Multiphase Flow
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Lecture No. # 11
Compressible Flow – A Recapitulation (Contd.)

Well. So, today we will be continuing our talk on compressible flows. I had the in the last class what I did was I had covered or rather I had identify what is compressible flow why mac number is important and we had also defined the stagnation properties and the sonic properties it is it not?. So, next what I had planned to do is just to show you the effect of area change on compressible flows. Because normally we see we have seen that for incompressible cases, we have seen that in area change it suppose there is a contraction it accelerates the flow when there is an expansion it is vice versa. So, in this particular change apart from velocity the certain other changes are expected. So, let us see what are the changes we will be starting from the basic equation of the energy equation the the final form that we had derived in the last class. The final forms which we are derived from where we had got that the enthalpy heat and the kinetic energy heat remains constant for an adiabatic isentropic situation.

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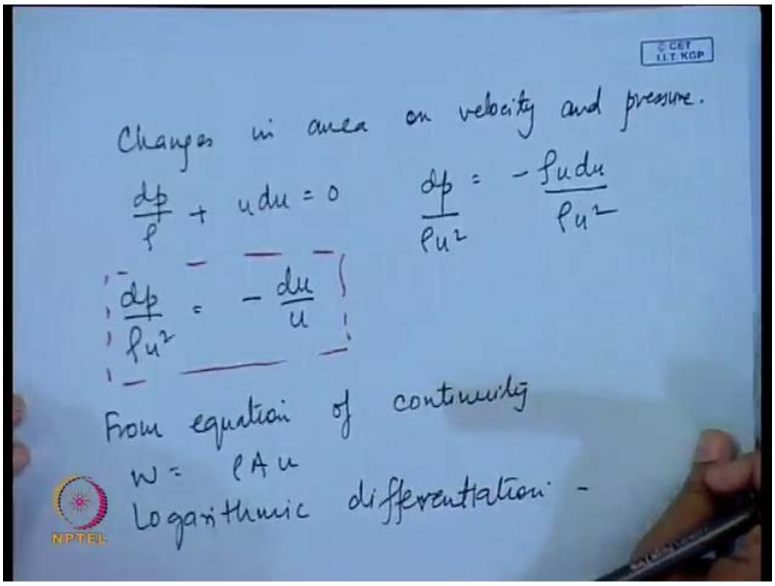

So, from there, we will start and we will see how area changes effect the compressible flow cases. Now the thing is its changes in area on the area change actually what it does it changes the velocity and it changes the pressure as well. Now let us start from the basic equation which we had derived in the last class dp plus rho dp by rho plus u du equals to 0 or in other words we can write it down as dp equals to minus rho u d is not it? Dividing both sides by rho u square what we get we get dp by rho u square this is equal to minus du by u - this is one equation.

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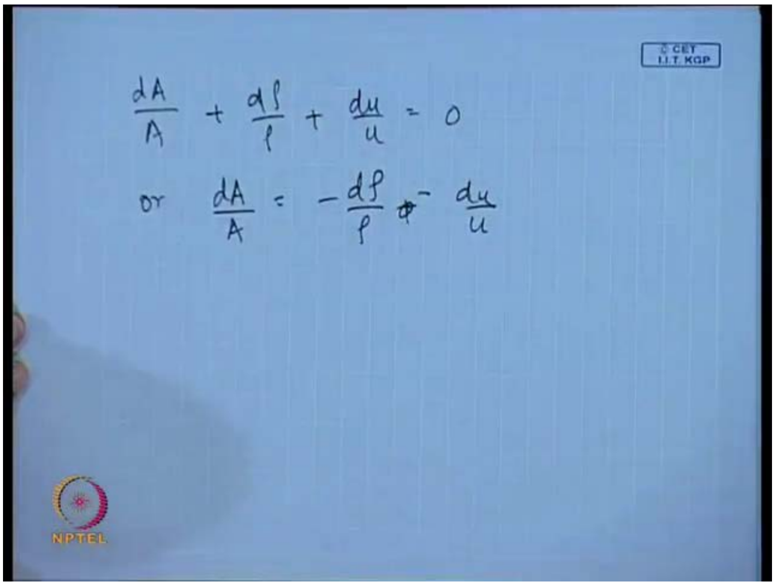

Now for continuity equation of continuity what do we get from equation of continuity we get we get you all of us know w equals to rho a u now if it take the logarithmic differentiation like taking log and then differentiating which we had already done in the last class I believe. So logarithmic differentiation what does it give it gives us an expression something of I hope you have noted this down this as d a by a plus d rho by rho plus d u by u this is equal to zero or in other words d a by a this is nothing, but d rho by rho sorry minus d u by u ok.

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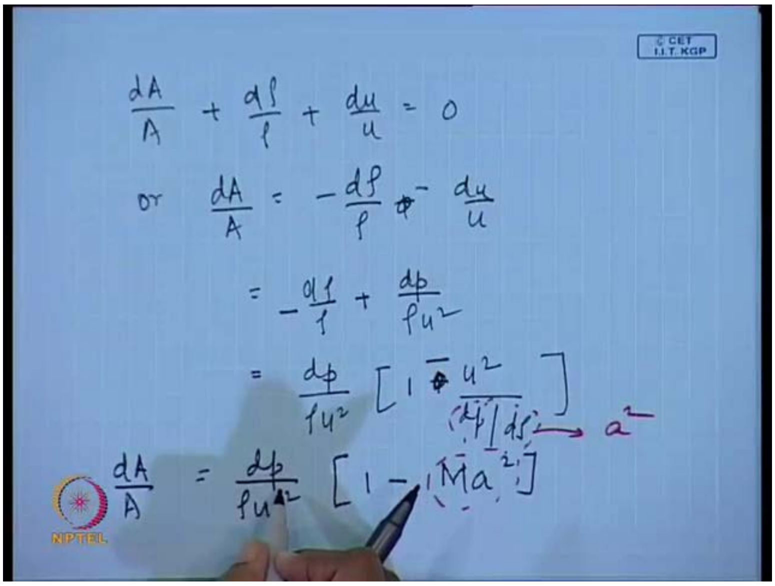

Now, from here what we had got we had obtained an expression for d u by u or minus d u by u. So we can substitute this this particular d u by u with d p by rho u square isn't it. So if we do it then in that case what do we get we get this as d rho by rho plus d p by rho u square right or in other words it can be written down as this can be written down as d p by rho u square 1 plus u square by d p d rho can we write it down in this particular form we can write it down in this particular form and what is this particular term equals to d p d rho d p d rho this is this is a the rather it is a square isn't it. So the velocity the acoustic velocity in that particular medium under that particular conditions no no not minus d rho by rho its sorry very sorry correct correct actually here only I had made a mistake here it is minus d rho by rho here only I made a mistake that is why it came in the. So this is the situation. So therefore this can then be written down as d p by rho u square this is nothing, but equal to 1 minus m a square isn't it. So therefore we find out that how pressure will change with area that depends upon the mac number isn't it. So we find that if this mac number is less than 1 then a positive area change will give rise to a positive pressure or a pressure increase a negative area change will give rise to a negative pressure and similarly the mac number is greater than 1 just the reverse takes place is is this part clear to you that for mac number less than 1, what we have in area change causes a pressure change of the same sign and for mac number greater than 1 and area change causes pressure change of the opposite sign agreed [FL].

