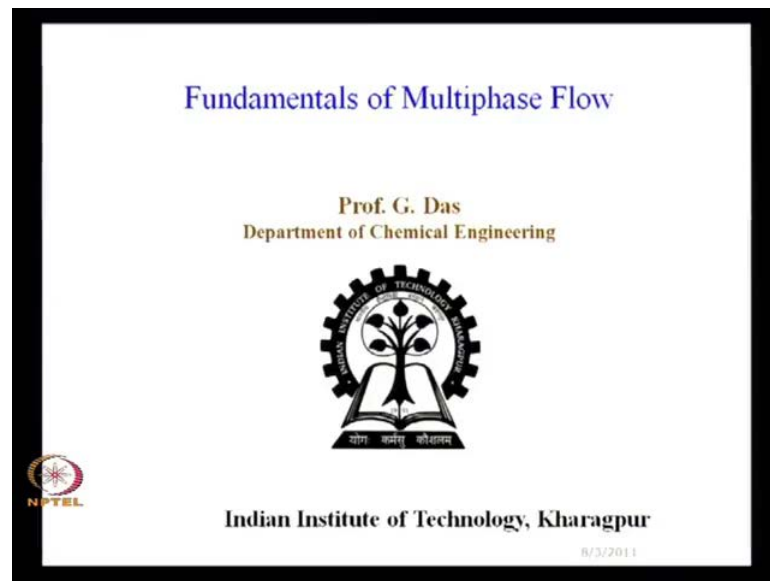


Multiphase Flow
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Module No. # 01
Lecture No. # 01
Introduction

Well, a very good morning to all of you. So, we have assembled here to study the fundamentals of multiphase flow. And, I begin with an assumption that you have already covered the basics of fluid flow at least to know, what happens when water or air flows through a particular pipe. So that now you can appreciate, what happens when both water and air are introduced in one in the same particular pipe. How the characteristics of flow change, how the hydrodynamics change? And how to quantify, and how to estimate those changes? So, these are the things that I would be covering in this particular subject of multiphase flow.

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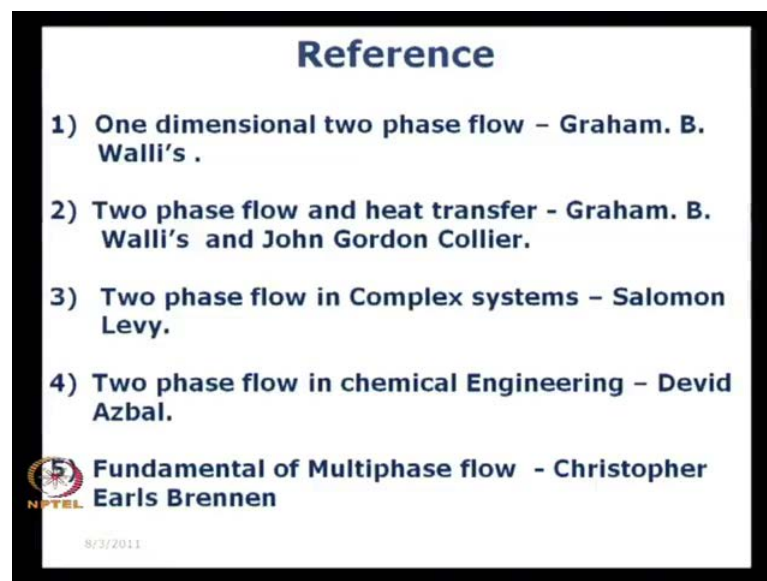


Now, before I start anything, naturally the, probably you are not very acquainted with this particular subject as you are acquainted with other common subjects like say pollution control, nanotechnology. So, these things are may be biochemical engineering. So, this is a comparatively new concept probably to you. But the first thing which I would like to tell you is, that multiphase flow it prevails in all other subjects that you

have come across like in pollution or say in the micro fluids or say in all the biochemical engineering in biotechnology. In all these things, we usually encounter two or more than two fluids or two or more than two phases flowing together. And, we hardly come across situations where only one fluid is flowing.

So, to be very honest multiphase flow is much more prevalent industrially as well as in your daily life as compare to single phase flow situations. So, in this particular class first thing which I would like to tell you is firstly, what is multiphase flow? Where do you encounter multiphase flow to be more specific? May be you know number of applications, but just to summarize the whole situation? And why do you need to study multiphase flow? How is it different from the basic fluid mechanics that you have already studied till now.

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So, when we talk of multiphase flow the reference books of course, I will be mentioning at the end of the class.

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Multiphase flow


Interacting flow of two or more phases, (solid, liquid, gas) where the **interface** between the phases is influenced by their motion

Three phase flow

- Oil+ water+ gas- Oil filled
- Sand+ water+ gas- Stowing of mine voids

Four phase flow

- Oil+ water+ gas+ solid – Oil exploration

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So, when we talk of multiphase flow, it is usually the interacting flow of two or more phases, where the interface between the phases is influenced by their motion. Now, just note certain things regarding this particular definition. When we give this particular definition, there are certain words. I do not say it is just the simultaneous flow of two or more phases. Now whenever we mean phase, it is just the phase which we have studied in thermodynamics it is that particular state of matter which is continuous throughout in chemical composition and physical structure.

So, when we talk of phases it is either a solid phase, a liquid phase or a gaseous phase. Now, remember it is just not the, simply the presence of two or more phases which guarantees multiphase flow. There are certain words here, the interacting flow of two or more phases, which means that we call it multiphase only when the phases, they interact among one another. And as a result of which the inter phase between the phases is influenced by this motion.

A very classical example which I would like to give you is, suppose you have flow through pack column, say water is flowing through a pack column or say air is flowing through a pack column. Is it multiphase? Is it two phase or is it single phase? Any idea? When water is flowing or say air is flowing through a pack bed, do we call the flow two phase or will be called the phase single phase flow situation?

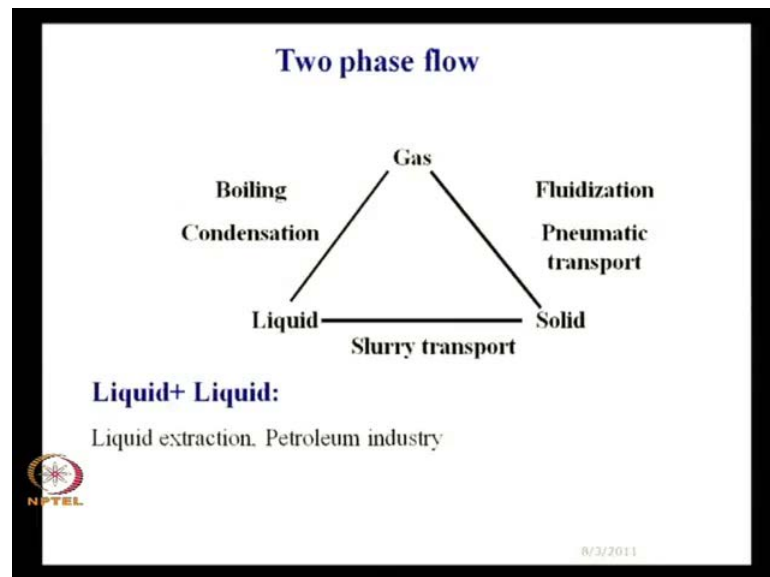
We call it a single phase flow situation because in this case, the inter phase between the solid and the air or the solid and the water does not change with the motion. And we find that the inter phase is not influenced by the motion. Simply the fluids which is flowing, it simply flows between the vertices which are available in this pact solid. Moment we keep on increasing the flow rate, a time comes when the entire thing become fluidized. Moment it becomes fluidized it becomes a two phase flow situation.

So therefore, remember one thing the simply the presence of two or more phases does not guarantee multiphase flow situation. It is that particular situation, where two or more phases they interact, the place of interaction is obviously the inter phase between them.

