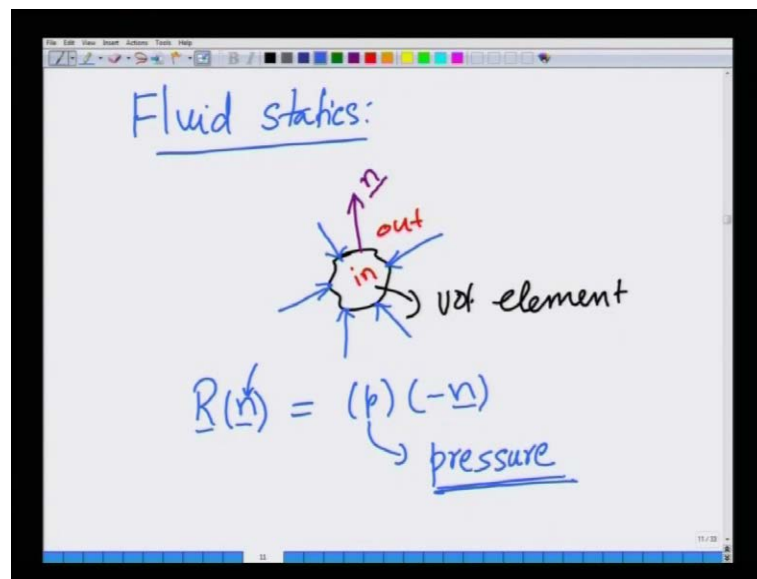


Fluid Mechanics
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Lecture No. # 08

Welcome to this lecture number 8 on this NPTEL course on fluid mechanics. For fluid mechanics; for under graduates students in chemical engineering and until the last lecture we discussed fluids under static conditions, the variation of pressure in a fluid under static conditions, the forces that are experienced by objects submerged in a fluid under static conditions and so on. Before, I move on to the new topic, I am going to quickly recapitulate the key results that we discussed so far.

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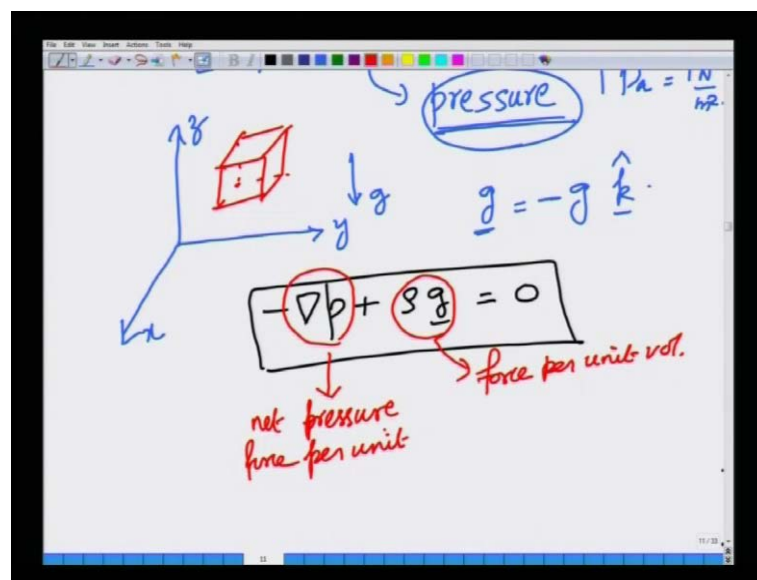


So, under fluid statics, we first saw that if a fluid is under static conditions, you should take any volume element. This is a volume element in a fluid. Then, the forces that are exerted by the fluid that is present outside on the fluid that is present inside is purely normal and it is acting in a compressive sense. So, the force that is exerted for the fluid that is present outside on the fluid is present inside is denoted by R . And if you take any particular point on this volume element, this force is a function of the unit normal. So,

since the; so, if you take any point on this surface, the unit normal point outwards that it is from inside to outside. The force on a static fluid element acts from outside to inside.

So, this force is written is written as p times minus n because minus n is the direction at which the force is acting and p is the magnitude of the force. Well, it is more appropriate and this is the magnitude of compressive force per unit area acted upon by the fluid outside on the fluid that is inside, on the surface that separates outside and inside. So, this compressive force per unit area, the magnitude of it is called the pressure in a fluid. And we saw that this pressure has units, pascals. So, 1 Pascal is 1 Newton per meter square.

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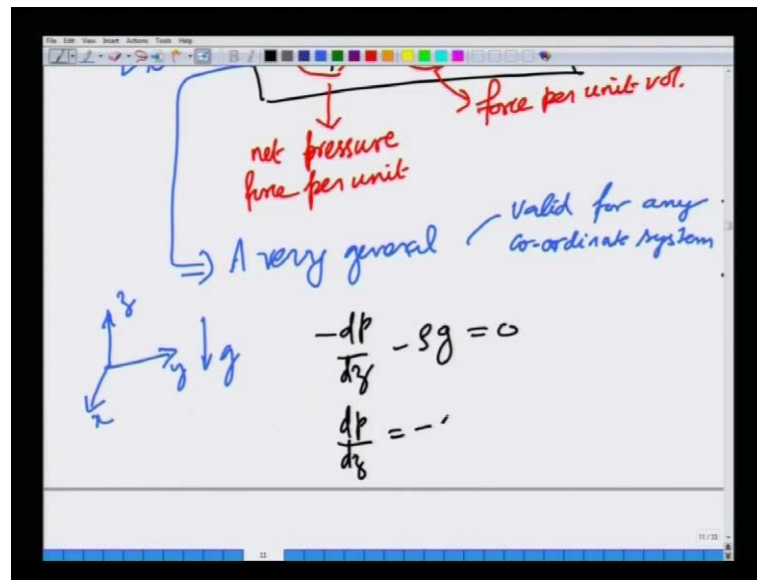


And then we said that if you have a fluid that is under the influence of a gravitational field. So, we took a coordinate system x , y and z and gravity acts in the direction of minus z direction. So, if you look at; if you want to write the gravity vector. It is minus g times k . The g is the acceleration due to gravity, it is 9.8 meter per second squared and the surface of the earth and k is the direction of the positive z axis. Minus k is the direction of negative z axis. So, g is minus, the g vector is minus g times k . So, we have a fluid that is present under the influence of a gravitational field and we said that by taking tiny volume element of a fluid. We can show that the fundamental equation of hydrostatics is minus del p plus ρg is equal to 0.

So, this is the fundamental equation for a fluid under static conditions. This the physical interpretation for this equation is as it is very simple ρg is the force per unit volume,

volume due to gravity. This is the weight of the tiny volume element and this is ρg is the net pressure force minus ρg . So, net pressure force per unit volume acting on the fluid element and since a fluid element is at rest the sum of all the forces must be 0 and that is what gives rise to this simple balance.

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Now, normally this equation, if you remember that this is very general equation in the sense that this is valid for any coordinate system. Not necessarily the rectangular coordinate system depicted in this cartoon valid for any coordinate system. So, (No Audio Time 04:47 to 04:52) but if you want to solve a problem you refer this equation with respect to this partition or a rectangular coordinates x y z and g pointing like this. So, we found that minus ρg dz minus ρg is 0 or ρg dz is minus ρg .

