Multiphase Flow
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Lecture No. # 14
Drift Flux Model (Contd.)

Well. So, good morning to all of you. We will be continuing our discussions regarding the drift flux model, the things which we were doing in the last class what we did? We discussed about the advantages of the drift flux model and the concepts of drift flux. How it modifies their different mixture parameters namely the void fraction and the mixture density, the local velocities and so on and so forth.

So, we understood that if we have some idea regarding the estimation of drift flux; then once we can incorporate drift flux into the equations which we had discussed in the last class. Then we will be in a position to predict mixture properties much more accurately and accordingly we can predict the hydrodynamics of two phase mixed flows as well as transitional flows in a much more accurate fashion.

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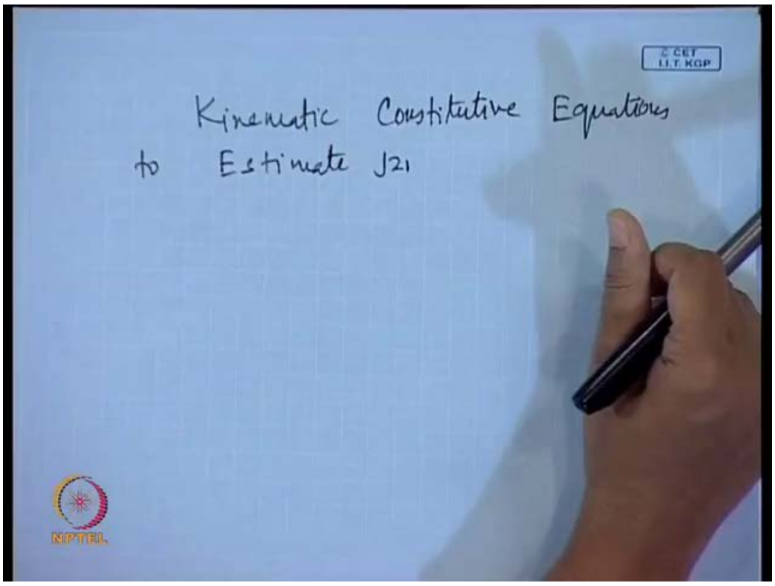

So, today we will be continuing our discussions regarding the different ways or rather the approach to estimate J 2 1 by using different kinematic constitutive equations. So, we will be discussing basically the kinematic constitutive equations to estimate J 2 1. Now, in this particular regard I would like to say one thing that there are basically two approaches in order to estimate these kinematic constitutive equations.

Now, one thing is for sure that this particular model is particularly more useful when the relative motion it can determined by a few key parameters and it is independent of the flow rates of each phase; then it is much more useful. Now, usually there can be two approaches to find out the kinematic constitutive equation in order to estimate J 2 1. Now, what are the two approaches?

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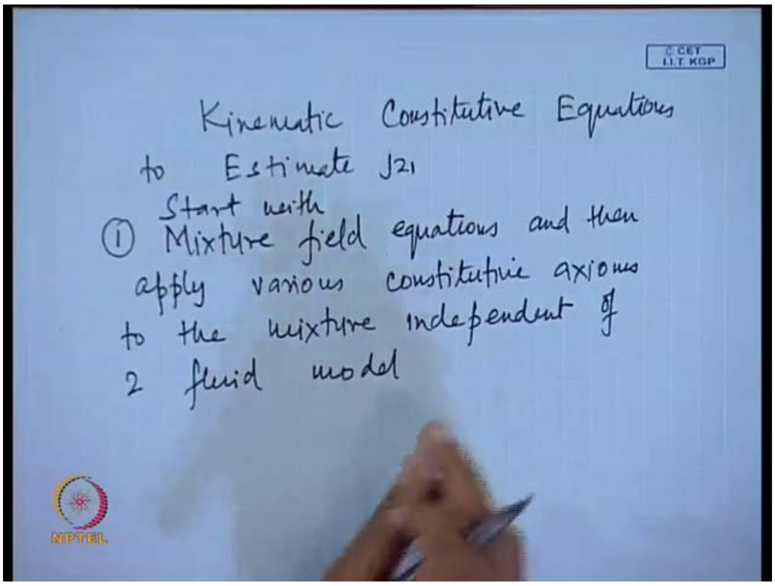

One can be that you consider the mixture as a whole; and then what we do is that we start with the mixture field equations. So the two approaches the first approach is that we start with the mixture field equations and then we apply various constitutive axioms or rather various constitutive laws to the mixture. Now, this is one approach and this seems to be quite logical. What we do? We consider the mixture as a whole, because in this case we haven't concentrated on the individual phases what we have done?

We have concentrated on the mixture as a whole. So, what we do? We concentrate on the mixture and then we try to apply different constitutive axioms to the mixture as a whole without considering their individual movements or rather without considering the two fluid model. Two fluid model means what? We will be dealing with it in much more details in the next chapter the separated flow model.

That means we would be considering the two phases separately and we would like to write down the momentum, the continuity and energy equations for the two phases. So, the first approach is we do not consider the 2 fluid model; we consider the mixture as a whole and in the mixture field equations; we use different constitutive axioms and which is applied to the mixture as a whole and it is independent of the two fluid model; this is one approach.

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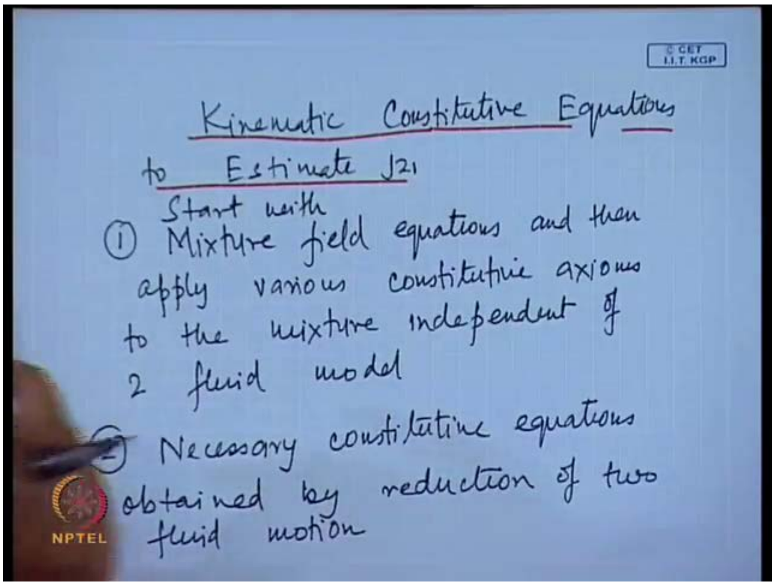

The second approach is that you consider the two fluid model; in this particular thing the necessary constitutive equations obtained by reduction of two fluid model. So, this is the second approach; what we do? We consider the two fluids separately in whatever way they are mixed.

And then considering the two fluid model we try to arrive at the necessary constitutive equations; this is the second approach. Now, logically when you think you will always be tempted to think that may be the first approach is much more logical and that should be used, because here we are considering the mixture properties as a whole. So, logically you will think that well we should consider the mixture field equations and then we should apply the various constitutive axioms to the mixture and it should be independent of the two fluid model. But if we notice properly you will find that there are certain problems in using the first approach. Now, what are the problems? The first problem arises from the fact that the two phases are generally not in thermal equilibrium. Now, when they are not in thermal equilibrium we cannot define a particular temperature for the whole mixture.

Now, if we cannot define a particular temperature for the whole mixture then in that case we cannot define the mixture properties as a whole. The second thing is that we will also notice that the kinematic and the mechanical state between the two phases that is greatly influenced by the interfacial structure and their properties. So, therefore, we find that since the interface properties they are changing.

Or in other words the point is under certain flow conditions you are having bubbly flow; and the certain under other flow conditions you are having flux flow. So, therefore, we find that whether the dispersed phase exists as bubbles or as Taylor bubbles; or may be as churns. This influences the constitutive equation; this influences the mechanical and the kinematic state between the two phases.

So, as a result if we consider the mixture as a whole then in that case we are not free to observe what is happening between the two phases? How they are distributed? So, therefore, we find that, this first approach using the mixture field equations and then applying the various constitutive axioms to the mixture that usually it is not very preferred. And for most of the cases we obtain the constitutive equations by reductions of the two fluid model.

Now, in order to use this or rather to incorporate these particular effects, the effect that the two phases may not be in thermal equilibrium the kinematic as well as the mechanical state is greatly influenced by the interfacial structure and their properties. In order to account for these particular factors, we find that it simpler as well as more realistic to obtain the equations from the two fluid formulations rather than the formal approach.

So, accordingly, what is usually done? Usually, the two fluid formulations are done; the momentum equations for the two phases are written down separately. And from that particular momentum equation naturally those momentum equations will be considering some particular term which arises due to the relative motion. From there the relative motion term is obtained and it is accordingly some constitutive equation is proposed for it, and that is used in the drift flux model; this is the approach which is used.