So, we call that particular situation multiphase, where the two or more phases they interact at the interface and as a result of this interaction the interface that changes. And that takes up a large number of forms and accordingly the study of multiphase proceeds by studying the different forms which the interface can take.

Now, we have solid phase, liquid phase, gas phase. So, from these combinations we can have either a two phase flow situation. It can be a solid, liquid or a gas.

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It can be either a solid or a solid liquid or a liquid gas or a liquid solid situation. It can also be a liquid liquid situation also, where two immiscible liquids are flowing.

We can have three phase flow situations also. For example, oil water gas phase flow in petroleum industries, sand water gas flows in the stowing of mine voids. We can also have a four phase flow even. Oil water gas solid, that is that we can encounter in oil explorations.

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Among the different multiphase flow situations, most common -

- **Two phase flow**

Interacting flow of two phases, (solid, liquid, gas) where the **interface** between the phases is influenced by their motion

Among different two-phase flows

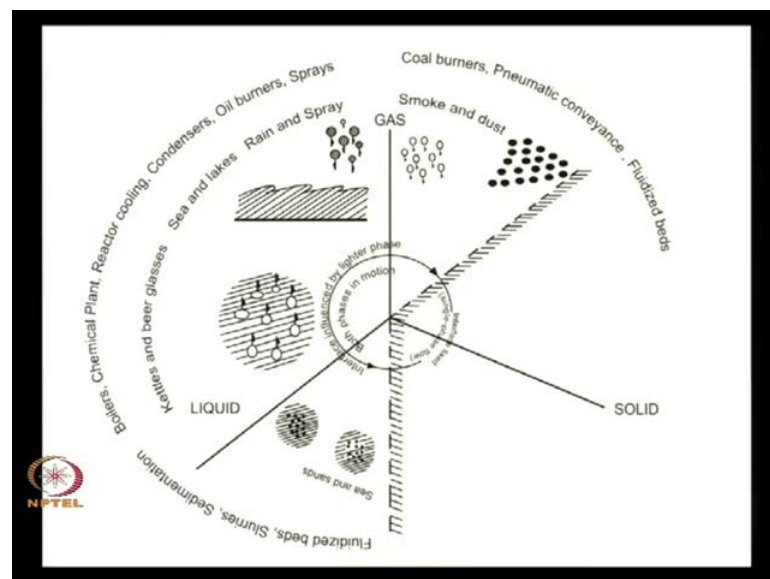
- Gas-liquid and vapor-liquid flows -commonly encountered



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But, the most common feature which we find is the two phase flow situations, where it can either be a gas liquid, liquid solid, solid gas flows or it can be a liquid liquid flows as well.

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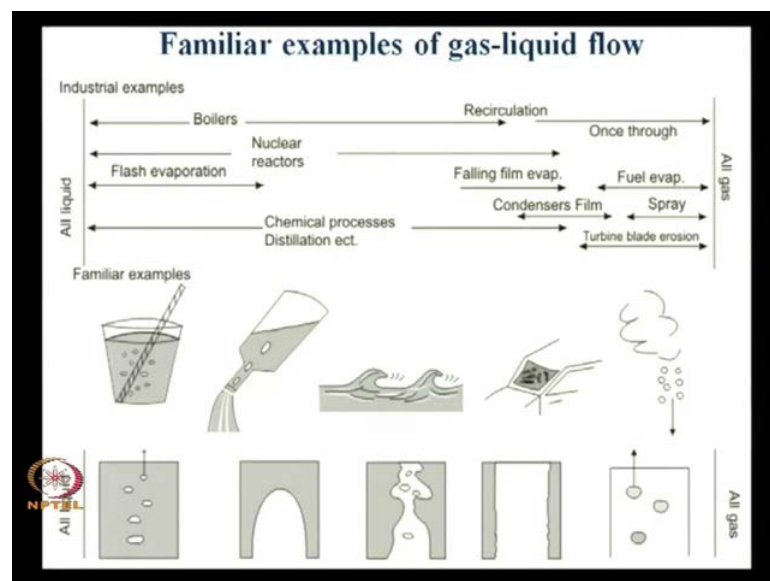


Now, whenever we talk of such flows this particular diagram it shows you, the entire range of two phase flow situation. We find that we can have a two phase flow situation for the entire range from gas to liquid. That means, from very low gas large amount of liquid to very high gas and to small amount of liquid over the entire range, we can have the two phase flow situation.

On the contrary for solid, liquid and gas solid phases we find, that more or less when the solid packing becomes much more dense, may be more than 50 percent solid loading can we have. Then under that condition the interface is not influenced by the motion of the phases and we have single phase flow situations. So, for liquid, solid and gas solid cases may be we have a two phase flow situation which is bounded by this particular line and this particular line over a limited range of flow conditions. And for the rest of the situation last just like the pact column which I has said, we have single phase flow situation.

So, there is limited two phase flow for gas solid and liquid solid cases file. For the case of gas liquid cases we have it over the entire range the of composition starting from very low liquid very high gas to very high liquid very low gas. Because in and this particular flows you will find, that this is most widely studied probably it is quite complex in the sense.

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Why it is very complex? Because in this case there are two things. Both of them they do not have a particular shape. Just like solid they have a particular shape, but gases and liquids none of them they have a particular shape. And therefore, they are characterized by a deformable interface combined with the compressibility of the gas phase.

So therefore, what happens in this case? The interface is deformed and the other case is, the gas is a compressible fluid. So, naturally some portion of compressible fluid flow also comes into picture under such circumstances.

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Most common occurrence in industry
- Two-phase flow

Types of two-phase flow

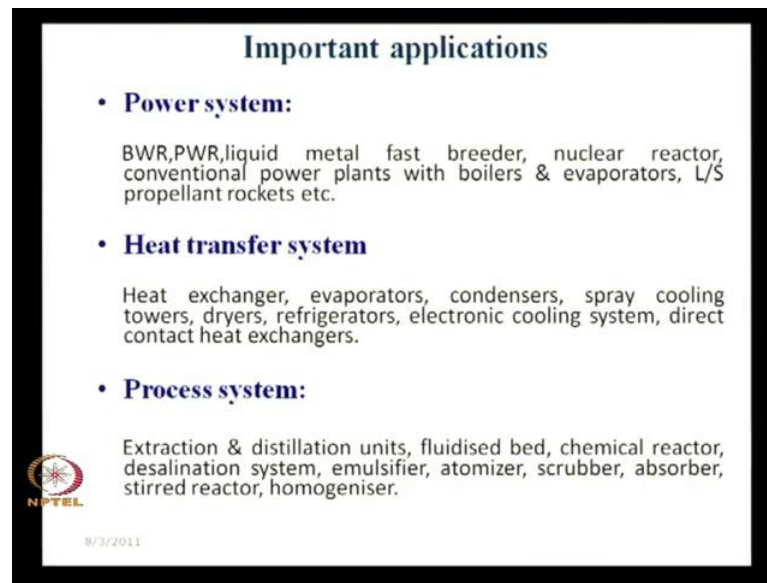
- a) Gas – liquid flow – boiling, condensation as well as adiabatic flow. common in power and process industries, refrigeration, air-conditioning and cryogenic applications.
- b) Gas–solid flow – pneumatic conveying, combustion of pulverized fuel, flow in a cyclone separators
- c) Liquid–solid flow – slurry transportation, food processing as well as in various processes in biotechnology.
- d) Liquid–liquid flow – petroleum industries, liquid-liquid extraction and chemical reactors

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Therefore, gas liquid flows they are slightly more complex as compare to or rather may be they have a little more variety I should say, as compare to gas solid liquid solid flows. And liquid liquid flows, it is a comparatively much more recent topic. And this has come up with the advent of say petroleum industries, liquid liquid extraction, chemical reactors etcetera.


