Multiphase Flow
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Lecture No. # 19
Separated Flow Model (Contd.)

So, we will be continuing with our discussions on the separated flow model as we were doing. So, till the last class whatever we have done, we have first taken up the two fluid model. And then, we had derived the continuity and the momentum equation for phase one, for phase two. And after that what we tried to do is, we tried to find out what happens, when there is mass transfer between the phases, what happens when there is some change of phase, may be some sort of boiling condensation whatever it is. So, under that condition, what happens?

What I forgot to (( )) derive was and then after that what we did? We derived the mixture sorry the momentum equation for phase one, momentum equation for phase two and then we combine both of them in different ways; one gave us the relative motion equation, the other gave us an expression of the pressure gradient. So, today what we are going to do is we will try to express the pressure gradient equation, the mixture pressure gradient equation in terms of known input parameters, as much as possible. We will see that we can substitute all the terms, if we look at the mixture pressure gradient equation, we find that I will just write it down once more, I do not know whether it is there or not with me.

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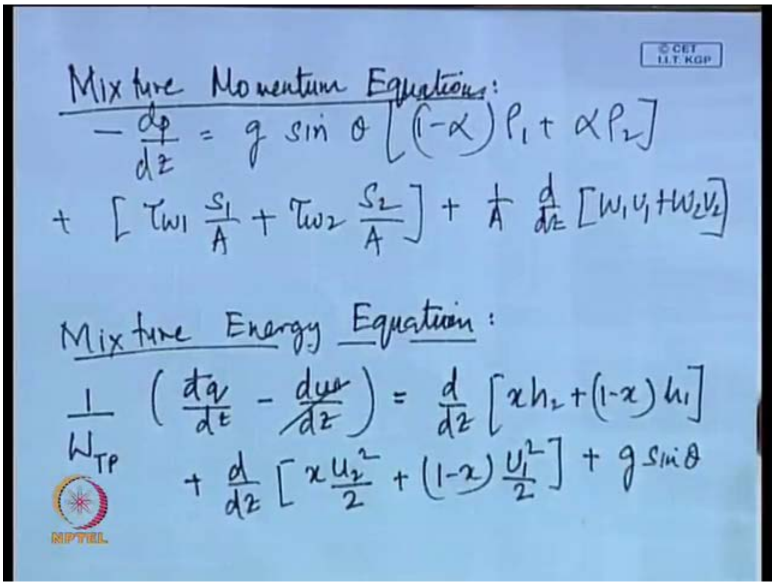

Anyhow I will just write it down if you just notice the mixture pressure gradient equation, it was something of this sort 1 minus alpha rho 1 this already you have it with you, because in the last class itself I had discussed this part tau w 1 s 1 by A plus tau w 2 s 2 by A plus 1 by A d d z of w 1 u 1 plus w 2 u two so, this was all that we had. So, therefore, we find that in this particular case, more or less alpha is also an unknown here, why? Because unlike homogenous flow model, we cannot assume that alpha equals to beta so, alpha is the in-situ volumetric composition.

When it is the in-situ volumetric composition, it is not within the control of the experimental or the designer, as I have already mentioned; it depends on a number of input parameters, but the dependence is not very straightforward. So, therefore, it has to be determined or estimated either experimentally or theoretically from some known input parameters. So, this is also not very straightforward. Other than that, this particular frictional pressure gradient of course, we have to discuss methods, and here we find w 1 w 2 are known, but u 1 u 2 are not non again they are the in-situ velocities of phase one and two. And they vary just, because this in-situ velocities, they are functions of specific volume, they are function of pressure gradient and so on and so forth.

So, we have to express u 1 u 2 in terms of known measurable parameters, or in other words we have to express the acceleration pressure gradient in terms of known input parameters. And once we can do it, then probably we can find out the pressure gradient or the pressure drop across a known length, provided we have some idea of how to estimate the frictional pressure gradient.

So, today what we are going to do is, we would try to do the pressure drop calculation from known input parameters, but before that yesterday, I had forgotten to mention the or rather I did not have time to mention the mixture energy equation. In the same way, you can derive the mixture energy equation, I am just writing the final form, the derivation is left as an exercise or as a home assignment for you.

This was the mixture momentum equation, which we obtained by adding up if you remember by adding the equations we had derived for the momentum balance of phase one and phase two, I have just added them up and obtained this particular equation. In the similar way we can find out the energy balance equation and for unit mass of the fluid 1 by W T p, it will be something of d q the heat flux d z minus d w the work done which is usually equal to 0. This is equal to d d z of x h 2 plus 1 minus x h one or in other words this is nothing, but the two phase enthalpy plus the two phase kinetic energy for unit mass of the fluid, unit mass of the mixture in other words U 1 square by 2 plus g sin theta. So, the; at this gives us the mixture energy equation, again from the first law you can find out first law for open systems this particular equation can be easily derived.

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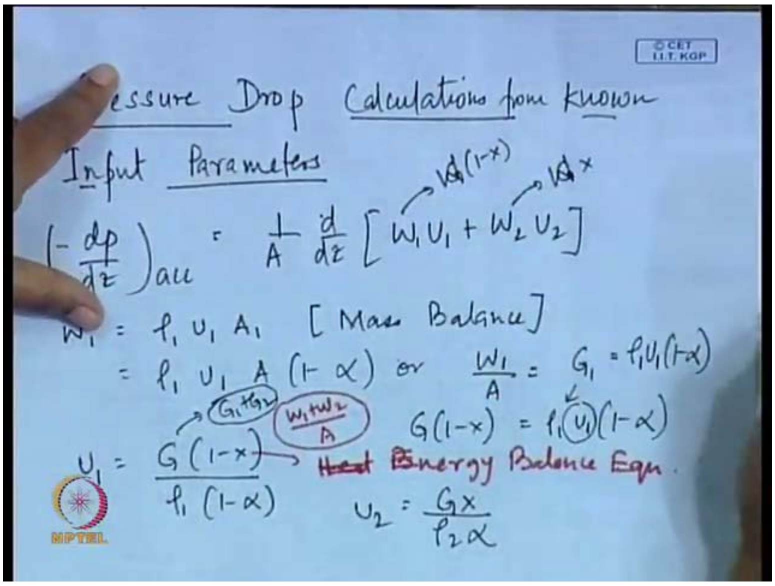