Now, is this portion clear to you or should I repeat this part once more? It is clear to you more or less. So, therefore, what we do remember one thing that in order to estimate J 2 1 there are usually two approaches that we can use; one is we consider the mixture as a whole and then from there we can find out the constitutive equations. Now, if we have to do this, then the mixture has to have more or less uniform properties throughout.

But what we find? We find that generally the two phases they may not be in thermal equilibrium and more over their kinematic as well as their mechanical states it is greatly influenced by the structure and properties of the interface. So, therefore, when the interface changes, this kinematic state that is also going to change. Now, if that happens then in that case we cannot derive an accurate constitutive equation by using the mixture field equation.

What is more accurate? We take up the two fluid formulation; we consider the presence of the two fluids separately. And, how will I consider that the two fluids? They can be in bubbly flow slug flow churn flow interaction parameters are going to change. So, we will be incorporating theses particular the effect of the flow patterns in the interaction parameters which shall be incorporated in the momentum equation written for phase one; in the momentum equation written for phase two. And from then we would like to see how the expression of relative motion can be derived or the physics behind deriving the relative motion between the two phases. Now, let us then start writing the relative or rather the momentum equation for the two phases.

Now, if we consider the two phases, I will just write down the momentum equation I will not go into much details; in the next chapter when we deal with the separated flow model I will just write down the basic equations and then see what best I can do with those equations in order to arrive at the kinematic constitutive equation to obtain J 2 1. Now, whenever we write down the momentum equations say maybe in the three dimensional form; well I have it here itself.

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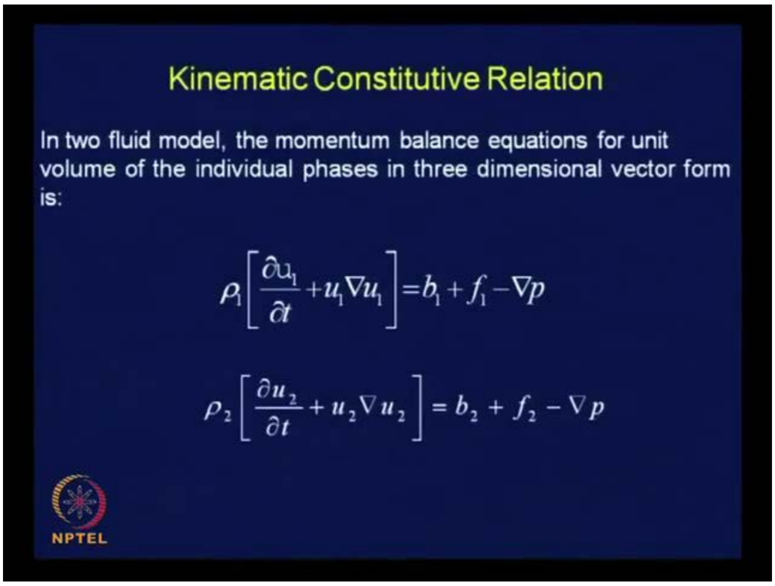

So, in the three dimensional form you find it is it resembles probably the navier strokes equation which you have already done. We have a time dependent term; we have an inertia dependent term and then b 1 and b 2 they are the body forces per unit volume of the fluid element; that means, this is written for phase one, this is written for phase two.

Now, for phase one naturally we consider unit volume of phase one and here we consider unit volume of phase two. So, therefore, b 1 is the body force which is nothing, but the gravitational force arising due to gravitational acceleration. So, b 1 and b 2 are the body forces per unit volume of components one and two which act on the respective components. Delta p this is nothing, but the pressure gradient. So, delta p it is the pressure gradient and it is the average pressure gradient or the bulk stress which is suitably defined, it is usually the pressure difference of one or both the phases. And what about this term f 1 and f 2?

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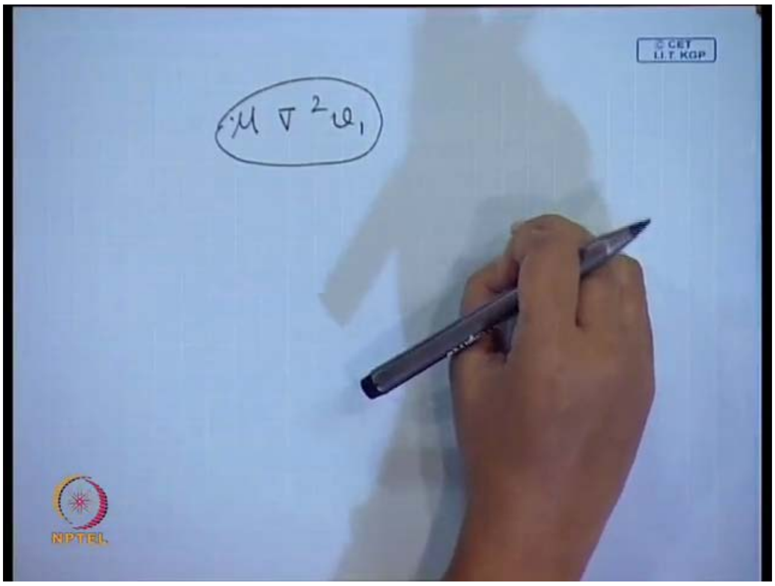

Do you remember in navier strokes what was the other term which was there was mu is it not? And therefore, for fluid one it will be this for fluid; it will something else?

So, this arose from the viscous terms. So, the point is in this particular case what is f 1? f 1, f 2 are simply what is left over which have to be incorporated in order to keep the account straight; this is what you should remember. Now, whatever, when you write down the momentum equation body forces are there, pressure difference is there apart from this whatever other term should come which is not included in the pressure gradient that is included in f 1 as well as f 2.

So, therefore, f 1 f 2 they are simply incorporated in order to keep the account straight. If you observe this particular equation you will find that f 1 and f 2; they have been incorporated just they are just left over forces per unit volume of the corresponding phase. They are simply incorporated to complete the momentum balance equation. So, therefore, when we were considering that it is incompressible Newtonian fluid containing only one component which is not undergoing phase change then f 1 will be equal to this.

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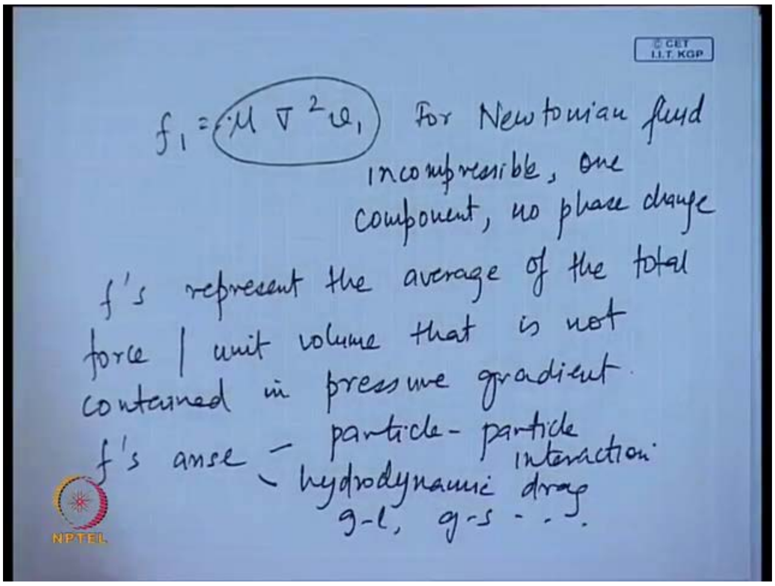

So, this you already knew for Newtonian fluid incompressible, one component and no phase change; this we have already derived. Now, usually what we find? We find that this f is they represent the average of the total force per unit volume that is not contained in pressure gradient. So, therefore, from where can f is arise? This f is therefore, they can arise in this particular case it was the wall shear stress. Apart from wall shear stress they can arise from particle to particle interaction if it is gas solid or liquid solid flow.

Then they can arise from particle to particle interaction; they can also arise from hydrodynamic drag. The drag between hydrodynamic drag and in other words it is the two face drag; it can be between gas liquid; it can be between gas solid; it can be anything. So, it can arise due to the hydrodynamic drag. Suppose, there is evaporation condensation etcetera; then what is happening? One particular phase it is shifting from say the liquid phase to the vapor phase or vice versa. So, therefore, due to this there is a momentum change of some portion of the fluid which is actually undergoing phase change. So, therefore, due to that there is some momentum change. So, that can also be incorporated in f.

