Multiphase Flow
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Lecture No. # 17
Separated Flow Model

Good morning to all of you. So, till the last class we had completed the analysis which we usually use for the mixed flow patterns, the transitional flow patterns and so on. We had dealt with the homogeneous flow model and we had also dealt with the drift flux model. So today, we are going to start the separated flow model and in other words it is better coined as the two fluid model. In this particular model exactly what we do is? This is usually applied for when we are having two phases, and the two phases they are separated by a distinct interface.

This can also be applied to mixed flow patterns as well. Here, the basic thing which we do is we consider each phase separately, and we write down the equations of continuity, momentum and energy for phase one and for phase two. And then, we consider in these particular equations itself we consider the interaction between the two phases. So, therefore for example, suppose we are considering say two phases flowing as separated flow in a particular channel. So, in that particular case I will just show you a figure.

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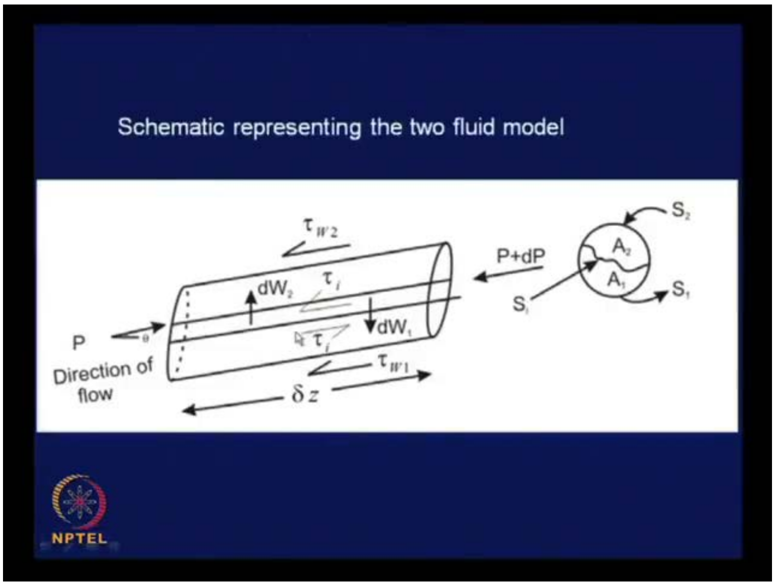

So, if this figure you see there is phase one on the top and there is phase two at the bottom. So basically, suppose it would have been a single phase flow then this particular fluid it would have interacted with the wall. Now, since there are two phases, there therefore, each particular phase say phase two that interacts with the wall with a wall shear stress which is tau w2 and it also interacts with the with the other fluid and where the interfacial shear between the two is tau i. Now this tau i has been wrongly written down in this particular case.

Now, this sign is correct. If the direction of flow is in this particular direction then, we find that naturally the interfacial shear will be opposite to the direction of motion. Now since the direction of motion it takes place from the left hand side to the right hand side. So, naturally tau i will be in the opposite direction for the faster flowing fluid and in the direction of motion for the slower flowing liquid or the slower flowing fluid. So, therefore, in this case what we do, we consider the two phases separately, say we consider phase two and then we consider phase one. For each particular phase what we do we write down the momentum equation, we write down the equation of continuity we write down the energy equation. When we are doing this then in that case, we have to consider the interaction between the two phases we have to consider that the two phases they interact and they will be interacting at the interface. Now remember one thing, this is a situation for a completely separated stratified flow situation.

So therefore, the interfacial shear it operates at a well defined interface where the defined interface is well defined between the two phases. Suppose, the other phase would have been distributed as bubbles or a slacks teylor bubbles etcetera then in that case also the same concept can be applied. That is why we had applied he same concept in the drift flux model in order to arrive at a proper expression of f12 the interacting force at the interface from there we could we had we tried to get some sort of a relationship between j12 and the input parameters. So, the only thing which is going to vary when the two phases are not so, well separated is the area over which tau i is going to act and the magnitude of tau I as well. So, in this particular case, it is pretty straight forward and evaluating Si the interfacial weighted perimeter along which tau i is acting.

So, this is much straight forward is in this particular case it will be much more complicated. In when the two phases are mixed together. But any how the two fluid model is a most fundamental model I should have started with this earlier. But I did not do this because I wanted to go step wise. Therefore, the naturally the first thing was consider the two fluid to be completely mixed and then you apply the mass momentum energy balance, or the equations of continuity momentum and energy across this particular two flow, the mixture of the two fluid and then you work accordingly the other extreme is you consider the two phases to be completely separated consider each phase separately apply the equation. So, in the homogeneous model we had three equations in this case we will be having six equations.

Apart from having six equations remember, that we got to include the interaction terms in the constitutive equations. Now how complicated these interaction terms will be that will govern on how complex the model will be and how accurate it will be for practical purposes. But this is the actual the most complex or rather the the accurate way of modeling two phase flows. But we always we do not use this the advantages and disadvantages will be discussed in shortly regarding this.

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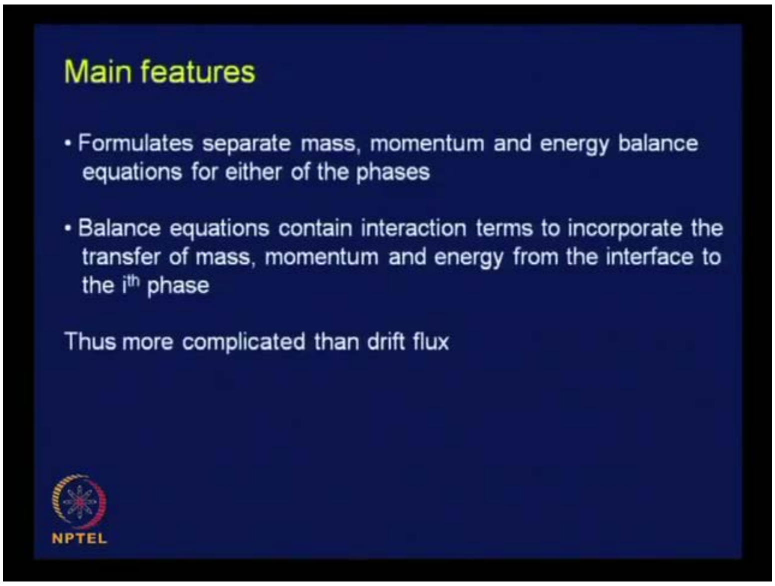

So, naturally what are the main features as I have told you that we consider the two phases separately, and to formulate separate mass momentum or energy balance equations for either of the phases. Now remember one thing the main difference between single phase and two phase flows are the interaction terms all these balance equations they must contain the interaction terms which incorporate the transfer of mass momentum and energy, from the interface to the i-eth phase this is something very important, that we must be considering both or rather. We must be considering the interaction terms if we do not consider the interaction terms then naturally this simply becomes the single phase flow equations.

Now, due to this the separated flow model is more complicated than the drift flux model this complicacy arises not only in the number of field equations there we had four equations, as I have told you here we have six equations. But this is not the only reason why two fluid model is more complicated, than the your drift flux model the main reason is that, in this particular case we consider or rather the the necessary constitutive relationships which have to be incorporated in order to account for the interaction between the two phases. This is what is important first is a number of equations have become more and Secondly, the constitutive relationships which have to be incorporated in order to account for the interaction between the two phases.