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$$\int dp = -\rho g \int dz$$

$$p_0 = \rho g(z_0 - z)$$

$$p - p_0 = \rho g(z_0 - z)$$

air
atm
 p_{atm}

free surface

water

$z = z_0$

$(z_0 - z) \equiv h = \text{depth of liq. from}$

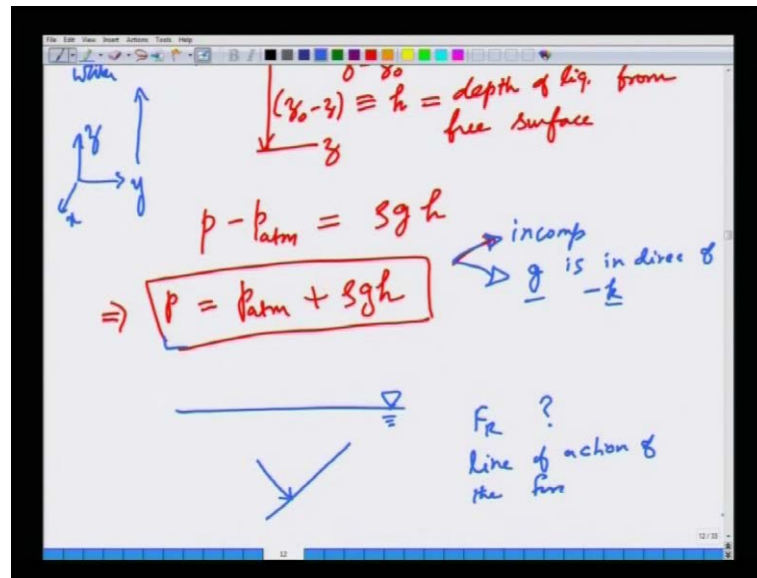
z

z
 y
 x

So, if you want to integrate this equation we have to assume certain things about density. If you assume incompressible flow, sorry, incompressible fluids for which density is independent of pressure, ρ is independent of pressure p . Then we can and since g is a constant both ρ and g are constants in this equation. So, we can easily integrate this. So, you get integral dp between any 2 points p_1 or p_0 and p is minus ρg integral z naught to z dz . So, p minus p_0 is ρg times z_0 minus z . Now, in most applications you have a pool of liquid that is water let say and that is open to an atmosphere, air at atmospheric pressure.

So, this is p atmosphere, this is the free surface that separates the liquid from air, water from air. For example, so at this point the pressure is p atmosphere, it is known. So, our coordinate system is align like this, x y z , where z is increasing in this direction. So, if you refer z naught in this equation as the free surface z equals to z naught and any point z , is this distance is z naught minus z and we can call that as h .

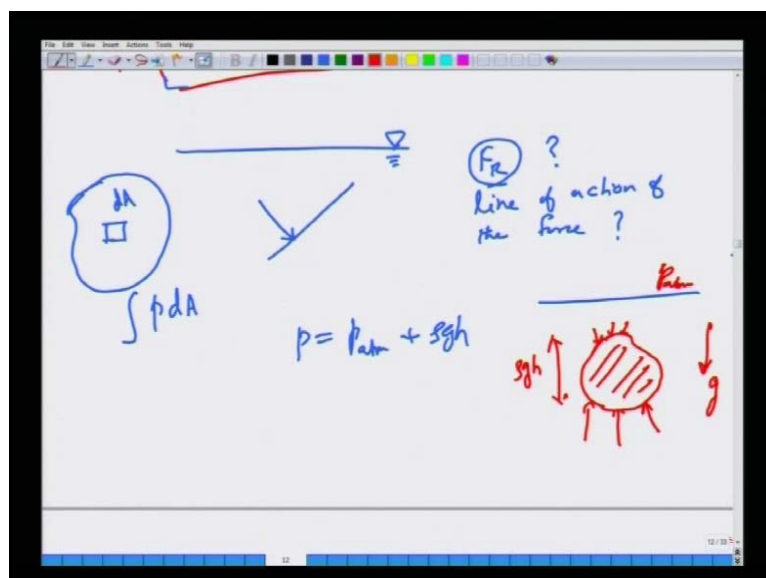
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h is the depth of the liquid from the free surface. In which case, we can rewrite this equation as $p - p_0$ atmosphere because that is the value the pressure at z equals to z_0 . $p - p_0$ atmosphere is $\rho g h$ or p is p_0 atmosphere plus $\rho g h$. This is a fundamental equation valid for incompressible fluids because we assume ρ is constant and it is valid when gravity is in the direction of minus k . So, these are the two key assumptions and then this equation is very useful in solving many problems.

One example is what we did was to show, suppose you have a free surface and then you have a solid surface, solid planar solid surface. And then we are interested in finding the force, net force, effective force on this solid surface and that force will act normally what is the magnitude of force which we call F_R and what is the line of action of the force?

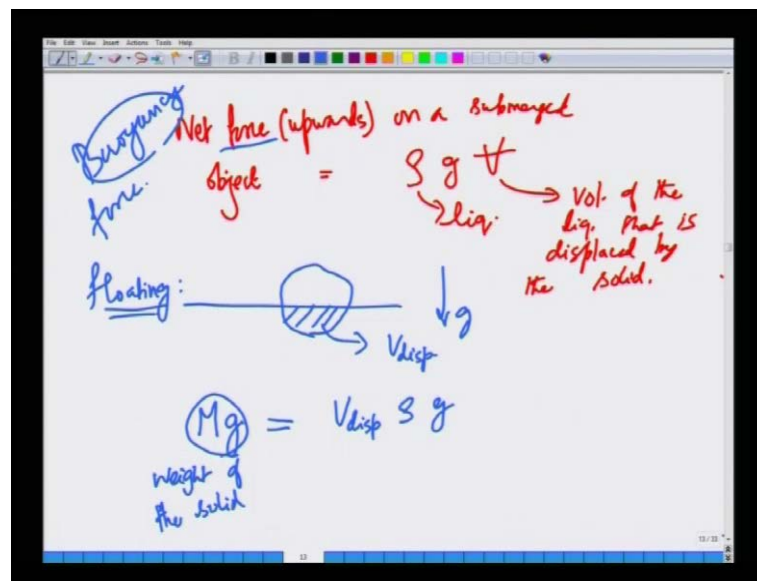
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So, both of this we calculated by simply. So, this is a cross section of a planar surface like this. So, this is the planar surface and you take a tiny area element and then we find what is the; that is called as dA . The pressure the force acting is $p dA$ and then you integrate this you get the total force FR . So, it is as simple as said and then by equating the moments we found what is the line of action the last lecture. The next application of this simple formula that p is p atmosphere plus ρgh was to calculate the force on a submerged object.

Suppose, you have a solid object that is submerged in a fluid and this is atmospheric pressure. The fact is that because gravity is acting in this direction, the force the pressure on this side will be smaller than the pressure on this side because of this column of liquid that increases the pressure by ρgh .

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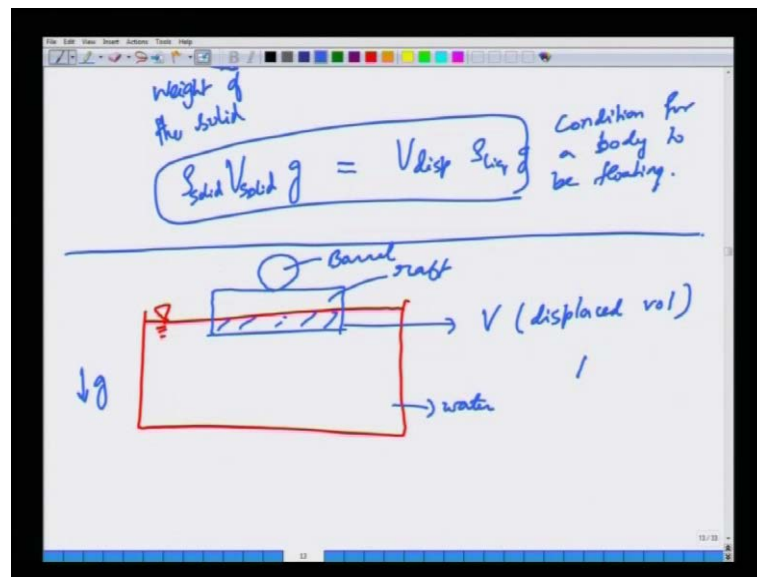


Then, we saw that by taking a tiny cylindrical volume element and we saw that the net force acts upwards because of the fact that the pressure is more than that the pressure here. And by taking many such tiny cylindrical volumes over the entire volume, we saw that the net force acting upwards, the direction upwards to the gravity vector. That is the upward direction upwards on a submerged object is simply ρ of the liquid, density of the liquid times g times the volume of the liquid that is been displaced by the solid. Here the entire volume is submerged. So, the entire volume of the solid itself volume of the

liquid that is displaced by the solid. If the object is partially submerged like here only this part is submerged, then is a floating object.

