**Multiphase Flow**
**Prof. Dr. Gargi Das**
**Department of Chemical Engineering**
**Indian Institute of Technology, Kharagpur**

**Lecture No. # 27**
**Analysis of Specific Flow Regimes (Contd.)**

Well, so we will continue with our discussions on the flow regime dependant analysis of two phase flow. So, in the last class, we had discussed the bubbly flow pattern and I told you the range of bubbly flow patterns that we can have. And definitely for each different ranges, we are going to have different values of u infinity and n; and according to that actually then what we did was we proposed different regimes based on some non dimensional stoops. And then based on those particular regimes, we proposed different values of u infinity and n, which can be incorporated in the basic expression to find out j 2 1 is it not.

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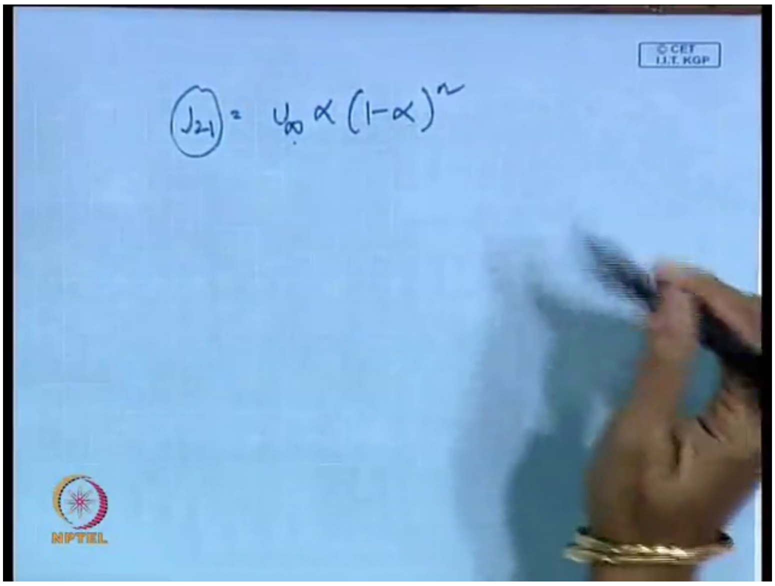


So, we knew already that j 2 1 this is equal to u infinity alpha into 1 minus alpha whole to the power n. So, therefore, in order to find out j 2 1, we need to find u infinity and n. So, what we had proposed was we proposed that this u infinity and n they are different for different flow patterns; and I had proposed the set of a particular in a tabular form I had proposed that if different regimes where different values of u infinity and n, they are applicable. And those particular regimes were based on the some particular non dimensional groups; one was definitely the bubble Reynolds number, the others were g 1 and g 2, two two particular dimensionless groups with liquid and liquid properties particularly incorporated in them.

So, after that what we did and once we could, so we found out that in order to find out j 2 1we need to know u infinity. So, therefore, and we found out that, the expressions of u u infinity that I put up on the table, they were mostly a function of their bubble radius or the bubble diameter, it was a function of r b.

So, we next when to discuss the different ways of finding out r b, we found out that the bubble equivalent diameter or the bubble equivalent radius that is a function of the way the bubbles are produced. They might be produced through an orifice, they might be they might be produced or they might be detaching themselves from a blanket of vapors, which has been formed over a porous plate or over a heated surface; and from there they can detach depend depending on the way they are formed; there where equivalent bubble radius can be determined once we can determine them, we can then again incorporate them in u infinity. Certain other small things also we had discussed in the last class.

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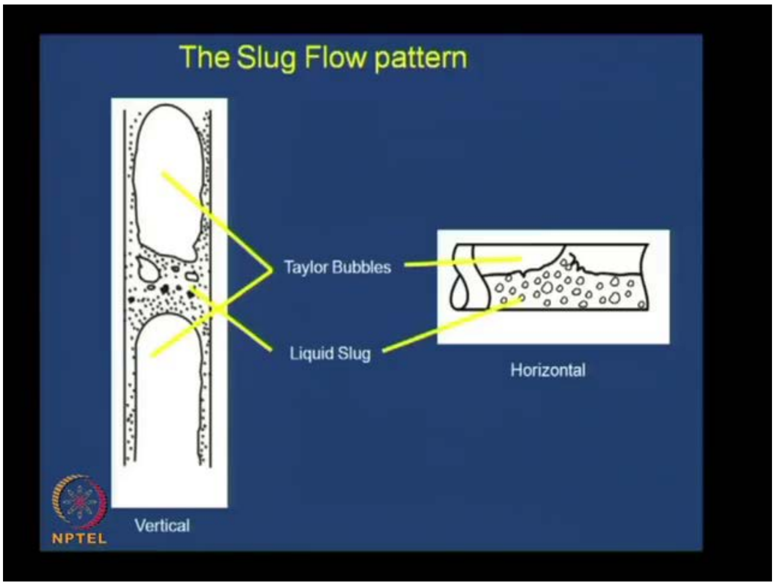


Now, today's class I will be taking up the slug flow pattern. Now, I have already discussed slug flow pattern in great details in the in one of the previous classes. And I just like to show you this schematic of the slug flow pattern which which we had already discussed.

We found out the distract flow pattern is a unique flow distributions, which is characterized by a periodic appearance. That means, if you look at any one particular cross sections say if you suppose look at this particular cross section we find that, at one particular point, it will be almost filled up with gases for a particular interval of time; and of that it will be filled up with a bubbly mixture and then again pure gas with a thin liquid film and again a bubbly mixture. So, this particular periodic appearance makes slug flow very unique in in several aspects.

Now, the best way to analyze the slug flow pattern firstly as we know that, it is I have already told you it is importance has been enhanced particularly with miniaturization, because in micro systems and in millimeter size systems we find that slug flow occurs over a very wide range of phase flow (( )).

Such conditions we usually do not get bubbly flow. So, therefore, as we reduce the pipe dimension we find that slug flow pattern becomes much more prevalent number 1. Number 2, we find that as we reduce the diameter, the tendency of the liquid slug to become aerated also decreases.