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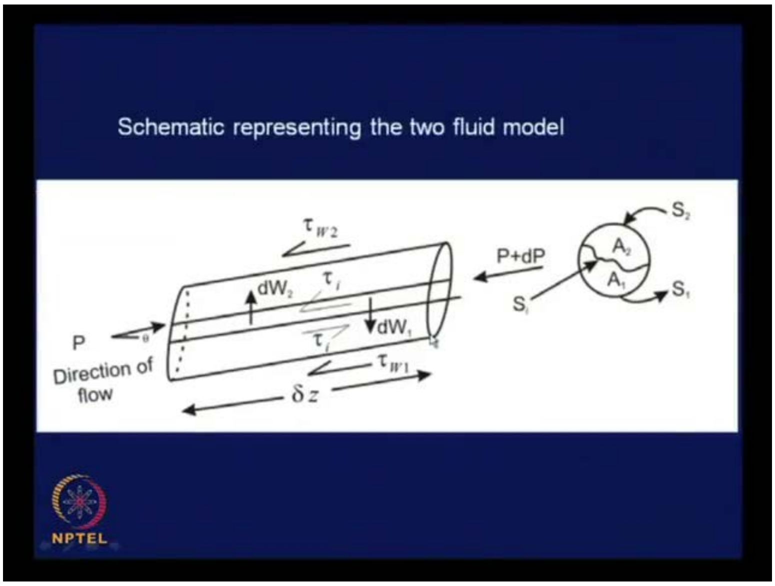

This is particularly, if it this is applicable when the degree of coupling is or rather the the the degree of coupling. It is not very high under that conditions this particular model is particularly applicable when a degree of coupling is very high, then finding out tau i finding out Si they become much more complicated. If you see this particular figure we find that finding out finding tau i and Si becomes much more complicated. When the degree of coupling is becomes very high in that case the constitutive equations they become much more complicated and the accuracy of the constitutive equations they govern the usefulness of this particular model. So, if those equations become very complex then naturally this particular model will have very limited applicability.

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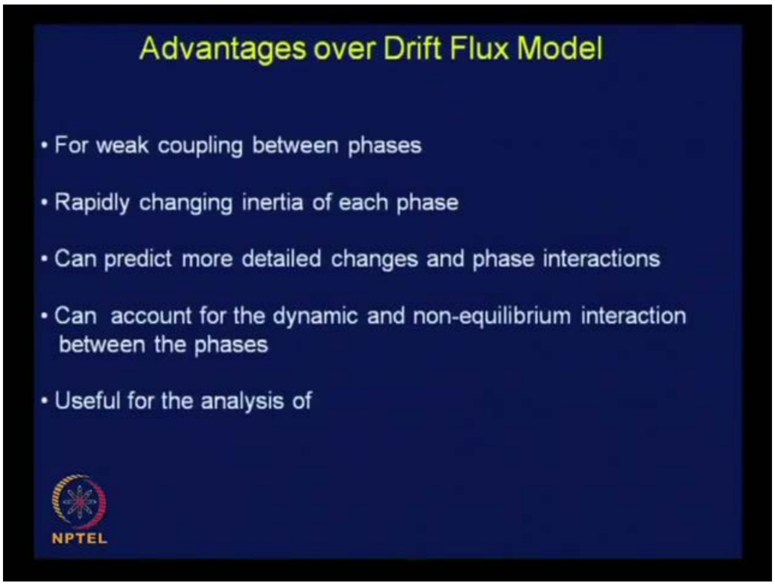

So, this is the thing now the advantages you have over the drift flux model now in this particular case. So, it is quite evident that the interaction terms they determines the degree of coupling and therefore, they determine the transfer processes in each phase. Now if this interfacial exchanges would not have been incorporated in the balance equation then the two phases would have been essentially independent and they could be analyzed by near single phase flow equations. When we start the analysis we will find that the only difference between single phase flow equations and the two fluid model equations are the interaction terms.

Now, when we should use it and when we should prefer the drift flux model. So, the advantages which we have over the drift flux model, the first thing is it is more useful when the two phases are weakly coupled and the inertia of each phase it changes rapidly. This is the first thing when the for weak coupling between the phases when the inertia of each phase they change rapidly under this condition finding out tau i Si is not very difficult and definitely, under this particular condition it is easier to use the two fluid model as compare to the drift flux model.

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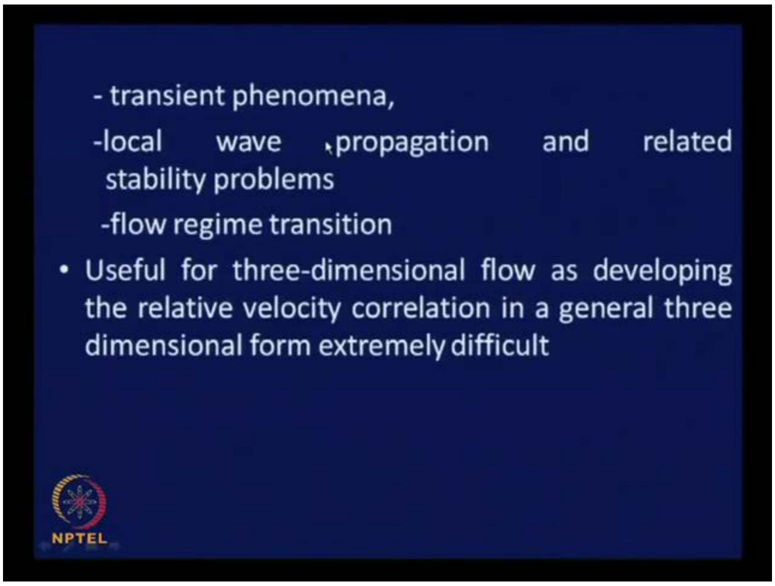

Because under when there is weak coupling then finding out j21 becomes much more complex. So, under those conditions we would prefer this next is due to the separate conservation equations. Therefore, since we are writing separate equations for phase one separate equations for phase two, what we do it can predict more detail changes and more detail phase interactions as compare to the drift flux model and it can also account for the dynamic and non equilibrium interaction between the phases, and this is particularly useful for the analysis of transient phenomena local wave propagation and related stability problems as well as for flow regimes transitions.

Therefore, we find that this particular model it is particularly useful for the analysis of transient phenomena local wave propagation, and related stability problems as well, as flow regimes transition cases and when we are having a general three dimensional flow naturally, under that condition the two fluid model is going to be much better than the mixture model. Why because for really three dimensional cases it is extremely difficult to develop the relative velocity correlation in a general three dimensional form is not it.

Whenever we are trying to derive the drift flux model we had derived it for the one dimensional case and then what we did we incorporated C0 to account for the three dimensional case with a supposition that C0 is not far removed from unity. That was when will it not be far removed from unity when more or more less the two phases they obey the one dimensional flow assumption may be there is a slight velocity or a void age profile which can be accounted for. But if there is really very remarkable three d effects then naturally C0 will be far removed from unity and then it is no longer a correction factor is not it, then we have to do a proper balance and then find out a proper C0.