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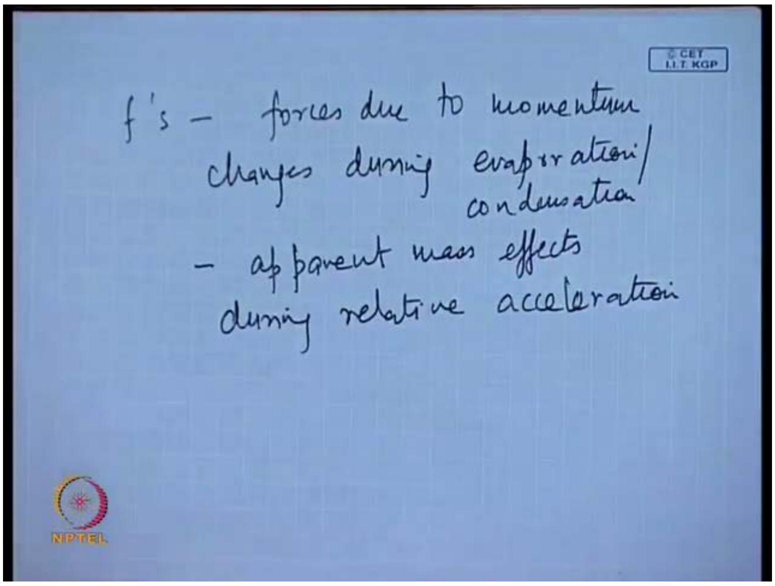

So, therefore, when there is a phase change then forces due to momentum changes during evaporation slash condensation; or it can also be the apparent mass effects during relative acceleration this can also happen.

So, therefore, we find that whatever the leftover force which is not accounted for the pressure drop thing, that particular force that is included in f 1. So, therefore, when it is a Newtonian fluid and incompressible in single phase Newtonian fluid flowing through a pipe then in that case you find that your f 1 is nothing, but it arises due to the wall shear stress it is this particular thing. Now, when there are two phases we find that it can arise due to number of situations.

One is interaction between the wall and the fluid maybe interaction between wall and fluid one wall and fluid two. So, therefore, wall and fluid 1 should be included in this f 1; wall and fluid 2 should be included here. It can be the hydrodynamic drag; that means, fluid 1 fluid 2 that will also be included; fluid 1 with respect to 2 will be included in f 1, fluid 2 with respect to 1 will be included in f 2.

Now, when there is some particular condensation, evaporation something then some portion of the fluid it is changing the phase. Now, we know that both the fluids are moving at different velocities. So, therefore, when you are changing the phase state and evaporation is occurring some amount of liquid is going into the vapor phase. Therefore, it is changing its velocity say from u 1 to u 2. So, due to this there has to be a momentum change due to this velocity change. So, that will be incorporated in f 1 and f 2. And apart from this of course, the apparent mass effects during relative acceleration that can also be included.

So, whatever is not included in the pressure difference that comes under f 1 and f 2. Now, remember one thing that quite frequently some portion of the effect is included in delta p; some portion is not included only that portion has to be included in f 1 and f 2. So, these things you have to be quite cautious about always it is not very easy to segregate the forces which is not included in delta p it has to be in f 1, f 2; these have to be kept in mind clear.

So, therefore, whenever you write down a momentum balance equation, what are the things? Definitely, there are the left hand sides and in the right hand side you have force due to simply the weight of the fluid which is included in b one b two; then the pressure gradient and whatever is left over. That left over thing that depends on the exact flow situation whether it is a change of phase situation; whether the hydrodynamic drag is important; whether the wall drag is important. So, f 1 f 2 they depend upon the actual flow situation and according to the flow situation f 1 f 2 is different. And that is why using the two fluid model we arrive at different equations for different two phase flow situations. It is just because of the incorporation of f 1 and f 2.

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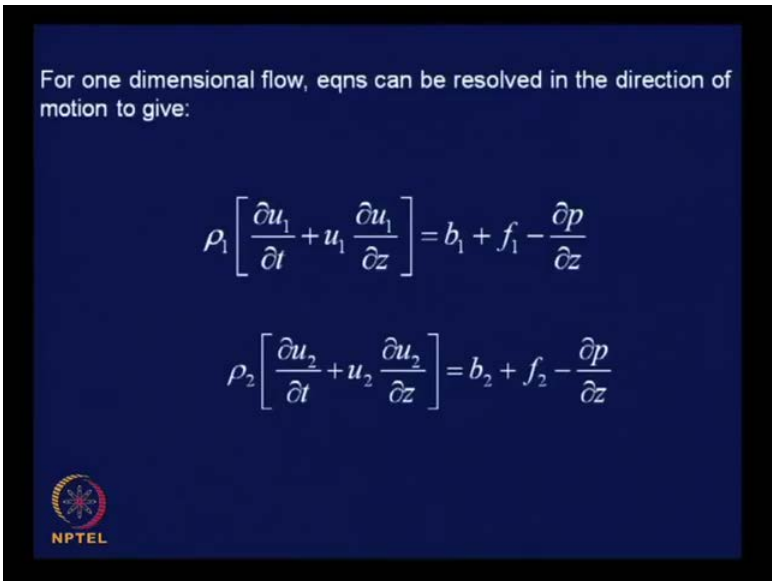

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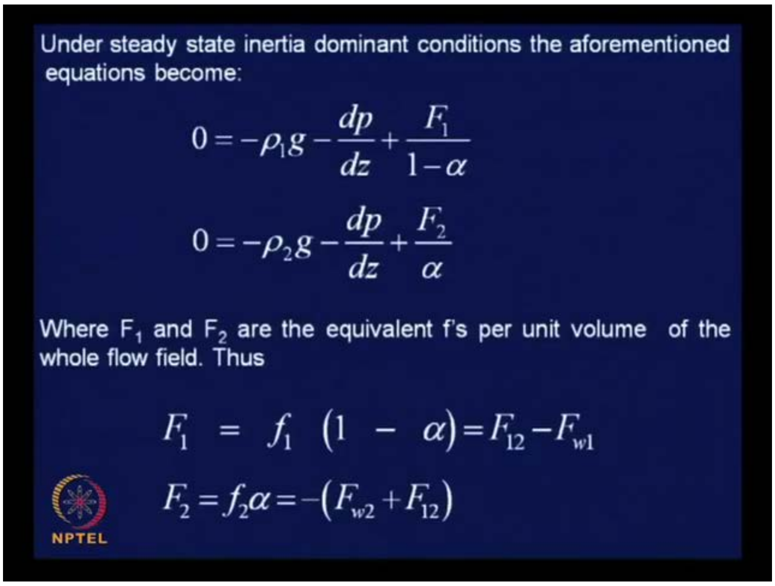

Now, usually as what we do? We usually take up the one dimensional approach. So, in the one dimensional approach, if we take equations reduced to this particular form which is quite evident to you. Now, in steady state conditions, if we find that for steady state conditions when the inertia dominant conditions are there; then naturally the left hand side it disappears of the left hand side which was here.

This disappears off and this portion becomes equal to zero which I have written down. B one is nothing, but minus rho one g. I have considered the direction of flow to be positive or the upward direction to be positive. So, naturally your b 1 becomes minus rho 1 g, b 2 becomes minus rho 2 g. One dimensional, therefore, they become minus d p d z. Now, in this particular case you tell me, what should be included in f 1? What should be included in f 2?

Two phases are flowing; how they are flowing? How they are distributed? We are not concerned about it, but we know in whatever way they are flowing, in whatever way they are distributed more or less what will happen? What will be the forces acting on fluid one other than the pressure drop force? It has to arise from the wall and it has to arise interaction between fluid one and fluid two; these two forces have to arise.

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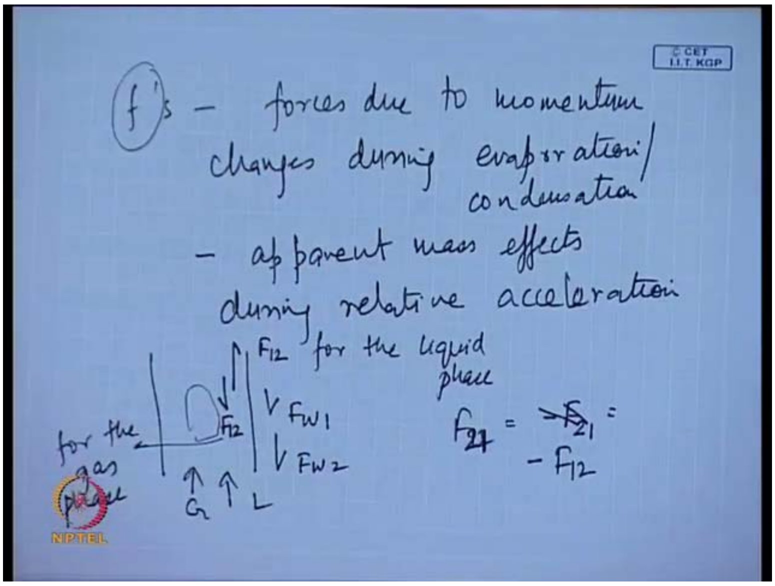

Now, considering the introduction of the fluid with the wall naturally, if the flow direction is in the upward direction thus interaction will be in the opposite direction. So, suppose you say that two fluids are flowing in this particular way since both the fluids are flowing your F W 1 and F W 2 they should be in the downward direction; quite natural. The other thing is there will be interaction between the gas phase as well as the liquid phase.