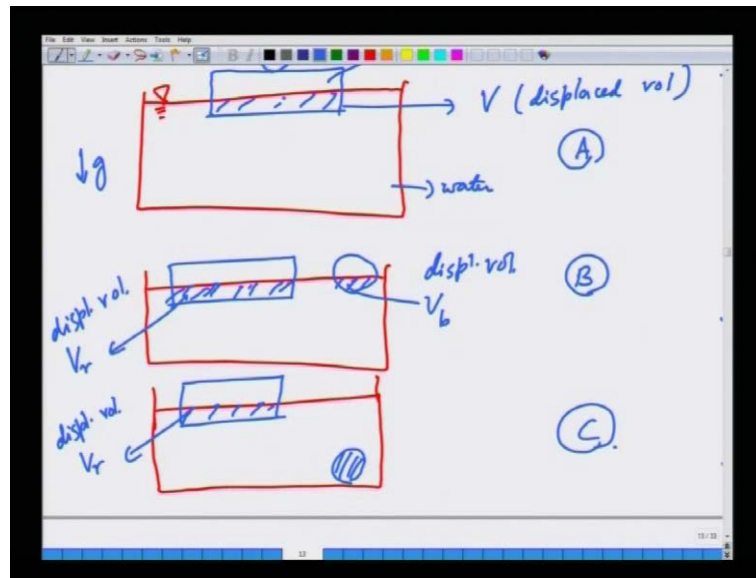
By the way this force is called the buoyancy force and this principle is Archimedes principle. So, for a floating object, the displaced volume is let us call it as $V_{\text{displaced}}$. So, if an object is floating that means that it is not completely sinking into the liquid. Gravity is acting down, if M is the mass of the solid and g is the acceleration due to gravity, Mg is the weight of the object, solid object, weight of the solid. This must be balanced by $V_{\text{displaced}} \rho_{\text{liquid}} g$.

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Now, the mass of the solid is nothing but, $\rho_{\text{solid}} V_{\text{solid}}$, the volume of the solid times g is equal to $V_{\text{displaced}} \rho_{\text{liquid}} g$. This is condition for floating, condition for a body to be at equilibrium for a body to be floating in a liquid surface. So, this is Archimedes principle. What I will do next is to illustrate this with a straightly more involved example. So, let us try to apply principle for a following problem. You have a huge you know swimming pool let us say huge body of water. This is the water level is water and you have an object let say a raft is floating and over the raft there is a barrel, a cylindrical barrel. This is a raft, this is a barrel. So, this is water so this is gravity is acting in this direction and let us say the displaced volume is V . In this case, let us call this situation case A.

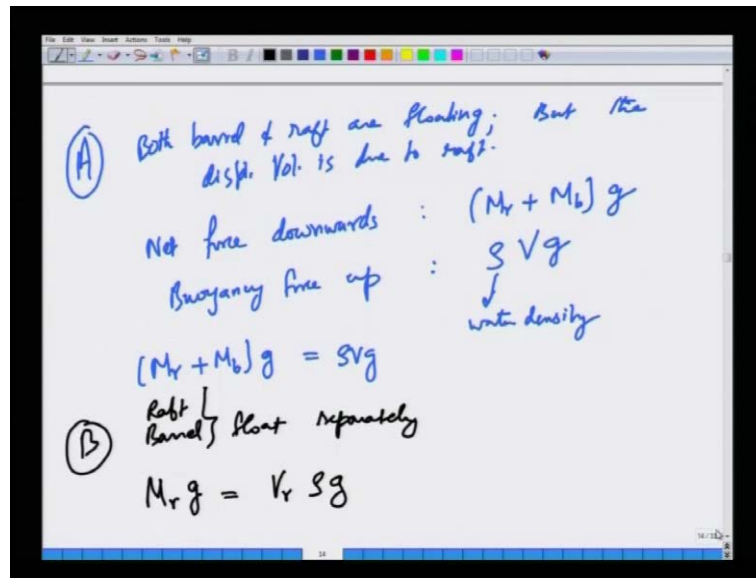
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Let us consider two other situations, a second situation is where you have same barrel and raft, but the barrel and the raft are floating separately. They are partially submerged, both are floating. So, let us call the displaced volume as V raft and this is V barrel. So, their floating separately let us call this case B. And the third a situation is like so, you have free surface of the water and then you have the barrel, but sorry, you have the raft and the barrel is such that it is completely submerged.

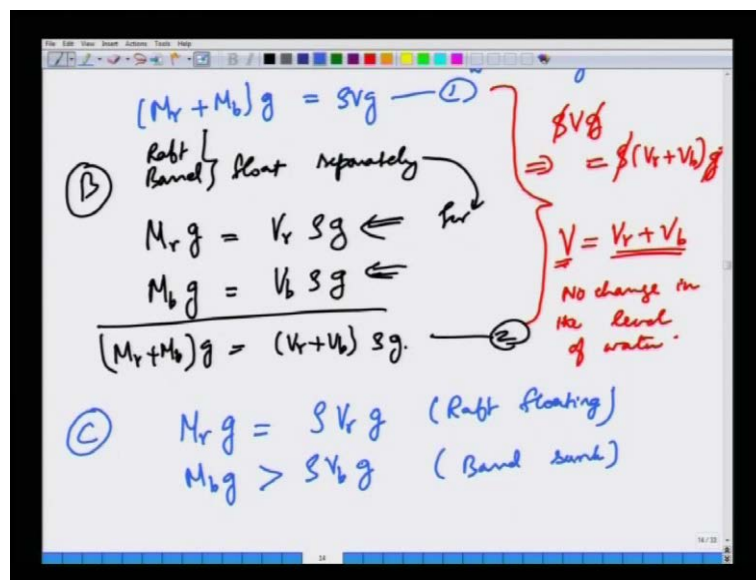
So, let us call this displaced volume. We are in the both cases of the raft. So, this is the displaced volume of the barrel, let us call this case C. Now, in comparison this three cases, what is the level of water of case B and case C with respect to the level of water in the swimming pool in case A? That is question that we are going to answer by using the Archimedes principle.

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So, let us consider case A. Here, the both in case A, in both barrel and raft are floating, but the displaced volume is only due to the raft. By using Archimedes principle, the net force downwards is given by the sum weights of the raft and the barrel times g.