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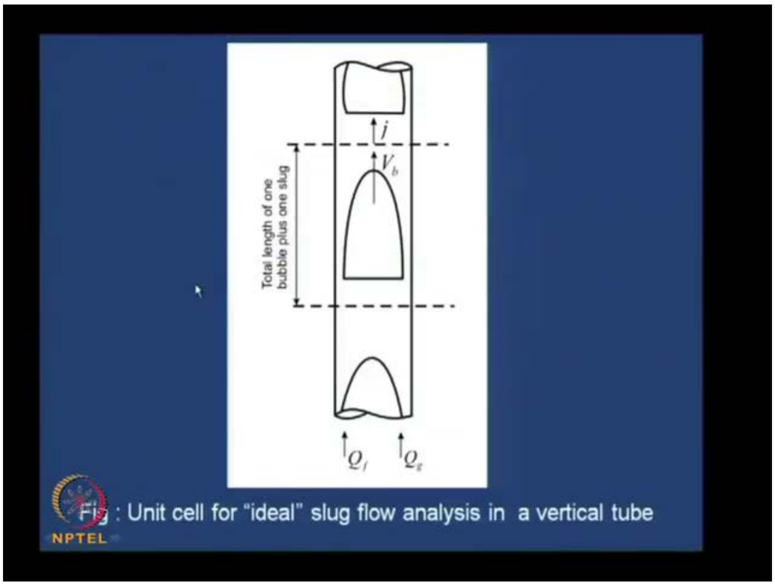


And more or less for very small pipe dimension say mill metric or the micrometric channels we find that more or less slug fluid is characterized by periodic appearance of Taylor bubbles and unaerated liquid slugs as I have shown in this particular figure. This is an ideal slug flow in a vertical tube and usually we find that, when we go for narrow channels we get something of this sort.

Now, for a vertical pipe the best way of analyzing slug flow is because, since lot of things are varying. What are the things that are varying? The gas phase it is it is in two different type of forms, one is the Taylor bubble, the other as small bubbles in the liquid slug. The liquid again in two two forms one is as a continuous liquid slug, which may or may not be aerated then, as a very thin liquid film, annular film between the Taylor bubble and the pipe (( )).

Again for what are the other things that we notice? The flow of liquid, the flow direction of liquid also changes at a particular cross section, why? Because, the liquid slug it is moving up, the liquid moves up in the liquid slug, but in the Taylor bubble regime, the liquid portion moves down.

So, therefore, we find that everything the void fraction, the pressure drop, even the velocities everything they change periodically with time; and therefore, it is quite it appears to be quite complicated; and therefore, for this analysis the most convenient things which is done since everything changes over particular cross section. So, therefore, if you concentrate on a particular cross section then, with time it keeps on changing, so we have to take into account the temporary variation.

In order to avoid this, what we do? We divide the entire slug flow passage or the entire flow passage into unit cells; just like I have done in this particular slide (Refer Slide Time: 06:31), we divide the entire flow passage into a large number of unit cells, where each unit cell it comprises of a Taylor bubble and the part of the liquid slug above it as well as below it.

So, therefore, what happens if we can find out the void fraction of this particular unit slug, we can say this is the void fraction of the entire channel is it not. And once you can find out the void fraction, we can find out the pressure drop, we can find out other hydrodynamic as well as other parameters of slug flow. So, therefore, we find that the analysis becomes considerably simplified, if we can assume that the entire flow passage comprises of unit cell, where each cell is made up of a single elongated bullet shape axis symmetric Taylor bubble and some liquid portions above and below it.

And naturally I I as I have already mentioned before, we have been also considering axis symmetric one dimensional flow with constant gas and liquid properties in this particular case. And then in that case, once we know the void fraction here and we know the total number of unit cells in the entire flow passage then, we can find out all the liquids are parameters.

The other thing to start with what we do is, we assumed pure liquid slugs pure liquid slugs means unaerated liquid slugs, which we usually do not get under ordinary circumstances; under ordinary circumstances as I have already mentioned what happens, just when the liquid it is flowing down for here and it meets the upward flowing liquid slug of wake region is formed. This wake due to this formation of this wake region, what happens is, good amount of gas is shared from the tail of the Taylor bubble and they distribute as small bubbles, and they resemble the bubble flow pattern in the liquid slug region.

Usually, we will find that the concentration of bubbles are more near the your just below the tail region and then, after that more or less we can assume a uniform distribution of the liquid bubbles sorry of the gas bubbles. What happens, when this particular liquid again begins to fall as an annular film between the Taylor bubble as well as the tube wall, then we find usually the liquid film dimension it is much less than the dimension of the dispersed phase. Therefore, usually bubbles are not carried in the liquid film.

So, therefore, pure liquid flows as a thin annular film between the Taylor bubble and the tube wall. And when this meets the liquid slug below, then a good amount of aeration occurs under normal circumstances. Now, to start with see this analysis has gone to a well researched extinct, because slug flow is so very important. Now, to start with since I do not have much time I cannot spend much time on this.

So, we will be doing the simplified the analysis and after that I will just discuss what are the ways by which we can incorporate the other factors, which will make the model much more realistic and much more applicable for practical situations. So, what is the thing that we sorry, the thing that we first consider we consider ideal slug flow, where we are broken up the entire flow passage into large number of unit cells.

Now, in each unit cell if you observe for to find you find a single Taylor bubble, this Taylor bubble is raising up at a velocity u u b, where u bubble is nothing but, the u gas, because the entire gas phase is confined as Taylor bubbles in the slug flow pattern. So, u b is nothing but, it it it is nothing but, u g. For the present case, where we are assumed unaerated liquid slugs remember this.

If we had assumed aerated liquid slugs, then definitely we could not have (( )) this. So, therefore, we find that this (( )) up at a velocity u b, which is equal to u g the (( )) gas velocity. Now, at the bottom of the passage air and water or rather gas and liquid are introduced at flow rates Q l and Q g. And this particular these two phases they keep on flowing up (( )) flow passage.

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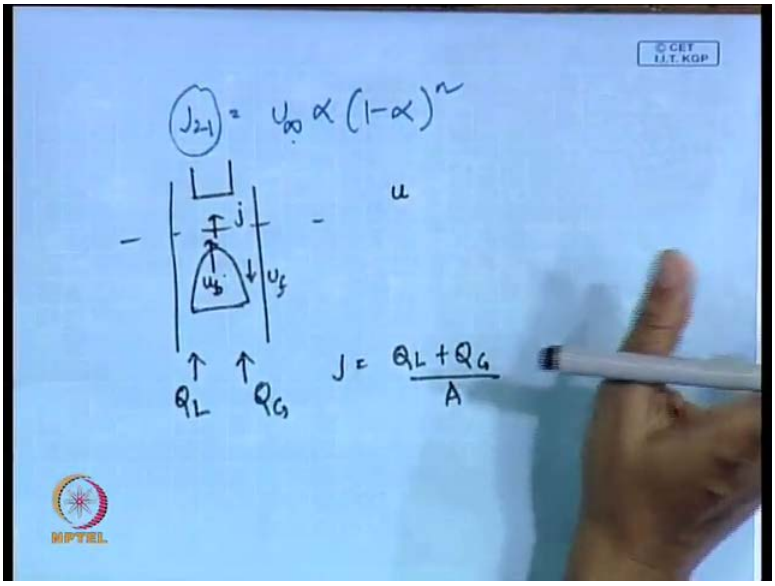


Now, try to understand one thing, since from continuity of mass what do we get, suppose we have a flow passage where this is introduced at a at Q L, this is introduced at Q G. So, at the entry we have j which is equal to the overall volumetric flux is equal to Q l plus Q G by A, inside the passage may be we have some sort of Taylor bubbles liquid slug and again another sort of Taylor bubble.