Now, assuming that the phase two is the gas phase of whatever it is the phase two is a discontinuous, it has a higher velocity as compared to the liquid phase. So, therefore, the hydrodynamic drag in which direction is it going to act? It will be acting in the direction opposite to the direction of motion for the gas phase and it will be acting in the direction of motion for the liquid phase; just like we decide here.

So, this should be the thing and this F 1 2 this is for the gas phase. You can take it as F 2 1 also, but if it is mutual hydrodynamic drag then in that case cannot we say F 2 1 is nothing, but equal to minus F 1 2. If you take F 2 1 then it is fine; you can take it in any direction.

But since, I think both these forces they are equal and opposite. So, therefore, F 2 1 will be equal to minus F 1 2; that is why I have not differentiated between these two. Otherwise, what I would have done? F 1 2 for the liquid phase and F 2 1 again upward direction for the gas flows; gas to liquid, liquid to gas. Now, we know that gas to liquid and liquid to gas the hydrodynamic drags are equal and opposite.

So, instead of F 2 1 in the upward direction I put F 1 2 in the downward direction for the gas flows. So, these are the forces which should act. Now, remember one thing, when I was defining this particular F repeatedly I have told you one thing this is per unit volume of that individual place. Do you remember this thing? That f 1 is the left over force per unit volume of phase one, f 2 is the left over force per unit volume of phase two.

Because both these equations if you observe this equation and this equation, this is written down per unit volume of phase one; this is written per unit volume of phase two. Now, if we combine the mixture as a whole, then this small f 1 and small f 2 these things if they have to be expressed in terms of per unit volume of the total mixture. Then in that case, in what way it should be expressed?

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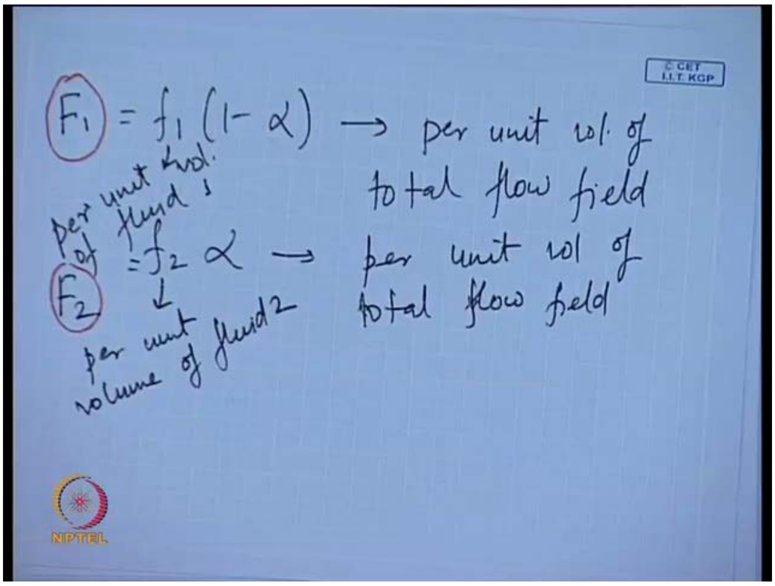

In that case, f 1 into one minus alpha is per unit volume of total flow field. Tell me if there is any doubt regarding this; F 2 alpha is per unit volume of total flow field where F 2 is per unit volume of fluid two; F 1 is per unit volume of fluid one. This particular portion, this particular transformation which has to be done and this is denoted as F 1; this is denoted as F 2.

Now, tell me whether this particular part is clear to you. What we did? First, we found out that finding out the constitutive equation it is much more advantageous to use the two fluid model. What is the two fluid model? We consider the two fluids separately; we write the momentum equation for fluid one; we write the momentum equation for fluid two. We did it in the three dimensional form in this particular way.

We are always considering one dimensional form. So, this is the equation that we get. Now, in this particular equation the only thing which has to be decided is regarding f 1 and f 2. Now, the point is this f 1 and f 2 they should consider for interaction of fluid one with the wall interaction of fluid, one with fluid two. For interaction of fluid two with the wall; interaction of fluid two with fluid one.

So, therefore, it should contain something like F W 1 and F 1 2; and the other one F W 2 and F 2 1. Now, we know that the drags at the interfaces they are equal and opposite. So, F 2 1 equals minus F 1 2. So, therefore, F 1 should contain F W 1 and F 1 2, and your F 2 should contain F W 2 and minus F 1 2. Now, what about the directions of these two

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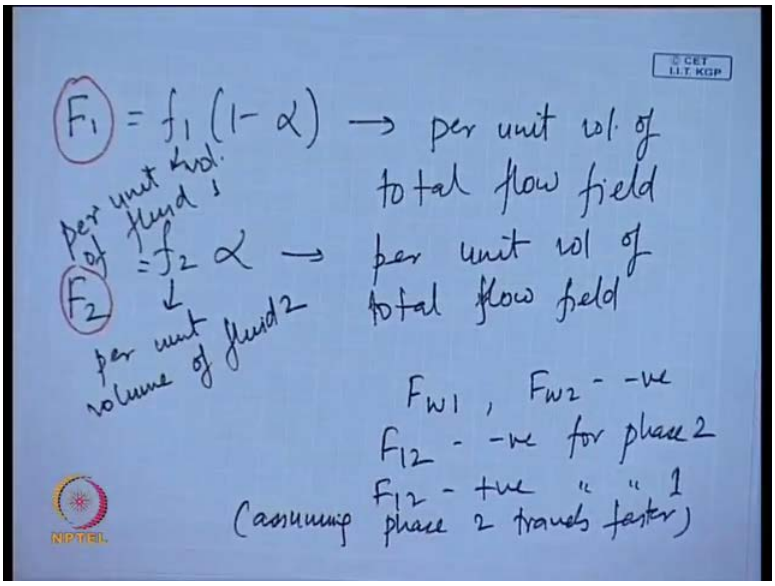

things remember regarding the directions we have considered the upper direction as positive.

So, then in that case your F W 1, F W 2 they will be negative. F 1 2, it will be negative for phase two and F 1 2 it will be positive for phase one; assuming phase one, assuming phase two travels faster. It is the lighter phase on this assumption these are the sign conventions which we can use. So, therefore, in F 1 and F 2 we have F W 1, F 1 2, F W 2, F 1 2 or minus F 1 2.

Now, you try to understand this F 1 and F 2 they were per unit volume of that particular fluid. Now, if we have to equate see F W 1, F W 2 or in other words F 1 2 and F 2 1; if we have to relate them, then they have to be expressed on one particular volume basis. It cannot be your volume of phase one per unit volume of phase one and per unit volume of phase two; in that way we cannot correlate F 1 2 and F 2 1. Can we? Both of them have to be expressed on the basis of the same volume element. What will it be the mixture volume? So, therefore, they have to be expressed in terms of per unit volume of the mixture. Do you agree?

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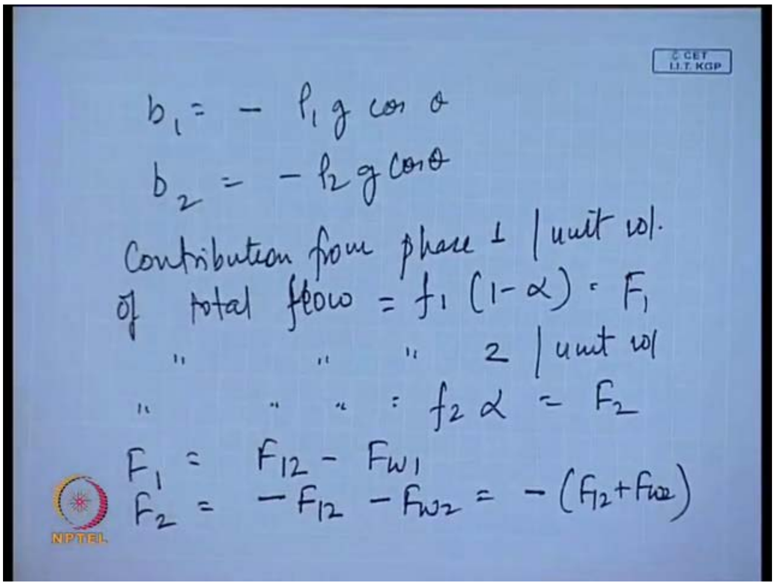

Now, if we take per unit volume of the mixture then also your b 1 that is going to be minus rho 1 g for vertical or else minus rho 1 g cos theta; or something in this particular case.

b 2, it will minus rho 2 g cos theta, I will write it down; it is cos or sin whatever the case may be. Even it is per unit volume of the entire flow field also these things are not going to change. Do you get the point? Because per unit volume these are the other things which are remaining, but if we consider per unit volume of the flow element then in that case the contribution from phase one per unit volume of total flow, then this will naturally become f 1 into one minus alpha. Do you get my point?