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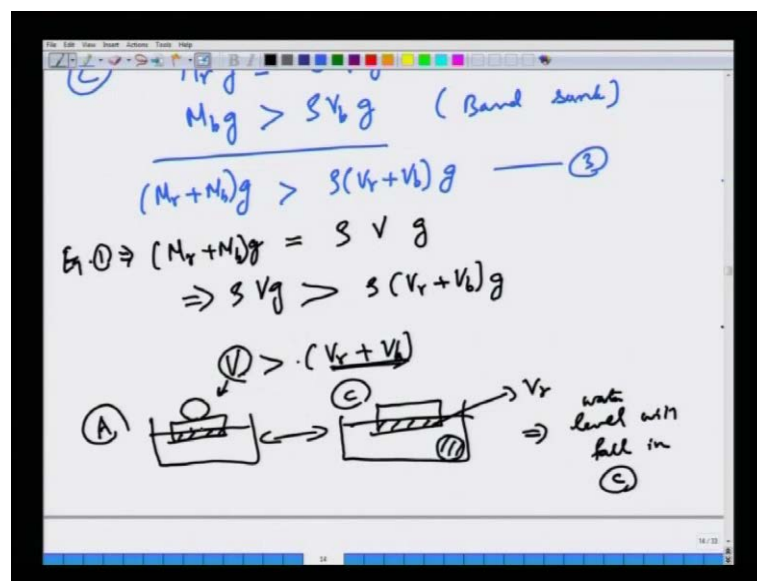
This must be balanced if this combination is to be floating, then this must be balanced by buoyancy upwards. The buoyancy force upwards which is nothing but, rho water, which I denote as rho. This is the density of water times Vr, let us call it just V that is the notation we used times g. So, condition for the floating in the first case, case A is Mr

plus $M_r g$, $M_b g$ is so $\rho V g$. In case B, barrel floats separately, the raft and barrel float separately. That means that $M_r g$ is $V_r \rho g$ and $M_b g$ is $V_b \rho g$. This two must separately hold for both these things to float separately. We can add these two equations $M_r + M_b g$ is equal to $V_r + V_b \rho g$. Let us call this equation 2, let us call this equation 1.

When I compare these two equations, this implies that $V_r M_r + M_b$ is same on the left side. So, $\rho V g$ must be equal to the right side must also be the same must be equal to $\rho V_r + V_b \rho g$. So, if I cancel ρ density of water and acceleration due to gravity. I get that V is $V_r + V_b$. That is in case B, where the barrel and raft are floating separately. Here, the net displaced volume is the same as the total displaced volume in case A. So, no change in the water height, in the level of water in the swimming pool. That is the conclusion; we can come to just by applying Archimedes principle to these two cases individually.

Let us go back to case C. Now, in case C should remember that the raft is floating. So, if the raft is floating then the net weight downwards must be balanced by buoyancy force acting upwards. ρ is a density of water, but this is raft floating condition, but the barrel as sunk. That means that the net downward weight must be greater than the buoyancy force, this is the entire volume of the barrel g .

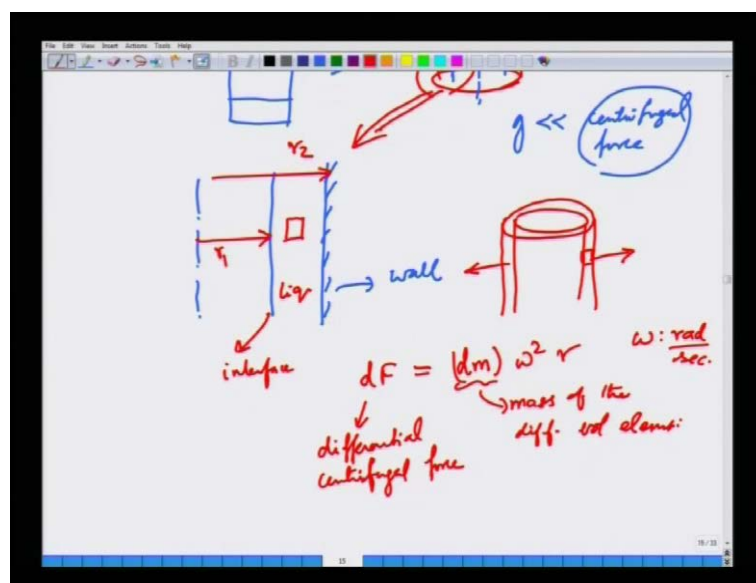
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So, if I add these two equations then I get M_r plus M_{bg} must be greater than or equal to ρV_r plus V_{bg} , let us call this equation 3. Let me rewrite equation 1, which is M_r plus M_{vg} , this is equation 1. That I just derive few minutes back is equal to ρV_g . This means if this is the case, this implies that if the left side of this is greater than this. This means that, this is identically equal to this. So, ρV_g is greater than; so, this is a g here greater than ρV_r plus V_{bg} or V is greater than V_r plus V_b . So, this is equation 1 from case 1, where this is the displaced volume when the raft and the barrel are floating in this is case A. Now, in the case C only the raft is floating. This is the displaced volume of the raft entire barrel is sunk.

So, this implies that the level will fall, this is case C water level fall in case C because displaced volume here is less than the total volume when the raft and barrel both were floating. So, compare to these two cases this water level will be smaller because a displaced volume here is small smaller compared to the displacement volume in case A. So, this is slightly counter intuitive a result because you have a situation where this barrel is completely sunk and you may intuitively or instinctively think that this water level will raise here compare to case here, but by careful application of Archimedes principle, we can show that it is not the case and fact it is opposite. Now, one last topic fluid statics, so far, I have been discussing fluids under static conditions under the influence of gravitational field.

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But in many chemical engineering applications, you encounter a fictitious body force, call the centrifugal force which happens; this force happens because of rotation. Suppose, you have a bowl of, a bowl of liquid like water and you rotate it. This whole bowl is rotated at a high speed. Let us say ω . Now, if you have liquid in it and it is not completely filled, then this liquid the centrifugal force if you remember from mechanics acts from the radius radial axis to outwards. So, this is the direction of centrifugal force. So, the centrifugal force on any fluid element tends to through the fluids towards outside.

So, you may imagine that if, the bowl is not fully filled, then this water, initially the water level will be like this. It is partially filled. Now, after rotating this water will just be thrown close to the surface and this situation will happen when gravity is very very small, a small compare to the centrifugal force. When the centrifugal force is so large, when the rotation speeds are so large, then this then this liquid will be completely thrown towards the end of the towards the rim of the container, the bowl. In this case, the entire mass of liquid will rotate like a fluid.

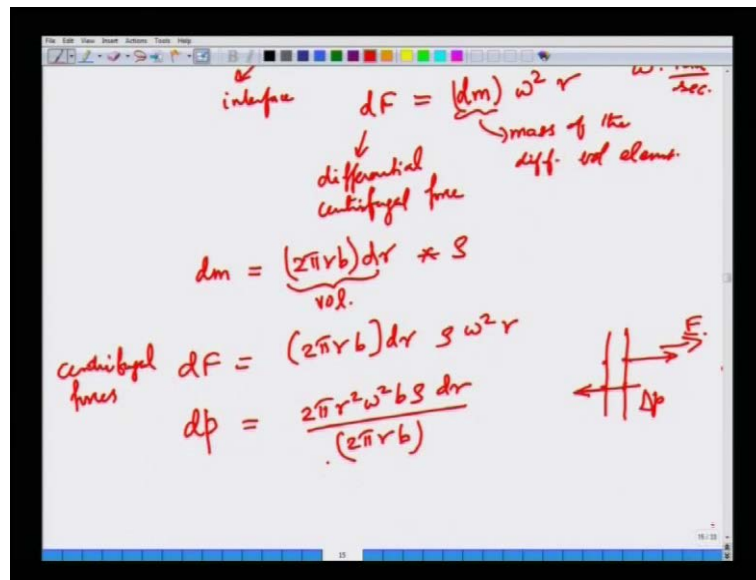
Now, imagine this is the axis and this is the wall. This is the wall of the bowl and let us call this radius as r_2 and this is the interface, this is liquid. So, I am just trying to blow this region up here and let us call the distance of radio distance of the interface from axis as r_1 . Now, the centrifugal force if you take any tiny fluid element. Well, it is actually annular, it is a annular element. So, because the geometry cylindrical. So, the fluid element will be annular, it is a ring like element. So, if you take any fluid element there is a unbalance centrifugal force acting readily outwards.