Now, suppose we take any particular cross section, the overall volumetric flux must be seen. So, therefore, the liquid in this particular portion this should also raise with a volumetric flux j, where j equals Q L plus Q G by A. And in this particular portion if we assume that the entire gas in the Taylor bubble raises at the same velocity and this will rise at a velocity u bubble and the liquid will be flowing here flowing downwards at a velocity u f or the u film.

Now, in this particular case try to understand one thing, now suppose we consider the drift velocity, remember one thing usually till now, we have been using u 1 u 2 subscripts 1 and 2; here since we are considering gas liquid slug flow we I will be using subscripts G and L, you can use whatever suits you best. If it is liquid liquid slug flow, then it that case definitely 1 and 2 has to be used. So, whatever suits you, you can use I will be just using G and L subscripts here, simply to show that we are discussing gas liquid slug flow.

Now, remember one thing, gas liquid slug flow in vertical and horizontal tubes have different characteristics, why? What is the basic difference, if you observe the slug flow which I have shown in the vertical and the horizontal tube? You find at the basic differences, here the slug flow pattern although it has temporal variations, it is axis symmetric. In this particular case, it is distinctly as symmetric.

So, therefore, the other thing is here the gas bubbles they raise due to buoyancy, here in the horizontal tube we do not expect any buoyancy (( )) exist. So, therefore, it is quite understandable that a different physics should govern, slug flow in vertical and horizontal pipes.

So, therefore, what we will do, we will be primarily discussing slug flow in vertical pipes, because slug flow is more commonly encountered in vertical pipes; and after we are finished discussing, we will take a brief discussion on horizontal slug flow as well. Remember, whatever we have discussed for vertical slug flow cannot be directly applied to the horizontal case yes, it can be applied where under micro in micro channels, why? Because, moment we go to lower and lower dimensions, surface properties becomes more important and gravitational force gets a back seat.

So, therefore, under that condition we find that the flow patterns are independent of inclination. And we find actually that there is not much difference in the flow distribution in a horizontal as well as the vertical tube. We find in both there is no satisfaction for micro channels are involve say 1 millimeter 2 millimeter channels, we usually get slug and annular flow patterns predominantly in both the cases.

So, under that condition it is different otherwise, the analysis for vertical slug flow cannot be extended in a straight forward manner for horizontal slug flow. So, to start with, we start the analysis sorry we start the analysis for vertical slug flow.

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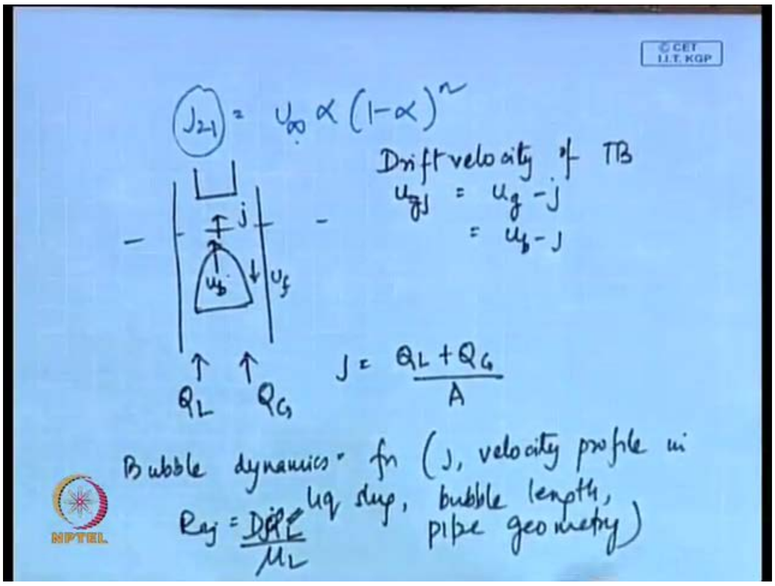


And this particular case what do we find, we find that the liquid ahead of the bubble that moves up at a velocity j, try to understand this concept very well. So, therefore, the drift velocity of the bubble say, drift velocity of the Taylor bubble which can be written as u g j or u 2 j whatever, that is equal to u g minus j; agreed all of you, again this is equal to u b minus j; assuming that, the entire gas phase raises up as the bubble and all the gas in the bubble raises at the same velocity.

So, from this particular expression what do we get, we find that bubble dynamics which is governed by u g j that is the function of what parameters, it is a function of j volumetric flux, it depends upon j is it not. If it depends upon j, it should also depend on the velocity profile in the liquid slug yes or no is it not, it should depend on the velocity profile.

And this velocity profile it is governed by which parameters governs velocity profile can you tell me? The velocity profile in that case, it is governed by what are the factors which govern the velocity profile? See, it can be parabolic it can be flat what decides the velocity profile. When do we, if I have a flat velocity profile, and when do we have a parabolic velocity profile and so on, and so forth. Yes does it depend upon Reynolds number by any chance, it should depend upon Reynolds number yes or no.

So, therefore, velocity profile in the liquid slug, this is again a function of R e j we write, which is D or rather D j rho L by mu L is it not. On what other factor it should depend, it should depend upon the conduit characteristics, it should depend upon the fluid flow properties and so on, and so forth, is it not.

So, therefore, we find that more or less your u j it should depend upon j the corresponding velocity profile in the liquid slug, then it should also depend upon bubble length and pipe geometry. Usually, it depends on these factors is it not, because bubble dynamics means the raise of bubble with respect to the surroundings. So, therefore, that should definitely depend upon the liquid, which is flowing ahead of it, because depending on that velocity, the bubble will raise faster or slower.

It should depend upon the profile, why should it depend upon the profile? Suppose, it is the parabolic profile, the bubble tip is located at the center. Then in that case, it will not rise at the average volumetric flux, but will raise or it will be effected by the center line velocity is it not. So, therefore, that depends upon the velocity profile. Then it should depend upon the bubble length, it should depend upon the pipe geometry, it can depend upon the fluid physical properties etcetera etcetera.