Because in that per unit volume there is alpha volume of phase 2; one minus alpha volume of phase one. So, therefore, the net contribution of one on unit volume of the total flow had to be f 1 into one minus alpha; this we simply denote as F 1. Similarly, contribution from phase two per unit volume of total flow, this will be f 2 into alpha which we denote as F 2. And where we find? What is F 1 equals to? F 1 2 minus F W 1 considering the signs.

What is F 2 equals to? This will be minus F 1 2 minus F W 2; in other words minus F 1 2 plus F W 2. So, therefore, instead of the F and the F 2 which we have got here, we have

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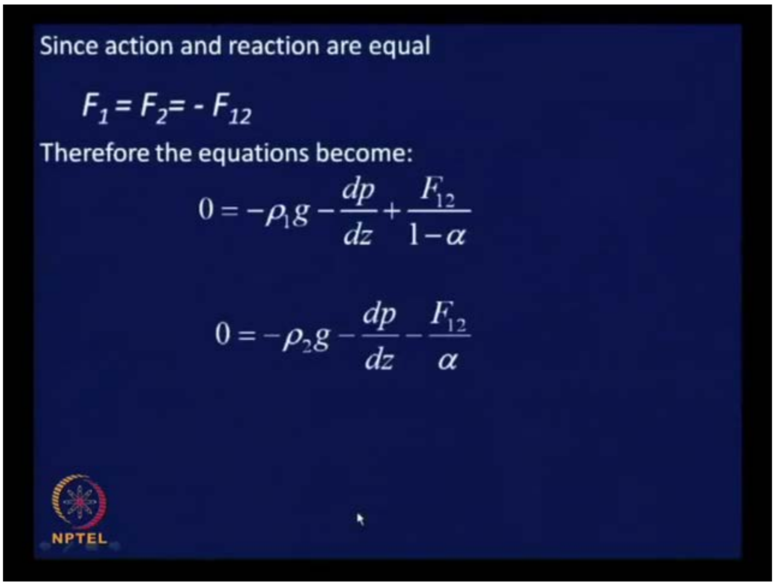

to substitute instead of F 1 we can substitute F 1 by 1 minus alpha and in that F 1 by 1 minus alpha we can substitute this thing by 1 minus alpha. Anything you do not understand; you tell me to repeat.

Similarly, instead of F 2 we have to substitute F 2 by alpha; F 2 by alpha means this by alpha or this by alpha. We have simply done that substitution and we have got this particular. Accordingly, I have written it down here also we can make these substitutions and finally, we arrive at these two equations for the two fluid model. Now, remember how have we accounted for the different interfacial phase distributions? By different expressions of F 1 2; that is the way we have tried to incorporate the different interfacial distributions. Now, if we subtract one equation from the other what do we get? If we subtract say equation two from equation one or something. Then in that case what do we expect to get? You just subtract it and then tell me. What is the equation that you are going to get on subtracting?

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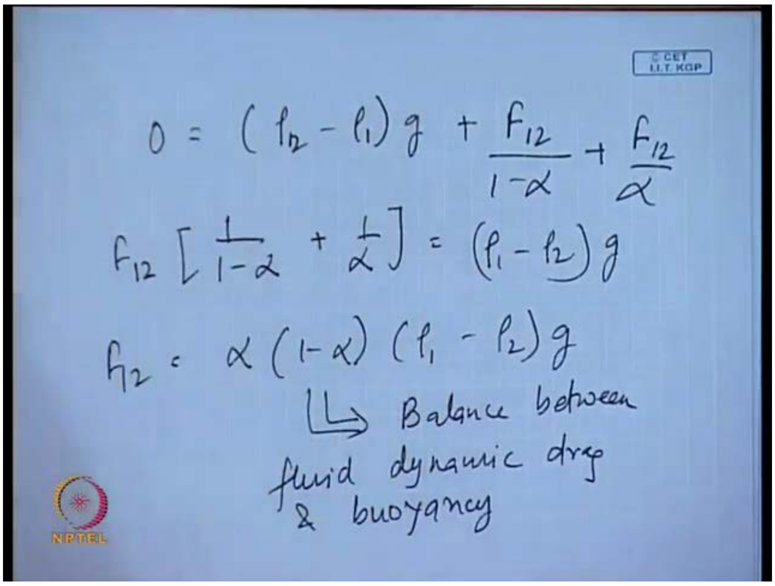

We will get something like zero equals to rho 2 minus rho 1 into g plus F 1 2 by 1 minus alpha plus F 1 2 by alpha; just by subtracting we get this.

Or in other words if we want to express F 1 2 then this become 1 by 1 minus alpha plus 1 by alpha; this is rho 1 minus rho 2 into g; or in other words F 1 2 equals to what it has to be alpha into 1 minus alpha into rho 1 minus rho 2 into g. And this particular equation what does it represent? It represents a balance between fluid dynamic drag and buoyancy.

So, thing is what we did first? We first wrote down the momentum balance equations and then we took it for steady state conditions under inertia dominant. If you see the p p t, you will notice that initially what we did? We did it for the steady state inertia dominant conditions we got this. Then we substitute F 1 and F 2. They will be having terms arising from hydrodynamics drag and wall shear stress. Now, usually what we have done is we have neglected the wall shear stress effects.

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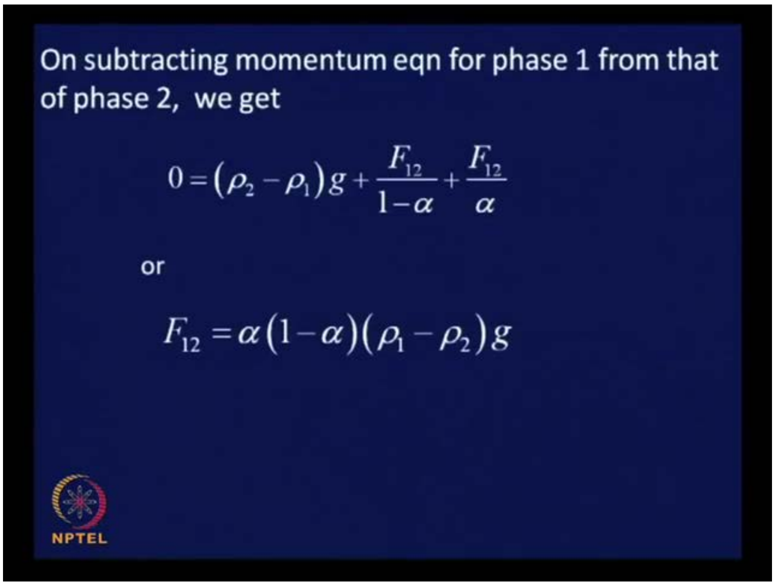

We have tried to consider that particular situation where hydrodynamic drag is much more important. Under such a situation we have written down the two equations. This is where we can neglect the wall shear stress and basically this gives a balance between your buoyancy as well as the fluid dynamic drag. Then we have subtracted one equation from the other and we find that in the absence of wall effects. Remember, this is something very important we have obtained this particular equation only under a situation where hydrodynamic drag is important and that is balanced by buoyancy.

We have neglected your wall interaction and definitely, if this is applicable for gas liquid cases; if it is particle fluid cases then we have also neglected particle to particle interaction; it is just hydrodynamic drag and your buoyancy. From that particular balance in the absence of wall effects under steady state conditions for inertia dominant cases we have got this particular equation. And from this particular equation what do we find? This particular equation which has been obtained as a balance between buoyancy and fluid dynamic drag what do we get?