Well now, we go to discussing the pressure drop calculations from known input parameters. So, in this particular case if you notice here so, we will be substituting your

U 1 and U 2 from here. So, we will consider the acceleration pressure gradient. Acceleration pressure gradient it is nothing but 1 by A, I would request you to do the derivations yourself and then compare with my derivations, otherwise it is going to be very dry. And finally, when you just see the derivation note it down and when you go to your hostels it is very difficult for you to derive the whole thing together, because lot of things have to be substituted. So, the best is, if you listen to my instruction do the derivation on your own and then compare with the final form I have got, that will be the best.

So, in this particular case we find that we know from mass balance W 1 equals to just from the basic equations I have started. So, from mass balance we know this or in other words this is equals to rho one u 1 A into 1 minus alpha. In your exams if you are supposed to do any derivations start from the basics and then start deriving, otherwise it is going to be very difficult or we can find out W 1 by A this is equal to G 1 agreed and this is equals to rho 1 u 1 into 1 minus alpha, this G 1 this is nothing but G into 1 minus x which is equal to rho 1 u 1 into 1 minus alpha. So, from here we can find out u 1, in terms of measurable properties or in terms of alpha, see alpha already you have this alpha variable in your mixture momentum equation. So, one if you can reduce the number of variables that is our target, is it not. So, therefore, we get u 1 in terms of alpha as well as other known measurable properties. So, u 1 it becomes G into 1 minus x by rho 1 into 1 minus alpha, G we already know it is nothing but G 1 plus G 2, is it not.

Or in other words this is nothing but w 1 plus w 2 by A, is it not. So, x we can obtained from heat balance equation or energy balance rather it should be we can get it from energy balance equations. So, this is also an input from heat balance, rho 1 we can find out from standard tables and; or in other words if it varies with pressure we can find out. How it varies with pressure either from thermodynamic equation of state, usually that happens for the gas phase rho 2, rho one is usually constant. In case it varies we can find out the variation either from the thermodynamic equation of state or may be when there is some phase change we can consider that. And therefore, u 1 it is obtained in terms of measurable parameters.

Similarly, u 2 proceeding similarly we can get it as u x by rho 2 alpha. Now, if we substitute these two here, then in that case what do we; if we substitute u 1 and u 2 just see. So, for u 1 we get g into 1 minus x by rho 1 into 1 minus alpha, is it not. It w 1 by A that is also G into 1 minus x, is it not.

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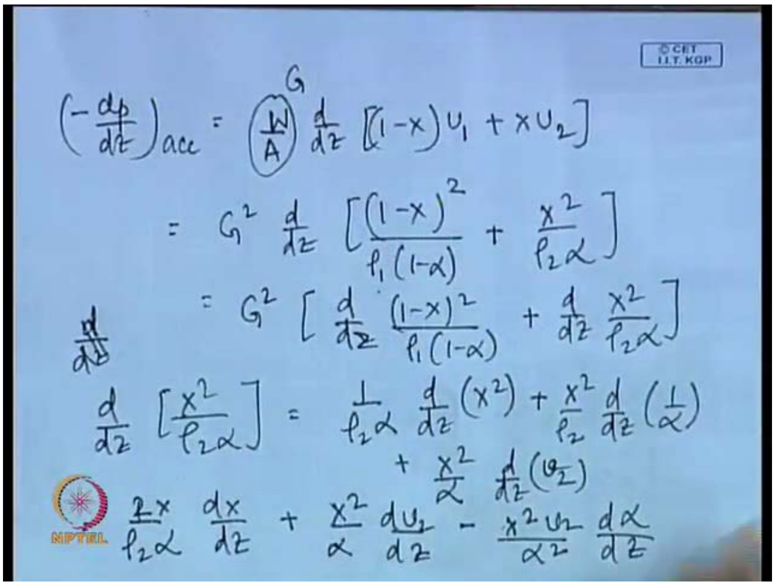

So, what we can do is, this w 1 it can be written down as G into 1 minus x, this can be written down as G into x sorry, very sorry w into one; very sorry, W into x agreed, yes or no. (( )) that can be done this w can be taken out and we can get W by A which is nothing but G fine. So, let us do this and see what we get? So, minus d p d z acceleration this is equal to; we have 1 by A, we have taken out the W so, it becomes d d z of 1 minus x U 1 plus x U 2, is it not, where this W by A this is nothing but G. Now, in this equation if U 1 and U 2 are substituted from these two equations, U 1 from this particular equation this is G into 1 minus x by rho 1 into 1 minus alpha and U 2 equals to G x by rho 2 alpha.

If these are substituted, then what do we get? Another G comes out from here and it makes a G square outside. So, therefore, from here we get G square d d z 1 minus x into the 1 minus x here. So, it becomes 1 minus x Whole Square by rho 1 into 1 minus alpha plus x square by rho 2 alpha, is it not? Now, in this particular case we find that what are the things which vary with z, x varies with the z if there is some heat flux. if there is no heat transfer, no heat it is under adiabatic condition x does not vary. Since we are dealing with a most generalized case we would like x to vary and if the problem you have to apply for an adiabatic case then in that case you can assume that x is a constant in that case you do not assume x 2 vary.