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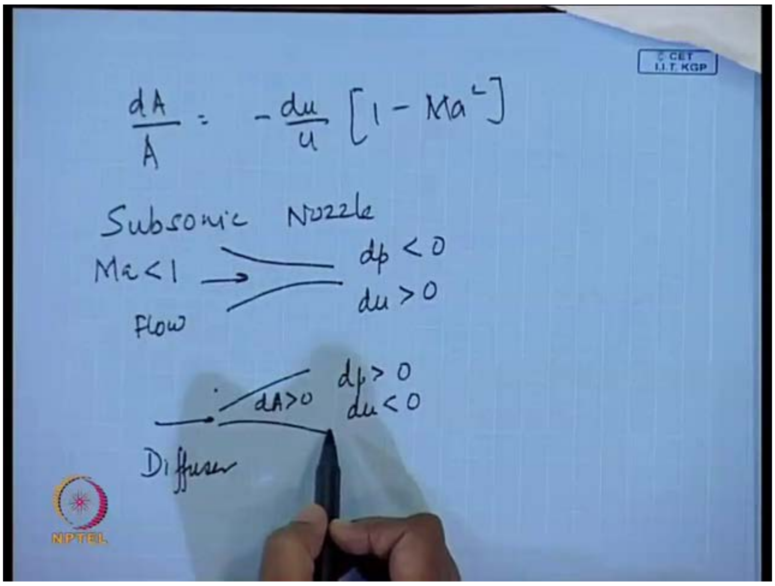

Now let us say see how area change affects velocity change now here I had already reduced a equation between d p by rho u square and d u by u yes. So therefore this particular relationship instead of d p by rho u square I can write it down as minus d u by u. So if I substitute it then what do I get I get the expression as d a by a equals minus d u by u into 1 minus m a square. So for from this what do I get I see the area change has an opposite effect on velocity change depending upon whether mac number is greater than 1 or less than 1 if it is less than 1 and area change may be the positive area change causes a negative velocity change or in other words when there is an expansion we have deceleration or we have a velocity decrease and when there is a contraction velocity or this the flow it accelerates. This is in agreement with what we have observed for incompressible cases, but when mac number is greater than 1 the reverse situation happens isn't it. You find that for this particular case for mac number less than 1 area change causes a change of velocity of opposite sign positive d u means negative d u for mac number less than 1 and for mac number greater than 1 an area change causes a velocity change of the same sign. So therefore from this what do we deduce we deduce that for the subsonic range for the subsonic range if mac number is less than 1 then in that case what happens the nozzle will be something on something of this sort flow occurs in this particular direction for this particular case we know d p less than zero and d u greater than zero isn't it.

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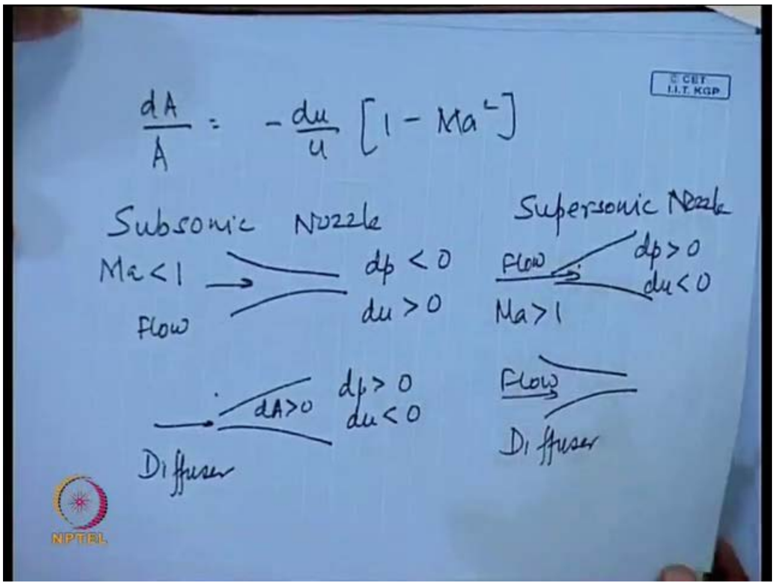

And what about a diffuser a diffuser will be something of this sort. So this is a nozzle and this is a diffuser again the flow is in this particular direction in this particular case we get d p greater than zero d u less than zero d a greater than zero agreed so. So therefore a nozzle it it it is a convergent profile a diffuser is a divergent profile this accelerates flow this decelerates flow just in the same way as we had studied for your incompressible cases agreed, but we find that when we go for the supersonic nozzle in this particular case we find that the nozzle will be something of this sort here m a is greater than 1 flow occurs in this particular direction and we find that for this particular case d p is greater than zero d u is less than zero. So therefore we find that for this particular case the supersonic nozzle is of this particular form and the supersonic diffuser has a convergent profile again the same thing flow is in this particular direction. So from from from these things what do we deduce we deduce the important observations are that at subsonic speed that means, when mac number is less than 1 these cases a decrease in area increases speed of flow a subsonic nozzle. Therefore it should have a convergent profile a subsonic diffuser should have a divergent profile isn't it.

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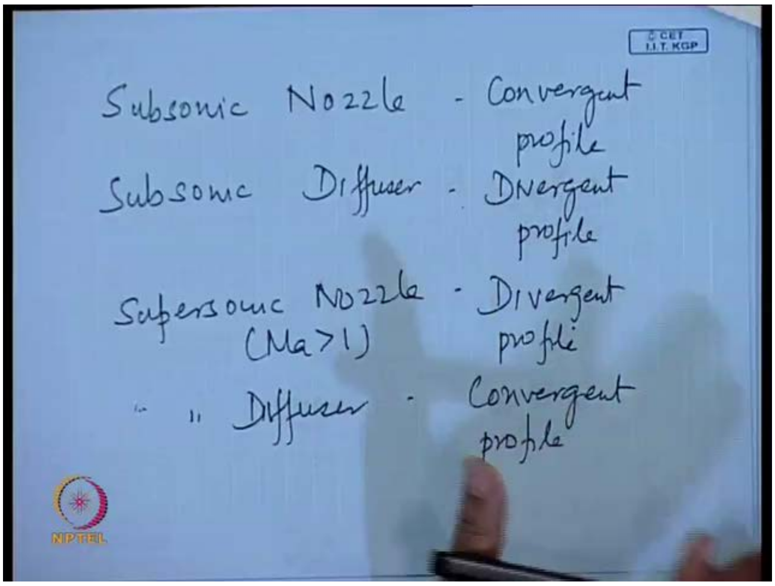