We find that the other things the other types of flow. Gas liquid flows, they are as you know these applications you already know boiling condensation, adiabatic flow, whenever they are in power plants we have such type of flows. Gas solid it is a pneumatic conveying, combustion of pulverized fuel, flow in a cyclone separators etcetera. Liquid solid slurry transportation at least chemical engineers have come across it I believe, food processing various processes of biotechnologies. Biotechnologies there in gas liquid, liquid solid in all such cases. Even for liquid liquid cases also.

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Important applications

- **Power system:**
BWR,PWR,liquid metal fast breeder, nuclear reactor, conventional power plants with boilers & evaporators, L/S propellant rockets etc.
- **Heat transfer system**
Heat exchanger, evaporators, condensers, spray cooling towers, dryers, refrigerators, electronic cooling system, direct contact heat exchangers.
- **Process system:**
Extraction & distillation units, fluidised bed, chemical reactor, desalination system, emulsifier, atomizer, scrubber, absorber, stirred reactor, homogeniser.


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Now, in a nutshell if you, I would like to show you the practical applications of different two phase or multiphase flow situations. Now, the idea of showing you is just to show you in the variety of industrial cases in which, we encounter such type of flows. We find it in power systems. Large number of examples I have given and heat transfer system, heat exchanges, evaporators, condensers, spray cooling towers, dryers and so on and so. Process systems almost everywhere you find two phase flows in, whether it is a distillation column, whether it is liquid liquid extraction, emulsifier, atomizers, scrubber, in everywhere we find process in different for different process systems. We find that several applications of multiphase flow or other multiphase flow occurs in a large number of applications.

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Important applications – contd...

- **Transport systems**
Air lift pump, pipeline transport of gas & oil mixtures, pipeline transport of slurries, pneumatic conveyors etc.
- **Information system**
Super fluidity of liquid He conducting/charged liquid film.
- **Lubrication system**
Two phase flow lubrication, bearing cooling by cryogenics
- **Environmental control**
Air conditioner, Refrigerator, cooler, Dust collector, sewage treatment plant, Air pollution control


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Transport systems definitely in air lift pipe, pipeline transportation very common in oil explorations, pneumatic conveying, pipeline transport of slurries we find it. Information system also super fluidity of liquid helium and the charged liquid films due to this two phase flow occurs. lubrication system definitely, environmental control as I have said in air pollution equipments, scrubbers, cyclone separators, most of the cases you find two phase flow situations to be there

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Important applications – contd..

- **Geo-Meteorological phenomena**
Ocean waves transport by wind & motion of rain droplets
- **Biological systems**
Cardiovascular system, respiratory system, Bronchus flow, Nasal cavity flow.

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Geo-meteorological phenomena is there and of course, if you see the biological systems your blood, your body fluids almost nothing is a single phase. Most of them are either emulsion or suspension or something on that sort.

When you look at the range of application you find, at one end you have air water or steam water flows. At the other end you have say suspended solid flows, you have slurries and all those things. So, it might occur to you that it is too vast and how can one particular subject or one particular principle govern all this type of flow phenomena? Now we will find that in all these types of flow phenomena what we find is, that certain initially what was the attempt initially the thing which was done is, that each and every system was taken up. And the approach was essentially empirical in nature.

So therefore, the study was primarily empirical and design equation was suggested or operating condition was suggested based on the system. So, people use to think that whatever is applicable for biological fluids, will definitely not be applicable for oil industries. So, therefore it was of a limited generality. There was another thing also. See, may be in the the early 21st centuries also, under that condition if we take the power plants, we find that in there mostly the designs were empirical. very low efficiency was there and a large amount of power rather large amount of fuel loss was there. And that time the performance probably was governed by the material constants. The material was not very well developed, so the boiler material etcetera. But the limiting constraints to decide how well the boiler is going to perform.

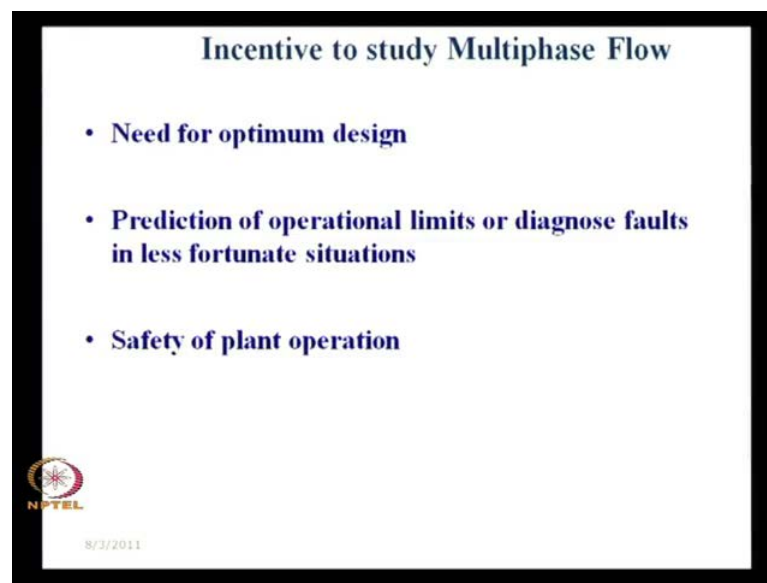
Now, what has happened? Now we find that under the present circumstances, meteorology has improved a lot lot of new materials have come up. And then the increase pressure for much better performance, more reliable performance that is number one. Number two, now safety has become very important. Now if safety has to become important, you need to know number of things, you need to know the safe operating limits, under which limit you are suppose to operate. And where you should not, in which particular territories you should not invade.

Then in that case, you have to know the limit of operations. Under first what range of conditions you should be operating? And in a less fortunate situation if there are faults, how to diagnose the faults? In case you have to know these things, you need a good amount of data on the hydrodynamics and the heat transfer characteristics of the two

phase flow situation or the multiphase flow situation, which is undergoing under that particular condition. Then you need to have, then in that case you need an optimum design. Now in order to have an optimum design, you need to know the proper design equations which will again depend upon the fluid flow characteristics. If there is a heat transfer, it will depend upon the heat transfer calculation. You need data on say two phase pressure drop, you need data on heat transfer coefficient for multiphase flow situations.

So, we find that now and there are certain other things. Now, probably we go for larger size scale of plants. Whenever we go for larger scale, we need much more accurate data now. And, on the other contrary we have come to micro fluids. So, for flow through micro channels as well as flow through a larger scale, for all these things the physics slightly becomes different. So, unless we understand the physics of multiphase flow, it becomes very difficult to go into areas which have not been studied previously or which were not very prevalent in earlier times.

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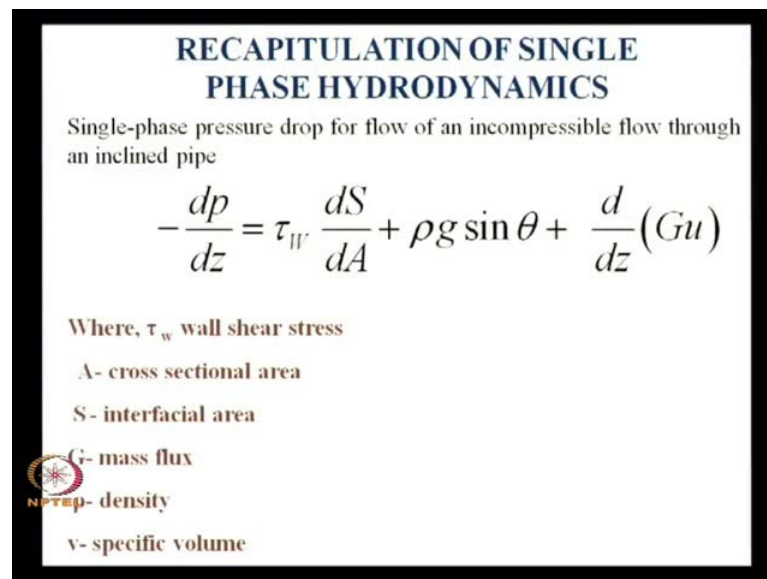


So, now therefore, we find that the incentives to study multiphase flow, that comes from three points of view. Firstly, it is the designer's point of view, next it is the operator's point of view and finally, it is the safety. From designer's point of view as I have already mentioned to optimize the design, to produce product which is much more competitive in today's market. We need a quantitative or rather detailed quantitative study of say heat transfer coefficient two phase pressure drops and things like that. For the operators, they

need to know the optimum operating condition. And in a less fortunate situation they need to diagnose faults which has already occurred due to departure from the ideal condition.

