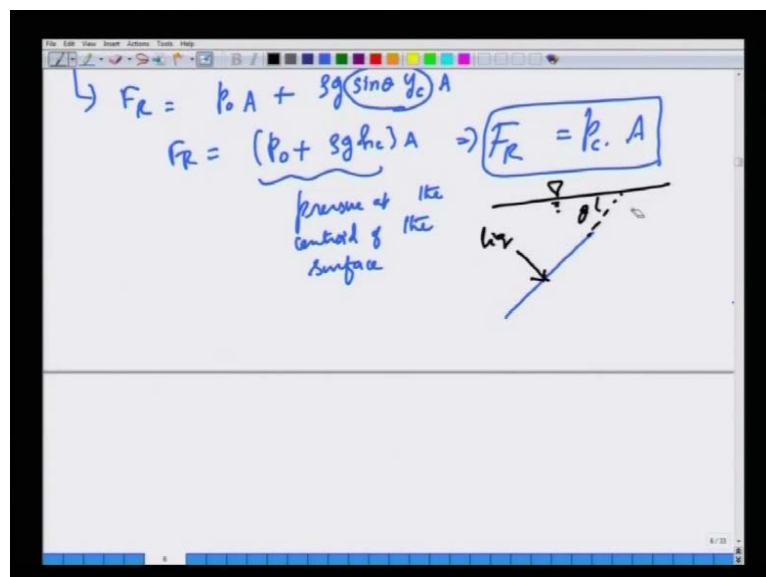
So the resultant force is nothing but you take the differential force $p dA$ acting on a tiny slice and integrate over the entire area that will give you the resultant force. Now, p is nothing but $p_0 + \rho g h$. So, F_R is nothing but $\int_A (p_0 + \rho g h) dA$. Now, h so this is y , this is h , this is θ , so h is basically like this angle, let me this rule slightly differently. So, this angle is θ so this is the surface, this angle θ , this is h , this is y , h is nothing but $y \sin \theta$. So, we want the value at point h . So instead of h , we will do y , because the co-ordinate is along the surface that is y . So, we will say F_R is nothing but $\int_A (p_0 + \rho g y \sin \theta) dA$. This is how one calculates, the force the resultant force on a surface that is submerged in a fluid. Well traditionally, the way this force is done, calculated is you could calculate either by just carrying out this integral or you define the centroid of a plane. A centroid of a plane, the plane is in the $x y$ this entered of a surface which is on $x y$ plane is $\frac{1}{\text{Area}} \int \text{Area} dA x y$.

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This is the centroid of the co-ordinates of the centroid of a surface. So, integral $y dA$ is nothing but y_c times A . So we have from here, F_R is $p_0 A$ upon integration, p_0 is a constant; it's usually an atmospheric pressure plus $\rho g \sin \theta$. Since, $y dA$ integral is y_c , so we will write this as $y_c A$ or F_R is nothing but p_0 plus ρg . This is sometime refer to us $h_c \rho g h_c A$, this is the pressure at the centroid of the area of the surface.

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So F_R , the resultant force is the pressure at the centroid times the area. So this is the simple result for flow R for the forces that have been exerted on a planar surface that is submerged into inside a liquid like water under static conditions. Now two comments, firstly, this force acts only on one side. So we are looking at, recall that the geometry is like this, this is θ , this is the free surface, this is liquid. This is the force only acting on one side, the other side is also comprise of the same liquid, a same amount of force will act on this side also. But if the other side is open to some other, it may be open to atmosphere, then this is the force that is because of the liquid that is present on one side of the surface. So it depends on the problem and context has to what the other side is. If it is open to atmosphere, then this will be the force due to the liquid that is present, and the pressure variation in the liquid under static conditions. What is the point of action of the force?