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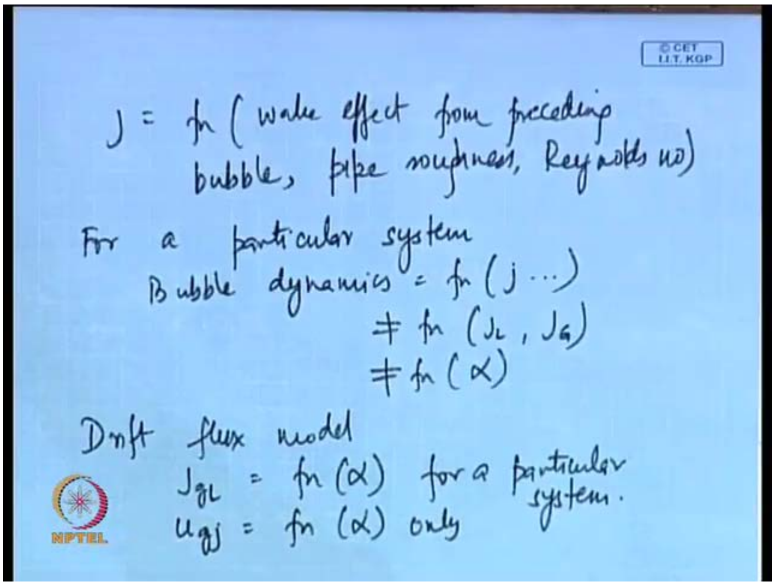


And again we find on what does this j depend, this j it depends upon the wake effect from previous bubble, because whatever the previous bubble was there, it there there is a wake behind the bubble due to the falling of this liquid film and mixing here there is a wake effect. So, therefore, if the liquid slug is not large enough, this wake effect will affect this bubble; and anyhow this wake effect will affect the liquid slug here. So, the j with which the liquid slug is raising that will be inference by here. So, therefore, j is affected by the wake effect from preceding bubble.

Then pipe roughness you can say, naturally pipe roughness it it gives the interfacial shear sorry the wall shear and the Reynolds number is it not. So, therefore, we find that if if we compare whatever we have written that, bubble dynamics is a function of volumetric flux velocity profile in the liquid slug, bubble length, pipe geometry.

And since, it depends upon j it will also depend upon the wake effect from the preceding bubble, pipe roughness, Reynolds number etcetera. Therefore, what do we find, for a particular system say when the fluid properties, pipe geometry everything it is constant under that condition your bubble dynamics, which is mainly governed by the drift velocity drift flux etcetera, that should be a function of j and certain other parameters; but primarily a function of j, it does not depend upon the individual fluxes j L and j G, do you agree with me? It depends upon the total j, because the liquid ahead is moving with j, it does not depend upon the individual flow rates j L and j G.

Again if you observe you find that, it does not also depend upon alpha, why does it not depend on alpha? Because, from here what you observe that, see the gas viscosity density it is much negligible as compared to liquid viscosity density. Therefore, gas inside this bubble can be assumed to be at a constant pressure, the curvature is constant more or less we can assume it to be a cylinder. So, therefore, it is a uniform cylinder the curvature is constant.

Therefore, this entire surface it can be assumed to be a region of negligible shear is it not. So, therefore, no matter how big the bubble is, it is not going to affect the bubble dynamics, because this entire region can be taken as a region of constant pressure, but negligible shear.

So, therefore, whatever shear is occurring that is occurring between the liquid film and the tube wall; and if it is bigger than in that case what we find, we find that may be less than number of bubbles will be a lesser number of unit cells will be there, but in each particular unit cell this this situation is the same is it not.

So, therefore, we find that considering the slug flow characteristics with (( )) aerated liquid slugs and axis symmetric vertical flow, we find at the bubble dynamics it is a function of j only, it does not depend upon the individual fluxes j L, j G. And it also should not depend upon alpha, it should primarily depend upon j and certain other factors for a particular system under particular flow conditions, it depends on j only well.

So, this we have got by considering slug flow characteristics, you agree with me all of you. So, again from drift velocity or from drift flux model what do we get, let us consider the drift flux model, now from the drift flux model if you remember what did we get from the drift flux model? We obtained, if you remember j 2 1 in this case, it is going to be j g L that was a function of alpha for a particular system. Do you remember this thing we had obtained there, and that is why from here we set that j g L equals to u infinity. And from there itself we had written down this particular expression (Refer Slide Time: 23:00), so it was a function of alpha only.

If j g L was the function of alpha, then naturally your u g j this should also be a function of alpha only yes or no? Try to understand I am speaking quite contradictory things. So, see if you understanding what I want to say or not, basically by considering the slug flow dynamics what do we get, we find that the slug flow dynamics is primarily governed by the bubble dynamics the raise the characteristics of the bubble.

Now, I considering the the ideal slug flow what did we get? We find that the bubble dynamics for a particular system it is governed by overall volumetric flux; it does not depend upon the individual fluxes, it does not depend upon alpha, because the bubble is considered to be region of constant pressure with negligible inter wall inter facial shear agreed with me.

So, if we it at all it has to be a function it is a function of j only. Now, if you consider the drift flux model, because we are considering your drift velocity and the drift flux. If you remember drift flux model for negligible wall shear that was one particular condition. For vertical slug flow, we can always consider that the gravitational pressure drop is much larger than the frictional pressure gradient that we can always assume.

So, therefore, for vertical systems with negligible wall shear non accelerating flows under such conditions if you remember we had obtained. We had proposed that, j g L is a function of alpha only for a particular system.

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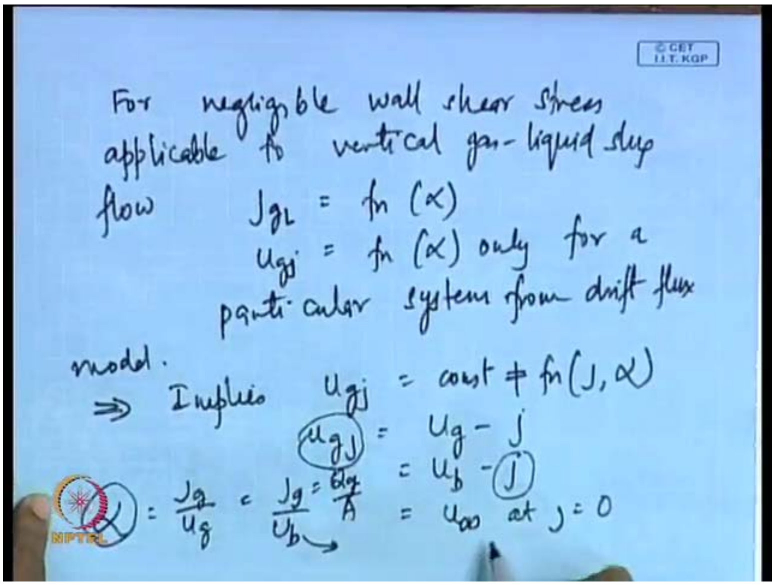


Remember that, this had come for I will write down the conditions here, for negligible wall shear if you remember I am just writing it down so that it gets into your head; for negligible wall shear applicable to vertical gas liquid slug flow, these are all with respect to gas liquid slug flow under this condition, from the flux we obtained j g L as well as u g j they were function of alpha only for a particular system is it not.