So, what we need to do? We need to perform some experimentally investigation in a particular geometry or we need to find out the two phase flow regimes which are prevailing. For example, say suppose a fault has occurred because the flow has become stratified. So, if you just change the flow condition we will see shortly and if you can break this stratification, probably we can have a problem free operation. So, those things we need to know. Next obviously, now safety has become very important. So, for these particular things, the incentive to study multi, the hydrodynamics or multiphase flow, the heat transfer characteristics that has become very important in modern times.

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RECAPITULATION OF SINGLE PHASE HYDRODYNAMICS

Single-phase pressure drop for flow of an incompressible flow through an inclined pipe

$$-\frac{dp}{dz} = \tau_w \frac{dS}{dA} + \rho g \sin \theta + \frac{d}{dz}(Gu)$$

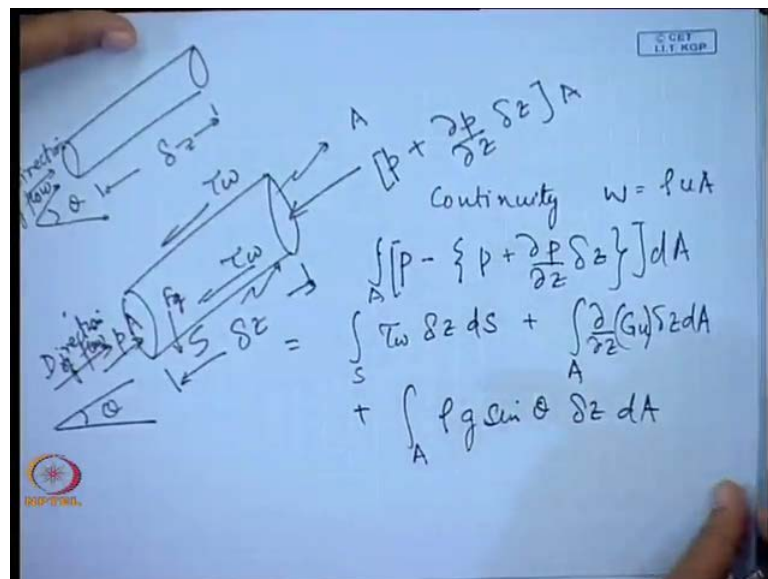
Where, τ_w wall shear stress
A- cross sectional area
S- interfacial area
G- mass flux
 ρ - density
v- specific volume

Now, if we have to study, well this I will come later. Now if we have to study multiphase flow then the first thing is you know. Now, what is multiphase flow? You find out that it is simply the flow, whether it is interacting flow or its non interacting flow. It is basically fluid flow. so therefore,our analysis should start from the basic equations which governed single phase flow in different conduits. We should start from there itself. Now for analyzing such fluid flow what do we do? We write down the equation of continuity equation of momentum, equation of energy. Then we have several constitutive relations, boundary conditions, certain constraints, we apply those and try to solve these equations.

Definitely for multiphase flow, we should approach in the same particular way and we should first write down the equation of continuity momentum energy. And then you would like to solve them, may be the constitutive relation will be slightly difficult or different for this particular case. Why? Because in single phase flow, the fluid is interacting only with the pipe wall. In this particular case, what happens? With two fluids they interact separately with the pipe wall, but at the same time they interact with one another as well.

So, let us first derive the momentum equation, the equation of momentum, the equation of continuity for single phase fluid. And then, we will see how that equation, just will be taking up case by case and doing it much more details later. Just at first glance, how that equation should change for multiphase flow? And in order to use the changed equation, what are the things that we require?

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So, suppose we take up simply say water flowing through a close conduit. See, it is inclined at an angle theta with the horizontal.

Now, in this particular case we say, this is the direction of flow. Now, this particular length it is a small length, say delta z. Now in this particular case, I think I should draw slightly bigger one that will be easier for you to understand. This is delta z, say the cross sectional area is say A and your interfacial area it is say S, the interfacial area. So, the direction of flow is this. If this is the direction of flow, then in that case the pressure

force will be applied, here its $P A$ here and in this particular case it is going to be P plus. Any doubts anywhere, you can just stop me and you can ask me. This becomes area.

Wall sheer stress, this you already knew it is just a recapitulation for your convenience. And the definitely there is a gravity force also, which is acting in this particular direction say f_g is the gravitational force acting. Interfacial area is s , s we have taken as the interfacial, it is just the waited area the perimeter in this particular case. It is just the perimeter, the area in which the sheer stress applies. Where does the sheer happen? The sheer happens at the at the wall. So therefore, where does sheer happen, it happens around the entire circumference of the wall. So therefore, the sheer stress multiplied by the entire circumferential area that is the, the waited perimeter area in this particular case.

Now, for this particular case suppose, we write the equation of continuity. So, this gives you w equals to $\rho u A$. ρ is the density of the fluid, u is the velocity and A is the cross sectional area. Now, if you strike a momentum balance or other force balance in this particular case, what we get? We get $\int p \text{ minus } p \text{ plus } \frac{dp}{dz} \Delta z$. If we assume that the area is not constant, then we take it as dA this is equal to \int over s this was \int over area. So, this is $\tau_w \Delta z d s$ plus \int over area $\frac{dp}{dz} \Delta z$ of $g u \Delta z d A$ plus \int over area $\rho g \sin \theta \Delta z d A$. So, we find that in this particular case we are having the pressure force, we are having the wall sheer stress or the sheer stress forces. And we have the gravitational forces and as a resultant of this, we find this is the rate of change of moment of the fluid ok.

So, taking the directions into consideration, assuming that direction of flow is the positive direction, we can write it down a this particular manner.

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for one dimensional assumption

$$[P - \{P + \frac{\partial P}{\partial z} \delta z\}]A = \tau_w S \delta z + \frac{d}{dz} (G_u) \delta z A + A \rho g \sin \theta \delta z$$

~~$- A \frac{dp}{dz} = \tau_w S$~~

Dividing throughout by δz and taking the limit as $\delta z \rightarrow 0$

$$- A \frac{dp}{dz} = \tau_w S + A \frac{d}{dz} (G_u) + A \rho g \sin \theta$$

$W = \text{Mass flow rate} \quad G = \frac{W}{A} = \text{Mass flux}$

Now, for one dimensional analysis, suppose usually we have one dimensional assumptions. So, for one dimensional assumption, how does the equation change? Under this condition, we find that all the things will change along the flow direction and normal to the flow direction it is constant. So, in this case we get p minus for one dimensional. For one directional assumption not dimensional, one directional unidirectional assumption we get p plus $\frac{\partial p}{\partial z} \delta z$ into A . This will be $\tau_w S \delta z$ plus $\frac{d}{dz} (G_u) \delta z$ into A plus $\rho g \sin \theta \delta z$ into A .

So, this is the thing which we get. Now in this particular case, we find that dividing throughout by or rather if we just solve it, we get minus $A \frac{dp}{dz}$. This will be equal to $\tau_w S$ dividing throughout by δz and taking the limit as δz tends to zero. Should I write it down? Dividing throughout by δz , these things you already know. You have done these things several times, but anyhow I am taking the limit as δz tends to zero, we get minus $A \frac{dp}{dz}$. This is equal to $\tau_w S$ plus $A \frac{d}{dz} (G_u)$ plus $A \rho g \sin \theta$. This is equal to $\tau_w S$ plus $A \frac{d}{dz} (G_u)$ plus $A \rho g \sin \theta$. Any doubts you can get it clarified from me plus [FL]. One thing I will just mention. This particular term probably you have not come across in single phase flow it is basically the mass flux.