So, if the fluid is rotating like a rigid object. There is no relative motion between two fluid elements, then you can think of the entire think as so it is a solid like motion. So, it is under static conditions even though it is moving like a rigid body, there is no relative deformation. So, there is no shear stresses. So, the only stresses are very normal to the fluid elements, so the pressure. So, the pressure must be vary accordingly to balance the body force due to centrifugal forces.

So, how does one calculate this? The differential force on an element, on a volume element differential centrifugal force. Centrifugal force on a volume element is centrifugal force goes as mass times the radial distances squared times, sorry, the angular

velocity square times the radial distance, $\rho r \omega^2$. So, if you take a differential volume element. So, differential mass dm times $\omega^2 r$ is the angular velocity square times r . This is the angle ω is the angular velocity in radians per second. So, this is the mass of the volume, tiny volume of the differential volume.

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If you take a cylindrical volume element its mass is dm its volume is $2\pi r b dr$. So, this is r any at any radial distance r . You consider a slice, cylindrical slice and at this height be b or the cylindrical element. So, $2\pi r b$ is the surface area of the cylinder at in a cylinder r times dr which is a thickness. This is tiny thickness we are considering. This is the volume times the density, this is the mass this is dm . So, dF is nothing but, $2\pi r b$ times dr ρ times $\omega^2 r$. This is the differential force on a volume element due to centrifugal forces due to rotation.

Now, this must be balanced by differential pressure. So, if you take a tiny slice, a cylindrical slice there is a differential force that is acting like this. So, the pressure build up must act such that it balances; this is the pressure acting in this direction. So, pressure must tend to counteract this centrifugal force that acts in this direction. So, the pressure must balance change in pressure must balance this centrifugal force which tends to; now, let me simplify this slightly. So, it is $2\pi r^2 \omega^2 b \rho dr$. That is the force. So, the pressure is force divided by area, area is $2\pi r b$. This is the force pressure is force divided by area. So, let me cancel $2\pi r b$.

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Centrifugal force

$$dF = \rho \omega^2 r b \delta r$$

$$dp = \frac{\rho \omega^2 r b \delta r}{b \delta r}$$

$$dp = \rho \omega^2 r dr$$

$$\int_{p_1}^{p_2} dp = \rho \omega^2 \int_{r_1}^{r_2} r dr \Rightarrow \boxed{(p_2 - p_1) = \frac{\rho \omega^2}{2} (r_2^2 - r_1^2)}$$

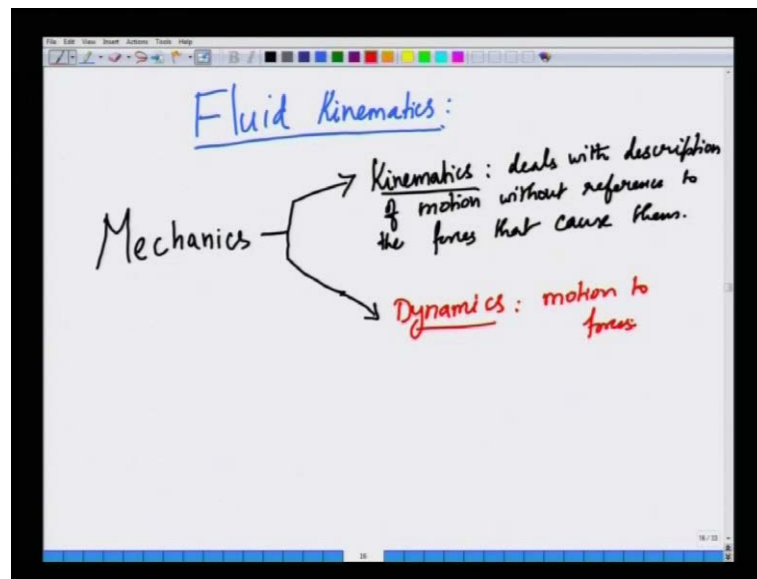
neglected g
compared to
centrifugal force

So, dp is nothing but, $2 \pi b$ then one r cancels is $\rho r \omega^2 dr$. Integral dp if I want to find the pressure distribution or difference between any two points and integrate this over the two points between p_1 and p_2 , between any two radial locations r_1 and r_2 . No longer small let us $r dr$ this implies p_2 minus p_1 is $\rho \omega^2$ by $2 r_2^2$ square for minus r_1^2 square. This is a kinetic decision. So, this is an important result where pressure variation is now, a happening in a rotating fluid where the body forces due to centrifugal forces and not due to the gravity. And this is in the limit when the centrifugal forces are large compared to the gravitational forces. So, we have neglected gravity compared to centrifugal forces.

So, just as gravity acts on objects. So, in a gravitational field centrifugal forces can also act on objects and the centrifugal force will tend to accelerate particles with higher density or element with higher density. And they will be thrown to the wall because their magnitude of the centrifugal forces are larger and these are used in many separations. So, if you want to separate two liquids, two immiscible liquids with different densities. One way is to take these two liquids and put them in a centrifuge which is basically a bowl that is rotated with very high speed. And then because of the centrifugal action the liquid of higher density will be thrown towards the wall and the liquid of a lower density will be more towards the center. And then you can simply separate these two away just based on density difference.

The same thing can be done due to gravity with the help of gravity also. That is called gravity base separation, but the driving force which is because of acceleration to gravity g is very small. It is fixed rather, but in case of centrifugal forces we can vary the centrifugal acceleration at our will by changing the angular velocity of rotation and you can achieve faster rates of separation between these two fluids. So, this a really completes my emphasis on fluid statics. So, the next topic I am going to discuss is Fluid kinematics. (No audio from 29:54 to 30:03)

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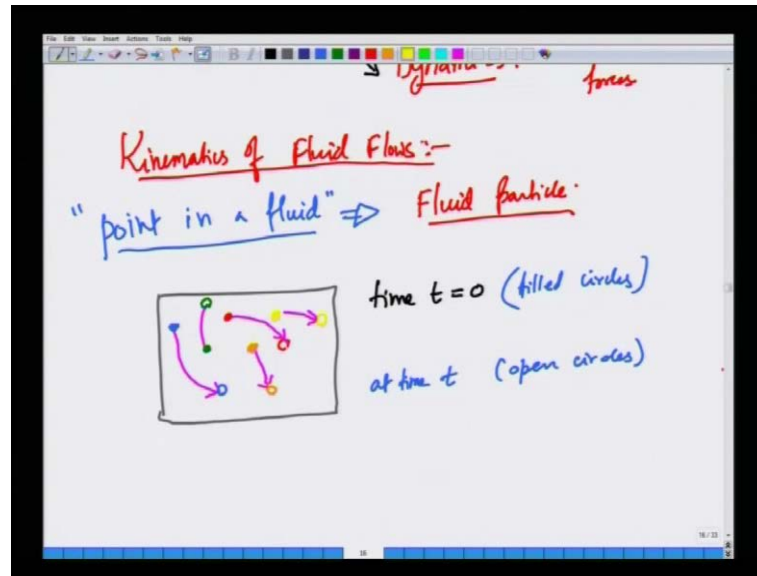


So, firstly what is kinematics? In general in any subjects; any subject that deals with a mechanics whether it is solid mechanics or fluid mechanics or particle mechanics whatever subject you have it. Mechanics deals with; as I told in the beginning forces and the motion that cause by forces. So, mechanics is broadly divided into two parts one is kinematics. Kinematics is the subject that deals with description of motion (No audio from 30:38 to 30:45) without worrying about or without reference to the forces that cause them. (No audio from 30:53 to 31:01) So, that is the first thing is to be able before understanding how forces cause motion?