Rho 1 usually it does not vary, but we will just keep it as d v 1 d p or something and then we will eliminate it for the as the case. Alpha definitely varies with z if x varies then also alpha varies if x does not vary then also alpha can vary. So, therefore, we find that all these terms they vary with z. So, therefore, if we have to find out these in other words can be written as G square d d z of 1 minus x whole square by rho 1 into 1 minus alpha plus d d z of x square by rho 2 alpha, we can write it down in this particular way. So, therefore, let us see this d d z of; say, let us take up this term first x square by rho 2 alpha this is no fluid mechanics, no multiphase flow this is basically differentiation, but very long long differentiations which will give you very long long equations and you might get confused and you might make a lot of careless mistakes.

So, that is the thing which you have to look out for, when you are doing particularly these derivation, these are naturally, they will be the most complex derivations as far as multiphase flow is concerned. And I would request that you would practice those derivations quite a number of times, otherwise you will not get yourself familiarized with this particular things. So, therefore, this is nothing but simply the differentiation I am doing d d z of x square, x is the quality plus x square by rho 2 1 by alpha plus x square by alpha d d z of U 2 sorry, v 2 rho 2 I have made it v 2 agreed any problems. So, therefore, simply; whatever you have learnt in your class eleven twelve nothing more than that.

2 x d x d z I am see the thing is the final expressions, we will find in several text books. But the total derivation you would not find and usually students they have a problem, it is just you have to sit and do there is nothing very great in it, but usually students they have a problem in deriving the whole thing. So, therefore, I will do the whole derivation I do not even have slides, because they may; they might become very dry means it will difficult for you, just to see a large number of equations without derivation. So, I am doing the derivation in the class so that you can actually follow the step wise things and you can actually arrive at the final expression from the basics so, just to avoid any sort of confusion from your side.

So, therefore, this becomes 2 x this plus it becomes x square by alpha if I do this part first d v 2 d z and please look out whether I am making any mistakes or not that can also happen, minus for this term if we do it is going to be minus x square v 2 by alpha square d alpha d z, just see whether all of these terms are correct or not. First term on differentiating gives you 2 x d x d z with A 1 by rho 2 alpha, the second term naturally it gives you minus 1 by alpha square d alpha d z and there is an x square by rho 2 or x square v 2 here. And this one definitely gives you x square by alpha d v 2 d z or d d z of 1 by rho 2 in whatever way you want to write it.

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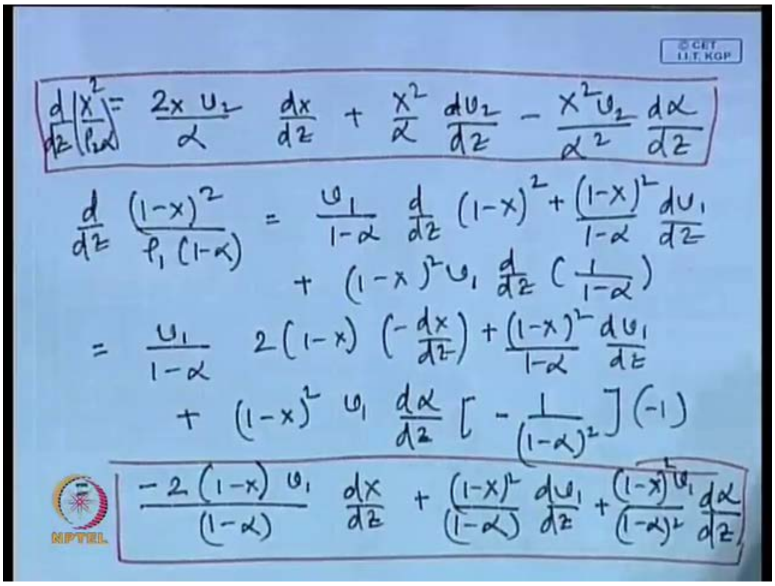

So, now from here, what do we get? From here we get, this is nothing but 2 x v 2 alpha 2; or in other words if we just transform everything into v 2 then in that case this simply gives you 2 x v 2 by alpha d x d z plus x square by alpha d v 2 d z minus x square v 2 by alpha square d alpha d z fine. Just the things which I had derived agreed, this is equal to I should write d d z of x square by rho 2 alpha fine. So, this was one particular term in the acceleration pressure gradient. Same way we can do for the other term as well, just you have to keep in mind that here it is 1 minus x square so, therefore, it will be minus 2 x etcetera etcetera. So, certain things you have to keep in mind.

So, in the similar way if we perform the differentiation of d d z of 1 minus x square by rho 1 into 1 minus alpha, what do we get? We get v 1 by 1 minus alpha same way, let us proceed so that you can understand it, or you can do the derivations yourself d v 1 d z plus 1 minus x whole square v 1 d d z of 1 by 1 minus alpha. So, this; if it is done this gives you v 1 by 1 minus alpha 2 into 1 minus x minus d x d z, is it not it. Plus 1 minus x whole square by 1 minus alpha d v 1 d z and again plus 1 minus x whole square v 1 d alpha d z, this gives you minus 1 by 1 minus alpha whole square into minus 1. Simply differentiation I have done, I am not jumping steps, because in case I also make some mistakes we will not be able to find it out.

So, therefore, from this particular term if you see if you see it is basically d d z or rather this term it can be written d d 1 by alpha 1 minus alpha of 1 by 1 minus alpha into d 1 minus alpha d z. So, d 1 minus alpha d z gives you minus one and the other part differentiation gives you this. So, these parts, these things please derive on your own and then you try to find it out. So, this gives you minus 2 into 1 minus x v 1 by 1 minus alpha d x d z plus 1 minus x whole square by 1 minus alpha d v 1 d z plus 1 minus x whole square v 1 by 1 minus alpha square d alpha d z. So, therefore, one particular equation for x square by rho 2 alpha d d z of this I have found out, the second expression is for this part d d z of 1 minus x square rho 1 by 1 minus alpha.