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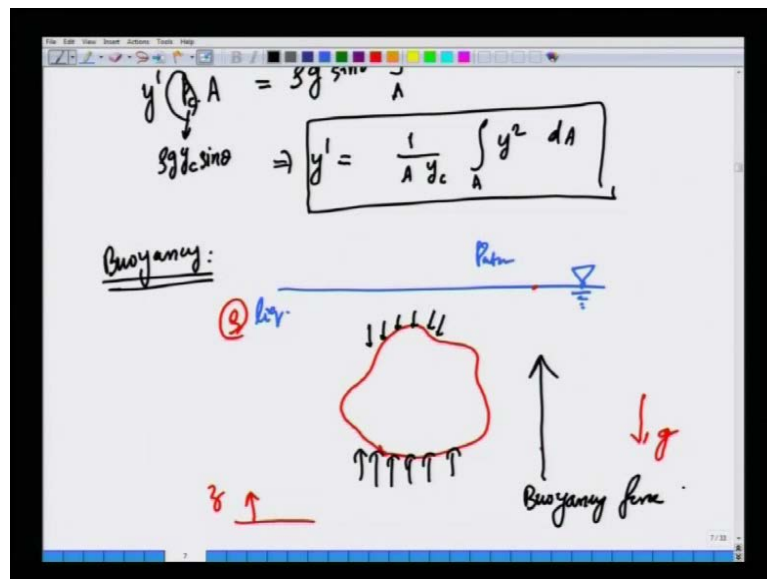
The image shows a handwritten derivation on a whiteboard. At the top left, it says "Point of action of force: (y') ". The main derivation starts with the equation $y' F_R = \int_A y p dA$. To the right, there is a note: "may be air" with a diagonal line through it, and "atm" below it. Below that, the pressure is given as $p = \rho g h = \rho g y \sin \theta$. The next step is $y' F_R = \rho g \sin \theta \int_A y^2 dA$. Then, a diagram shows a circular area A with a centroid C at a distance y_c from the point of action y' . The force F_R acts at y' . The final boxed equation is $y' = \frac{1}{A y_c} \int_A y^2 dA$.

What is the point of action? Well, in order to find the point of action, we simply take the moment. Let us call that point of action as y prime. So y prime, the moment of the force about the point of action must be equal to the distributed moment y times p times dA over the entire area. So, y times F_R is nothing but y is $\rho g \sin \theta$. Now, if on one side you have liquid and the other side you have atmospheric air, then you need not be worry about the atmospheric pressure. So, p is simply written as the gage pressure, because the atmospheric, the contribution due to atmospheric pressure on this side and this side will cancel. So we can neglect by atmospheric pressure and write only the gage

pressure. So this is $\rho g h_c$ times A , so p is $p g$ is ρg times h which is $\rho g y \sin \theta$. So, $\int \rho g \sin \theta A y^2 dA$.

But $F R$ is nothing but $p c$ times the pressure at this centroid times A . So this is $\rho g \sin \theta$ times $\int y^2 dA$. So $F R$ is nothing but $p c$ times A which is $\rho g y_c \sin \theta \int A y^2 dA$. So $p c$ is nothing but $\rho g y_c \sin \theta$, if the other side is surrounded by air. So this implies y' is nothing but 1 by $A y_c \int$, so there is this A here is coming, their denominator $\int y^2 dA$. This is the line of action of the force that is going to act on us submerged planer surface. So this is of use in several applications where you are interested in at the forces that have been exerted by the fluid under static conditions on solid surfaces. And this is primarily of interest in storage of water in dams and so on. We can also generate, generalize these two curved surfaces, but I will not go through this for want of time.

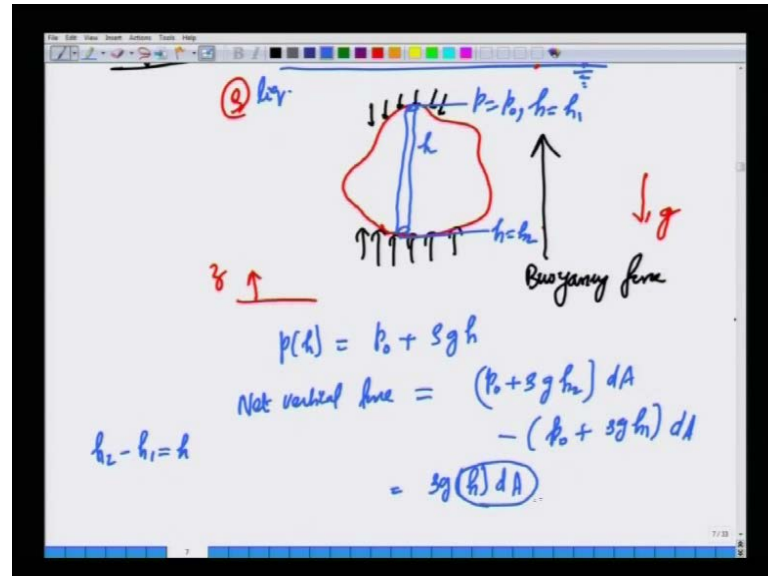
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So I will go to the next topic, which is buoyancy, which is also related to forces exerted on curved surfaces. Suppose, you have an object that is a solid object, that is completely immersed in a liquid. So you have a free surface that is atmosphere, you have a liquid like water, so you have a solid object is completely immersed. Now, let us say you are coating a co-ordinate z like this and gravity is acting like this. Now, this density has this liquid has density ρ . Now, because of the fact that is this liquid has a density and acceleration due to gravity is acting downwards the pressure here is p atmosphere, the