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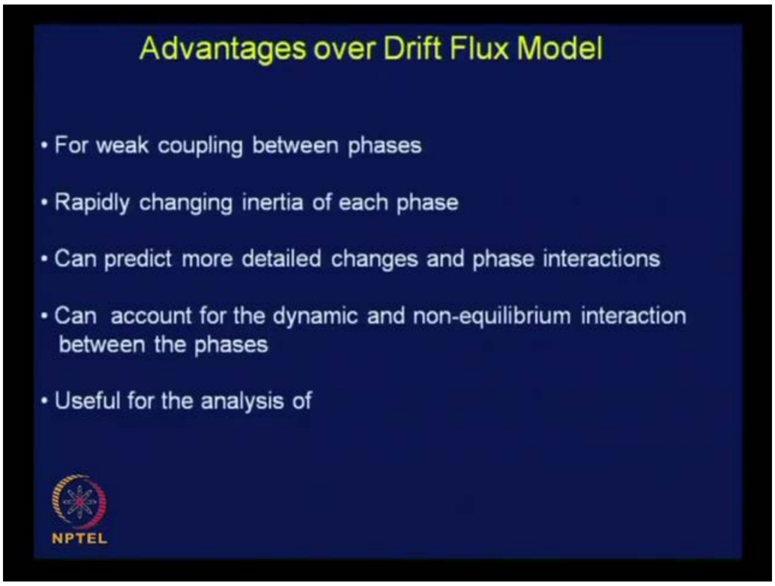

Under, that circumstance it is much more easier to consider the two fluid separately and write down the equations write down find out the interaction terms and then do it accordingly. So, remember one thing, whenever the two phase are weakly coupled whenever we have some transient phenomena local wave propagation and things like that. Whenever there are rapid changes in inertia of the two phases and when we want to predict more detailed changes and phase interactions when the flow is truly three dimensional under such circumstances generally, we preferred or rather it is better to use the two fluid model.

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And under what circumstances we should be going for the drift flux model when the total response is more important than the local behavior of the individual phases. Naturally, when the total response is important then in that case writing it down for phase one writing it down for phase two and then adding the two and then finding out the mixture response it is not worth. It is better that you find the mixture response directly and there we work with four equations and we can predict the mixture properties or the mixture characteristics as a whole by just incorporating the relative velocity between the two phases.

So, under that circumstances the drift flux model is going to be better and the other thing naturally, when there is a strong coupling between phases.I that case finding out tau i Si they become very complicated and the and such cases we would prefer to go for that drift flux model., when it is more of a mixed flow see you cannot definitely say that this is totally dispersed flow this is totally separated flow, under normal circumstances for the the range of flow conditions that we encounter in industries there always be a good amount of interaction.

When the interaction is such that one phase tends to get completely mixed in the other phase then we prefer to go for the drift flux model, because even when they are mixed also there will always be a relative velocity between the phases and if we just consider the relative velocity and modify the mixture equation it is going to be better. But when we find that the interaction is not such that one phase mixes in the other may be the interface can be very much wavy or may be there are well defined cross sectional areas occupied by phase one, well defined cross sectional areas occupied by phase two, even for the slack flow model also may be at times the two fluid model can be much more useful.

So, depending upon the flow situation you have to decide whether you will go for the two fluid model or whether you will go for the separated flow model. Now, if we go for the different equations what is the simplest the equation of continuity. If we start writing the equation of continuity or. In fact, the mass balance equations we will first try it and more or less lot of things are written down here, but it is better that we derive those.

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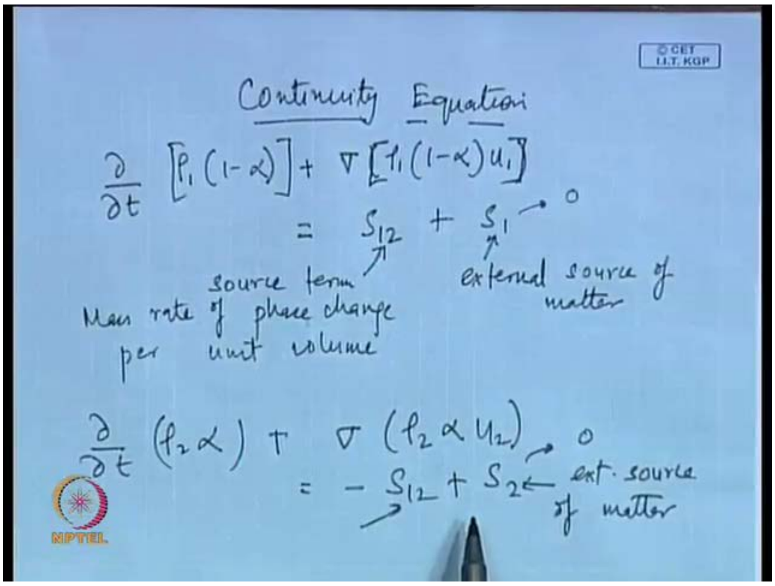

So, what is the general continuity equation see we will first do it for phase one and then we will write it for phase two. So far phase one we will have a navier stoke sort of a expression rho 1 into 1 minus alpha plus differential rho 1 into 1 minus alpha u1 let us do it like this, this is equal to S12 plus S1 now what is this S1 this S1 is external source of matter. Suppose, some particular amount of phase one is entering the channel other than the flow condition. Usually this term can be taken as 0 and what is S12 this is known as a source term represents mass rate of phase change per unit volume that means, this tells us the amount of mass which goes from phase one to phase two or vice versa as a result of phase change as a result of mass transfer it can be anything.

And S1 this is the external source of matter if any amount of phase one comes from outside into this usually under the conditions which we consider S1 can be taken as 0. Similarly we can write it down for phase two it is going to be rho 2 alpha better I put a bracket over here rho 2 alpha u2 this is equal to minus S12 plus S2, this S2 is again external source of matter usually this is taken as 0 and naturally the this represents as I have told you it is a source term which represents the mass rate of phase change per unit volume.

So, naturally if mass is moving from phase two to phase one. So, therefore, S12 is positive mass is entering and S12 is negative since mass is leaving. These signs they will be adjusted according to the situation. So, this is the most generalized three dimensional form of continuity equation. It simplifies to several simpler equations under different conditions, for example, if we have steady state flow what happens? for steady state flow what do you expect is going to happen your del del t terms these two terms they should cancel out and under that is condition we get for steady state flow.

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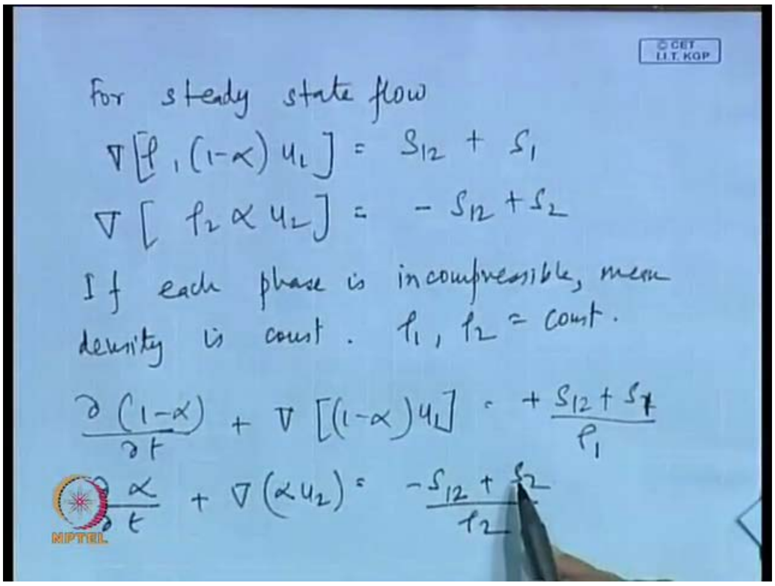

What is the thing we get it is delta rho 1 1 minus alpha u1 this is equal to S12 plus S1 and here it is going to become rho 2 alpha u2 minus S12 plus S2. Next, If each phase is incompressible what do you expect then what do you expect your mean density becomes constant, or in other words your rho 1 rho 2 they are constant. Sothey come out of thisfrom this particular equation what we get this rho 1 and rho 2 they will be coming out of this equation is not it. So, under that condition what type of continuity equation do you expect it should be del into 1 minus alpha del t plus delta 1 minus alpha u1 equals to minus S12 plus S2 by rho 1 plus S1.