The first step to first understand how to describe the motion per say the motion itself and once we have the necessary tools to describe the motion, then we can go ahead and study how forces cause motion. That subject is called dynamics. Dynamics relates motion to

forces. So, there are these are two branches of mechanics and so, firstly we will have to understand how to describe fluid motion? So, we will do kinematics of fluid flows.

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Because in this course, we are interested in motion of fluids. So, we will first discuss kinematics of fluid flows, then we will proceed to dynamics. Now, a useful, so firstly, let us try to understand how to how; what are the various options we have to describe fluid flows? Now, remember that we have already taken the continuum route or the continuum approach where in we are saying that the fluid is a continuous medium. And you can identify each and every point in the fluid and ascribe unique properties to points such as velocity, pressure, temperature, density and what not. All kinds of properties can be attributed to each and every point in the fluid and these various variables such as pressure density and so on. They are smoothly varying functions of spatial coordinates and time.

This is the essential crux of continuum hypothesis. Now, how do I, what do I mean by a point in a fluid? (No Audio Time: 32:58 to 33:04) So, in this context the notion of what is called a hypothetical fluid particle helps in the continuum picture. What is the fluid particle? Well, imagine let say you have a box containing a liquid like water and it is stationary initially. Let us say, we take a colored dye and then mark a fluid here and then take another color dye mark a fluid here. Take another color dye and mark a fluid here, take a dye of another color and mark a fluid here and so on.

Imagine that you are marking fluids with colored dye and let us assume that dye molecules do not diffuse. So, that the dye stays good I mean it does not if you drop a color. Color liquid like ink of course, you know that it is going to dissolve and diffuse in water, but let us assume the diffusivities are so small for our timescales of interest here that you can imagine that the dye does not dissolve at all. We can dye; imagine the dye to be insoluble in the liquid that we are considering. Now, this is the time initially at time t equal to 0. So, you are imagining that at each and every point that you can resolve in your scale of measurements. You can ascribe you can point a dye and you can visualize its motion.

So, this is the time t equals to 0. At a later time all these points may move. For example, this is blue color dye, let say this blue color dye element will move. So, let us use pink color for; at a later time. So, I will draw the later time picture with an open circle. So, time t equal to 0 is closed or filled circles. At a later time at time t it is open circles. That is I am going to indicate the motion of various points in the fluid and the fluid is moving due to presumably due to application of some forces. So, essentially we have a bunch of points that are marked by dye, color dye and we are assuming that the dye molecules do not diffuse and so, that they just stay put wherever they are and at time t equals to 0. When there is no force forcing on the fluid and at a later time due to application of some force the fluid is under motion and then the fluid is under motion all these points will move to some other points, some other locations.

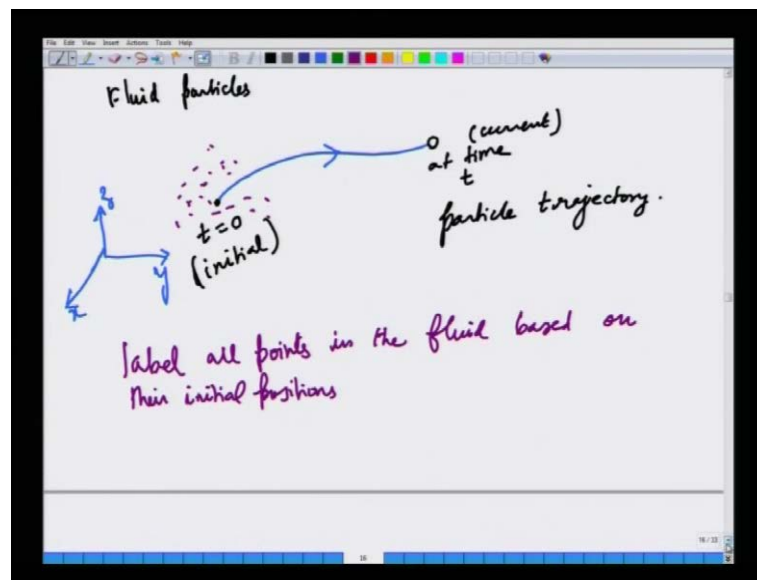
All these colored points will move to some other locations for example, this may go to here and so on. And red dot, sorry, red dot will move here. This orange dot will move here and this yellow dot will move here. So, these are the locations of these various points. Notice that I am using closed circles for initial time the location is initial time of various dye point, dye elements and I am using open circles for just for the sake of clarity exaggerating the size of these points, so that you can see them easily. So, this can happen that various points at were the initially located. At some locations denoted by closed circles we will move eventually upon fluid motion to some other locations.

Now, these can be thought of as the location or this let me just. These points can be thought of as fluid particles. Such dye parts spots which can be used to identify location of fluids at various points in the fluid at initial time and the subsequent motion can be thought of as a fluid particle. And within the continuum hypothesis a fluid element can

fluid can be compressed of infinitely a fluid is infinitely smooth. There is no discreteness in a fluid. So, you can resolve a fluid to any link scale you want. So, there are infinitely large numbers of a fluid particles correspond to each and every point in space within the continuum hypothesis.

And just by way of an illustration. I use the notion of coloring the fluid with a dye element. A dye, a substance to visualize the motion, but idea is you can think of it as a mathematical framework where at time t equals to 0. You have various locations in the fluid which are marked by their initial locations and at a later time due to application of forces all these points will start moving and they will tend to occupy various different locations in general.

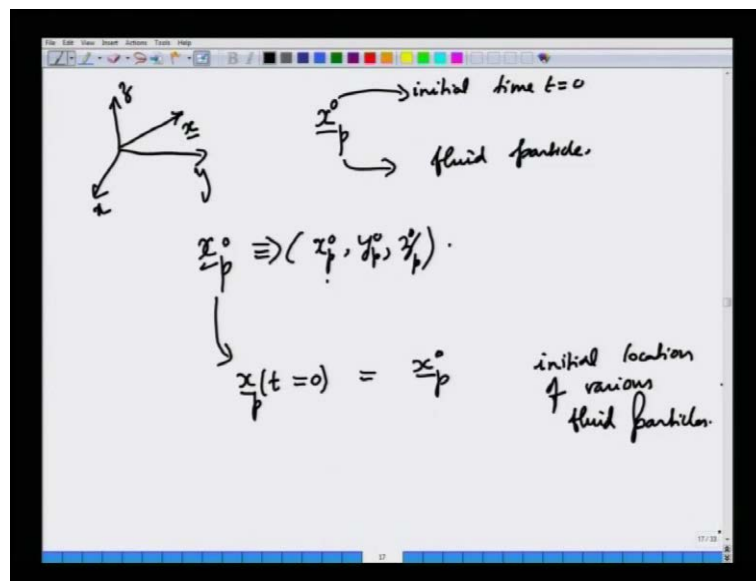
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So, this is the notion of a fluid particle. So, in general at time t you have a fluid particle which is moving at a later time. This is time t equals to 0. Initially, that is there is no motion initially and upon application of some forcing. This point moves in spatial coordinate you always have let say an x y z coordinate with respect to which we are describe in a motion that that implicit. So, just to be complete let me just put coordinate system there. So, there is a point that is located at time t equals to 0 and it is moving due to application of forces to some other location. This is the current location let us say at time t . This is what is called a particle trajectory.