So, this is one particular expression this is other. Now, once I substitute both these equations here. I get the final expression of acceleration pressure gradient in terms of known measurable parameters it will just contain alpha extra and x that also we considered to be a known measurable parameter. So, if we substitute this particular equation and this particular equation in the acceleration pressure gradient equation, the final expression which we get is something of this sort.

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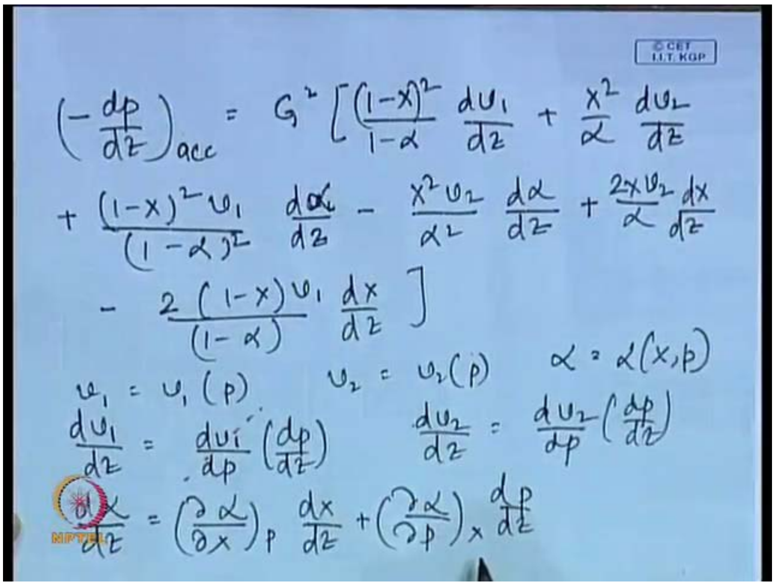

It is minus, simply we substitute it we get something of this sort this is equal to G square 1 minus x whole square by 1 minus alpha d v 1 d z plus x square by alpha d v 2 d z plus

1 minus x; simply I am writing down whatever I have got. So, so far by 1 minus alpha whole square d v 1 d sorry, d alpha d z minus x square v 2 by alpha square d alpha d z plus 2 x v 2 by alpha d x d z minus 2 into 1 minus x v 1 by 1 minus alpha. So, therefore, there are two d x d z terms two d alpha d z terms and one d v 1 d z and one d v 2 d z this is all. Now, here we find that why does v 1 or v 2 vary with z, any idea why does v 1 v 2 vary with z, just because pressure varies with z they are a function of pressure how to know; in what way they vary with pressure one is the p v t behavior of gases or simply from the equation of state we can find it out, is it not.

Or suppose there is a change of phase then in that case we can also find it from thermodynamic tables as well. So, we know that d v 2 d z and d v 1 d z they are simply; these terms arise simply because v 1 is a function of pressure, v 2 is a function of pressure fine. So, if that is the case, then in that case d v 1 d z this can very well be written down as d v 1 d p into d p d z yes or no, same way d v 2 d z this can be written down as d v 2 d p into d p d z agreed. For most of the situations we will find that, this term is not there this cancels out. Now, we are doing it keeping everything intact, but remember for most of the situations this term is going to cancel out, depending upon the problem you will be given in the mid sem; or in your examination.

You will eliminate them from the very beginning and you can start doing no marks will be deducted. In fact, your derivation will be slightly simpler in that particular case so, remember this thing. So, therefore, we find d v 1 d z that arises or that can be substituted with d v 1 d p d p d z similarly, this d v 2 d z that can be substituted with d v 2 d p d p d z fine. Now, you tell me why does alpha d alpha d z from where does this come see d x d z, they are inputs this it just depends upon the amount of heat flux. If we have a constant heat flux there are situations where more or less d x d z is a constant or in other words x varies linearly with z such situation. So, we need not bother about d x d z terms they are simply inputs.

Now, d alpha d z can you tell me why this d alpha d z term arises, on what does your void fraction alpha depend any idea under this situation. Just like v 1 v 2 on what does your alpha depend, it depends upon; if there is a phase change definitely it will depend upon pressure x, plus it will depend upon pressure as well. Because depending on the pressure the velocities etcetera are going to change so, alpha just like v 1 was a function of pressure only, v 2 was a function of pressure only, we find alpha is a function of x as well as p agreed. So, therefore, d alpha d z that can be written down as del alpha del x at constant p d x d z, again this becomes an input plus del alpha del p at constant x d p d z this again depends on pressure.

Now, we can substitute instead of d v 1 d z, we can substitute this particular term instead of d v 2 d z we can substitute this and instead of d alpha d z we can substitute this particular term. That will give us the final acceleration pressure gradient, is it clear to all of you? That will give us the final acceleration pressure gradient fine. You just substitute and find out the acceleration pressure gradient and then what; see finally, what we want to do is we would like to express the pressure gradient here in terms of known measurable quantities. So, once you substitute these particular terms in acceleration pressure gradient and then we can substitute this acceleration pressure gradient in the total pressure gradient expression. In that case what we do? When we substitute it in the total pressure gradient expression we find that even on the right hand side, we have some d p d z terms, is it not; which again, we have to take to the left hand side.