And this is going to be del alpha del t plus delta alpha u2 this is equal to minus S12 plus S2 by rho 2. So, depending on the condition if it is steady state flow you get this. If each phase is incompressible you get such a type of situation now in this particular case if we say that there is no phase change. Then what do you expect suppose, there is no phase change both the phases are incompressible and there is no phase change under that condition what do you expect, this particular the right hand side they disappear off S1 S 2 are any how 0 is not it. So, the right hand part they disappear off

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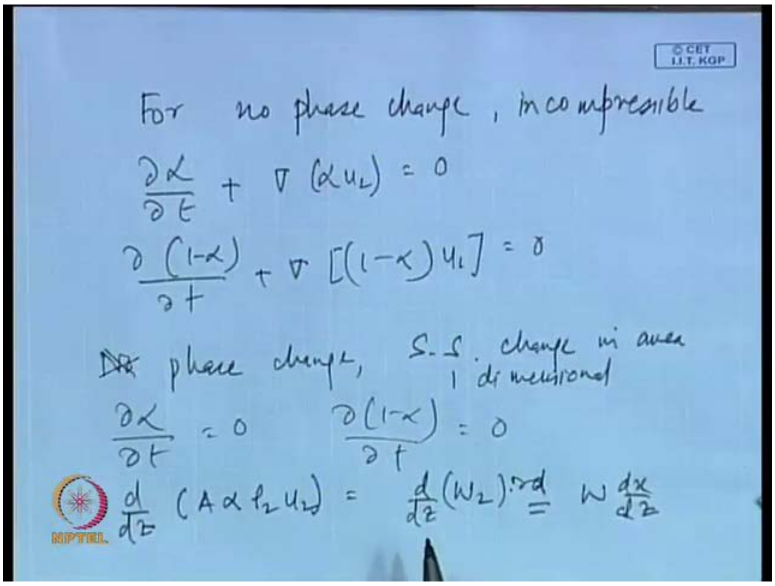

And under that circumstances what do you get we get del alpha del t plus delta alpha u2 equals to 0 1 minus alpha delta t plus delta 1 minus alpha u1 this is equal to 0. Next suppose, there is no phase change incompressible and steady state. but there is change in area then what do we get say suppose it is no phase change steady state So, naturally these terms they cancel out and change in area. For that particular circumstancewhatdo we expect these become equal to 0 and if it is one dimensional sorry, one dimensional I forgot to write steady state change in area one dimension.

So, in that case what do you get under such circumstances these two things they become d dz alpha rho 2 u2 this is naturally equal to d dz of W2 or in other words equal to W dx dz sorry, phase change steady state change in area one dimensional is not it. This is the equation that is expected similarly, you can write down the equation for phase one as well. So, therefore, these were the different ways I think I have got a slide for these also.

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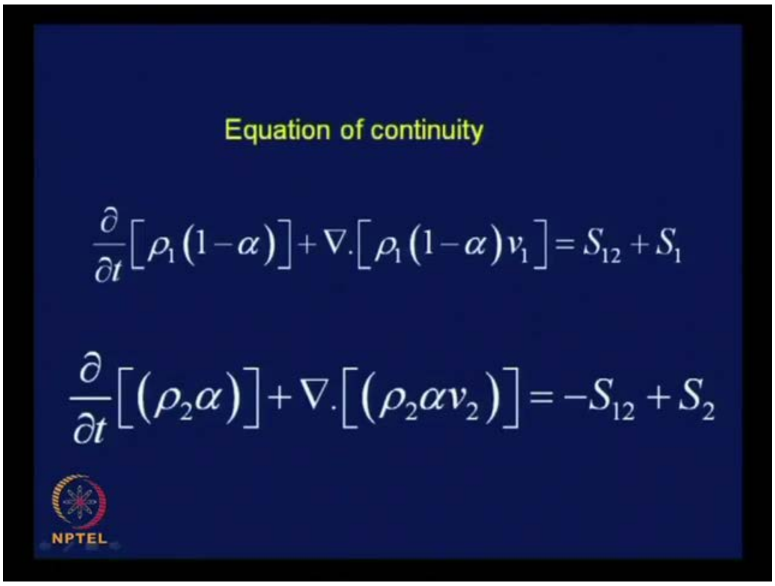

This is the general equation of continuity, if you notice my ppt you are going to see this is general equation of continuity then for incompressible phases. We have just taken out the rho part rho 1 rho 2 out and we get this and then after integration across the duct what do you get once you integrate this across the duct you take a one dimensional case and then you integrate it across the duct. This general particular thing if it is for steady state and to integrate it or rather unsteady state also it is one dimensional flow and integrate it across the duct then in that case what do you get, you get an expression which is written down in this particular slide.

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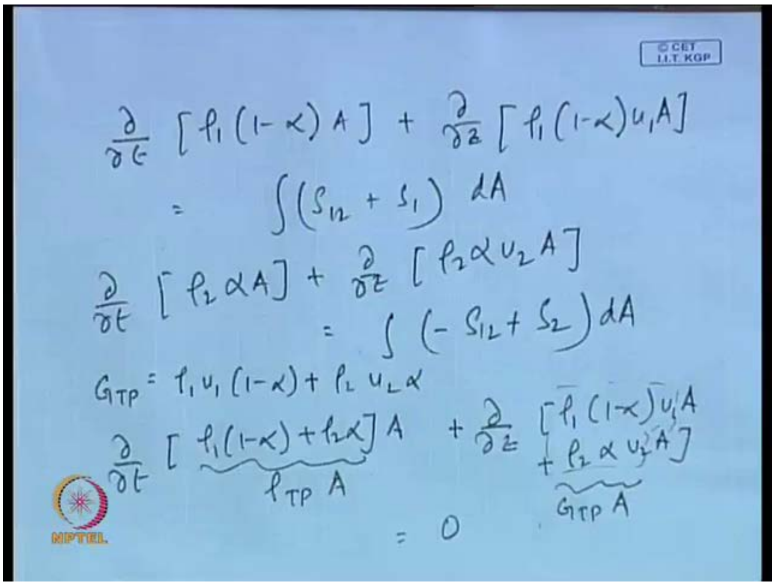

Now, in this particular expression if we add up the two expressions we can do it and then with some sort of manipulations we get the separated flow equation for the mixture, now let us see how to arrive at that this particular equation was del del t rho 1 into 1 minus alpha A plus del del z rho 1 into 1 minus alpha u1 A this is equal to integral of S12 plus S1 dA is not it. Similar way we can write del del t rho 2 alpha A plus del del z this will be equal to integral of minus S12 plus S2 into dA.

In the integrated form this is the case now in this particular equation if we add the two and we consider that GTP this is equal to rho 1 u1 1 minus alpha plus rho 2 u2 alpha yes, or no GTP it is nothing but rho TP utp what is rho TP? rho 1 1 minus alpha plus rho 2 alpha uTP u1 plus u2 is not it So, therefore, GTP is something of this sort. So, in this integrated form if we if we add each of them and if we substitute this just do it and see what you are going to get simply, do it and then you see what you are going to get in this particular situation what we get here we get rho 1 into 1 minus alpha plus rho 2 into alpha alpha into A.

So, therefore, here we get del del t of rho TP A it is rho 1 1 minus alpha plus rho 2 alpha into A and this part is nothing but rho TP into A plus this portion will be del del z of rho 2 alpha u2 A plus this part. So, if we add this up rho 1 into 1 minus alpha u1 A plus rho 2 alpha u2A. So, the rho 1 u1 1 minus alpha plus this whole thing this particular portion this becomes GTP. So, therefore, this is GTP into A is not it. So, this will be equal to this

S12 S12 they will cancel out S1 and S2 they are almost equal to 0. So, this becomes equal to 0 is this part clear to you or should I explain this part once more?