So, therefore, for vertical gas liquid slug flow we can always say from drift flux model from drift flux model. So, this we can always say yes or no agreed. So, therefore, we have we have said two contradictory things, once we have said bubble dynamics is a function of j only ok from slug flow. And again from drift flux we say bubble dynamics is a function of alpha only, it cannot be a function of j.

Now, if both of this has to be true then, in that case what can happen? Both of this can be true, when u g j is neither a function of alpha nor the function of j it is a constant. So, since the both the things your slug flow model as well as the drift flux model, they contradict each other.

So, therefore, from these two it implies it simply implies that, u g j equals to constant, it is not a function of either j or alpha only under that condition only, the entire thing can be applicable, do you agree with me. So, so once you can find out u g j then we can proceed now. What is this u g j? u g j is nothing but, u g minus j. On other words, it is the in this case since the entire thing raises up as a Taylor bubble it can you can write it down as u t b also, u t b minus j.

Therefore, we find no matter if whatever will be the value of j, your value of u g j has to be constant; that means whenever we increase j, your bubble velocity also has to increase. So, that the difference is a constant. If that is case then in that case, this should be equal to the bubble velocity at j equal to 0.

That means suppose, in a tube we have just water, the water is not flowing we have just filled the tube with water and we introduce a Taylor bubble inside the tube, then the velocity with which the Taylor bubble is raising in single Taylor bubble raising through a tube; then that should give me the velocity u g j the drift velocity under all conditions of slug flow is it correct.

So, therefore, we find out a method of finding out the drift velocity, why drift velocity important or why the bubble velocity important for that matter? Because, whenever we want to analyze anything of slug flow, the first thing anything of two phase flow for that matter, first we would like to know the (( )) composition that is the most important thing. One for two phase flow we know the (( )) composition we can find out all other parameters.

Now, what is alpha for slug flow? It is nothing but, j g by u g is it not, which is nothing but, j g by u b in this particular case. So, therefore, if you have to find out the alpha j, j g is very simple it is nothing but, Q g by A, we need to know u b. What is u b we do not know. So, therefore, we find that u b, this u g j that becomes equal to u infinity at j equals to 0.

So, therefore, once we know this, we can find out u g j from there if you know j, we can find out u b; once you can find out u b, you can find out alpha, do you agree with me all of you agree. So, therefore, and we find that finding out u g j the drift velocity is very easy, why? Because, this particular velocity it is constant. So, therefore, if we can measure the raise velocity of a single Taylor bubble in stationary liquid for that particular conduit geometry for that particular conduit orientation for that particular conduit dimension; then, that will give me the corresponding u g j for under that conduit characteristics and that particular fluid physical properties fine.

So, therefore, the way of finding out u infinity is now we find it is very simple in this particular case. So, therefore, we find that u g j is nothing but equal to u infinity at j equals to 0. Now here, I would like to mention one thing very categorically. So, long we had been talking about u infinity; and in drift flux model also I had shown you that, that the expression can be expressed as a unique function of alpha in this particular form.

What was this u infinity that I had mentioned there? It was the velocity of a single bubble in an infinite medium. Remember, this u infinity and u infinity which I have used here, both of them are not the same; this was the velocity of a single bubble in a infinite medium (Refer Slide Time: 30:32). If you get confused you can you can use some other nomenclature for this.

This is the velocity of a single Taylor bubble in that particular conduit characteristic. There is nothing infinite medium here, because remember one thing in an infinite medium we will not have Taylor bubble. Taylor bubbles are formed just because the gas is confined or (( )) by the surrounding walls to flow through a definite space; that is why we found Taylor bubbles agreed.

So, u infinity in slug flow and u infinity in bubbly flow and other cases are completely different just remember this. So, therefore, in this particular case, what we find? That, we can find out u infinity; and if you find out u infinity, then we are going to find out u g j; and once we find out u g j we can find out alpha and so on, and so forth ok.

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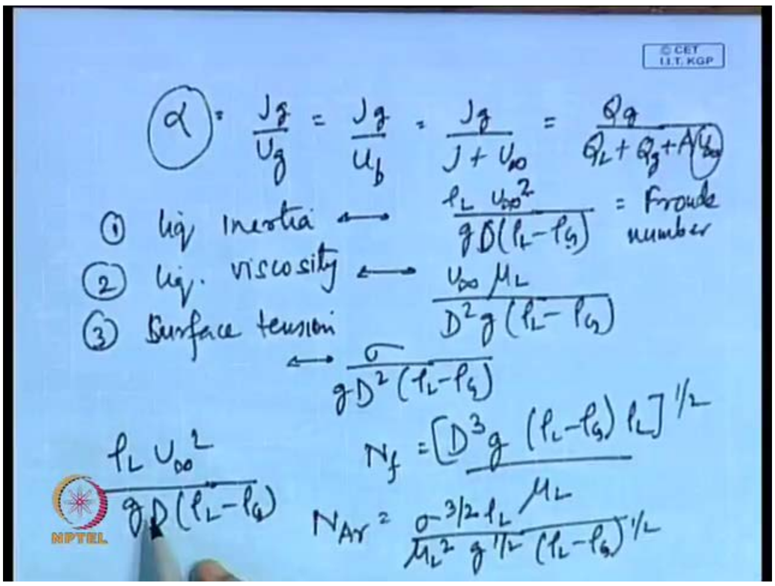

So, therefore, what is the (( )), so for finding out alpha then what do we need? We find that, for finding out the alpha it is nothing as I told it is just j g by u g as I had written down; which is nothing but, j g by u b the bubble velocity. And what is the bubble velocity, bubble velocity is nothing but, u g j plus j is it not. So, therefore, this is equal to j g by j plus u infinity, agreed fine. Because, u g j equals to u infinity and it is easy to find out u infinity.

Now, in terms of volumetric flow rates if you write this will simply be equal to Q g by Q L plus Q g plus A u infinity fine. So, therefore, for ideal slug flow what do we know? We know that, if we find out alpha, Q g Q L are easily (( )) (( )) parameter, A is also a input parameter, we need to find out u infinity, this is the first thing.

So, therefore, once we can find out u infinity, which is very simple to find out the experiments, are very easy. So, once we can find out u infinity, we can very well find out alpha; once you can find out alpha, we can find out the pressure gradient and other parameters accordingly.

Now, what are the different methods of finding out u infinity? Now, in order to find out u infinity remember certain things that, it is the bubble raise velocity in an infinite medium sorry very sorry it is the bubble raise velocity in a stationary liquid, why does the bubble raise due to its (( )). So, therefore, the interaction of the (( )) force with the other prevalent forces should give us the measure of how to find out u infinity is it not. And what are the other forces which are acting on the bubble, what are the other forces which will be acting on the bubble? We assume for the time being, the bubble density and the bubble viscosity, they are negligible compared to the liquid density and liquid viscosity.