So we can deduce it or we can just it down in this particular fashion subsonic nozzle convergent profile just I am writing down whatever I have drawn and shown you subsonic diffuser divergent profile on the other hand if we take a supersonic nozzle; that means, for mac number greater than 1 we find that the effect is just opposite we find that supersonic nozzle it has a divergent profile and a supersonic diffuser it has a convergent profile and this particular thing the supersonic the divergent nozzle to produce supersonic flow that is very frequently its used in missiles and launce vehicles in aerospace we use we as chemical or mechanical engineers we do not come across this particular situation ok. So this is something very important that you should be notice that whenever we are the behavior of compressible flows they are not consistent over the entire range of mac number one type of flow characteristics are exhibited for mac number less than 1 or under subsonic conditions just the reverse step of phenomena is observed for supersonic conditions and that explains why this particular fact mac number equal to 1 is. So very important and for mac number equal to 1 this is usually it is termed as choked flow condition or the condition of choking is when mac number equal to one.

Now let us see that when we have a flow through a convergent convergent nozzle for that particular case let us see what what what is the maximum flow rate when we get the maximum flow rate and So on. And so forth [FL] before that I would like to show one more thing to you I had expected this particular question from you that see in this particular case what do we find we find that for subsonic cases what do we get we suppose we start from a very low velocity and we keep on increasing the velocity the profile is convergent, what happens gradually with increasing velocity the mac number keeps on increasing till it reaches mac number equal to one. If you keep on increasing the mac number further we cannot do it in this particular case we can just come till mac number equal to one we cannot accelerate a flow further under these particular circumstances is it clear to you. So therefore if we want to suppose we we want to start or we have started with a very low velocity and from that particular velocity we would like to accelerate the flow to subsonic conditions that means, you would like to go from mac number less than 1 to mac number greater than one.

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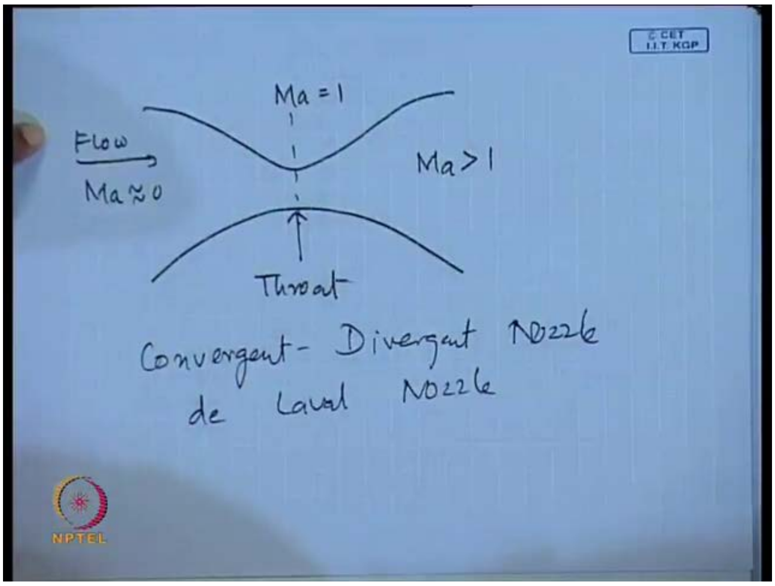

Can we do it by either a subsonic nozzle or a supersonic nozzle can can we do it we cannot do it. So therefore for that particular case what do we need if we start from a very low velocity and we have to go to the supersonic range we have to use a convergent divergent nozzle otherwise it is not possible. So for that particular case what we need is it is something of we have to do something of this sort suppose if we want to obtain supersonic stream starting from very low speeds at the inlet. So here its say mac number its almost equal to zero flow occurs in this particular direction this is known as the throat. Now as we keep on increasing the velocity we find we come to m a equals to 1 here and then in this divergent section we get m a greater than one. So therefore we find that in order to accelerate flows from the subsonic to the supersonic range what we need is we need a convergent divergent profile of a nozzle it has to converge in the subsonic portion it has to diverge in the supersonic portion such type of nozzles are known as convergent divergent nozzle or the its also known as the de laval nozzle named after the person who had first used it and designed it.

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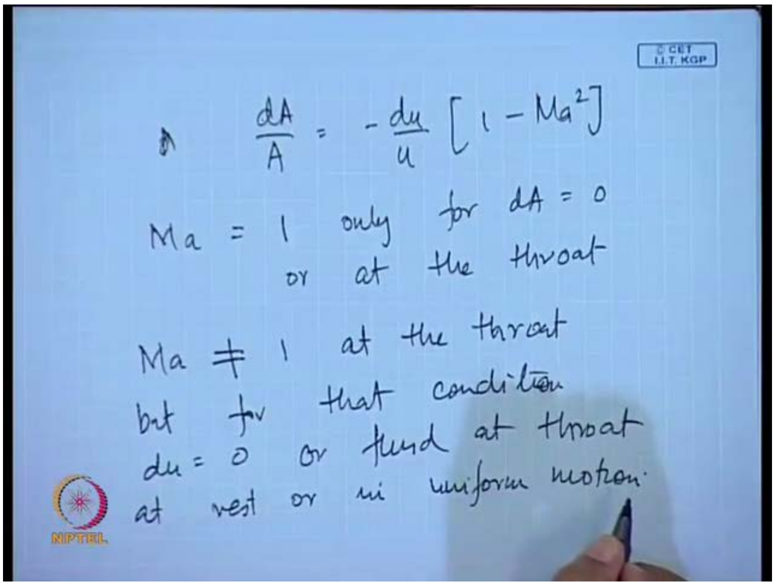

So therefore there are certain things that we can note if we take up the expression relating d a by a with this what do we find we find that mac number can be equal to 1 only when mac number can be equal to 1 only when d a equals to zero or in other words where is d a equals to zero at the throat at that particular portion where the area does not change any further. So therefore certain observations we get from this particular equation the observations are. Firstly that I will write it in a separate page that I will write down the equation once more that will be easier from this equation we get from this particular equation we get that mac number can be equal to 1 only for d a equal to zero or at the throat it cannot be equal to 1 under any other condition agreed with me Again the same thing does it imply that mac number is always equal to one at the throat because d a equals to zero can be obtained either by m a equals to 1 or d u equal to zero. So therefore mac number might not be equal to 1 at the throat, but for that condition d u equals to zero or fluid at the throat has to be at rest or in uniform motion it cannot accelerate at the throat you get my this point this point is extremely important first thing we get is that if we have we want choked flow condition if we want m a equals to 1 that can only happen only at the throat where d a equal to zero we cannot have mac number equal to 1 under any other condition number one. Number two that does not imply that mac number has to be equal to 1 at the throat or in other words flow has to be under choked flow conditions at the throat we can have a nozzle nozzle or a converging diverging nozzle also where the flow throughout is subsonic that means, mac number is less than 1 at the converging section it continues to be one at the diverging section it never becomes equal to 1. We can have a condition where mac number is greater than 1 throughout the entire region we can have that it is not that mac number has to be equal to 1 at the throat, but if we are not having choked flow conditions at the throat then the fluid either has to come to rest or it has to be under uniform motion at the throat or in other words if m a not equal to 1 then d u equals to zero at the throat clear to all of you.