We will be using it quite frequently in this particular case, it is just if we take the mass flow rate as w . Usually we are used to take it as \dot{m} , but in our case I will prefer to take it as w . Why? You will understand later. Then in that case, G equals to the, it is the

mass flux of that particular fluid. So, here instead of w by A I have taken it as g , in this particular case.

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The whiteboard shows the following derivation:

$$dp = \tau w \frac{s}{A} + \frac{d}{dz} (G u) + \rho g \sin \theta$$

$$= \left(-\frac{dp}{dz} \right) F + \left(-\frac{dp}{dz} \right) a_c + \left(-\frac{dp}{dz} \right) g$$

$$\left(-\frac{dp}{dz} \right) a_c = \frac{d}{dz} (G u) \quad u = \frac{G}{\rho} = \frac{w}{\rho A}$$

$$= G W \frac{d}{dz} \left(\frac{1}{\rho A} \right) = G W \frac{d}{dz} \left(\frac{u}{A} \right)$$

$$= \frac{G W}{A} \frac{du}{dz} - \frac{G W u}{A} \frac{dA}{dz}$$

$$= G^2 \frac{du}{dz} - \frac{G^2}{A} u \frac{dA}{dz}$$

So therefore, we find that dividing throughout by A , we find the expression of pressure gradient which we get, that is minus $d p d z$. That is equal to $\tau w s$ by A plus $d d z g u$ plus $\rho g \sin \theta$. Agree? This particular definition probably you have come across earlier as well.

So, this is the thing which I have already written it down here. So, there we find that more or less, whenever you have been solving you have found out that horizontal pipe, just this particular term which you call as the frictional pressure gradient was prevalent. When we go to vertical pipes, we find along with frictional pressure gradient, we have a gravitational pressure gradient. Is it not? So, usually it is since for vertical pipes are gravity, gravitational pressure gradient is much less as compare to, the frictional pressure gradient is much less as compare to gravity gravitational component. So, usually for vertical pipes we just use to say that minus $d p d z$ equals to $\rho g \sin \theta$.

But, definitely there is a frictional component as well in under that condition. What probably you have not come across much is, this particular pressure gradient. Any idea of what this particular pressure gradient it signifies? Change in velocity and mass flux. So, it is the acceleration pressure gradient. So, we find that the pressure gradient it comprises basically of three components. It is the frictional pressure gradient and the

acceleration pressure gradient as well as the gravitational pressure gradient. Certain things are important under certain conditions. For example, horizontal pipes this one goes away in vertical pipes usually this one, this one is much more important and. So, on and so.

Now, acceleration pressure gradient under normal circumstances the fluid does not accelerate we do not get it. We get acceleration pressure gradient under certain conditions. Any idea under what conditions the acceleration pressure gradient is encountered?

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Very correct. In case of nozzles and diffusers we can have acceleration pressure gradient. Why? What accelerates the rather, what causes the acceleration pressure gradient for the case of nozzles and diffusers? The change in cross sectional area is definitely one. Under any other circumstance, we can have this particular pressure gradient. Other than that any other, under any other condition we can have it? We can have it. When? Say there is a when we have compressible flows, when the density is changing with length or when there is a change of phase.

Whenever there is something which brings about acceleration, we have the acceleration pressure gradient. So therefore, usually this particular component the acceleration pressure gradient this is much more important for compressible flows. So, for compressible flows, what is the situation? We will derive it and then I will show you the actual derivation. So, we find that normally under normal circumstances we do not have this pressures gradient. probably you have, I do not think you have come across this in your under graduate classes. But definitely it should have been there and then you should have cut it out. Unless there is a change area or a change in phase or flow is compressible under, without if these things are not there, then usually this is cut out.

So, this should be there. Now this pressure gradient is nothing but $\frac{d}{dz} \rho g u$. Where we find, what is this u ? This u is nothing but g by ρ or in other words this u is nothing but w by ρA . To make it more general to include all sorts of variations. So, we find that the total mass or at the mass flow rate is remains constant what can change? In order to bring about a change in velocity, either the density can change or the cross sectional area can change. So therefore, this can be written down as $\frac{d}{dz} \rho w$ can be taken out one by

ρA or in other words this can be written as $d\tau$ of ρ is density. So, what is the reciprocal of density? Specific volume.

So therefore, it can be written down as $d\tau$ of v by A . Now this can next be written down as so therefore, it is $g w$ by A . If you take up the entire thing if, I have to take out g out. This is all right and so therefore, if we take out g here and if we deal just with u . So, then it becomes $g w$ by $A d v d z$ minus $g w v$ by square $d A d z$ which can then be written down as $g^2 v d z$ minus $g^2 v$ by $A d A d z$. Is it not? Now tell me one thing, $d A d z$ is fine you know the the the change in area whether change in distance and you can find it out.

Now, why does the specific volume change with distance? Why does this $d v d z$ happen? This $d v d z$ happens just because the specific volume or the density changes with the axial distance z , is it not?

So, therefore, instead of this $d v d z$.

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$$\frac{du}{dz} = \left(\frac{du}{dp}\right) \left(\frac{dp}{dz}\right)$$

$$-\frac{dp}{dz} = Tw \frac{s}{A} + \rho g \sin \theta - \frac{G^2 u}{A} \frac{dA}{dz} + G^2 \frac{du}{dp} \frac{dp}{dz}$$

$$-\frac{dp}{dz} = \frac{Tw \frac{s}{A} + \rho g \sin \theta - \frac{G^2 u}{A} \frac{dA}{dz}}{1 + G^2 \left(\frac{du}{dp}\right)}$$

We can write it down as $d v d z$ is nothing but $d v d p$ into $d p d z$. Yes or no? So, this can be substituted in the acceleration expression that I have put up here. And then this finally, this acceleration expression can be substituted in the total pressure gradient equation that I have written down or that I have derived in other words.


So, if we write it down, then in that case what do we get? If we write it down the finally, we get minus $d p d z$ this is equal to $\tau_w s$ by a plus $\rho g \sin \theta d A d z$ plus g square $d v d p d p d z$. Is it not? or if we bring this particular term on the left hand side we get minus $d p d z$ this will be equal to $\tau_w s$ by A plus $\rho g \sin \theta$ minus g square v by $A d A d z$ divided by one plus g square $d v d p$. This is the thing which we get and as we go too further, we can find out that what is this? How can this g and this $d v d p$ they can be represented.

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RECAPITULATION CONTINUED

For compressible flows:

$$\rho = \rho(z)$$

$$-\frac{dp}{dz} = \tau_w \frac{dS}{dA} + \rho g \sin \theta + \frac{d}{dz}(Gu)$$


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And finally, we can get it here. This entire thing has been done in this particular case for compressible flows, just whatever I have done I have just written it down.

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$$\frac{d(Gu)}{dz} = G^2 \left(\frac{dv}{dp} \right) \left(\frac{dp}{dz} \right) - \frac{G^2 v}{A} \frac{dA}{dz}$$

$$-\frac{dp}{dz} \left[1 + G^2 \frac{dv}{dp} \right] = \tau_0 \frac{S}{A} + \rho g \sin \theta - \frac{G^2 v}{A} \frac{dA}{dz}$$

$$-\frac{dp}{dz} = \frac{\tau_0 \frac{S}{A} + \rho g \sin \theta - G^2 \frac{v}{A} \frac{dA}{dz}}{1 + G^2 \frac{dv}{dp}}$$

$$\frac{dv}{dp} = -\frac{1}{\rho^2} \frac{d\rho}{dp} = -\frac{1}{\rho^2 a^2}$$

$$1 + G^2 \frac{dv}{dp} = 1 - \frac{\rho^2 u^2}{\rho^2 a^2} = 1 - M_a^2$$

$$\frac{dp}{dz} = \frac{C_f + C_g g \sin \theta + C_A \frac{dA}{dz}}{1 - M_a^2}$$

So, that you can see that finally, from this particular denominator we find that this denominator, we will be doing compressible flows a little later. So, I will be during that time I will be covering this portion. But for the time being you can very well see that this denominator corresponds to something like one minus m square. So therefore, we find that for compressible flows the pressure gradient comprises of the frictional pressure. It has a frictional component C_f , then it has got gravitational component. Then a component due to area change and finally, the mac number also comes into this.