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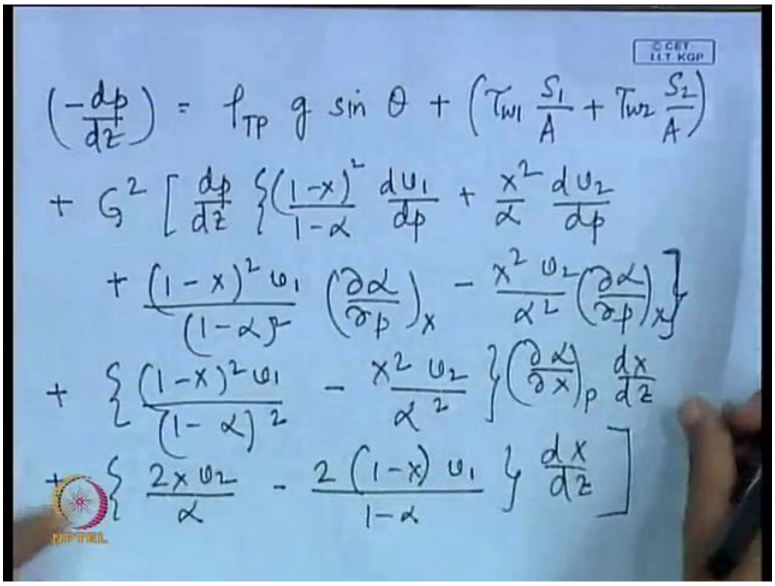

So, what you are required to do is you first substitute your these equations in your acceleration pressure gradient, in order to get the final expression of the acceleration pressure gradient. And after that this acceleration pressure gradient expression that has to be substituted in the total pressure gradient expression. Once this substitution is done then what do we get? On substituting this particular way we get minus d p d z this is equal to; the first term if you see here this is G sin theta into this whole thing 1 minus alpha rho 1 plus alpha rho 2 which is nothing but this term; this whole term this is equal to rho t p, is it not.

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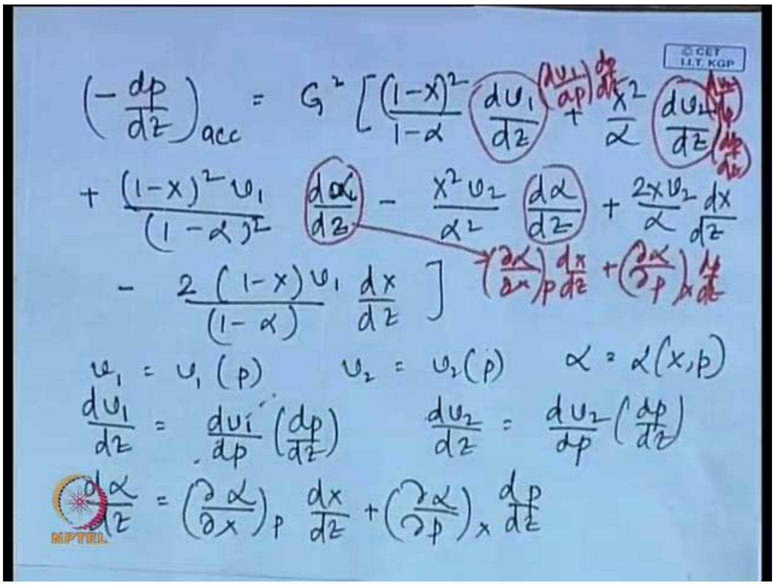

So, therefore, we can write it down as rho t p g sin theta plus the frictional part tau w 1 S 1 by A plus tau w 2 S 2 by A plus if this acceleration pressure gradient is substituted we get G square taking the d p d z terms together. What we do? We first once we substitute this, we find that there is a G square and then here instead of this d v 1 d z, we can write it down as d v 1 d p into d p d z. So, this becomes a d p d z term instead of this we can write down d v 2 d p into d p d z this can also be written and instead of this d alpha d z I can write down this whole thing del alpha del x at constant p d x d z and again another d p d z related term. So, therefore, one two three d p d z related terms we get from here.

So, therefore, if we write them down, we find (( )) this gives us 1 minus x; you just compare with the thing which you have got, one term associated with d v 1 d p then there is a second term associated with d v 2 d p. And there is one more term which is associated with plus 1 minus x square v 1 by 1 minus alpha square del alpha del p constant x minus x square v 2 by alpha square del alpha del p at constant x agreed. This term I missed out, this term also gives you the same thing. So, therefore, four terms associated with d p d z, one term contains d v 1 d p other term contains d v 2 d p and the rest two terms they contain del alpha del p at constant x.

And along with that whatever the rest of the terms containing d x d z and then all those terms are remaining. So, therefore, we get sorry, this will be like this curly bracket and then along with that what we get is plus 1 minus x whole square v 1 by 1 minus alpha whole square minus x square v 2 by alpha square del alpha del x at constant p d x d z terms containing d x d z and the term containing del alpha del x at constant p. Plus we have 2 x v 2 by alpha minus 2; please do this practice these derivations number of times or else it will simply be difficult for you that is the only point.

So, this is the entire total pressure gradient equation that we have, first term it is the gravitational pressure gradient, the frictional pressure gradient. And then a G square term associated with d p d z this entire portion will come to the left hand side and the remaining terms they contain d x d z. Just substitute and see whether you have got these terms or not.

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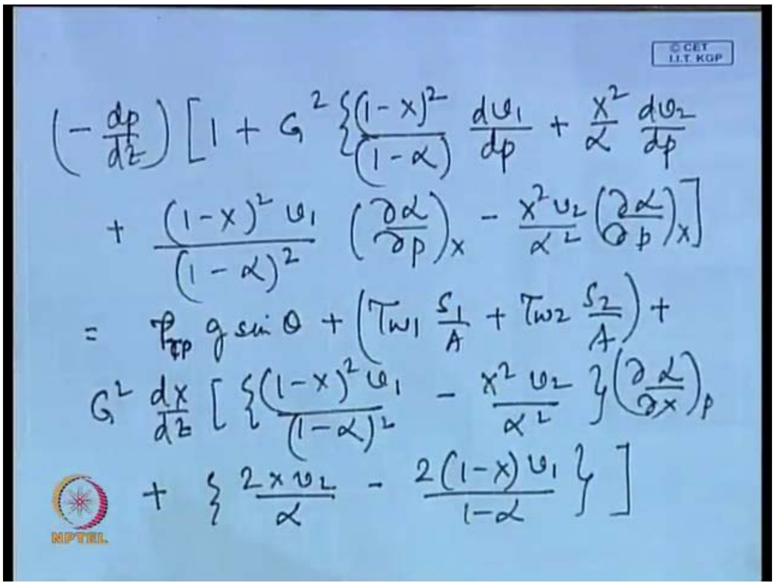