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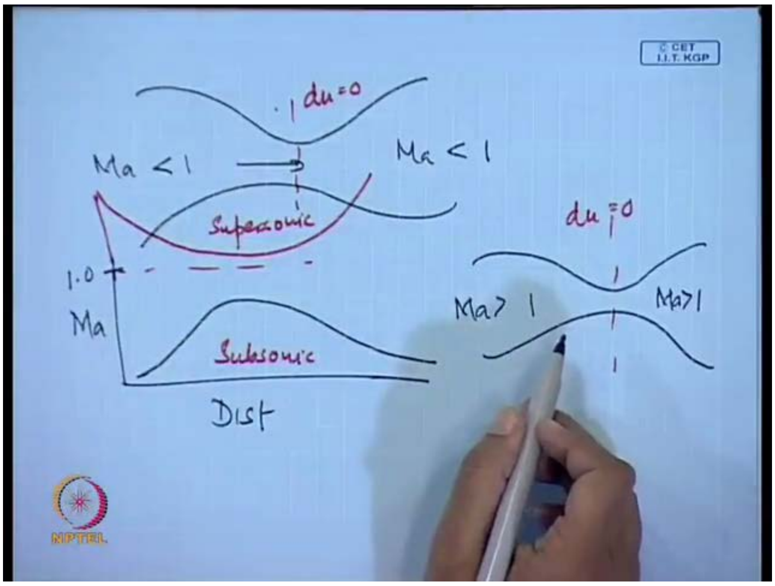

So therefore this is the thing which which you have to keep in mind that for this particular case if we got to have mac number equal to 1 it can be under this particular condition, but we can also have situations where we find that the flow throughout is here it is mac number less than 1 flow occurs in this case also mac number is less than 1 or in other words if if we plot mac number with distance say this is mac number equal to this is one. So we can we can always have something of this sort its starts in this way it ends, but always it is subsonic through out we can also have a situation may be where mac number is greater than 1 throughout the case or in other words we can have a situation where it is something of this sort it it never comes the flow is supersonic throughout and in this particular case it is subsonic throughout. You can also have these two conditions, but we need to remember that for this two occur d u has to be equal. So zero under this particular condition in this particular case d u has to be equal to zero this is the thing that you have to remember. So these are the two cases where mac number is not equal to 1 at the throat, but yet the flow can occur because the fluid at the throat is either at rest or in uniform motion at the throat. So this is very clear that if you want to accelerate the flow from supersonic to sorry from subsonic to supersonic conditions you cannot do it for a convergent nozzle we have to go for a convergent divergent profile where the mac number has to be equal to 1 or the choked flow conditions can be approached only at the throat ok

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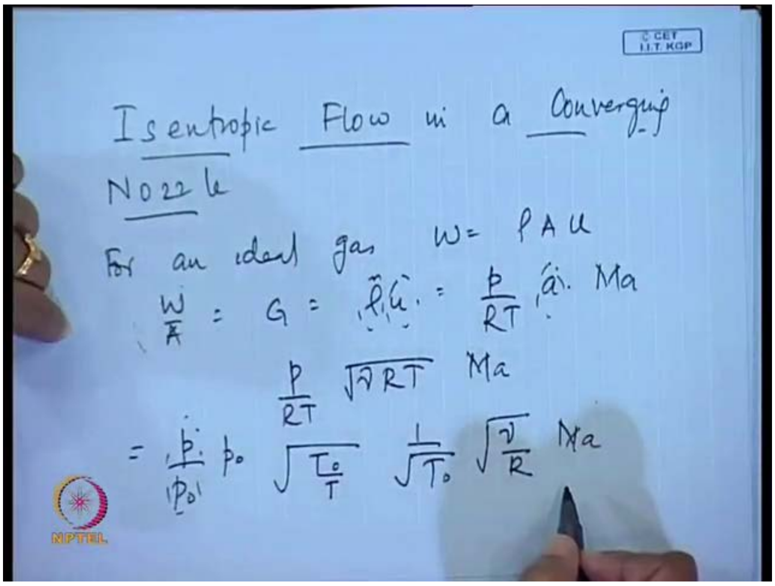

Now let us see what about the flow rates and what about the flow under choked flow conditions. So let us we are going to do say isentropic flow. So for this particular case we know that for an ideal gas in a converging nozzle for an ideal gas in a converging nozzle what do we know w equals to this all of us know or in other words w by a this is nothing, but g this is equal to rho u this also this we know now since assuming we have already assumed it to be an ideal gas. So therefore this rho can be written as p by r t yes or no this u can be written down as a into m a agreed this a can be written down as root gamma r t we can do whatever we are doing ok. So therefore into m a of course, so therefore, after this we can just rearrange and we can write p by p zero into p zero we can do this yes or no root over of t zero by t root 1 by t zero fine root gamma by r into m a can this we done and just introduce the stagnation properties why because I know p by p zero in terms of t zero by t by t zero and I can easily substitute them. So that I can my target is to express this particular mass flux or the mass velocity in terms of certain easily measurable measurable parameters and to find out on what this flow rate depends if I can know on what the flow rate in a converging nozzle depends or on what parameters the flow rate depends then I know how to manipulate those particular parameters and control increase or decrease my flow rate as I desire ok.