So, what are the other forces that should be acting on it? Inter facial shear it surface tension forces you can see, because generally we have already assumed that the bubble surface is at constant pressure with negligible inter facial shear; because the curvature is constant and the bubble the density viscosity is negligible. So, therefore, viscous forces are definitely one, surface tension forces are definitely one and definitely gravitational forces are there.

So, these so, therefore, a balance of your (( )) forces with liquid inertia, the viscous forces and surface tension forces should give you an idea of how to find out u infinity. Now, when all the forces are important, naturally the calculation becomes slightly more difficult. But, fortunately for most of the situations we find that, one of these forces are usually important.

So, first we will discuss what are the forces, then we will discuss how to correlate these forces where there is velocity taking into the consideration, the dimension less groups, what are the different dimension less groups which you can use to find out the bubble velocity. And then we will discuss the limiting cases, when one of the forces becomes important. So, we find out the things which happen is, we know that the bubble raises due to buoyant forces.

And the other forces which are important are, one is liquid inertia, the other is liquid viscosity, the other is surface tension. Now, balance of liquid inertia of an (( )) what does it give, if we balance these two, we get a dimensional groups something of this sort g D rho L minus rho G. We get buoyant force and liquid inertia if we balance, we get something of this sort which I am very sure you have come across the the across this number earlier; this is nothing but, the Froude number.

Remember one thing, when you had talked about Froude number, definitely we had talked about u square by g D or u by root g D is it not. We had neglected the density terms, why? Because, you always talk about gas liquid or air water systems and for such systems, rho G is negligible as compared to rho L and therefore, they cancel out.

So, a balance between buoyancy and liquid inertia gives you this. Suppose, balance between buoyancy and liquid viscosity, this gives you a number which can be represented in this particular form D square g into rho L minus rho G. And if you consider this surface tension and buoyancy balance, this gives us something of this sort, I do know whether you have come across this numbers or not; this is more or less known as an Eotvos number, so this is known as an Eotvos number.

Now, for a general solution what what is there? For a general solution we find that, the buoyant force is a function of liquid inertia, liquid viscosity, and surface tension; when it is a function of all of them, then all these three dimension less groups they become important. And when all the dimension less groups they become important under that circumstances, what we can do is? We can simply solve it by using a graphical technique, where may be we plot one group against the other whether third group has the parameter, this can be done.

Now, usually what we would prefer we want to find out u infinity. So, if u infinity is there in both the x axis and the y axis, then it becomes a problem is it not, then it it (( )) for a trial and error sort of an technique. So, in order to avoid that technique, what we do? We would like to element u infinity from the groups. So, that u infinity is there either in the x axis or the y axis.

And then you would also like to get a group, which is completely a function of liquid properties and accession due to gravity which does not have u infinity d anything on that sort. So, usually what we will do? We select one group as this one group rho L u infinity square by g D into rho L minus rho G this is one group that we will select.

The other group we eliminate u infinity from both of these; and once we eliminate u infinity from both of these, we get the second group which is usually denoted as N f. This particular group is obtained by eliminating u infinity from both of them, we get D cube g rho L minus rho G into rho L by mu L this one whole to the power half, and this is the second group that we get.

And again we eliminate both u infinity and D from these groups and the third group I do know whether you have come across that, that is the Archimedes number. This Archimedes number is sigma to the power 3 by 2 rho L divided by mu L square g to the power half rho L minus rho G whole to the power half. So, therefore, what do we get? We get one particular group, if we plot this verses N f here, with the Archimedes number as the parameter then in that case we get a family of curves from where we can find out u infinity. This is one such type of curve.

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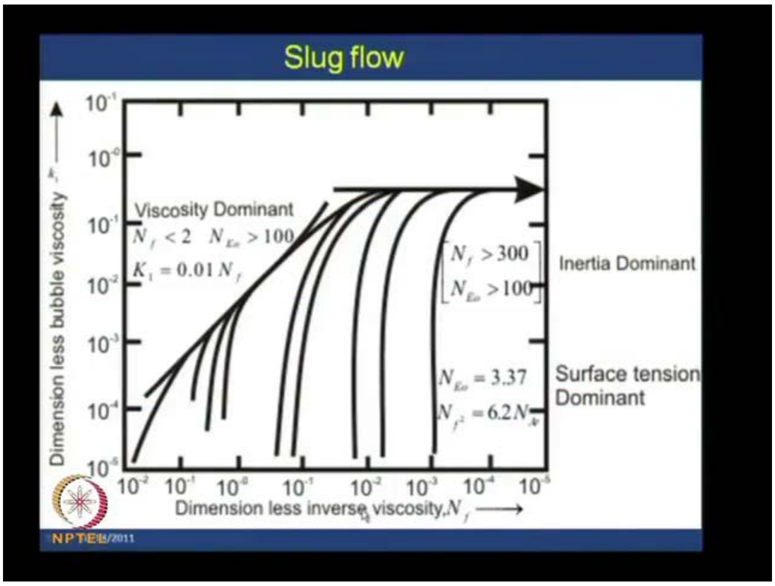


This is dimension less bubble velocity, this is the Froude number; in this case, the dimension less inverse viscosity number N f which have already written down and this has been plotted with your Archimedes number as the parameters. So, from this particular case, if we know the Archimedes number, you can locate the particular curve; you know the dimension less viscosity number, you locate the point; and from there from the y axis, you can find out the dimension less bubble velocity which is nothing but, the Froude number; once you can find out the Froude number, you can find out the u infinity as well.

Now, remember one thing see you can combine the the the the different forces in different ways, and you can generate a large number of dimension less groups according to your particular convenience. So, one particular way of generating and combining them is the once which I have shown.