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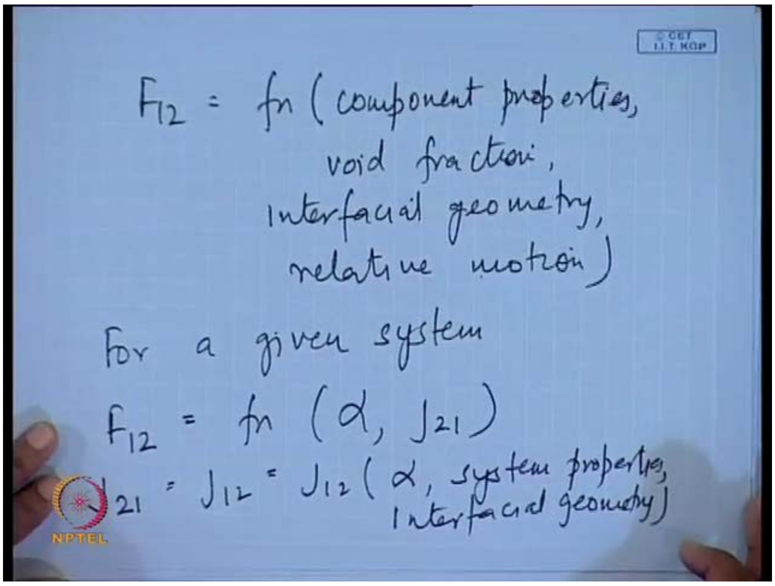

We find that F 1 2, it is a function of component properties; it is a function of void fraction and naturally void fraction is a function of interfacial geometry, and it also has to depend upon the relative motion. So, from this particular equation what we get? We get F 1 2, it is a function of one is component properties if you see it logically you will find that these are things on which F 1 2 depends.

Then it has to be void fraction. Now, void fraction for all flow patterns the relationship or variation of void fraction is not the same; void fraction depends upon interfacial geometry and definitely relative motion; on these things F 1 2 has to depend. Now, if we consider a given system what do we find for a given system? Component properties they become constant and void fraction is dependent upon the interfacial geometry; your relative motion is dependent upon your interfacial geometry.

So, therefore, for a given system F 1 2 that becomes a function of alpha and the relative motion. Do you get my point? Accordingly, we can also write therefore, if F 1 2 is a function of all these things, then J 2 1 or J 1 2; in whatever way that should also be a function of your alpha and your system properties as well as interfacial geometry. Tell me if any questions they cannot interact much with you. So, you should be telling me to repeat the things or whether you have understood or not understood; you should communicate with me. So, therefore, from the basic equation which we had got, from this particular basic equation what do we get?

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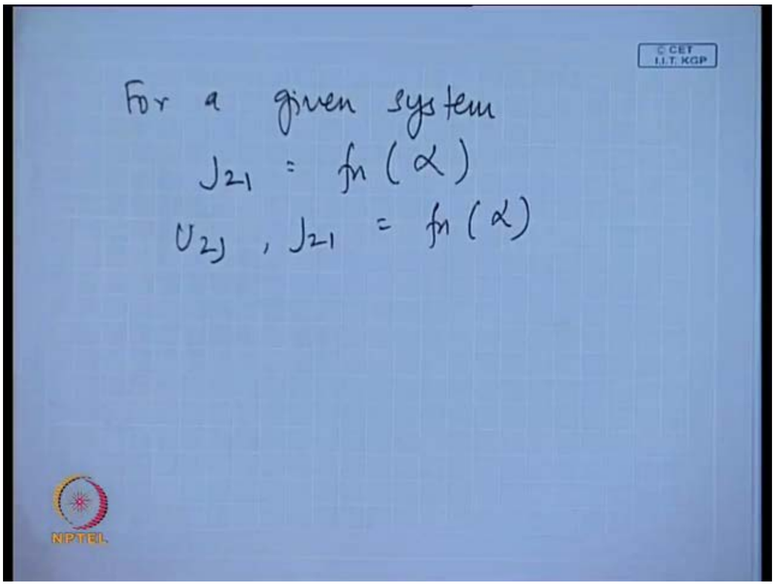

F 1 2, it is a function of component properties, void fraction interfacial geometry or interfacial configuration and your relative motion. Relative motion in whatever way you can express it. It can be relative velocity; it can be drift flux whatever it is. So, then automatically from this particular equation what do we get?

Then J 2 1 or J 1 2 whatever that should be a function of component properties, void fraction and interfacial geometry. And therefore, for any particular given system, if a system becomes fixed then naturally your system properties become fixed. And alpha and interfacial geometry they are dependent on one another. So, therefore, for a given system J 1 or J 2 1 that is a function of alpha only. Did you get my point?

So, what do we find? We find that the relative velocity or the drift flux both of them U 2 J as well as J 2 1, they are function of alpha only; they depend upon the drag forces acting at the interface as well as the interfacial geometry. So, therefore, your relative velocity as well as your drift velocity they are a function of alpha only, but this functional form it should be different for different interfacial structures.

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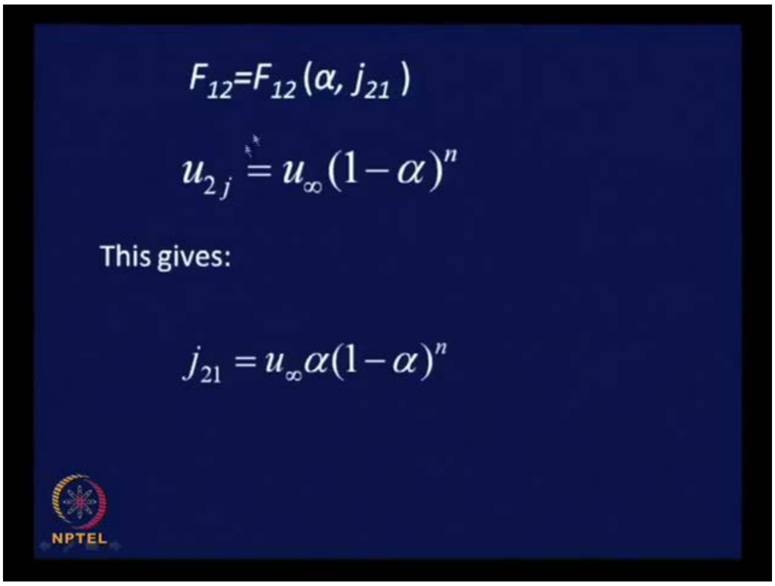

Is this part clear to you? What did we deduce? F 1 2 is a function of component properties I think I have got a slide over this. F 1 2 we found out that it is a function of component properties then void fraction, interfacial geometry, relative motion etcetera; from this p p t also I have written it down. Now, if that is true then in that case we find J 2 1, that should also depend upon void fraction system properties etcetera.

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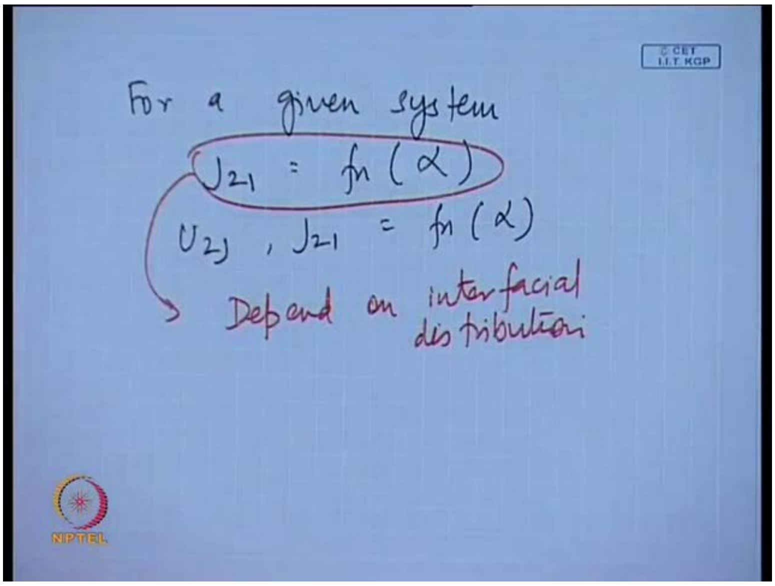

Now, moment we fix up the system then J 2 1 should be a function of alpha only. And the functional form of this particular equation that should depend upon interfacial distribution. Or in other words the type of equation which will describe the relationship between J 2 1 and alpha that should be different for different flow patterns. And I will be giving you the set of equations which are used for different flow conditions.

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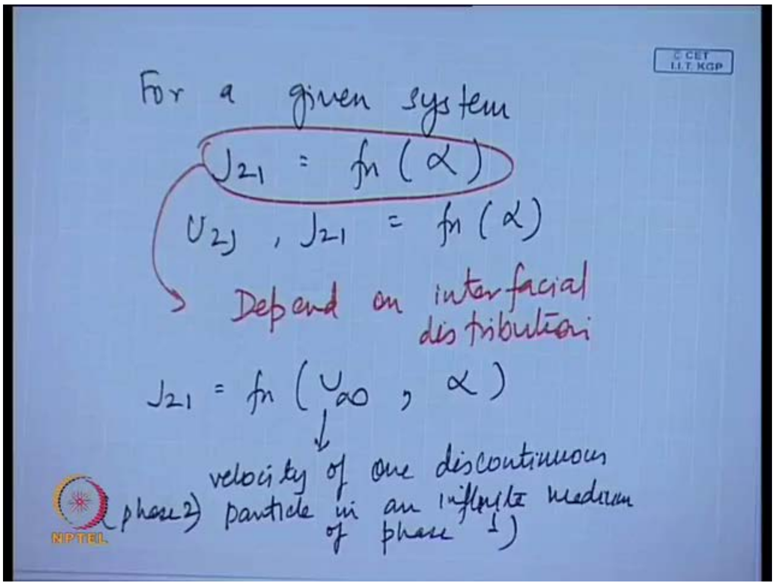

But in general, with lot of experiments what people have found? People have found that more or less your J 2 1 that depends upon usually two things; one is it can be expressed in terms of say there is a discontinuous phase and a continuous phase. So, it depends upon the velocity of one discontinuous particle in an infinite medium of phase one.