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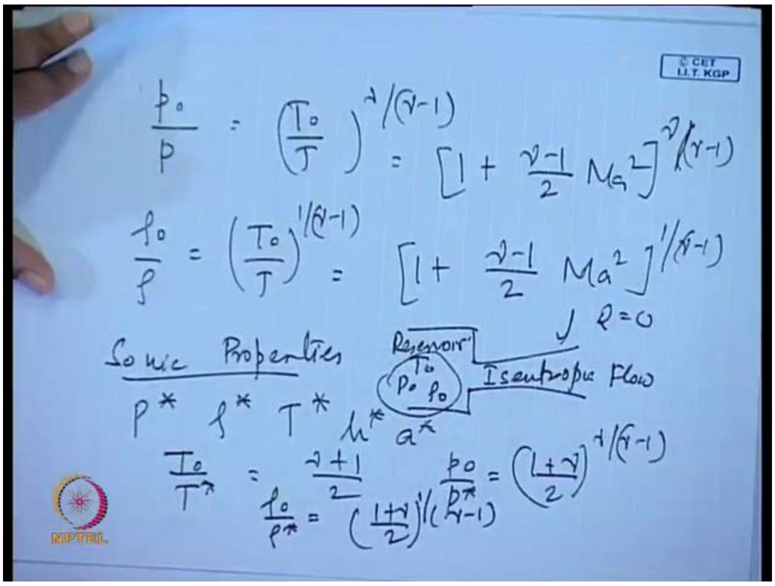

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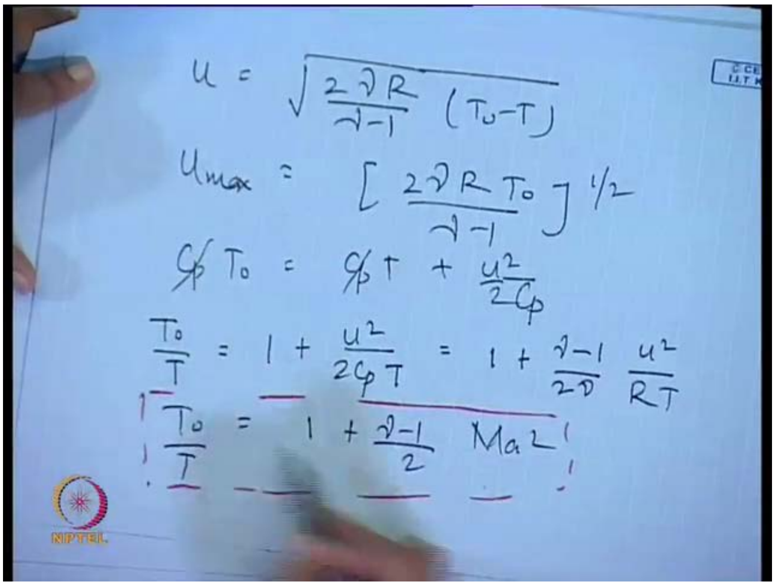

So therefore I would like to find out on what which measurable parameters my flow rate depends just for that I would like to express this expression in terms of certain known input or certain measureable parameters right. So therefore so after this what I can do I can just substitute p by p zero in terms of t zero by t I think in the last class I had defined those particular terms p zero by p equals to this particular term isn't it and I had also defined t zero by t I have that also thank god here also I have defined t zero by t isn't it. So instead of t zero by t I can substitute this instead of p zero by p I can substitute this just let us do these two substitutions and then let us find what do we get.

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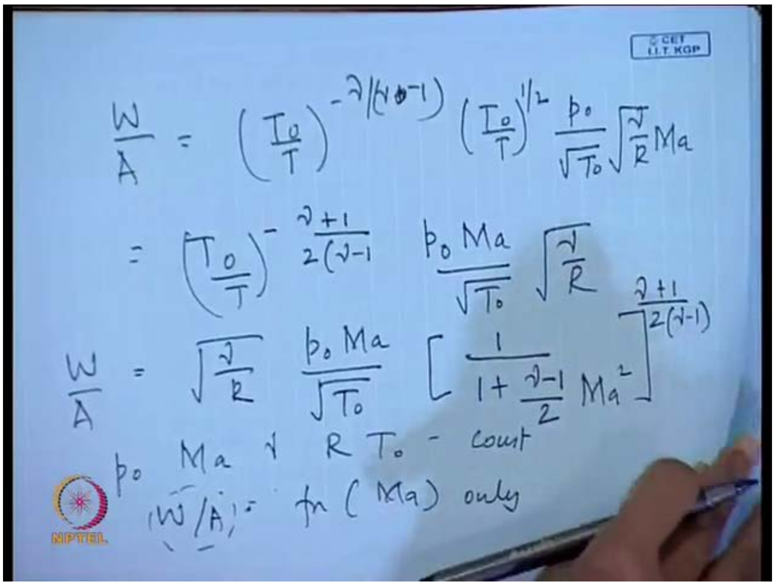

We get w by a this is equal to in that particular case t zero by t instead of p zero by p I am writing gamma by gamma sorry gamma minus 1 and t zero by t whole to the power half p zero by root over t zero root gamma by r into m a and then I can write this as t zero by t whole to the power minus of gamma plus 1 by two into gamma minus 1 I have combined these two terms then it comes as p zero m a by root t zero root over of gamma by r or if I arrange it slightly more it is almost the same thing we are arranging and writing I am just bringing all my known parameters to the towards the left this whole thing t zero by t I will substitute I get 1 by 1 plus gamma minus 1 by 2 m a square this is gamma plus 1 by 2 into gamma minus 1. So this is the final expression now in this particular case w by a equals to. So in this particular case what do I find I find gamma r p zero m a t zero all these parameters they are constants what is the only variable in this particular equation which can vary my mass flow rate or mass flux the only variable is p zero m a gamma r t zero constant. So the only thing which which influences w is mac number. So gradually I would like to show you the importance of mac number and why mac number has equal to 1 is. So, very important and why mac number is. So, very important that is what I would like to show you ok.

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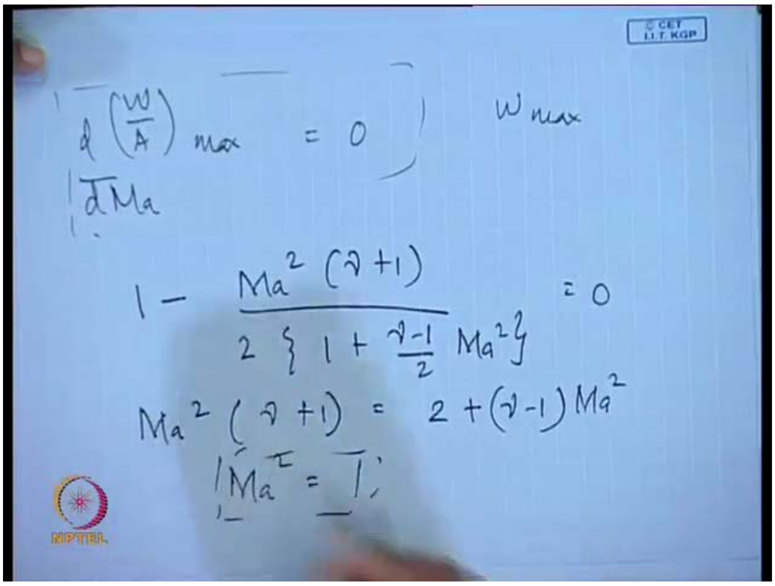