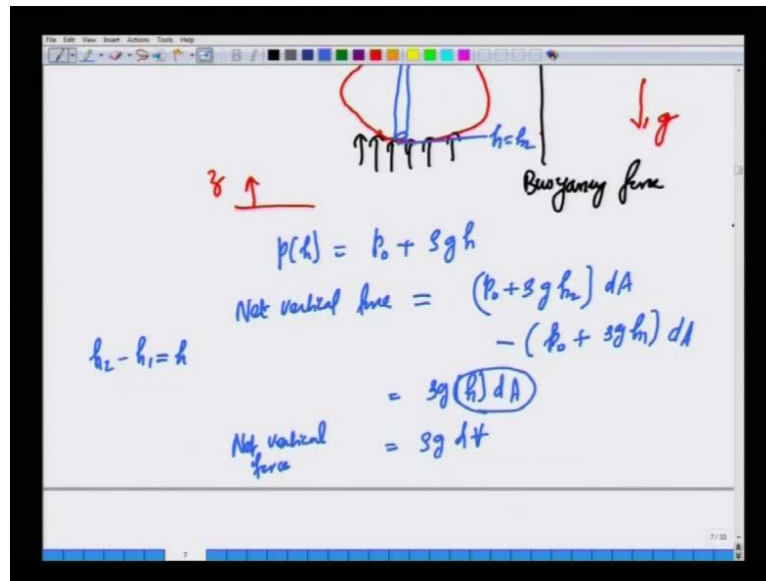
pressure here is more. So, the pressure exerted on this side on this submerged solid surface is more than the pressure that will be exerted on this side, because the pressure is less. So, this net force will act up wards is called the buoyancy force.

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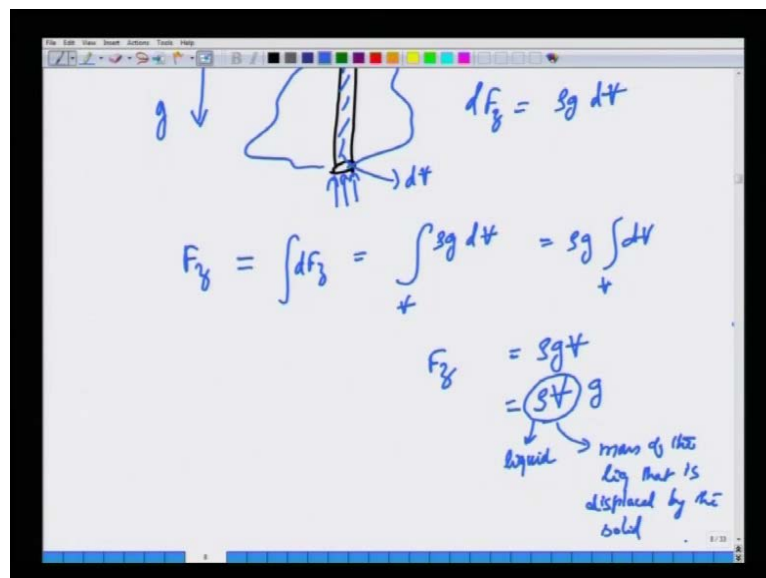
So, how do we estimate or derive an expression for a buoyancy force? It is not very difficult. Simply we have to take a thin cylindrical volume element, let us call this height as h . Now, p at h at the bottom is basically p naught, the pressure at the top plus this is p naught let us say plus $\rho g h$. This is the fundamental equation of hydrostatics. So, the net vertical force on this volumes cylindrical volume is nothing but p naught plus ρg , let's call this h equals h_2 and lets call the top surface h equals h_1 so p naught plus $\rho g h_2$ times dA minus p naught plus $\rho g h_1$ times dA , so h_2 minus h_1 let us call it h so its ρg were $h dA$. This is $h dA$ is the differential volume of the cylindrical volume element.