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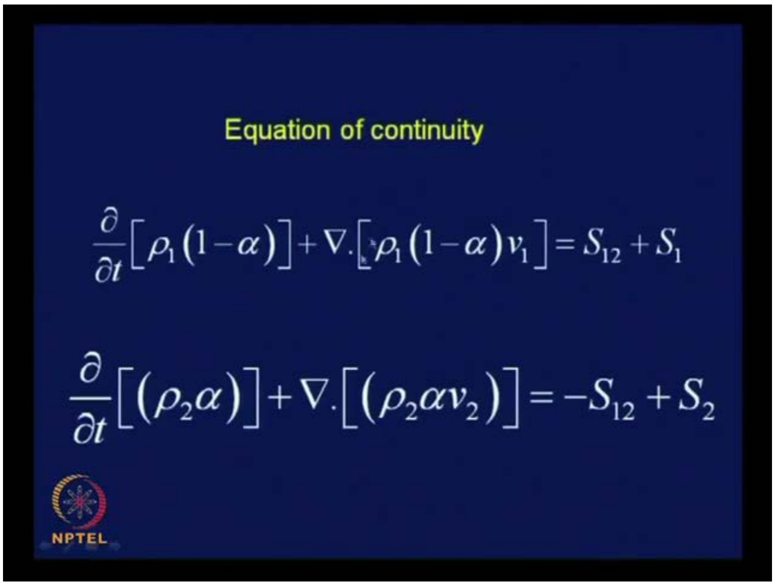

Thing is what we did first we took up the continuity equation just if you notice the ppt this was the equation of continuity, for one dimensional case what we have we simply have del del z because we have considered the direction the of flow as z. So, for one dimensional case it becomes del del t rho 1 1 minus alpha plus del del z into this. Now if we integrate this one dimensional equation on integrating what do we get this is for incompressible fluids and leave that. So, if we integrate it on integration we get something of this sort this equation and this equation.

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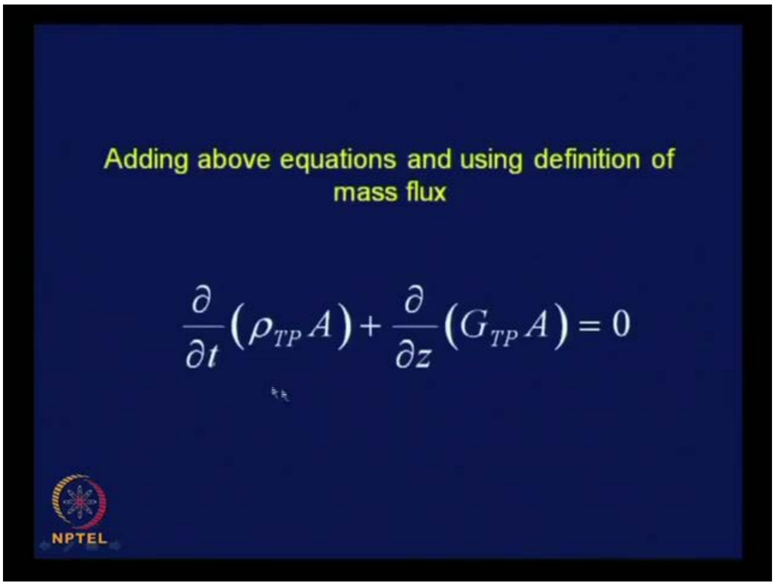

Now, if we add the two what do we get for this derivative of t it is del del t rho 1 1 minus alpha A plus rho 2 alpha A and what is this part rho 1 into 1 minus alpha plus rho 2 alpha this is rho TP into A then del del z of rho 1 1 minus alpha u1 A plus rho 2 alpha u2A. Now we know rho 1 1 minus alpha u1 plus rho 2 alpha u2 this becomes GTP. Therefore, this particular term becomes GTP into A if we add these S12 minus S12 they cancel out S1 S2 I have already told that we never have external sources of matter entering. So, this cancels out.

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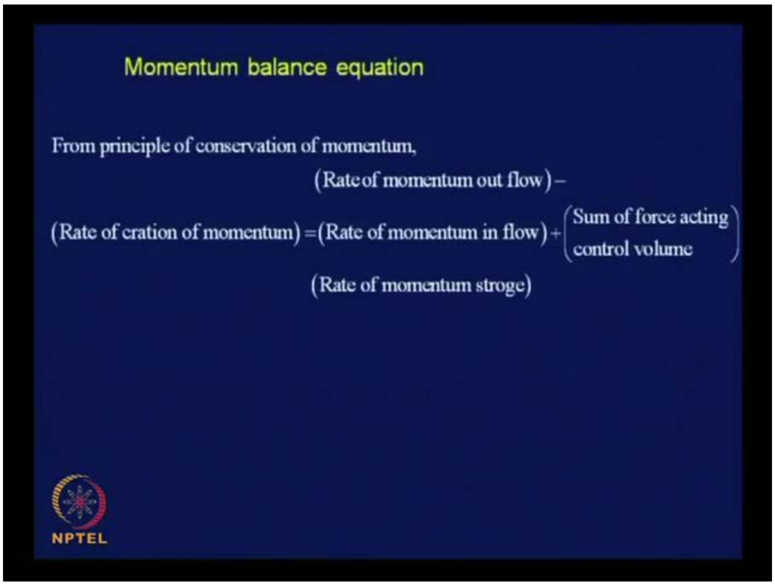

Therefore, this plus this finally, it gives this particular expression this is the equation of continuity for the mixture agreed. Now, if we go for the momentum balance equation. what is the principle of momentum balance? all of us know it is simply the rate of creation of momentum rate of momentum out flow minus in flow plus the amount of rate of momentum storage and here the there is a little mistake it is sum of forces acting on the two. if I just correct it plus rate of momentum this will be any how we will correct it later leave it now this will be storage this has to be equal to the sum of the forces there is a slight mistake here which I am going to correct.

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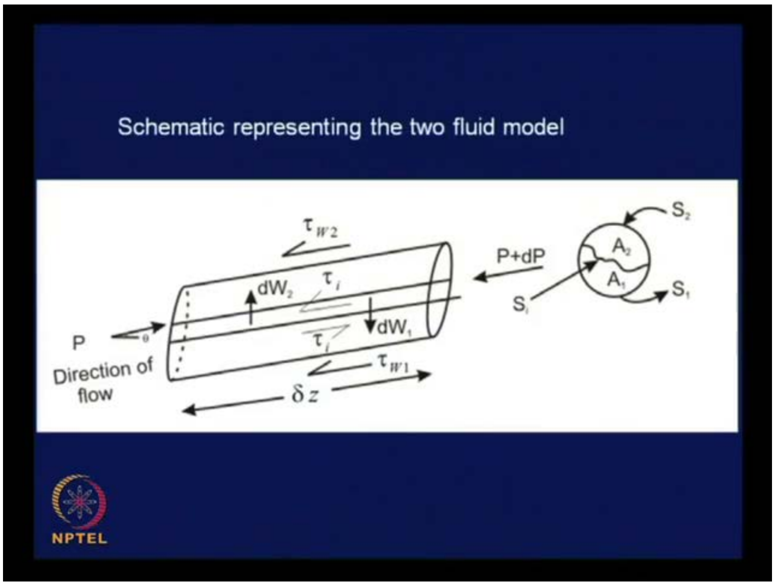

The rate of creation of momentum is nothing but rate of momentum out flow minus rate of momentum in flow plus rate of momentum storage and that is nothing. but equal to the sum of forces acting on the control volume. What is my control volume? control volume I had already shown here this is my control volume it comprises of a cross sectional area A and a small length of the flow passage delta Z. So, therefore, this is the control volume now in this particular control volume what are the forces that are acting can you tell me? pressure forces in this direction it is p here what it is p plus dp or in other words what is this dp del p del z into delta z is not it.