So therefore we know w by a this is a function of mac number only it does not depend on anything. So therefore by varying mac number we can vary this and under what condition will we get a maximum of w by a or under what condition do we expect to get w by a max this will depend definitely only on the mac number and the condition which will give you w by a max can we obtain as d of d d m a equals to zero yes or no from this particular condition I can find out I can get an estimate of w max isn't it now let us differentiate and let us find what is this expression. So what basically we require to do this whole thing can be taken as a constant k you need to perform [FL] 1 m a is there. So, you need to simplify perfrom sorry very sorry p zero gamma r t zero very sorry. So, you need to perform this particular differentiation. So you can take this portion as say k 1 a constant and here also this particular portion can be taken as k 2. So, therefore, mac number is the only variable and you can perform the differentiation and on differentiating what do we get we find that if we differentiate it the expression reduces to 1 minus m a square just perform it in your hostels and then you see whether you are getting it yourself or not this equal to zero isn't it this is the thing which we get or in other words we get just on substitution m a square into gamma plus 1 equals to 2 plus gamma minus 1 into m a square. And if if you solve it what do you get we get from this particular condition m a square equal to one. So therefore we are talking about the choked flow conditions or rather we are talking about the we will d a equals to zero when d a equals to zero m a equals to 1 and so on. And so forth and we find why is m a equal to one. So, very important because that gives us the maximum flow rate from the converging nozzle under the present circumstances. So therefore we find that for mac number equal to 1 we get the maximum flow rate under the present circumstances the discharges maximum ok.

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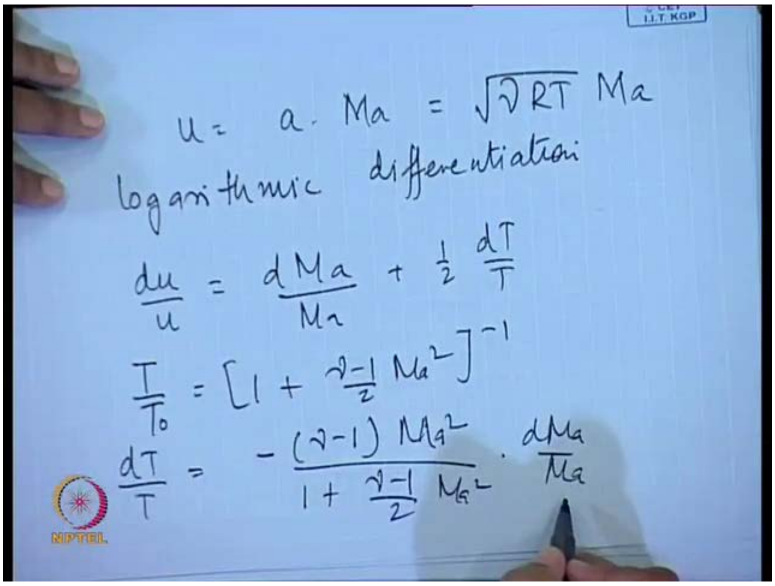

Now let us see that where do we get we have already seen that we get mac number equal to 1 where d a equals to zero. So just a little more substitution if we do then in that case we get we know u equals to a into m a you know this now again if we or this can be written down as root over of gamma r t into m a fine now again if we perform the we take the log of both sides and if we perform the logarithmic differentiation. So by performing this logarithmic differentiation what do we get we get d u by u equals to d m a by m a plus half d t by t isn't it and from the previosu previous expressions where we had obtained t by t t zero by t from this particular expression also we can perform the logarithmic differentiation ok.

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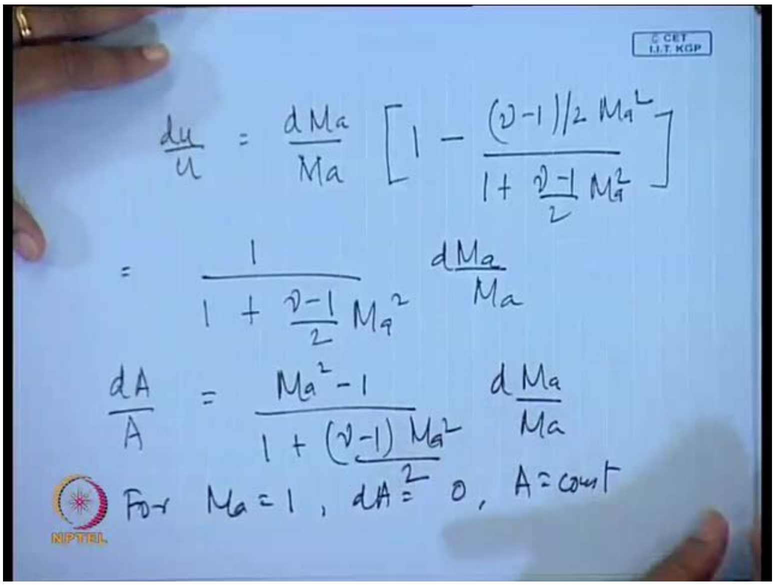

So that we get two expressions of of d t by t and we can equate them. So that particular expression 1 plus gamma minus 1 by 2 m a square whole to the power minus 1 isn't it. So in this particular case we get d t by t this is equal to minus gamma minus 1 m a square by 1 plus 1 by 2 m a square d m a by m a. So now what we can do instead of this d t by t we can substitute this particular expression yes or no just do it and see what we are getting if we do it we find that d u by u this becomes d m a by m a 1 minus gamma minus 1 by 2 m a square by 1 plus gamma minus 1 by 2 m a square or this can be written down as 1 by 1 plus gamma minus 1 by 2 m a square this is the thing that we can get from this particular condition. So therefore we find d a by a if we write then in that particular case we have already got d u by u in this from here we already had a equation of relating d u by u and d just a minute we had one particular equation yeah we had d a by a equals to this. So in place of d u by u we can substitute this particular equation and then we get this is m a square minus 1 by 1 plus gamma minus 1 m a square by 2 d m a by m a.

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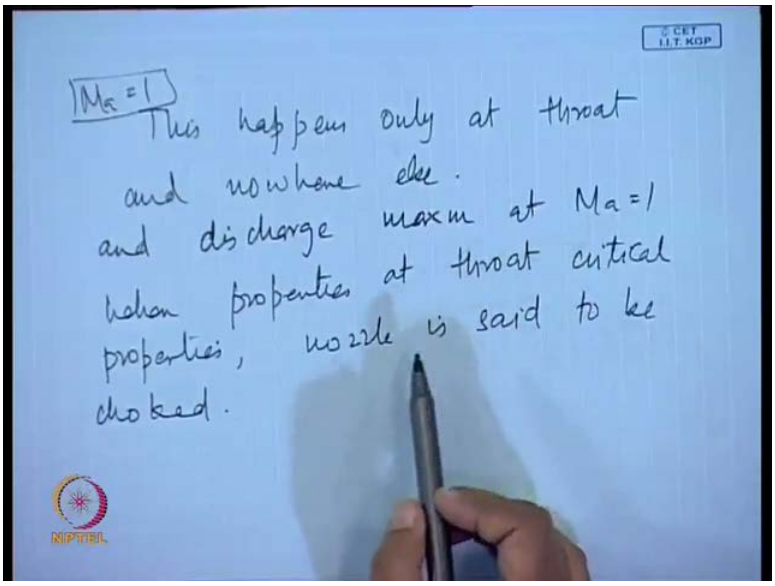

So therefore we had obtained that we get the maximum flow rate for m a equals to 1 and for m a equals to 1 d a equals to zero or a has to be equals to constant or in other words we observed that this happens only at throat and nowhere else. So this means m a equals to 1 as I had told you this happens only at throat and nowhere else and discharge maximum at m a equals one. So therefore we find that properties at the throat are no are can be the critical properties and when properties at throat critical properties nozzle is said to be choked it delivers the maximum flow rate this portion is clear to you ok.