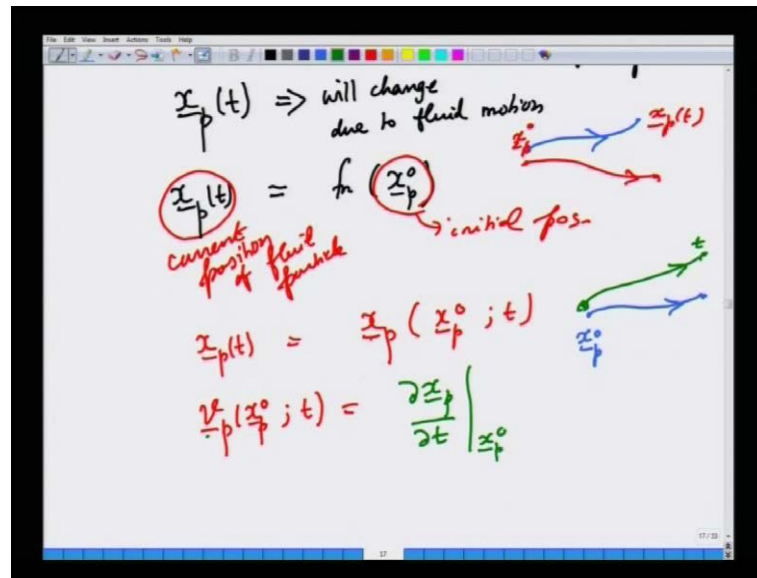
But, there are infinitely large number of such points. Let me just show it some other color. So, there are large number of such points. Because you can assign that to each and every point and space a point and all these points will start moving upon application of forces. So, one way to describe fluid motion is to consider all these points, label all these points at their initial location before application of forces. So, let us label all points in fluid based on their initial positions. (No audio from 40:38 to 40:46)

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So, let us call the initial positions of a point as \underline{x}_p^0 , if you put a coordinate system any point will be denoted by a vector \underline{x} . So, this is x , y and z coordinate system. So, \underline{x}_p^0 stands for initial time t equals to 0 and p denotes the fluid particles the subscript p denotes a fluid particle. So, \underline{x}_p^0 this a vector. So, it is comprised of three co-ordinates x_p^0 , y_p^0 , z_p^0 in a Cartesian coordinate system. So, this is nothing but, the location of the trajectory of the location of the particles position of the particle at time t equal to 0 is \underline{x}_p^0 . This is the initial location of various particles, various fluid particles.

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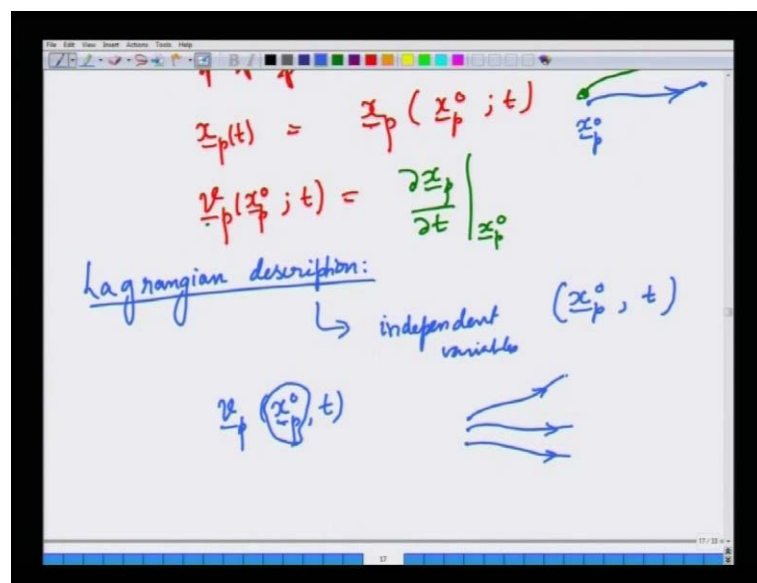


Now, upon influence of forces this the location of these particles at time t will change in general of various particles will change due to fluid motion. Now, this x_p of t the current location is a function of the initial locations. So, once I tell you what are the initial locations various particles, the current location of the various particles will be a function of the initial location. Because a point that was here will move here, a point that was here, will move to some other location. This point can never move here, unlike this point will never move here in a given set of fluid p in a specified flow. Each point which started out initially, at x_{p0} will eventually, lead reach a unique x_p at time t . So, the current position of various fluid particles fluid particles will be function of the initial positions.

Now, if I know this functional form sometimes I just people just write this as x_p of t is x_p of x_{p0} and time that is the function itself is denoted by the same symbol. Now, if I have this functional form how the current portion varies with initial portion and time then I can calculate the velocity of a fluid particle at time t . Velocity of a fluid particle x_{p0} at time t a fluid particle is identified by its location at time t equals to 0 . So, we use some other color here. At time t equal to 0 , a fluid particle is at x_{p0} and this particle moves here, some other particle which was here would move there at time t . So, each particle, this particle time t here is labeled by it is location at time equals to 0 . So, each particle is labeled by their initial locations.

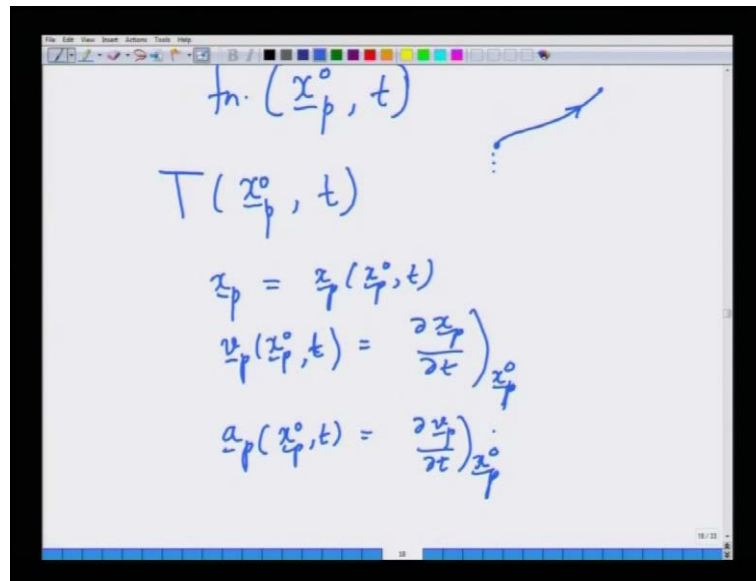
So, this is the velocity of the particle which was at time $x = p_0$ at time $t = 0$ and the velocity of that particle at time t is nothing but, the rate of change of its position dx_p by dt keeping $x = p_0$ constant. That is you are fixing the same particle, that you are following the same label in some sense and you are asking what is the velocity at time t valid is simply the rate of change of its position vector which is x_p . Now, such a description is called; such description of fluid flow where in you are identifying various particles by their locations at time $t = 0$. And merely following the positions of various particles as a function of time is called the Lagrangian description.

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It is called the Lagrangian description in fluid flow. What is the Lagrangian description? Well, here the independent variables are the position the initial position of various particles that is the label and time. So, all properties such as suppose velocity of a particle is a function of its initial position and time. So, what is this? This is the velocity of a particle which was at $x = p_0$ at time $t = 0$. At and as it particle moves at a later time what is its velocity? You are following various particles and then your enquiry what is the velocity? What is the acceleration? What is the density? What is a pressure? What is their temperature and so on?