Now, once we get this, what we can do? We can simply bring the all the d p d z containing terms to one side so that it becomes easier for us. And if we do that, what do we get? We get minus d p d z into 1 plus G square 1 minus x whole square by 1 minus alpha plus x square by alpha d v 2 d p plus 1 minus x square v 1 by 1 minus alpha square del alpha del p at constant x minus x square v 2 by alpha square del alpha del p at constant x. This whole thing this is again equal to sorry, rho t p g sin theta plus frictional component of it plus G square d x d z, because all the other terms they contain d x d z 1 minus x whole square v 1. Please do these derivations otherwise it will be slightly difficult for you del alpha del x at constant p plus 2 x v 2 by alpha fine.

This is the total pressure gradient expression that we have here a huge expression which might frighten you, but if you do it slowly if you try to do everything understanding everything and do not make mistakes then, I do not think it is going to be very very difficult for you. So, from here, what we can do? We can; you can just note down the derivations which I have made it and that is going to be easy for you. It simply d p d z into this whole thing this is equal to this particular portion. The final expression definitely you will be getting in text books, but arriving at the final expression that probably you have to do it on your own. So, all of you have copied down the expression fine.

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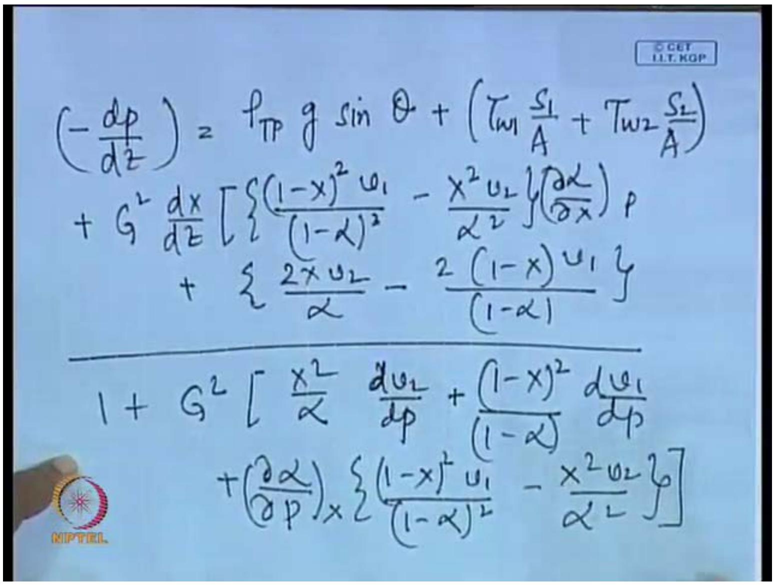

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So, from the here, what do we get? From here we get that the final expression of minus d p d z in terms of known measurable parameters that is simply; just I bring the left hand side, the multiplicating factor of minus d p d z to the right hand side simply nothing else. So, this is going to be rho t p g sin theta apparently it is slightly boring, but definitely when we do problems we find that we deal with certain simpler situations, because certain things tend to be negligible under certain conditions. Then situation becomes easier, but you should always know the actual expression so that you can know where and how the simplifications have to be done.

Again the same thing, I am just writing it down so that may be if I have made a mistake or you have made a mistake in the previous expression at least you can see and you can correct it here. Del alpha del x at constant p plus 2 x v 2 by alpha minus 2 into 1 minus x v 1 by 1 minus alpha, this divided by 1 plus G square whatever was there and if you remember in the homogeneous flow model as well, we had this d v 2 d p and d v 1 d p term there. In addition these two terms we already had in the homogeneous flow model is it not. And then this denominated term we compared it with the compressible flow situation and then we said that in the compressible flow situation it corresponds to 1 minus m a square, where m a is the Mac number which is nothing but the ratio of the velocity of flow to the velocity of sound under the same conditions of temperature and pressure as that also flow, is it not.

So, therefore, we find out that; accordingly we had told that in the homogeneous flow model definitely this would correspond to the Mac number. Or rather this would correspond to 1 minus m a square and 1 minus m a square from here we can get the criteria for Mac number equal to one which is nothing but the condition of choked flow. So, therefore, in this particular case also if I complete the denominator, in this particular case also this denominator it should correspond to 1 minus m a square, this whole thing it should correspond to 1 minus m a square. And therefore, this is; this should correspond to the Mac number for two phase flow under separated flow situations.

So, if this whole thing this is equated to 1 minus m a square what two phase flow under separated flow conditions then from there what are we supposed to get. We are; suppose to arrive at some particular condition which will give us. The condition of choked flow what two phase flow when the two phases are flowing under separated flow conditions, is it clear to you. The logical thing which should have been was that see we should have started with the basic mixture; the basic equations for component one and component two. We had written down the basic momentum equation in that particular basic momentum equation what we should have done was, we should have written down or we should have expressed it in terms of d p d z.

That would have given us the condition of choking for phase one for phase two, for the two different mixture momentum equations, which we have derived initially. If we apply in the same way see the there also we had d v 1 d z d v 2 d z there in the same way if we would have applied d v 1 d p d p d z etcetera then from there also we would have arrived at conditions of choking for phase one, from the momentum balance equation of phase one, is it not. And for phase two from the momentum balance equation of phase two. So, this would have given us the condition of choking for phase one and for phase two. But we if we will do it in the next class and then you will find that this will in no way guarantee us the condition of compound choking, because the because that; in the next class what I plan to do is, first I will be deriving the condition of choking from this particular denominator, the denominator which I have written down here.