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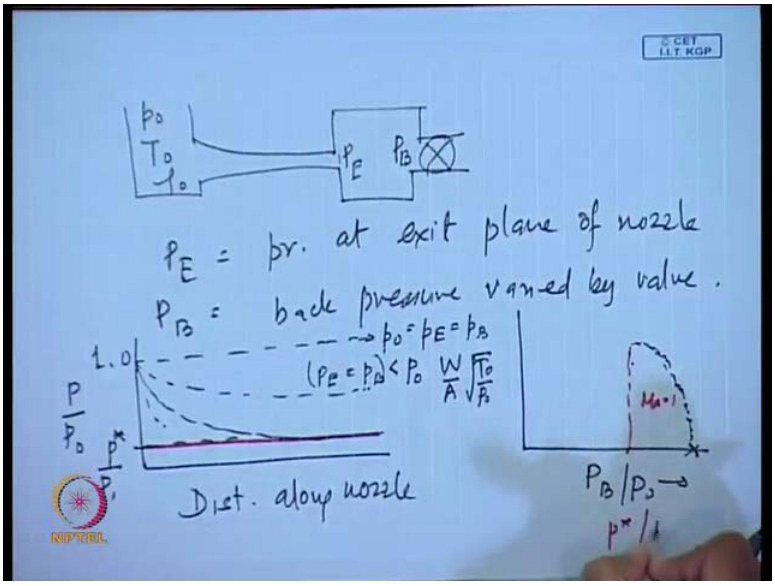

So we find that for m a equals to 1 that happens only at the throat and nowhere else under this particular condition the discharge is maximum and under this conditions the nozzle is said to be under choked flow conditions agreed. So this shows why m a equals to 1 is. So very important for all our purposes now let us see one more thing let us see that suppose from a very large reservoir say the flow has initiated in a nozzle and this is connected to a container which has a valve now since the flow has initiated from say a tank and flow is occurring under isentropic conditions. So what do we know conditions at the tank are the stagnation conditions right. So here the properties are p zero t zero rho zero etcetera here just at the exit of the nozzle p exit or p e and in this particular tank we have a back pressure which is p b this back pressure is regulated by means of opening or closing this particular valve. So p e is the pressure at the exit plane of the nozzle if I write it down p b it is the back pressure varied by valve now let us see what happens. So, initially what we can do we can gradually initially this valve is closed and the entire system is at a constant pressure no flow occurs under that condition right.

So if we plot say we plot say p by p zero as a function of distance across the nozzle distance along nozzle and along with this pressure we would also like to plot the flow rate which is w by a usually we take the entire that constant thing which was there this is as a function of. So initially what do we find initially we find that everything is equal and therefore, the curve is something of this sort where this is p zero equals to p e equals to p e under that condition there is no flow then what we do we gradually try to open this particular valve. As we open this valve p b reduces isn't it p b and p e are equal. So, therefore, when p b reduces p e also reduces and gradually p becomes less than p zero right p becomes less than p zero and when this happens due to this pressure some flow starts occurring. So gradually if we keep on opening this is for 1.0 and p p equals to p zero now when this happens we get something of this particular case where gradually we are reducing it and here in this case p e equals to p b and this is less than p zero agreed and gradually we find that the flow starts why we keep on continuing this we find that we get reduced and reduced pressures here by keep on opening the valve p b becomes equal to p e and as we get such type of curves the flow keeps on increasing in this particular way.

Now this continues till we obtain choked flow conditions here it it keeps we keep on reducing the pressure and we find that the flow rate keeps on increasing. So this goes on till the pressure here is under choked flow choked flow means we denote the properties by means of asterisk. So therefore this continues till we get p star by p zero this I different line. So this is till this portion we get the maximum flow rate in this particular case and this occurs for m a equals to 1 and this this particular point is p star by p zero ok. Now after the this what happens is after this we again keep on reducing the pressure now when we again keep on reducing the pressure what happens is we find that p cannot be reduced any further that becomes constant p b can be reduced, but p b is not equal to p e p e cannot be reduced any further since p e cannot be reduced any further the flow rate cannot be increased any further the flow rate becomes constant after this this is what we find when p b is less than p e this is this particular condition we find that we cannot reduce the flow rate any further ok. And but here what happens is when we are we are keep on reducing it the flow is coming after here what happens the compressible fluid it expands isentropically in order to match this reduction pressure is it clear to you what happens once choked flow condition is arrived or choked flow means p e equals to p b equals to p star till that portion we can p e become is equal to p b and your flow keeps on increasing moment p e p b becomes equal to p star after that if we reduce the flow rate further then what happens p cannot respond anymore ah.

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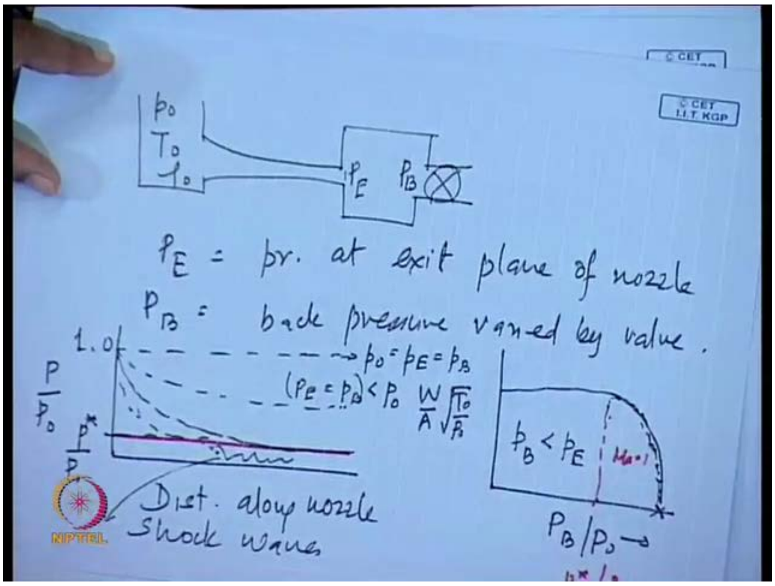