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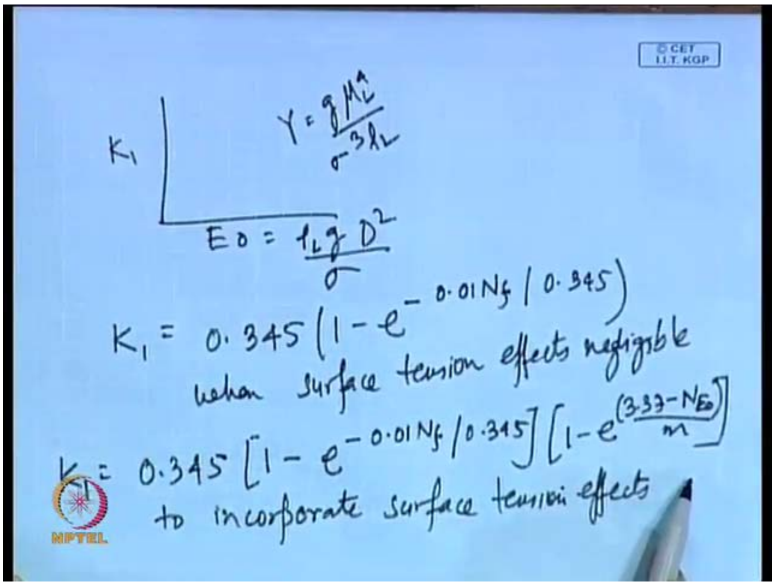


The another very well known way of combining them is again same thing the Froude number is always used; and that is plotted against the Eotvos number with a liquid parameter group y as the parameter it is g mu L to the power 4 by sigma cube rho L. So, these graphs have also been generated in fact, these graphs are much more widely used as compared to the previous one just because, it is much more useful to use this graphs.

In this particular case, this is rho L g D square by sigma, this is the dimension less Froude number; we have obtained them from the same previous groups which I have already shown you from these particular group itself by different combination of these three groups itself.

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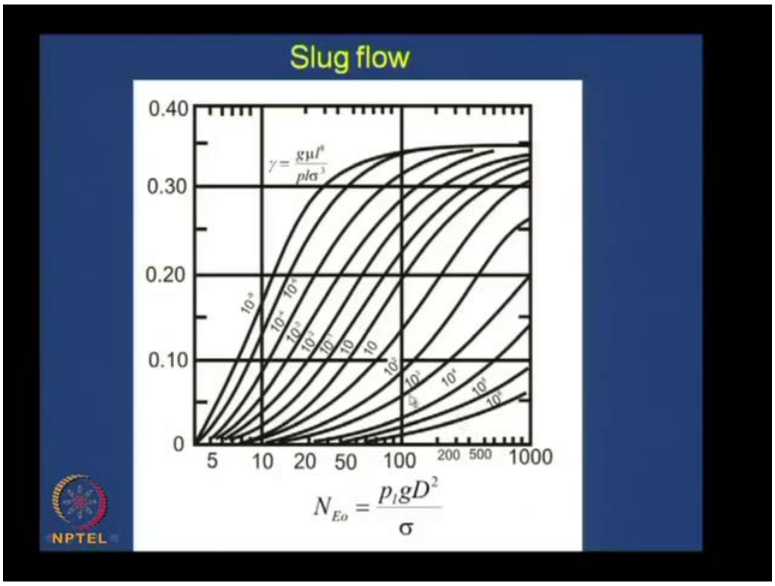


We have obtained the set of curves, which I have shown here in this particular transparency in this particular slide. So, here I have shown we can either use these curves, but they are much more spread out they are not very conveniently used. Usually, we use this set of curves, where your Froude number has been plotted as a function of Eotvos number with the liquid property group as the parameter.

Now, we generally find that there is (( )) that I will discuss later. So, there is one particular analytical expression which can be used, if the graphs are not available with you. This analytical expression is something of this sort k 1 which is nothing but, this particular group is is known as k 1. So, this k 1 the analytical expression, which correlates k 1 your Eotvos number y and all those things or which correlates the liquid viscosity term, the surface tension term etcetera.

This is something of this sort k 1 equals to 0.345 1 minus e to the power minus 0.01 N f by 0.345 this is used when surface tension effects are negligible. We can use instead of going for the graphical calculation and we can go for this particular analytical expression as well. And when surface tension effects are not negligible, then this can be written down as 0.345 1 minus e to the power minus 0.01 N f by 0.345 into 1 minus e to the power 3.37 minus N Eotvos divided by (( )).

So, if an all effects are are there to incorporate surface tension efforts. So, the entire set of the curves which I have plotted down in the transparencies and shown you a single analytical expression to merge all the curves is this particular expression. And usually surface tension effects are negligible then, we can use this simplified expression, when inertia and viscous forces are important.

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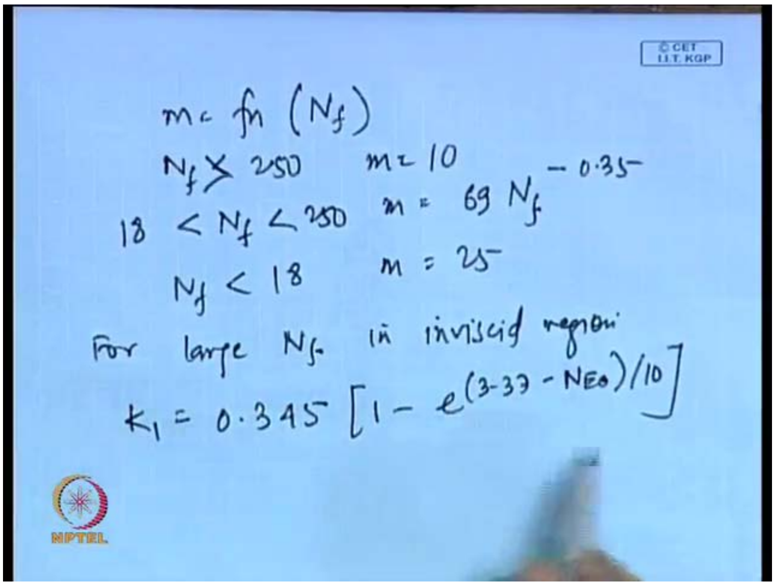


When inertia viscous and surface tension all are important, then this particular expression should be used, where m can be take up different values; these particular values they are more or less available in text books. We find that, m it is a function of your inverse viscosity number, which has been named.

And we usually, when N f is less than sorry greater than 250, m equals to 10 you will find this in text books. When N f lies between 18 and 250, m equals to 69 N f to the power minus 0.35. And when N f is less than 18 we find m equals to 25. So, therefore, their large number of expressions text books give you this expressions (( )) surface tension effects are negligible I have told you this is the expression.

Again when viscous effects are negligible that means for large N f inviscid region inviscid region under that condition what happens, naturally this term becomes important, and this term becomes less important is it not in the inviscid region. So, for under that conditions, we can write down k 1 it is equal to 0.345 1 minus e to the power 3.37 minus the Eotvos number divided by 10; we directly take up the value of m as 10, because for large N f means N f greater than 250 m equals to 10.

So, therefore, for large N f in the inviscid region k 1 can be taken as this. Now remember certain things. So, therefore, what do the what did we do from the beginning, from the beginning we were trying to find out alpha for the slug flow pattern. We found out that, the alpha can be written down in as this particular expression. In order to find out the alpha, we need to find out u infinity.