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If you see my; the thing which I have written down in this particulars in the written matter then you will find, that from this denominator which I have written down. Equating this to 1 minus m a square we will be getting a condition of choking, this condition of choking we are getting from mixture momentum equation. There we will find that there is some sort of anomaly in the expression. And that had a reason just because there were certain reasons since we had construct the mixture momentum equation we did not consider the interaction between the phases.

So, after that what we will do in the next class is, after that we will take up the two phases separately. The basic equations which I had written down, the momentum equation for phase one the momentum equation for phase two. And then, from each of the momentum equations I would like to derive the condition of choking that will give me the condition of choking for phase one condition of choking for phase two. If I combine the two I will find that there will be one Del alpha Del z or something term. Which shows; which would show us that just ensuring that flow phase one is under choked flow just ensuring phase two is under choked flow it does not guarantee that the two phase mixture is under choked flow, because what happens that whenever you are for this two phase flow is occurring the alpha is adjusting itself accordingly.

So, therefore, just phase one being in choked flow phase two being in choked flow will not guarantee that the two phase mixture is in choked flow for that we have to consider the variation of alpha with z or the variation of alpha with p. So, initially we will be doing the condition of choking from mixture momentum equation, or in other words we will be considering the denominator which we have derived in this particular case. After that we will be considering the condition of choking by considering the two phases separately. And just to make a vey generalized thing we will be considering the component moment equations, which we had derived by considering change of phase as well. If there is no change of phase simply the change of phase terms will be cancelling out, or in other words the terms containing eta if you remember.

In the last class we had derived there were some terms containing eta where eta was the fraction of the force which arose due to change of phase. So, out of the total force that arose due to change of phase, we had ascribed eta fraction of the phase of the force to phase two 1 minus eta fraction of the force to phase one. So, if you do not have a phase change simply those particular terms containing eta will be cancelling out. So, just to avoid doing the derivation once for normal two phase flow under separated flow under adiabatic conditions. Then again considering change of phase, what we will be doing is after we are; we have considered the mixture momentum equation we will be considering the; or we will be finding out the condition of choking by considering.

The separate equations of momentum for phase one and phase two keeping in mind that change of phases also occurring. Naturally this will give us to much more complicated equations, but this will be a generalized case, did you get all of you. So, accordingly what we will be liking to do, we will be doing that and naturally please come prepare otherwise tomorrow's equations will be slightly more complex. I am not going to start it today just because you need to prepare the whole thing. So, that see complex means basically we have to use the continuity equation and we have to use other equations and substitute them. So, that we can arrive at an equation, which can be expressed in terms of known measurable quantities as far as possible so, this is the thing which we will be doing in the next class.

Now, after that what we would like to do is that if you see, what we have done basically our intention is simply to find out or our intention is simply to predict the pressure drop for two phase flow under separated flow conditions. This is our intention, for that we had written down the momentum equations and after writing down what we did we combined them to find out the mixture momentum equation.

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And the final expression, we have got is this particular whole equation which has; enables us to calculate pressure gradient from known input parameters. If you go through this particular equation term by term, you find that most of the things you can obtain from input conditions as well. For example, x can be obtained from heat flux equation v 1 v 2 these things you can get from standard conditions etcetera, etcetera. The things which are unknowns in this particular equation first thing is your frictional pressure gradient. Now, this had also created a problem in our homogeneous flow model, if you remember there also we did not know how to express the frictional pressure gradient.

Why it becomes a problem, because how the fluid will interact with the wall that depends upon the nature of the fluid that depends upon the composition of the fluid if it is a two fluid or if it is a multiphase mixture. So, therefore, under that condition, under homogeneous flow condition the situation was simpler. Why it was simpler, because we had considered that the two phases were intimately mixed and when they are intimately mixed what happens they flow as a pseudo fluid. So, therefore, we assumed that well this is a single fluid which is flowing a pseudo fluid with; which is flowing with suitable average properties and this particular fluid as a whole is interacting with the wall accordingly the wall shear stress can be calculated.

But, under that circumstance also we found out that even if we use simply the single phase equations which are available under that condition also we found that, we had to encounter certain unknowns; we had to resort to certain empirical correlations. Why because in single phase flow we found out, how do we find out frictional pressure gradient? We find out frictional pressure gradient in single phase flow from moody's plot or in other words we find out the frictional pressure gradient by using a friction factor definition, where this friction factor it is a function of Reynolds number fine. Finding out Reynolds number for single phase flow it is very easy it is just based on measurable properties it is nothing but r e equals to d g by mu.

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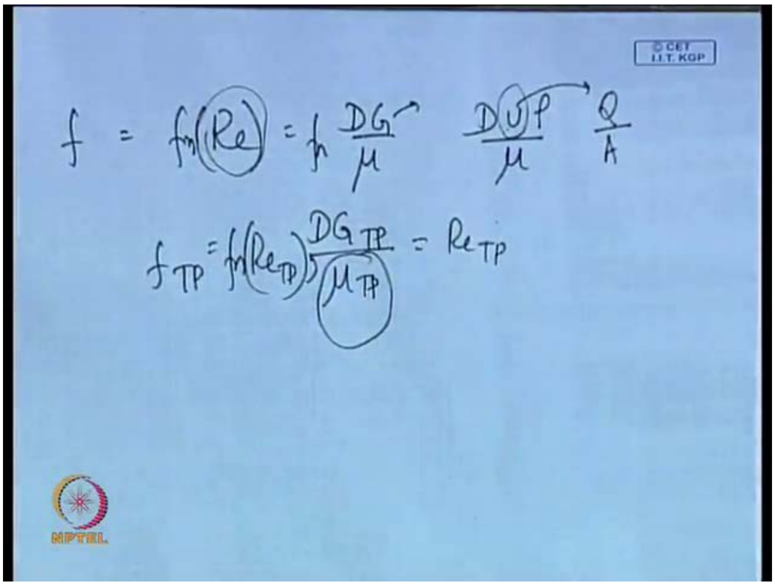

So, therefore, finding out friction factor it is just friction factor for single phase flow it is a function of R e where R e equals to D G by mu, this G; or in other words this is better written down as D u rho by mu. There see; there was no chance of u varying within the flow there was nothing like in-situ and input velocities, is it not. This u was simply you measure the volumetric flow rate divide by the cross sectional area you get u, D is the pipe diameter, rho and mu these are simply fluid properties. So, therefore, this Reynolds number it was an input there. Similarly, if we had considered the homogeneous flow model is the same way we could write D G T p by mu T p this I had discussed in the last class as well this is f t p, this is a function of R e T p where this equals to R e T p.