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So, the net vertical force on this infinitesimal cylindrical volume element is rho g times the differential volume.

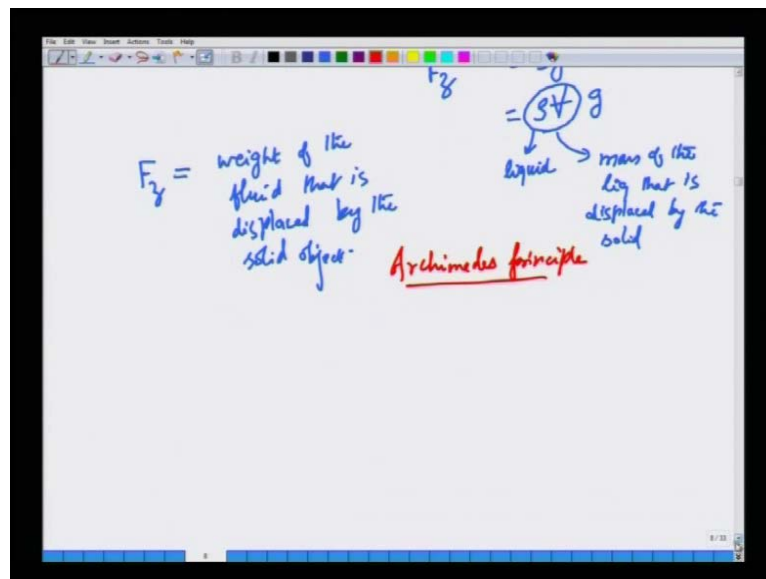
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So the net vertical force on a cylindrical volume element that is present in a solid that is submerged in a fluid the differential force is called dF_z is rho g times dV . This what we just derive, where dV is the differential volume of this volume element. And this force is precisely, because of the fact that the pressure here and the pressure here are different. And because of the fact that fluid is under a gravitational feel and the pressure varies due

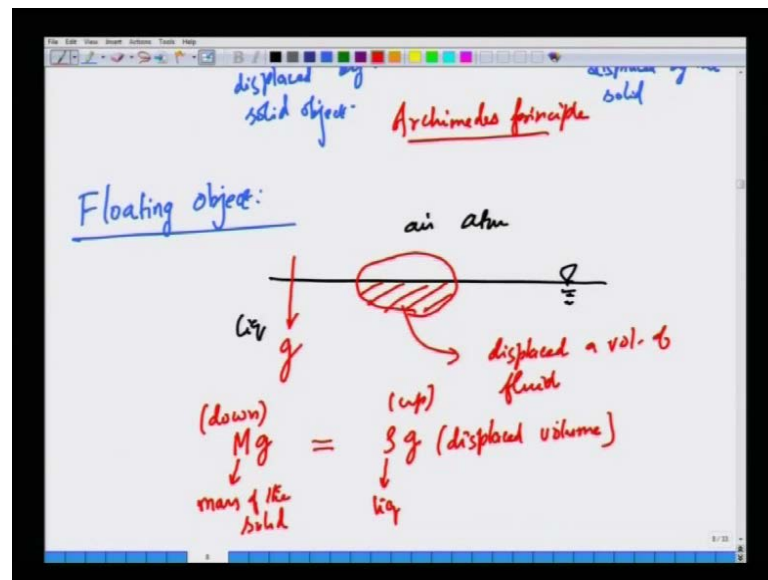
to hydrostatic equation, a hydrostatic force balance. Now to get the force on the entire object, we simply have to integrate this differential force over the entire object. Which is nothing but integrating over the entire volume $\rho g dv$. Since, ρg is constants, so you get ρg integral of dv which is nothing but ρg times the volume of the object. But let us try to understand this slightly differently, this is ρ times v times g , here ρ is density of the liquid in which this object is present. So, this is the mass of the liquid that is displaced by the solid.

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So the net vertical force on a solid substrate solid object that is completely submerged under in a liquid which is present under gravitational field, this is called the buoyancy force, and this is nothing but the weight of the fluid that is displaced by the solid object. This of course, the famous Archimedes principle, so this is called the Archimedes principle. This is again a consequence of a basic equation force balance in hydrostatics. And we merely have to apply this to the context of object that is immersed. So this is for a fully, what we derived is for a fully submerged object.