So, this was just a recapitulation for your single phase compressible as well as incompressible flows.

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**RECAPITULATION OF SINGLE
PHASE HYDRODYNAMICS**

Single-phase pressure drop for flow of an incompressible flow through
an inclined pipe

$$-\frac{dp}{dz} = \tau_w \frac{dS}{dA} + \rho g \sin \theta + \frac{d}{dz}(Gu)$$

Where, τ_w wall shear stress
A- cross sectional area
S- interfacial area
G- mass flux
 density
v- specific volume

Now in that case, let us go to the incompressible flow situation. Now, just by looking at this particular equation, this we have already derived for just one particular fluid and incompressible fluid. Sorry, even if it can be a for any particular fluid you have derived this particular general equation. Now the same equation must be applicable for two phase flow situation. I will generally be talking about two phase flow situation, because this is the simplest kind of multiphase flow. When we go to three phase situation it will just be the addition of parameters, but the basic physics more or less will remain constant.

So, if you are dealing with two phase flow can you tell me in what ways this particular this basic equation how should we change or what are the additional terms or different terms that should come. So that, this particular right hand side, this gives us a representation of the actual pressure gradient which occurs under two phase flow situations.

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$$\left(-\frac{dp}{dz}\right) = \tau_{w1} \left(\frac{dS_1}{dA}\right) + \tau_{w2} \frac{dS_2}{dA} + \rho_M g \sin \theta + \frac{d}{dz} (G_1 u_1 + G_2 u_2)$$

$$\rho_M = \alpha \rho_1 + (1-\alpha) \rho_2$$

Annotations:
 - α is labeled as "In-situ void fraction" and "Inlet void fraction".
 - ρ_M is labeled as "Input".
 - A note "Use in the denominator" points to ρ_M .
 - A note "Not control or d" points to ρ_M .

Any idea, what it should be? What are the terms that should come in these particular cases? We find that the terms which will come is, firstly there are two phases. Two phases are in contact with the wall. So therefore, instead of $\tau_w s$ by A , we should have one τ_w , one just to write it down. In this particular way to in order to keep it for length we can keep $d s$ one $d A$ or we can keep it as s one by A plus there has to be $A \tau_w$ two in this particular case.

Next if we take up the gravitational pressure component we had $A \rho g \sin \theta$ in that particular case. Now in this particular case it has to be ρ mixture, is it not? It has to be the effective density of the two phases, it has to be either ρ mixture or $\rho t p$ in whatever way you want to define it. And then there we had $d d z$ of $g u$, in this particular case there are two phases. So, there has to be $g_1 u_1$ plus $g_2 u_2$ in this particular case got it. So therefore, area in this case if you remember the original definition the original derivation, what we do? Did we took the entire area the pressure drop?

If you see the basic things which I had drawn in this particular case, you find the cross sectional area was there. The entire pressure was being applied along that particular cross sectional area. So, that area had actually come here.

So, that area. So, I am $g_1 u_1$ $g_2 u_2$ into that entire area. So, from so therefore, area in this case is not going to be A_1, A_2 . It is being conducted over a control volume which extends over the entire cross sectional area on the pipe.


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In Two-phase flows

$$-\frac{dp}{dz} = \tau_{w1} \frac{dS_1}{dA} + \tau_{w2} \frac{dS_2}{dA} + \rho_M g \sin \theta + \frac{d}{dz} (G_1 u_1 + G_2 u_2)$$

ρ to be replaced by ρ_M and $\rho_M \neq \frac{1}{V_M}$

- No obvious relationship between the wall shear stress in single and two-phase flow
- Information needed about the interfacial shear stress τ_i
- Moreover, S includes S_1 , S_2 and S_i
- Estimate of void fraction necessary
- A includes A_1 and A_2
 - ❖ 1 and 2 are the two-phases
 - ❖ By convention, 2 is dispersed phase lighter phase
 - 1 is continuous heavier phase



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So therefore, we find that if we take this particular, this particular situation this is just to start with, just at the first first glance, what appears to you now?

If the equation it gets modified in this particular way.

So, we find what are the things that we need to know? We need to know tau w one. In this particular case, we need to know tau w 2 into this particular case now if we have to know tau w 1 and tau w 2. Then in that case number of things, we have to know what are the things? First thing is how much of the other, what fraction of the pipe wall is covered by fluid one? What fraction of the pipe wall is covered by fluid two? And we do not even know whether the relationship between tau w and your frictional factor etcetera. If it is applicable for single phase flows whether that is applicable for two phase flows or not.

So, the first thing is there is no obvious relationship between the wall sheer stress in single phase and two phase flows.

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Difficulties in analysis of two phase flows

- Distribution of the two phases cannot be manipulated by control of input parameters.
- Slip between phases

➤ Implies different in-situ and inlet composition

➤ varies with phase physical properties, their flow rates and interfacial distribution.

For gas-liquid flows in-situ composition commonly expressed as :

void fraction

or liquid hold up α

$H_w = 1 - \alpha$

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The other things which comes into the other thing is that, in this particular case if you observe we find that there should be interfacial shear also. Apart from $\tau_w 1$ and $\tau_w 2$ there should be a τ_i as well for the total balance it does not come. But definitely there is a τ_i same thing just as you were telling instead of one A , A is a measurable parameter in this case we need to know a one and a two if we have to know $A_1 A_2$. We have to know, what is the distribution of the two phases in the conduit?

Then remember here we have one instead of s the waited perimeter we have s_1 and s_2 along with that we have an s_i as well. So therefore, we find what are the problems which are coming up first is no obvious relationship between wall shear stress and single and two phase flow information is needed about the interfacial shear stress τ_i as includes $s_1 s_2 s_i$ then a includes a one and a two these things are coming up. So therefore, in order to know a one a two in order to know $s_1 s_2$ and things like that first thing what we have to know is the distribution of the two phases.

Now, remember one thing this distribution is not in our hands completely we can just control the amount of the two fluids that we can introduce in the conduit, but whatever we introduce in the same proportion the two fluids might not flow inside the conduit suppose you have introduced one l p m of air or one l p m of water or ten l p m of air and ten l p m of water now. So, you know what is the inlet composition 50 percent 50 percent you have introduced a mixture with fifty percent air and fifty percent water now when it

flows inside the conduit remember both of them have different densities. So therefore, there will be a slip effect one particular fluid will try to slip past the other fluid.

And therefore, in that case what we have we find is just because of that slip the composition of the fluids at the inlet will not be equal to the composition of the fluids in the inside the pipe inside the pipe there will be a slip between the phases just because of the slip what happens the lighter phase it will tend to slip past the heavier one as a result of which the in-situ composition and the inlet compositions are going to change this is another problem in this particular case.

So, therefore, unless you know the in-situ composition can you find out a proper definition or a proper expression of ρ_m mixture.

What is this ρ mixture suppose we deal with a gas liquid mixture and suppose we express the composition in terms of void fraction which you have already done. So, we take α as the void fraction. So, what is this ρ mixture it should be $\alpha \rho_2$ where ρ_2 or rather $\alpha \rho_{\text{gas}}$ let me take it plus one minus α into ρ_{liquid} correct. So, in this particular case unless you know α the void fraction you cannot proceed to do anything this is number one so. Firstly, you have to know α what is α this is the in-situ void fraction remember and this is not equal to the inlet void fraction. So, this is the thing that you are **suppose** in this part is your input you can control it you can regulate it, but this part is not within the control of any experimenter or designer that in the control of any experimenter or designer.