Velocity of one discontinuous particle means velocity of one discontinuous particle of phase two an infinite medium of phase one and it also depends upon alpha.

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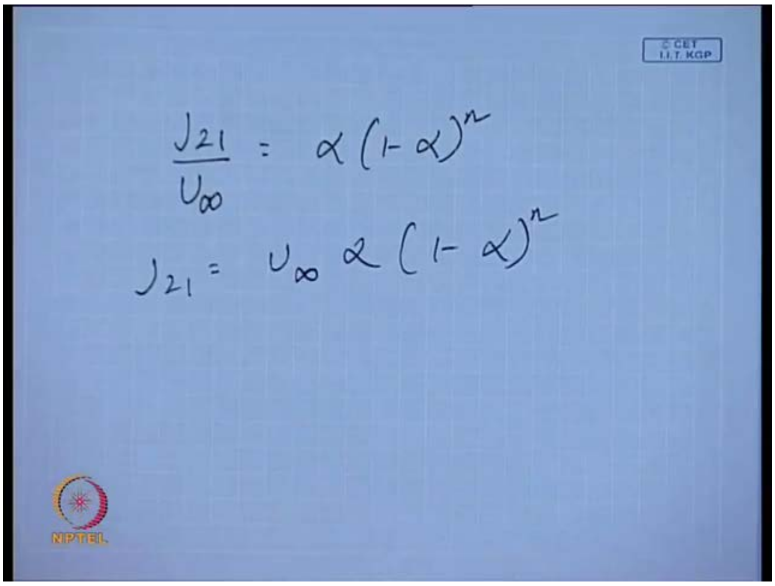

And usually this particular functional form that people have obtained as J 2 1 by U infinity equals to alpha into 1 minus alpha whole to the power n; or in other words you can also write it down as J 2 1 equals to u infinity alpha into 1 minus alpha whole to the power n.

So, what people have done? People have tried to find out some particular relationship between J 2 1 and alpha. And what did people find? People found out that usually for a wide range of flow conditions maybe for bubbly, for slug, for churn, for fluidized bed, for a wide range of conditions people have found that a generalized functional form which is given in this particular way. The generalized functional form can be used for different flow conditions. What is the difference?

When we take different flow patterns only the value of u infinity and n are different for each of the flow patterns. If you take a fluid particle system, you will have some value of u infinity and n; for bubbly flow some value of U infinity n; for slug flow some value of u infinity and n. In this way we account for the influence of the different flow patterns on J 2 1 by using this particular equation. So, therefore, for all flow conditions we find J 2 1 is a function of alpha.

The functional form can be represented by a generalized equation given in this particular functional form. And this is a general equation for all types of flow patterns which can be predicted by the drift flux model, but for different flow patterns this value of u infinity and n are different. What is then? It is simply a function or constant which varies with flow patterns; what is u infinity? It is the velocity of a single discontinuous phase in an infinite medium of the continuous phase.

If it is gas liquid bubbly flow, it is the rise velocity of a single bubble in an infinite liquid medium; if it a fluidized bed sort of a system then in that case it is the terminal velocity of a single solid particle falling in an infinite medium of the fluid. If it is slug flow then u infinity is the velocity of a single Taylor bubble without the wall effects. So, accordingly, U infinity is different for different flow situations, n is different for different flow situations and accordingly by incorporating different values of U infinity, and n we can find out J 2 1 for different flow situations correct.

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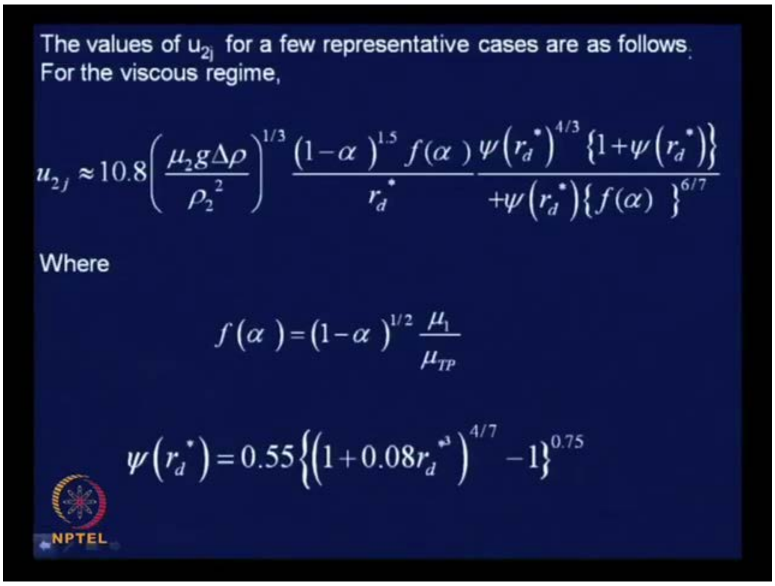

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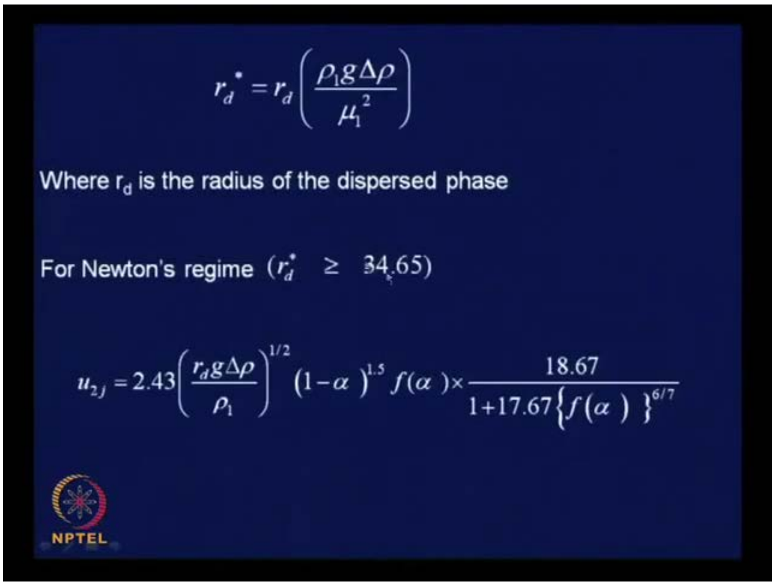

Now, for certain case the value of U 2 J and J 2 1 has been proposed in several text books I have just written down these particular equations for your convenience. So, for the viscous flow regime these equations they are just for you to note, you need not memorize them or you need not remember them; for the viscous regime this is the equation. Then for the Newton's regime this is the equation and then for distorted fluid particle again we have different things.

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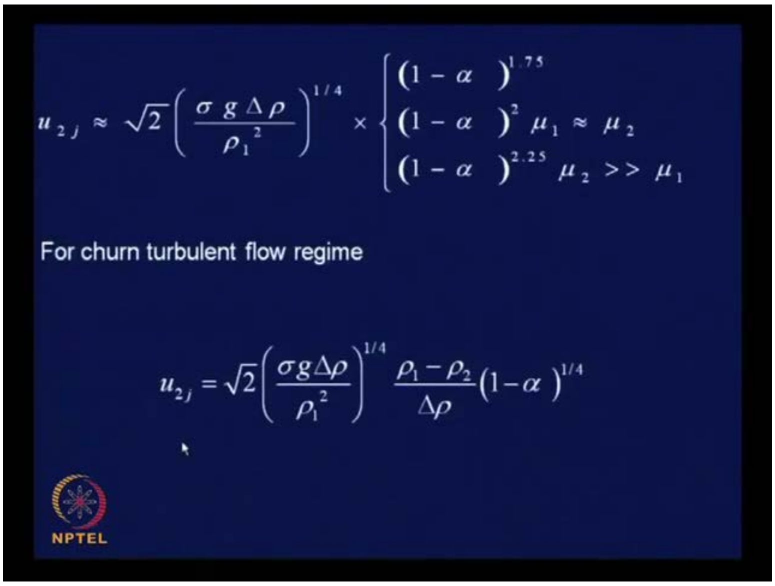

For the churn turbulent flow regime these are equations for U 2 J, we know that alpha into U 2 J is nothing, but J 2 1. And just I would like to mention what is this churn turbulent and for the slug flows probably this is the equation. For the churn turbulent flow regime, it is a bubbly flow pattern where the bubbles are can be of different sorts of sizes and shapes. It is just a transition between the bubbly flow pattern and the slug flow pattern.