So, there is a pressure force acting what else is acting here wall shear. So, there is going to be tau W2 into your weighted perimeter or the perimeter which is in contact with phase two then there is tau W1 into the perimeter in contact with phase one. So, therefore, two wall shear stresses will be coming unlike one wall shear stress for single phase flows and then there has been additional wall shear stress tau i. So, this tau i will also be coming in this particular case in any other and of course, gravity is definitely there the gravitational force. these are the total forces which will be acting and what is the rate of momentum in minus rate of momentum how. So, therefore, we assume that W1 amount of fluid one is entering W2 amount of fluid two is entering.

And here what is happening we assume a general case some amount of change of phase change takes place naturally whenever we talk of change of phase immediately it appears that may be some amount of evaporation occurs it will always do the reverse is that is the case then what happens some amount of say phase one it shifts to phase two it is a dW2 or dW1 whatever that amount it shifts now whenever that happens there is also a momentum change. Why? Because some portion of the fluid which is undergoing change of phase that actually changes it is velocity from that of one phase to that of the other phase since both the phases are flowing at different velocities.

therefore, that also takes place. So, therefore, there is a change of momentum due to phase change as well. . we will be equating we will be writing down the total number of forces which are acting then we will be writing down that momentum change we will be equating it. We will be doing it once for phase one then for phase two and then if we add the two we can we will find we can combine it in different ways if we add the two we will get the mixture momentum equation.

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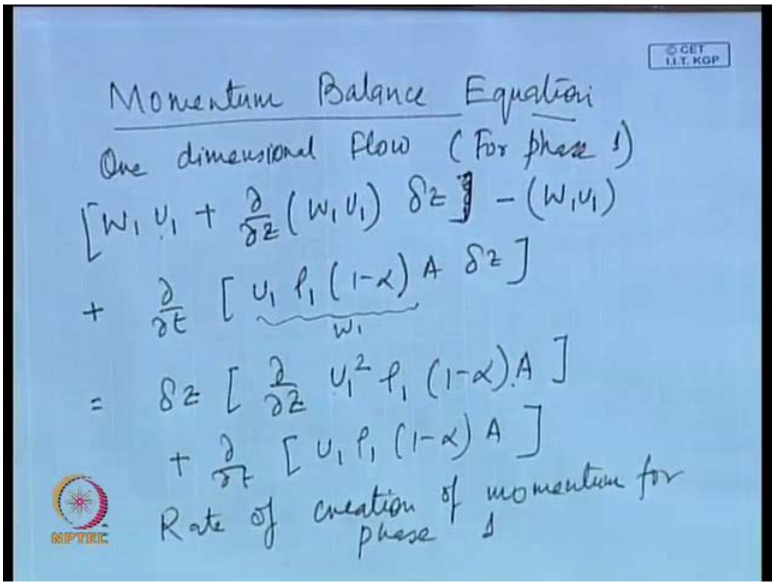

So, let us do it and then let us see what we are going to get for this particular purpose. This is the momentum balance equation now, in this momentum balance equation what is the rate of momentum, one dimensional flow this is definitely we are we usually we consider one dimensional flow now if we refer to this figure we find what is the rate of momentum out flow minus the rate of momentum in flow. This is naturally for phase one. So, this is W1 U1 plus del del z of W1 U1 into delta z agreed? minus W1 U1 sorry minus W1 U1 plus del del t of U1 rho 1 1 minus alpha A delta z where this is nothing but W1 this part this is W1.

So, this is the rate of momentum out flow minus the rate of momentum in flow plus the rate of momentum storage here, and what is the rate of or in other words this becomes this is equal to delta z into U1 square rho 1 1 minus alpha into A plus del del t U1 rho 1 1 minus alpha into A. Where how did we get this thing this is W1 equals to U1 rho 1 1 minus alpha into A into U1. So, we have got this part. Is this portion clear to you? this is the rate of creation of momentum for phase one agreed this portion is it clear to all of you?

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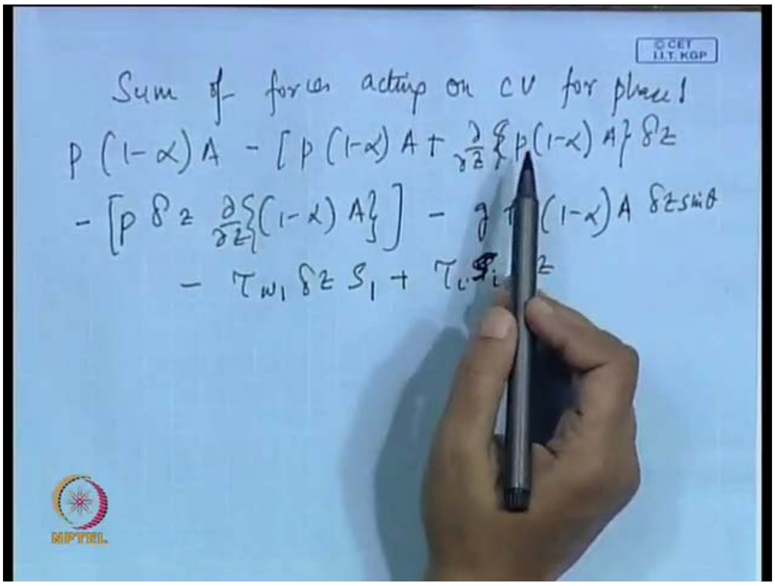

Now, what are the sum of forces I believe I have the ppt's for this on this control volume of phase one sum of forces acting on control volume for phase one. What are the sum of forces P into 1 minus alpha into A minus P into 1 minus alpha A plus del del z of P into 1 minus alpha A delta z then this gives minus P delta z del del z of 1 minus alpha into A to account for the force on the curved portion. Then of course, minus g rho 1 1 minus alpha A delta z sin theta minus tau W1 delta z S1 plus tau i Pi, Si delta z. See if this portion is clear to you? this is a force which is acting on one part, and then this portion is a force acting on the other portion if you see this ppt this is P and this is P plus dp.

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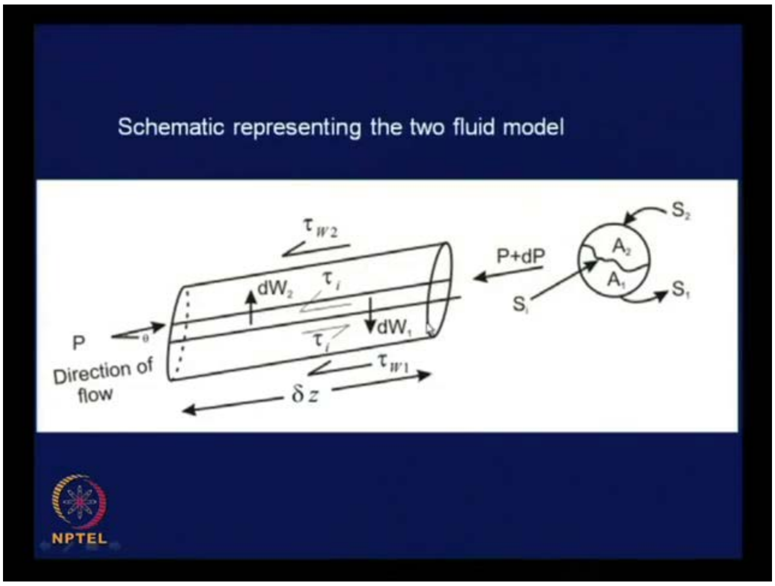

This is basically P plus dP where we have written P plus dP as del del z into P into 1 minus alpha into delta z and then this , tau w1 we are considering this particular phase delta z into Si tau i Si delta z and of course, there is one particular force which is pressure force which is acting on the curved surface area. So, therefore, the first two terms if you take the first two terms they refer to the pressure forces on the ends of the element the third term it is a pressure force on the curved surface which occurs when there are changes in the cross sectional area only. Otherwise this particular thing it does not arise.