But we found that even for such a simple case also, we found that finding out mu T p is a challenge. Because for density we can very well say, that well density it is a function of the component densities as well as the void fractions or the volumetric proportion of the two phases inside the flow. And since it was a no slip condition the volumetric proportion with which we had introduced the flow was equal to the volumetric proportion with which the two fluids were flowing inside the pipe. With what volumetric proportion, we had introduced them into the flow that is already known to us.

What we cannot control is what will be the proportion inside the pipe for homogeneous flow they are the same. So, finding out the rho was no problem, but finding out mu t p how this viscosity, the two phase viscosity that varies with the composition was very; not very well known to us. So, therefore, there also we phased a problem.

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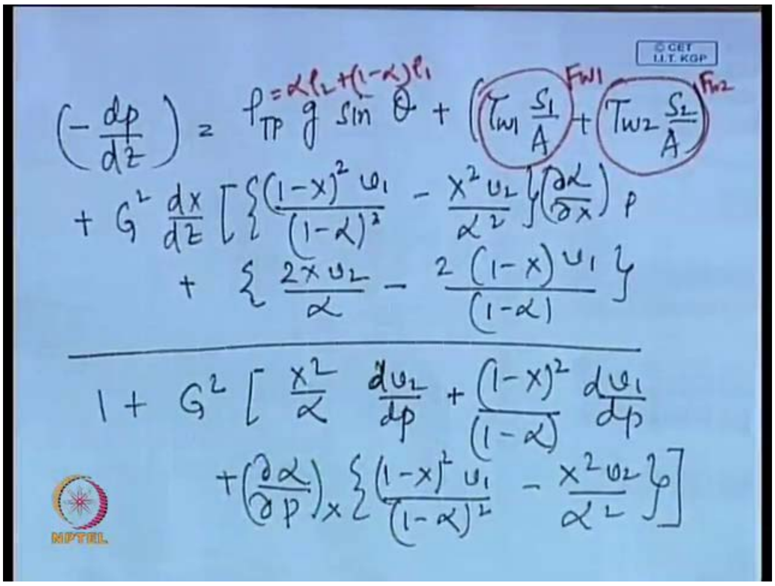

Now, in this case also we find that the frictional pressure gradient this is an additional problem. Basically they can be written down as, this can be written down as something like F w 1 which is the interaction of phase one with the wall, this can be written down as F w 2 which is the interaction of phase two with the wall. So, therefore, to make matters simple we can simply write the frictional pressure gradient, as the summation of the frictional contributions from phase one and phase two. When we consider the mixture equation then naturally the interfacial shears they cancel out and this becomes a summation of the frictional contribution from phase one plus the frictional contribution from phase two.

So, this F w 1 plus F w 2 this has to be found out. After we discuss the conditions of choking and all those things, we will next concentrate on how to find out the frictional components in this particular case. This we had also to discuss when we were discussing the homogeneous flow model also. But in addition to homogeneous flow model, we find that there is an additional factor also which was not there in that particular case and that is this particular del alpha del x or in other words if you expand rho t p it is alpha rho 2 plus 1 minus alpha rho one so, this alpha term is also not known here. For homogenous flow alpha was equal to beta we could find it out, in this particular case we do not know alpha.

So, therefore, we need methods to calculate the frictional pressure gradient component of the total pressure gradient expression and we also need some expressions to find out the void fraction. So, therefore, apart from whatever is there, apart from this particular equation in order to find out pressure gradient, we need two additional equations one for frictional pressure gradient and one for void fraction. And of course, apart from that we also need relationships between the thermodynamic properties and so on and so forth. Now, we can derive these particular equations by analyzing each component separately, that we can definitely do, but usually the physics is not very well understood.

And so, what we prefer when we do not understand the physics, we take a large amount number of data and we try to do some empirical correlations. So, here also in order to find out void fraction and the frictional pressure gradient in terms of known input parameters, we would like to use some empirical correlations in the absence of any further information, this is what we would like to do. So, therefore, usually for finding out this alpha as well as the frictional component, we would like to derive the equations or rather a common technique is to use the empirical correlations to predict the frictional pressure gradient as well as to predict the void fraction.

And of course, certain simplifications come up for example, when only one component is in contact with the wall that happens for annular flow. Under that condition, we have just f w 1 there is no f w 2 so, such a simplification can also come.

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Sometimes what happens we find that the drag force it is a function of the relative velocity. So, depending upon the relative velocity, we can find out the drag force accordingly, may be the drag force is more important than the wall track. So, depending upon the situation we will be modifying it, and we will be getting a much more simplified expression from this generalized expression, from which we can calculate the pressure gradient, when two phases are in separated flow conditions.

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Depending upon the actual flow situation, certain terms will be cancelling out, certain terms will be remaining and finally, you will get different final expressions in order to calculate the pressure gradient for the particular flow pattern, you have in question. So, tomorrow we will be dealing with the conditions of choking and after we finish that, we will be going for the different empirical approaches, which are there for finding out the void fraction as well as the frictional pressure gradient, thank you very much.