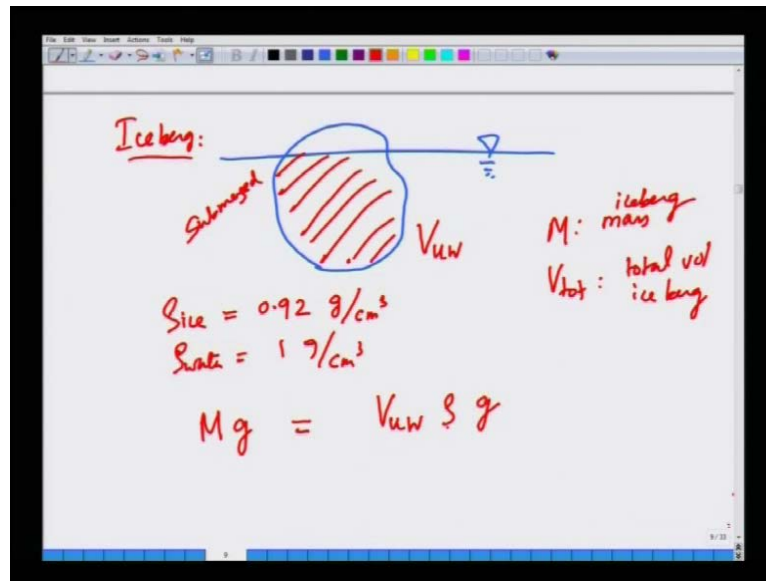
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Suppose you have a floating object, suppose you have an object that is partially submerged, this is floating. So you have this is atmosphere, air at atmospheric pressure, this is liquid. Suppose, you have partially submerged objects, so only this part is submerged. So this portion has displaced, this submerged object has displaced a volume of fluid. And the buoyancy force will be because of the fact that of due to that weight of the displaced fluid, so this is gravity.

Now this solid object is under stable equilibrium that is its not sinking, it is not moving down. That means that, the net downward force on the solid object, this is mass of the solid object, times acceleration due to gravity must be equal to the buoyancy force, which is acting upward. This is the downward force, this is the upward force, which is buoyancy, this is the density of the liquid, times acceleration due to gravity, times the displaced volume. Because only the displaced volume will contribute to the net upward force. So, this is the condition for floating. That the net downward force must be equal to the net upward force, which is the buoyancy force which is nothing but ρ times the density of the liquid times acceleration due to gravity times at displaced volume.

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Now, let us consider a simple example of an ice berg. Ice icebergs are found in oceans, these are huge chunks of ice that are present in sea, in ocean. So this is the water level, and this whole thing is submerged under water, this whole thing is submerged under water. So, let us call this submerged volume V under water $V_{u w}$. Let the mass of the ice berg be M , this is the ice berg mass and the total volume of ice berg is V_{total} , this is total volume of ice berg. Now density of ice is smaller than ρ_{ice} is smaller than density of water. Density of water is 1 gram per cc and density of ice is 0.92 grams per cc, centimeter cube. So, if this ice berg is under stable equilibrium, then the mass of the ice berg times is the acceleration due to gravity. The weight of gravity by is nothing but the buoyancy force; this is the weight of the displaced fluid which is nothing but the volume that is submerged under water of the ice berg times density times acceleration due to gravity.

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$\rho_{ice} = 0.92 \text{ g/cm}^3$ Total ice berg
 $\rho_{water} = 1 \text{ g/cm}^3$

$$Mg = V_{uw} \rho g$$
$$\rho_{ice} V_{tot} g = V_{uw} \rho_{water} g$$
$$\frac{V_{uw}}{V_{tot}} = \frac{\rho_{ice}}{\rho_{water}} = \frac{0.92}{1} = 0.92$$

In a floating iceberg 92% of the solid mass is under water!

What is the total mass of the ice berg? it is nothing but rho ice times V total volume times g is V under water times rho liquid, which is let us say water here of course, times g, g cancels. So V under water by V total, this the fraction of volume that is under water is nothing but rho ice divided by rho water. This is nothing but 0.92 divided by 1 is 0.92. So, what this simple example is telling you is that. When our ice berg is floating, in a floating ice berg, 92 percent of the solid mass is under water. So this is straight forward consequences the buoyancy principle, the 92 percent of the ice in an ice berg is completely under water.