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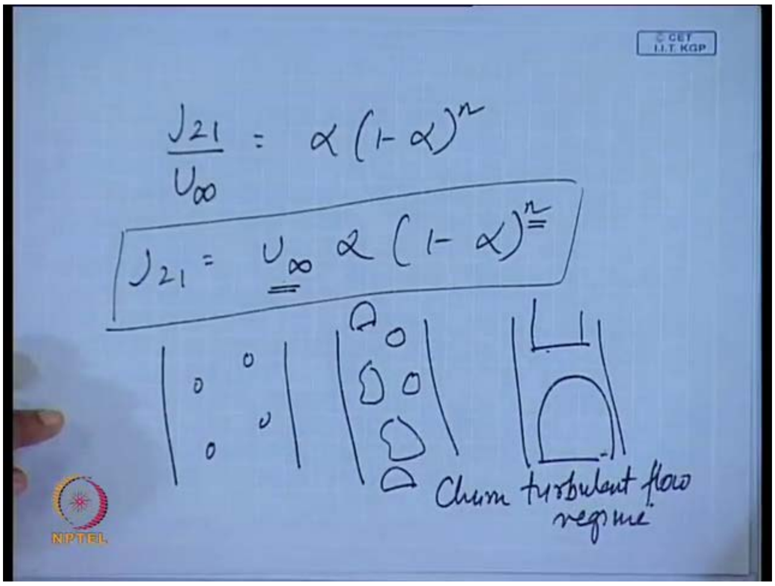

Normally what do we say? We say that for the bubbly flow pattern we have bubbles of this sort. For the slug flow pattern we have something of this sort. Now, for the churn turbulent flow pattern we can have a wide type of bubbles, we can have cap bubbles it can be a totally erratic distribution resembling the churn flow regime to some extent. So, therefore, usually this particular flow pattern which is the transition between these two this is usually known as the churn turbulent flow regime.

And, we find that for number of situations we neither operate here nor operate here you operate in the churn turbulent flow pattern. This is one type of bubbly flow pattern which marks the transition between the bubbly and the slug flow patterns. Now, for this particular case people have proposed this equation for U 2 j and people have said that when it is gas liquid system root two is fine, and when it is liquid to liquid system then instead of root two, 1.57 is better.

So, these are simply empirical equations in case you have to sort out any problem with drift flux model. Depending upon the situation you select a particular U 2 j; from this U 2 j, you find out a particular J 2 1; that J 2 1 you apply and then you find out alpha rho mixture, and whatever other things are there.

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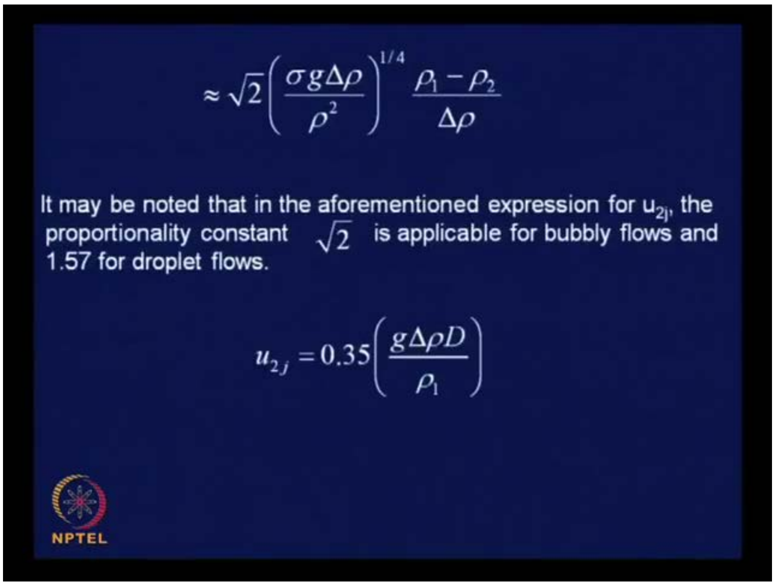

And this particular equation is for the slug flow pattern. So, for different flow patterns we have different particular flow equations or rather different expressions of drift velocity.

Depending upon your case you are suppose to select it and you are suppose to do it. But just remember whatever equation you use, whatever more or less we find that this particular equation can be used. So, naturally u 2 j becomes this equation; u 2 j people have proposed, it is a function of u infinity and the hold up of the continuous phase. From there people have found j 2 1 can be obtained from this particular expression and where u infinity and n depend upon the different flow conditions.

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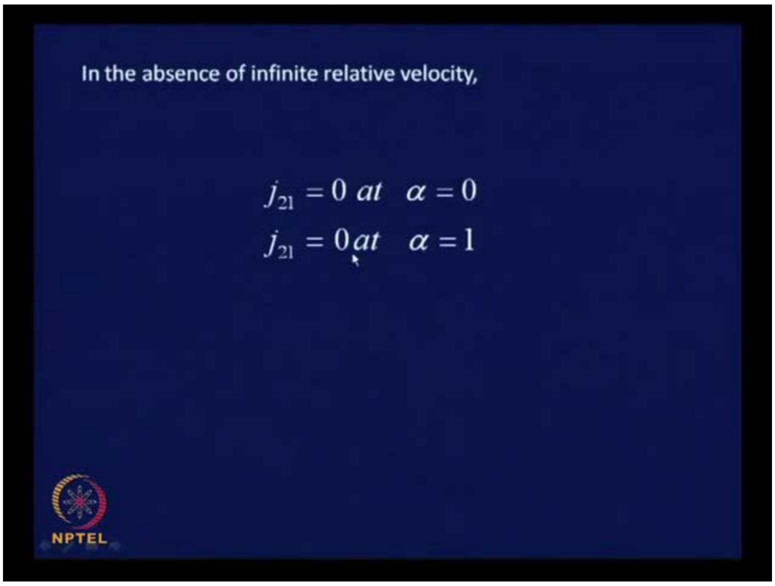

The only two limiting conditions which have to be taken into mind while using this particular equation is that j 2 1 has to be 0 at alpha equal to 0, j 2 1 has to be 0 at alpha equals to 1. These are the two limiting conditions which have to be agreed upon by all equations which we used to find out j 2 1. So, this was all about how the kinematic constitutive equation has been proposed to find out j 2 1. So, now, what we have? We have two equations; we have two unknowns.

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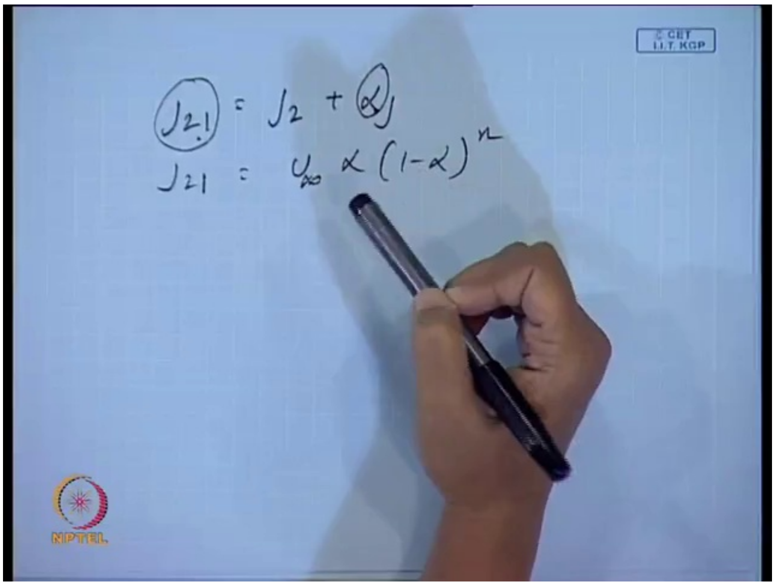

What are the two equations that we have? One equation was one which we derived from the drift flux model which gave us J 2 1 equals to J 2 plus alpha J. And the other equation we have J 2 1 equals to u infinity alpha into one minus alpha whole to the power n. There two unknowns one is J 2 1, one is alpha and we have two equations. So, we can solve them simultaneously and we can get a value of alpha; we can get a value of J 2 1 and from there we can get a value of different mixture properties.

Now, how we will solve them; simultaneous solution is definitely one, but we would prefer a graphical solution. Because graphical solution will enable us to take into account the different flow directions of the different flows it will also help us to account for the effect of varying the phase flow rates. So, in the next class we will take up these two equations, we will try to solve them or rather the simultaneous solutions will be done by a graphical technique. And we will see what is the different information we can from those graphical technique? What are the different ways of representing the two equations graphically and accordingly? We will proceed. Thank you very much.