And as a result the flow rate becomes constant the flow rate cannot be increased any further by reducing the back pressure or the exit pressure then in that case what the flow does the flow then what it does it comes here and then in order to to compensate for this mismatch in pressure it expands isentropically from pressure p star to p b you understand and this is in the form of three dimensional waves we cannot approximated by one dimensional flow theory is this portion clear to all of you. So therefore if we reduce the pressure below this then we find that here under this particular condition we have sort of a pressure waves or shock waves yeah these are known as the shock waves shock waves generate. And we cannot reduce the pressure any or rather we cannot reduce the pressure any further and here also the flow rate it becomes constant. So therefore we find that the pressure distribution here at g equals to g g max when p e becomes equal to p star further reduction is not going to be effect the flow any further even though p b can be reduced, but it does not give anything it does not what happens is the flow which leaves the nozzle here it has to expand to match the lower back pressure. And in order to match we find that this expansion which occurs that is three dimensional in nature and the pressure distribution under this condition cannot be predicted by one dimensional theory as a result we we we we cannot predict the pressure we cannot plot it properly here it cannot be done by one dimensional theory is this part clear to all of you well. So therefore remember one thing that for this particular case then what do we get for this particular case therefore, we have we had seen that what what will be the flow rate under this condition we had got the expression for mac number equals to w by a for maximum conditions where did I keep the slide yeah. So we find that for the maximum conditions we can get it for mac number equal to one isn’t it and see when it kept the final slide where I had deduced this particular condition d u d w just a minute this is the case yeah. So what did we find we find that w by a was given by this particular expression and if we put m a equals to 1 here then we are going to get the maximum flow rate and that occurs under choked flow condition where a equals to a star do you get my point.

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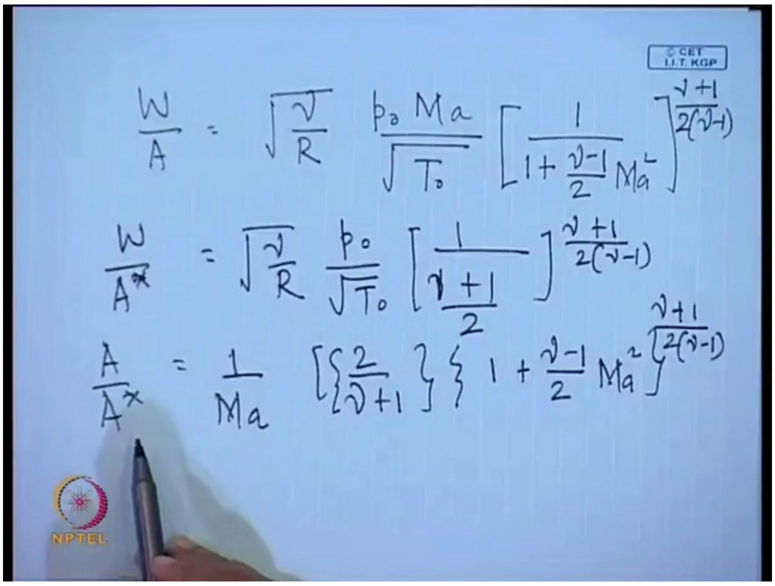

So therefore what do we get is I write down w by a once more this is root gamma this is what that we had obtained now w by a star that is nothing, but equal to root gamma by r p zero by root t zero 1 by gamma plus 1 by 2 gamma plus 1 by 2 into gamma minus 1 isn’t it. So if we divide this by this what do we get we get a by a square isn’t it what is a by a star then we can write it down as 1 by m a plus 1 into 1 plus gamma minus 1 by two m a square please perform these derivations otherwise it is going to be I am just deriving it and we are just coping it down that is not going to help you much please derive these derivations and then do them accordingly. So therefore we find that a by a star that means, area divided by the choked flow conditions this can be obtained from a expression again we find that a by a star is function of mac number only gamma is a constant that may guess is 1.4 and etcetera etcetera we know it. So therefore a by a star is again a function of mac number correct. So therefore since it is a function only of mac number we can plot or we can represent graphically the variation of a by a star with mac number yes.

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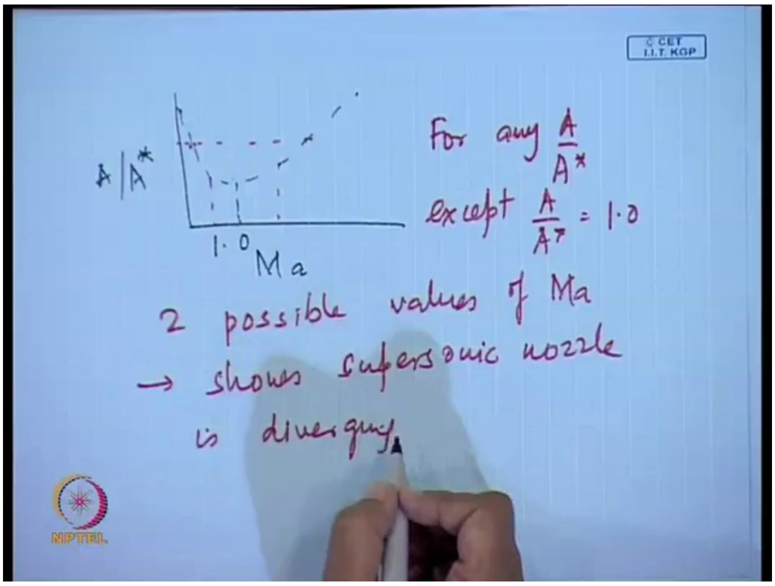

So if we do it if we plot a by a star as a function of mac number what do we get we get something of we get a graph or curve something of this sort where this is obtained at m a equals to 1.0. So therefore what do we find from here we find that for every mac number there is a unique a by a star listen to what I am telling very carefully for every mac number if you take up any particular mac number you find a unique value of a by a star and this is very much evident from this particular equation also one mac number you get a unique a by a square agreed, but if you take for any a by a star corresponding to any a by a star we get two values of mac number 1 is for less than 1 and 1 is for greater than 1 taking into account the existence of subsonic nozzles and supersonic nozzles ok. So therefore please remember that for any a by a star except your a by a star equal to 1.0 there are two possible values of m a. So this shows supersonic nozzle is divergent. So this is clear to you. So therefore we find. So from here we find that why your a by a star is so very important and we find that for any particular mac number there are two possible sorry for any particular mac number there is one unique value of a by a star, but for any a by a star there can be two values of mac number except for the condition of mac number equal to one. So therefore we conclude here and in the next class we will just be referring to the denominators which you had obtained for your compressible flows and to homogeneous flow and then we will try to derive certain things from it thank you very much.