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$\rho_{ice} V_{tot} g = V_{uw} \rho_{water} g$

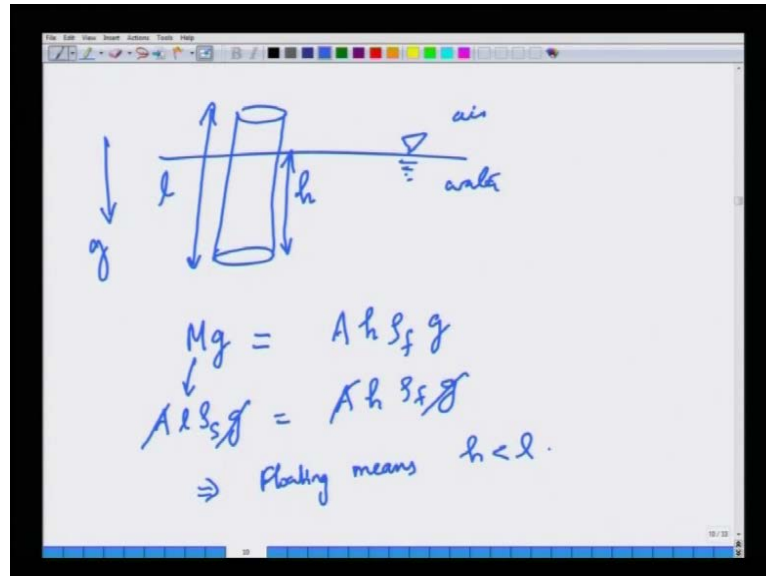
$$\frac{V_{uw}}{V_{tot}} = \frac{\rho_{ice}}{\rho_{water}} = \frac{0.92}{1} = 0.92$$

In a floating iceberg 92% of the solid mass is under water!

When does an object float?

We can also derive simple criteria for floating, when does an object float? When can object float? Let us consider a simple geometry here, let us consider a very simple geometry to get this result.

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Which is, take a cylindrical object and this is water, let us say this cylindrical object is floating, this is water, and this is air. So floating happens, when the weight downward force that is weight acting due to acceleration due to gravity on the solid object is equal to the buoyancy force. which is nothing but so let us call this height that is submerged as h , so the volume of cylinder that is submerged is h times the density of the fluid is ρ_f times g . Now m is nothing but suppose let us call this whole height as l A l times ρ_{solid} times g is nothing but A h times ρ_{fluid} times g . So, if you cancel A and g so floatingly can happen. By definition, floating means h is less than l otherwise the object will completely immerse under water.

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The image shows a whiteboard with handwritten notes. At the top, the equation $A l \rho_s g = A h \rho_f g$ is written. Below it, an arrow points to the text "Floating means $h < l$ ". Another arrow points to the text "if $h < l$ ". Below that, the equation $\rho_s l = \rho_f h$ is written, followed by an arrow pointing to a boxed equation $\rho_s < \rho_f$. Below the boxed equation, the text "necessary condition for floating." is written. At the bottom of the whiteboard, the equation $-\nabla p + \rho g = 0$ is written.

So floating means h is less than l , so if h is less than l then this equation tells you that $\rho_s l$ is $\rho_f h$, this implies that ρ_s is less than ρ_f . This is a necessary condition for floating. So, this completes the basic concepts that are that can be obtained by simple considerations of a fluid under static conditions. So just to recapitulate, we first derived the governing equation for fluid under static conditions, which was simply minus ∇p plus ρg is 0. And using this, in this lecture we derive the fundamental equation for manometers, the principle of manometry. And then we introduce a notion of atmospheric pressure and barometers.

Then we proceeded to derive the forces that are experienced by a planar surface that is submerged inside a liquid. And we found that it can be very easily obtained by integrating the pressure on a small area element. And by integrating this over the entire area we can get the force and we can also find the line of action of this resultant force on a solid surface. Next, we proceeded to discuss the notion of buoyancy on a completely submerged solid surface. And we can derive the Archimedes principles from the basic equation of hydrostatics. By realizing that, the net force downward on a on the solid surface is greater than the net force on the upper surface of the solid. So, this results to the buoyancy and we derived the Archimedes principle from the basic equation of fluid statics. And we also saw, when can an object float and what are the necessary conditions under which an object floats.

So, this completes the discussion on fluid statics, which is one of the simplest topics in fluid mechanics, because the subject of fluid mechanics deals with fluid flow, but fluid statics is an integral part, because even under static conditions, the forces that are being exerted are not simple, because of the fact that the pressure in a fluid varies with vertical distance. So, we will stop here, and we will continue with a new topic in the next lecture. So, we will see in the next lecture.