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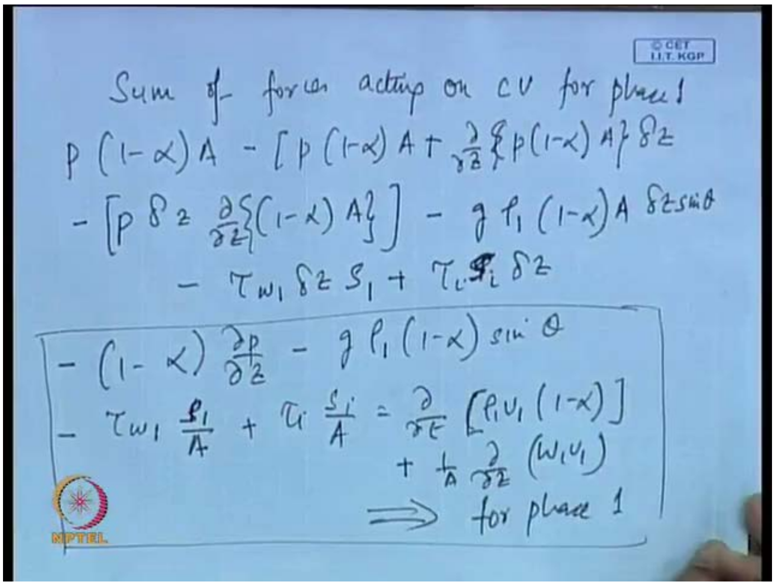

See, here I have not shown a change in the cross sectional area, but if this would have been a gradual expansion or a contraction thennaturally, that particular force would have come and this is naturally the gravitational force and this is the shear stress forces wall shear stress and this shear stress. Now, if we equate this particular expression with this particular expression then we get the momentum balance equation, therefore, if we equate it what do we get on equating it if you write it down and equate it we get minus 1 minus alpha del p del z minus g rho 1 1 minus alpha sin theta minus tau w1 Si by A plus tau i Si by A this will be equal to del del t of rho 1 u1 1 minus alpha plus 1 by A del del z of W1 U1. This is the thing that we get for phase one .Please do the derivations on your own. So, that you are in a position to do it or to repeat it.So, this is the situation for phase one that we get.

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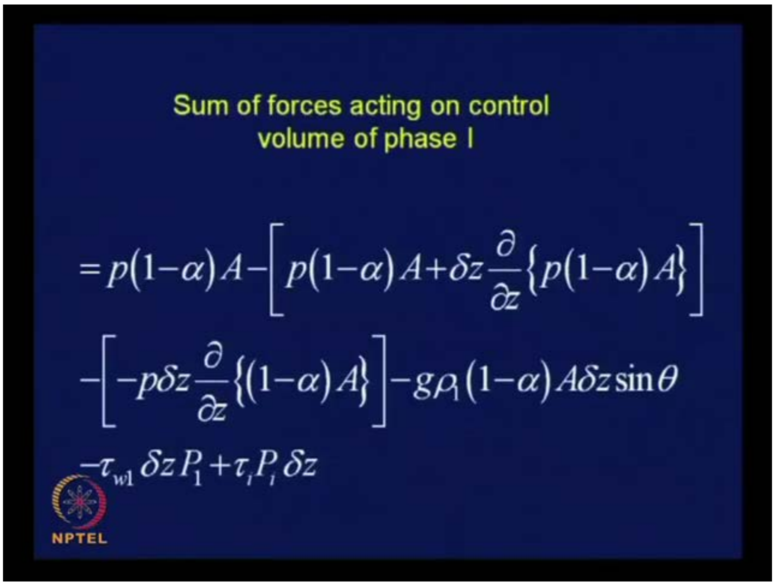

Similarly, for phase two also you can write such an identical thing write it down for phase two just write it down for phase two let me see what you get for phase two? if you write it down what do we get I think it is already written down. So, I will not bother to write it down once more if you see the ppt's more or less rate of creation of momentum of phase one this is sum of forces acting on the phase one which I have said you add it up you get something of this sort then for phase two also you have the you can have the similar type of equation. So, these are for one dimensional flow with everything else remaining..

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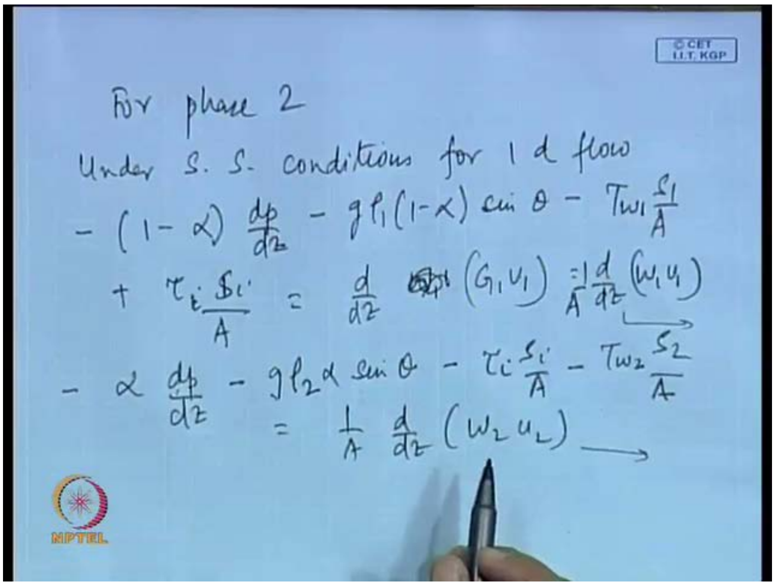

So, under steady state conditions what do you get? Under steady state conditions your this term and this term they disappear off. So, therefore, under steady state condition the equations it reduces to. So, you can write down the equation for phase two and then write down because usually we deal with steady state conditions. In steady state conditions we get steady state and one dimensional conditions for one dimensional flow. So, this becomes minus 1 minus alpha dp dz minus g rho 1 1 minus alpha sin theta minus tau W1 S1 by A plus tau i Si by A this is equal to d dz of W or rather g rho 1 or in other words it is W1 U1 by A. In whatever form you can write it down this is basically d dz of W1 U1 into 1 by A if we consider A to be constant. So, this is steady state conditions one dimensional flow for phase one.

Similarly, for phase two also we can write down remember all the deltas has become d please remember these things because this will be considered g rho 2 alpha sin theta minus tau i Si by A minus tau W2 S2 by A this is equal to 1 by A d dz of W2 U2. Now, these are the two equations that we get both these two equations and I have just written down the original form here cutting out this del del t terms we can get this now these two equations they can be combined in several ways to give us the mixture momentum equation one is you simply add them up, the other is if you subtract one equation from the other.