So, therefore, this this will depend on what this will depend upon the physical properties of the fluids it will depend upon their flow rates and most importantly it will depend on how they are distributed because the slip depends on how they are distributed for example, I give you another example suppose again air water let us state so.

In-situ means.

In-situ means inside the conduit at that particular that particular position where you want to find out the composition.

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$$\begin{matrix} 0 \\ 0 \\ 0 \\ \dots \end{matrix} \quad \begin{matrix} \vdots \\ G \\ \vdots \end{matrix}$$

$$\frac{d}{dz} (Gu) = G^2 \frac{d}{dz} U \quad u = \frac{G}{P}$$

$$= G^2 \frac{d}{dz} \left(\frac{1}{P} \right) = G^2 \frac{dU}{dz}$$

$$\left(-\frac{dp}{dz} \right)_{acc \text{ single phase}} = G^2 \frac{dU}{dz}$$

$$\left(-\frac{dp}{dz} \right)_{acc \text{ two phase}} = G^2 \frac{dU_M}{dz}$$

For example, suppose see this particular in-situ composition how it changes suppose we are having air water. So, we are having a large amount of water a small amount of air what do you expect when they are flowing air will be distributed as bubblys we will study this in much more details on the contrary you have large amount of air small amount of water how will they be distributed they will be distributed as there will be gas in the center and liquid will be flowing as films at the wall do you mean to say the slip will be same for both these cases it cannot be same. So, it depends upon the interfacial distribution as well.

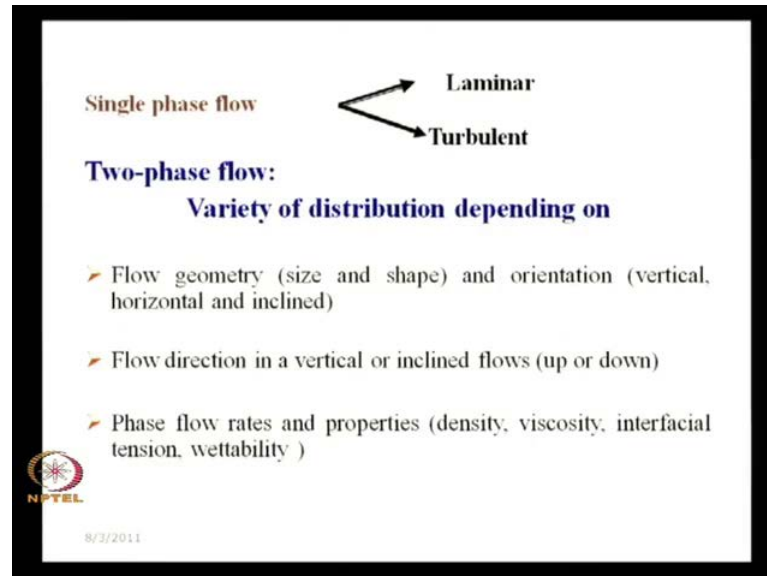
So, therefore, what we find is that why is see we have started with the single phase equation what we did we simply added up it is not as simple as as I tell you, but just at one go you.

Simply try to add up a few extra terms instead of tau w we had your tau w one and tau w two then instead of rho naturally we are going to have rho m. So, just by these simple things what we find is that in this particular case the situation becomes much more complex as compare to single phase flows.

Why it becomes much more complex because. Firstly, two things the two phases can distribute themselves in a large number of phase and that is not within the control of the experimenter or the designer.

And the other thing is there is a slip between the phases due to which the in-situ composition and the inlet compositions they are very different.

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Now due to this what happens for single phase flow what do you need to know you just need to know whether the flow is in laminar or turbulent if you do not do that you cannot find out that frictional pressure gradient can you you cannot use laminar flow equations and expect to get accurate results for turbulent flow conditions can you do it you have to find out whether the flow is under laminar or turbulent flow conditions.

For that what you have to do it is a very well defined process there is no argument about it go to find out the reynolds number refer to the modis plot or whatever it is and do it why cannot you use laminar flow equation for turbulent flow just because the distribution is completely different is we have a very ordered type of flow in laminar flow and a very chaotic random flow for turbulent flow same thing in this particular case also it is not it is just not sufficient to know for the individual fluids are in laminar or turbulent flow, but, we need to know whether the variety of distribution we have to know the distribution of the two phases just like we we cannot use laminar flow equations to predict turbulent flow pressure drop.

In this particular case also we cannot use bubbly flow equation to predict the pressure drop of annular flow can we do it suppose we have derived an equation for this particular bubbly flow case.

So, for this particular bubbly flow case suppose we have derived an equation we cannot use the same equation to find out the pressure drop in this particular case the total flow distribution is different. So therefore, what we find is that we have to know. Firstly, the distribution of the two phases and interestingly for single phase flows what we have whether the pipe is inclined it is vertical it is horizontal it hardly matters the same Reynolds number we use to find out whether the flow is laminar or turbulent.

But interestingly in this particular case

We find that the distribution it depends upon the geometry of the conduit whether it is circular its size its shape whether it is micro channel or it is a mini channel or it is a large channel on that it depends and it depends upon the orientation whether it is vertical whether it is horizontal whether it is inclined why whenever it is horizontal what happens there is a stratification tendency gravity acts on the two phases differently and therefore, the lighter phase would like to stay on the top the heavier phase on the bottom.

This is not going to happen for vertical cases agreed. So therefore, in this particular case finding out laminar and turbulent were much easier in this particular case we find that flow geometry becomes very important the flow orientation becomes very important flow direction becomes very important in the same vertical pipe whether it is up flow or down flow it is very important whether it is co-current flow counter current flow that is a becomes very important and definitely phase flow rates and properties it goes without same.

Phase flow rate and phase property it is important for both single phase and two phase situations in addition to this particular case we find that we have to know the distribution of the two phases for any particular hydrodynamics study and if we have to know the distribution of the two phases we have to take into consideration the flow geometry the orientation the flow direction and. So, on and. So, forth there is one other very interesting thing which I would like to tell you with respect to this particular situation.

So. Firstly, we find out we found out that find if to found out ρ_m we need to know the in-situ composition or the α there is another very interesting thing which I would like to show you see in this particular case this acceleration pressure gradient

we have expressed it as $\frac{d p}{d z} = \rho g$ what is this ρ this is nothing, but, ρ by ρ isn't it. So therefore, this can be expressed as $\frac{d p}{d z} = \rho g$ or in other words this can be written down as $\frac{d p}{d z} = \rho g$. So therefore, minus $\frac{d p}{d z}$ acceleration for your single phase cases this becomes ρg it can also be written in this particular form yes or no you can write it in this particular form.

Why could we write it because we could just take one by ρ equals to v similar way we can write down $\frac{d p}{d z}$ acceleration for two phase flow conditions as ρg $\frac{d p}{d z}$ of v mixture all of you agree we can write this down,

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$\rho = \frac{1}{v}$
 $\rho_m \neq \frac{1}{v_m}$
 1 kg mixture = x kg gas vapor + $(1-x)$ kg liq.
 $v_m = x v_g + (1-x) v_l$
 1 m^3 mixture = x m^3 gas + $(1-x)$ m^3 liq.
 $\rho_m = x \rho_g + (1-x) \rho_l$

But here comes a catch in single phase flow what we could write down we could simply write down $\rho = \frac{1}{v}$ tell me for two phase cases can we straight away write down I have already put not equal to can we straight away write down that your $\rho = \frac{1}{v}$ equals to ρ_m we cannot write it down why cannot we write it down any idea anybody.