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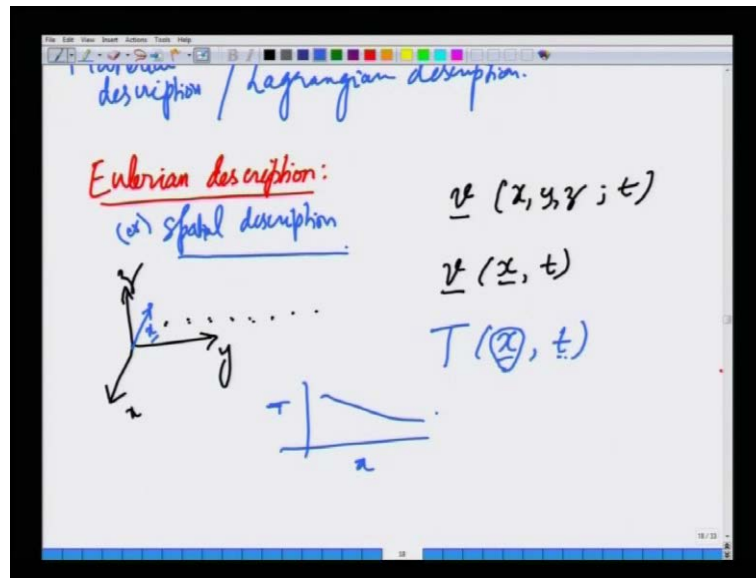


The image shows a whiteboard with handwritten mathematical expressions. At the top, there is a function $f_n(x_p^0, t)$. Below it is $T(x_p^0, t)$. The next line is $x_p = x_p(x_p^0, t)$. The velocity is given as $u_p(x_p^0, t) = \left. \frac{\partial x_p}{\partial t} \right|_{x_p^0}$. The acceleration is given as $a_p(x_p^0, t) = \left. \frac{\partial u_p}{\partial t} \right|_{x_p^0}$. There is a blue arrow pointing from the first equation towards the right.

So, independent variables in a Lagrangian description are, so, if you have any function any property it is given as a function of the initial position of the particle which is essentially serving as the label of the particle and time. Now, so not just for flow variables, even if can think of temperature in a fluid, in a moving fluid. This is a temperature of a particle which was at a time t equal to 0 at x_p^0 at a later time t . So, various particles as they move their property such as density, temperature, concentration and velocity and many other things will change, but you can describe the change based on their initial co-ordinates. This is the essential idea behind Lagrangian description.

So, once I have this motion, how the trajectory of the particle changes various particles change as function of time? Then I get velocity of a particle which was at x_p^0 at time t equal to 0. At a later time t is nothing but, the rate of change of it is position by keeping x_p^0 same that is your following the same particle. Now, acceleration of a particle is nothing but, the rate of change of its velocity keeping the same particle such a description is also called sometime as the Material description or the Lagrangian description.

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Now, this is a one of way of describing the fluid motion, but this is not the only way or this is neither is this is the most useful way. What is normally done in fluid mechanics is what is called the Eulerian description? (No audio from 49:07 to 49:15) So, what is a Eulerian description? Here, we place a lab coordinate frame in our lab x, y, z and measure various properties such as velocity in the fluid as a function of three spatial laboratory coordinates and time. So, velocity is measured not as a function of various particles not by following the particles fluid particles.

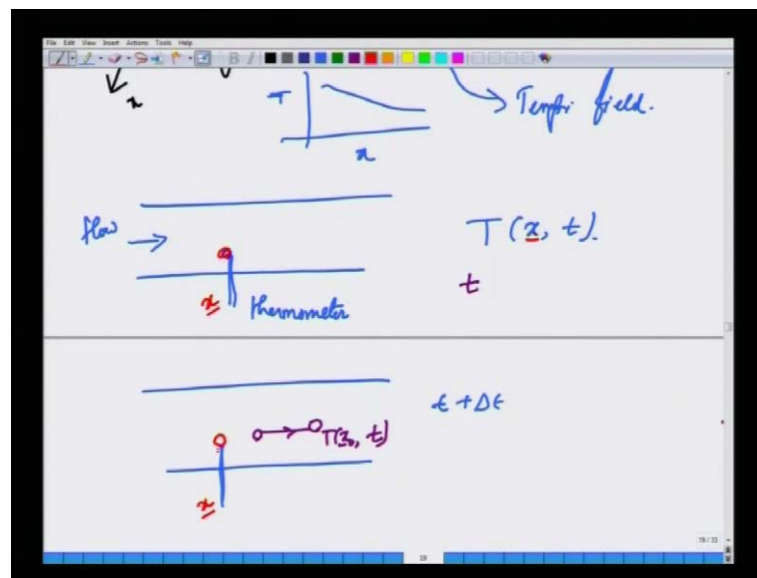
But by merely saying what is suppose I put flow measuring velocity meters at speed various points in space and then you measure velocity at each and every point as a function of time. So, that description is the Eulerian description. This is also called a Spatial description for obvious reasons because we are describing a part, a properties of the fluid such as velocity and acceleration, pressure, density, temperature as a function of by following the various particles. So, in the Lagrangian description by labeling a particle, when I say temperature of a particle, what I mean is that as I follow the particle how is the temperature changing with time. That is the Lagrangian description.

In the Eulerian description, we are not following the particle any more, fluid particles any more. We are simply keeping a stationary frame of reference, a lab frame of reference and this could be stationary or it could move with a constant velocity that depends on the nature of the problem. For simplicity, let us keep a fixed coordinate

system in our lab. And then we can measure various quantities such as velocity or temperature or pressure at various points with respect to this coordinate system as a function of time. And report quantities like how does a temperature change at various points in the fluid as a function of time.

So, what is T of x, t ? At a given location, if I fix x . So, if I have a 3 Cartesian co-ordinate system. I can fix x . At a fixed x , how does a temperature change as a function of time? Or at a given time how does a temperature vary as a function of x, y and z ? So, this is called the spatial description in fluid mechanics. Now, such quantities are called fields. This is called the velocity field, where the velocity is expressed as a function of a 3 spatial coordinates and time. This is called the temperature field.

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Now, what this description does is that suppose, I put a thermometer; suppose, I have fluid that is flowing, I put the thermometer. So, this a thermometer and I measure T at a given location let us call this x and time. What this is measuring is at a time T is measuring; whatever so, the fluid is continuously flowing. A fluid particle with will occupy this spatial location x at a time T and T of x, t will be the temperature of that fluid particle denoted by this pinkish violet circle which happens to occupy this spatial location at time t .

At a later time, t plus delta t a slightly later time at the same spatial location. This point would have moved let us say here, this point which was here at x , that are moved here

and some other point would come and occupy the location x . This is the same location x , so, let us call this vector x . So, different fluid particles the key to understand here is the different fluid particles will occupy spatial positions at the same spatial position at different times by virtue of fluid motion. Because a fluid is continuously moving or flowing. So, what the thermometer will measure at a given spatial location it is merely a record of the temperature values at that location as a function of time. And this does not correspond to the temperature of fluid particles. So, the Eulerian description gives a very very completely different view point compared to Lagrangian description. In the Lagrangian description you would follow the same particle as a function of time.

So, whereas in the Eulerian description, you are not following the same particle. You are merely sitting at a same point and space and you are recording various properties such as temperature in this particular instance. So, we are measuring different the properties of various fluid particles not the same fluid particle. So, we will stop here and will continue in the next lecture further. We will see you in the next lecture.