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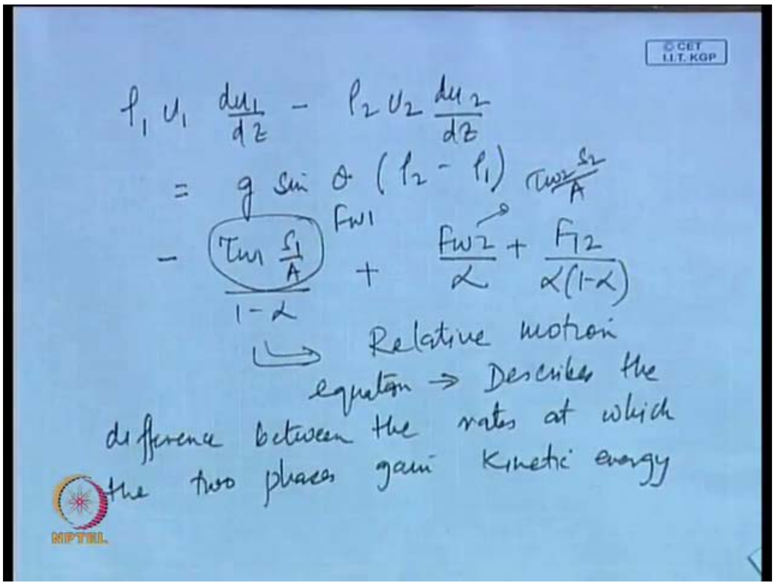

What do we do if you simply subtract if we divide this particular equation with 1 minus alpha we divide this particular equation with alpha and then we subtract one equation from the other what happens dp dz term it cancels out is not it. Just do it and tell me what you are going to get? We divide one particular equation with 1 minus alpha and divide this equation with alpha and then you subtract one equation from the other let us see, what you get? we are expected to get something of this sort rho 1 u1 du1 dz minus rho 2 u2 du2 dz this will be equal to g sin theta rho 2 minus rho 1 minus your tau w1 s1 by A divided by 1 minus alpha this can be taken as FW1 the interaction of phase one with the wall plus FW2 by alpha where this FW2 is nothing but tau w2 s2 by a plus F12 by alpha into 1 minus alpha.

Just see these two equations and see whether you can get what I have written down. Subtracting these two terms d d z of 1 by A this particular term this W2 is nothing but rho 2 u2 alpha into A. So, A cancels out therefore, you get rho 2 u2 du2 d z here rho 1 u1 du1 dz we have subtracted the two then we have divided by alpha. So, alpha 1 minus alpha goes off. So, g sin theta into rho 2 minus rho 1 in the same way if you do you find that you get a equation something of this sort.

Now, from this equation you find that it does not include the pressure gradient. And this can be considered as a relative motion equation. What does it do? it describes if you see these two particular terms you find they basically describes the difference between the rates at which the two phases gain Kinetic energy. See, if you have understood what I have written down basically if you find this is the difference between the rates at which the two phases gain Kinetic energy is not. So, this minus this the rate at which they gain Kinetic energy can be obtained from g sin theta rho 2 minus rho 1 minus the interaction of phaseone with the wall by 1 minus alpha the volume which is occupied by phase one plus the interaction of phase two with the wall divided by the volume occupied by phase two plus your interaction between the two phases by alpha into 1 minus alpha.

This is one way of combining these two equations for steady state conditions which we have obtained. What is the other way? you simply add the two equation do the addition part and see what you get you just try to see you get 1 minus alpha dp dz plus alpha dp dz. So, 1 minus alpha plus alpha it gives you dp dz is not it. So, in that way you get dp dz here what do you get g rho 1 1 minus alpha sin theta g rho 2 alpha sin theta it is simply g rho TP sin theta in this way if you keep on adding each and every term you finally, get a mixture momentum equation which is of the same form that we had obtained for single phase flows the same form that we had obtained for homogenous flow model. It will be dp dz equal to one particular frictional term plus one particular gravitational term plus one particular acceleration term.

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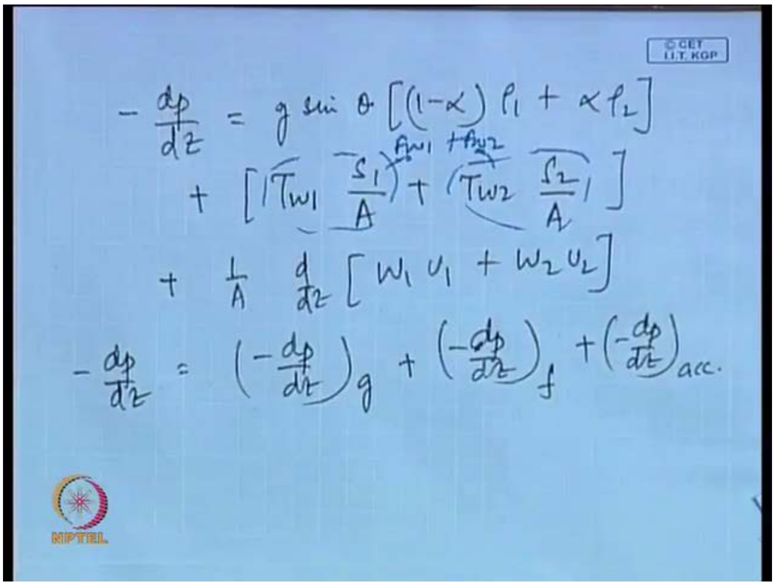

Only thing is they are the frictional term it was just with the wall and nothing else in this particular case the frictional term will consist of phase one with wall phase two with wall phase one and phase two. So, just do this addition and then you see what you are going to get? simply do this particular addition and then the thing which we are going to get is it is minus dp dz equal to g sin theta into 1 minus alpha rho 1 plus alpha rho 2 or in other words this is rho TP plus you get tau W1 S1 by A plus tau W2 S2 by A. We can also write it down as FW1 plus FW2 where this is FW1 this is FW2 which FW1 FW2 denoting the interaction of phase one and phase two with the wall .Plus 1 by A d d z of W1 U1 plus W2 U2 this is the acceleration pressure gradient. So, therefore, from here we get minus dp dz this is minus dp dz gravitational plus your dp dz frictional plus dp dz acceleration. So, this is the final form which we get. So, what we basically did in this particular case we first considered the two phases separately we wrote down the momentum equation for phase one, we wrote down the momentum equation for phase two, and then we tried to combine them in different ways in one particular way we tried to combine them in such a way that they give us the rate or the difference between the rate at which the two phases gain Kinetic energy.

And the other one the simple addition term what we got typical momentum balance equation for two phases when they are considered separately that comprised of the gravitational pressure gradient, the frictional pressure gradient, and the acceleration pressure gradient. Remember in the final expression there is no tau i here because the two tau i’s they cancel out fluid two on fluid one fluid one on fluid two the two tau i’s they cancel out So, therefore, we do not have a tau i in this particular expression

Now once we have got this particular expression the next endeavor is how to find out the pressure gradient from it. Now, similarly just we as we have done in the previous case for homogeneous flow we find that U1 it should be varying with z why? because you’re the specific volume it varies or the density it varies with the axial distance just because of the pressure drop is not it. So, U1 it is going to be W1 by rho 1 A1 or rho 1 A alpha.

So, we find alpha varies with z we find U1 varies with z A also may vary with z. So, accordingly we have to write down this term and then if you simplify the acceleration pressure gradient and we incorporate that particular term in the total pressure gradient. We would get the total expression comprising of a denominator and a numerator where the denominator will give us the choked flow condition for two phase flow under separated flow conditions and the numerator it will have three or four particular term denoting the gravitational pressure gradient, the frictional pressure gradient, and the pressure gradient due to the area change and such other terms.

So, in the next class we will be writing the particular momentum balance equation that we have got in this particular equation by substituting the acceleration pressure gradient. So, that we get the final expression by from where we can find out the pressure drop from known input parameters and then we will try to define the conditional choking for two phases under separate flow conditions. So, that is all for today, thank you very much.