Let us go to the basic definition of density and specific volume. What is the specific volume it is the volume of 1 kg of the substance agreed now when it is a single phase thing then in that case the ρ is the weight of 1 meter cube or unit **weight** sorry unit volume of the substance do you agree. So, again listen to it ρ is the weight of unit volume of the substance v is the volume of unit mass of the substance say our unit of volume is one meter cube s I units and this is 1 kg now when you take say two phase flow situation what happens in 1 kg of the mixture we have x kg vapor the x is the

quality remember quality and void fraction you have already come across and we have one minus x k g liquid agreed.

So, therefore, in this particular case what is v m equals to x v vapor if you take or its better I put it gas x v gas plus one minus x v liquid agreed now if I take one liter mixture or say one meter cube of the mixture what it has it has α meter cube gas and one minus sorry α meter cube liquid do you agree with me. So therefore, what is ρ t p in this particular case it is α into ρ gas plus one minus α into sorry ρ liquid this ρ gas is equal to one by v gas ρ liquid is equal to one by v liquid, but, definitely ρ t p cannot be equal to or rather sorry let me make it mixture.

ρ m cannot be made equal to one by v m the basic definitions are completely different is it clear to all of you. So therefore, what we find is. So, just try to understand the **complications** which I have been introduced here just by the introduction of an additional fluid in this particular case.

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Hydrodynamics manifolds more complex compared to single phase flow

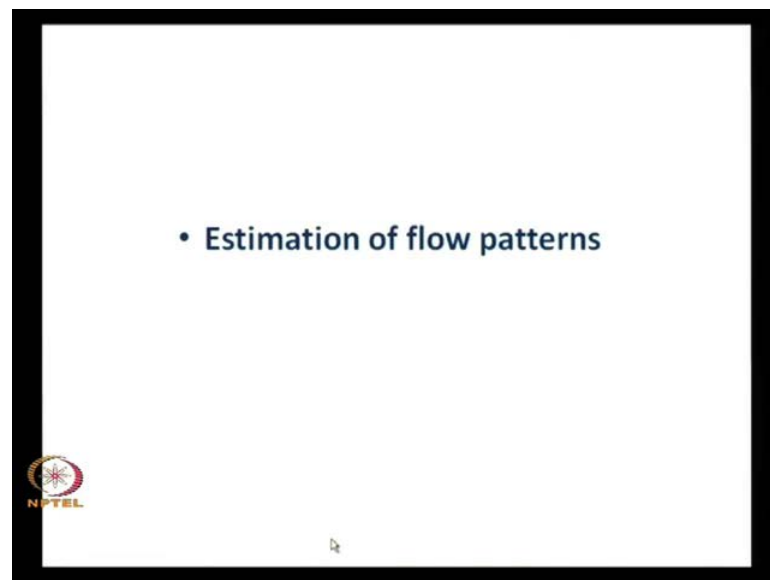
- Existence of multiple, deformable and moving interfaces
- Multi scale physics of the flow phenomena
- Significant discontinuities of fluid properties and complicated flow field near interface
- Compressibility of the gas phase (For gas-liquid and vapor liquid flows)
- Different wall interactions for different fluids

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So, in this particular case I would like to conclude this particular lecture by telling you that the hydrodynamics in this particular in two phase flow situations is many folds more complex as compare to single phase flow why existence of multiple deformable moving interface try to understand.

It is a multiple deformable moving interface particularly for gas liquid cases multi scale physics of the flow phenomena next near the interface what we find there is a significant discontinuity of the flow property instantly shifts from the property of one particular phase to the other other phase and therefore, there is complicated flow field near the interface for gas liquid vapor liquid cases there is a compressibility of the gas phase then as I was telling you τ_w one τ_w two they are they may not be appropriately predicted by single phase equations because we may have different wall interactions for the different fluids and therefore, for all these things we find that the two phase flow situation becomes many folds more complex as compare to single phase flow condition.

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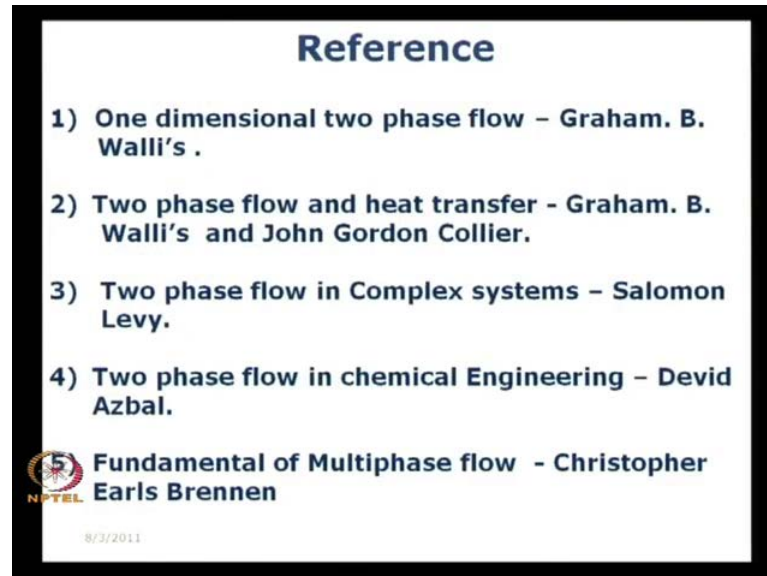


And therefore, in order to study two phase flow even if we have to make this particular equation our starting point even for.

That particular case also we what we have to do first we have to know the distribution of the two phases once we can know what are the distribution of two phases how the two phases can be distributed only after that what different distributions we can find out that what can be the generalized analytical models that can be proposed accordingly we will be going for the analysis of those particular we assume our type of flow and we try to analyze that particular flow in that by or rather following that we would like to propose some simple analytical models and after that we will be taking some flow patterns specific models.

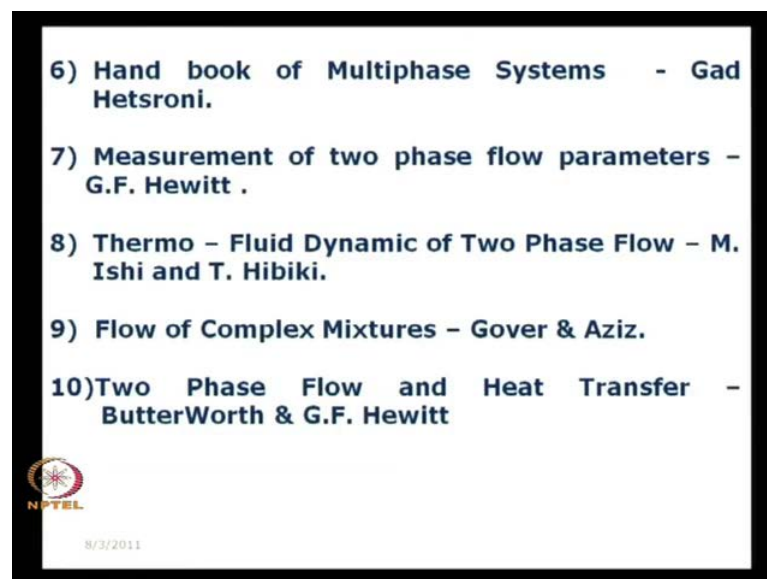
Now, before I conclude I would like to mention the text books which you can take up for this particular case.

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The text books large number of text books are already available this is wrong this has to be wallis this inverted comma should not be there now among these large number of text books which are available.

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What I would like to say is our library has got a number of text books we I mostly be discussing from one dimensional two phase flow by g b wallis we will also be be taking

up certain aspects from the hand book of multiphase systems measurements of two phase flow parameters and may be two phase flow in chemical engineering two phase flow in complex systems.

So, as the case may be you please try to remember that since two phase flow it is not it has still not become a very well prevalent text book material till now. So therefore, good amount of things might also be taken from different papers from different research findings, but for the time being initially probably wallis govier and aziz butterworth and hewitt the book by hewitt the hand book of multiphase systems these books will more or less suffice to do with you.

So, we conclude the session today with this particular note and from tomorrow we are going to start a discussion on the estimation of different flow patterns under different flow conditions for different fluid types. Thank you very much.