Now, in order to find out u infinity, naturally we have to go for a (( )) (( )). What are the (( )), which are important here, definitely your buoyant force is important gas rises to the buoyant force. Apart from that, liquid inertia is important, liquid density is important, liquid sorry liquid densities inertia, liquid viscosity is important, surface tension is important; but, gas density, gas viscosity we can neglect these.

Now, if these forces are important, then naturally the buoyant force will be interacting with these forces and then enable the bubble to rise is it not. When we find that inertial forces are important, then naturally we find that a balance of buoyant force and inertial force is represented by a Froude number type of expression.

When viscous forces are important, then the balance of buoyant and viscous forces gives you a sort of an Eotvos number or or something, it is not exactly Eotvos, it is the inverse viscosity sort of a number, which becomes important. And when surface tension is important, then this particular number it becomes important.

Now, there are circumstances usually for air water systems in ordinary pipe, liquid inertia is important comparatively viscous and surface tension forces are negligible; when that happens, what happens only this particular group is important (Refer Slide Time: 47:33), the other groups are not important. In that case, what happens? Naturally, this must take up a definite constant value is it not?

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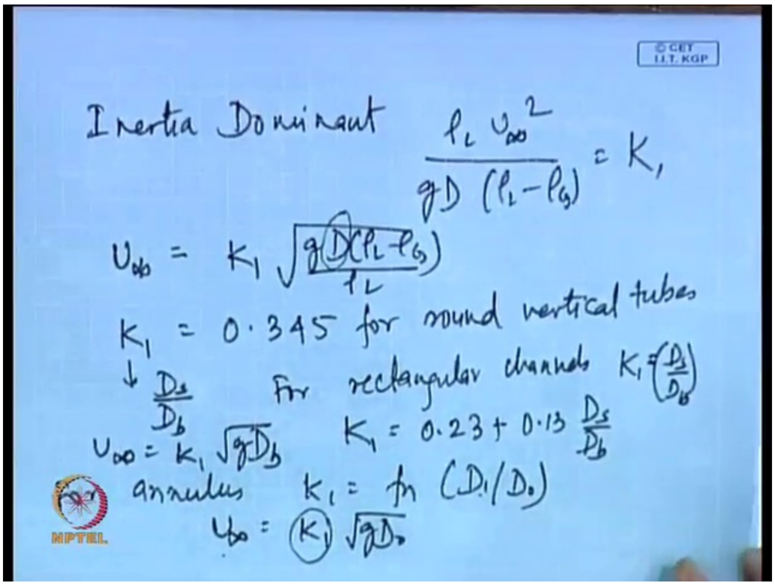


So, for only liquid inertia being important what do we have? When only liquid inertia is important under that condition, inertia dominant under that condition your rho L u infinity square by g D rho L minus rho G. This naturally becomes a constant which is given as k 1 in this particular case.

Now, usually people have found that, this particular k 1; so, therefore, u infinity it can be obtained as k 1 root g D rho L minus rho G by rho L. And usually when when gas liquid systems, these two cancel out, it is just k 1 root g D, unless you know k 1, you cannot find out u infinity. People are found out that, k 1 is a constant and for round vertical groups of value of 0.345 can be taken. This particular value of k 1, this depends upon the naturally, it should depend upon the conduit shape for round vertical tubes it is 0.345. For rectangles for annular some work has been done.

People have found that, usually for rectangles your k 1 it is a function of the larger shorter length by the larger length; and (( )) that condition this particular diameter has to be replaced by the larger length. These things people have found out and they have they have found out for a rectangular channels say for rectangular channels; k 1 it is a function of D s by D b and under that condition, it the the it can be written down as 0.23 plus 0.13 D s by D b.

Again for annular what people have found, for annular people have found k 1 it is a function of the again the inside diameter by the outside diameter. And u infinity is given as k 1 root g D 0. In this case also, your u infinity is given as k 1 root g D bigger.

Usually, people have found those things and this particular k 1 is given by this expression, here also the k 1 it it is the function of D 1 by D 0; and certain expressions have been proposed for people. And interestingly the annular for people have found, people have found that as your D 1 increases k 1 increase; or D o decreases, k 1 increases that means as the passage becomes more constricted, your k 1 increases. Or in other words, the bubble rises at a faster raise velocity, when my the annular gap it decreases. So, this was for inertia dominant situations.

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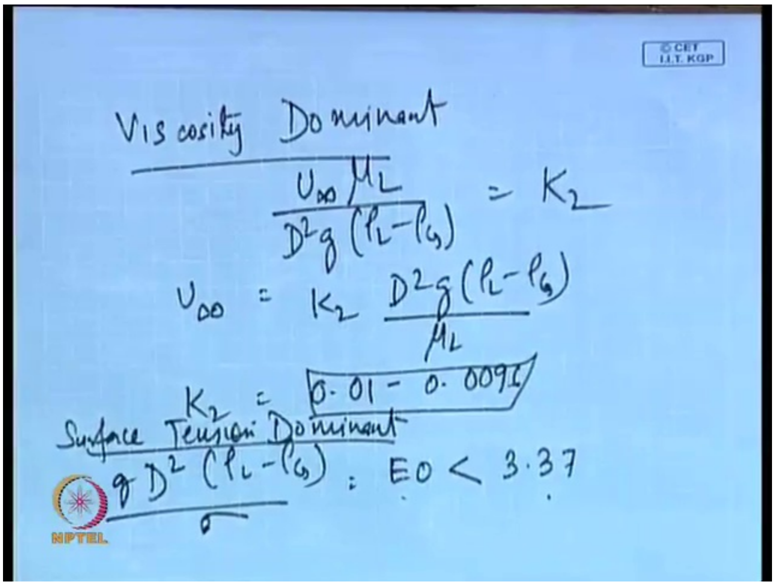


Now, when viscous dominant situations are there for viscous dominant situation, we find when viscosity is dominant under that condition, naturally the second group is going to be important and that is going to assume a constant value is it not. So, far such situations what do we find? We find u infinity mu L by D square g rho L minus rho G that becomes a constant, which is usually given us k 2.

Or in other words, for such situations where u infinity can be obtained from k 2 D square g rho L minus rho G by mu L, where this k 2 value it is almost equal to 0.01 or maybe say 0.0096 some sort of this value is obtained for k 2 correct. And when surface tension is important what happens can you tell me; inertia dominant I have already told you this particular group becomes important, I have already told you, this is a Froude number this becomes important.

When viscosity is dominant, this particular group becomes important. When surface tension is dominant, naturally the last group has to become important, what happens when surface tension becomes important, will the bubble move at all; see, one things its shape is governed by the containing cylinder. If particular the conduit is quite small, the bubble volume is very large it cannot be a sphere under that particular conditions.

So, therefore, what will happen if the surface tension is large; it will prevent the bubble from rising. So, the bubble does not rise at all under these conditions. So, when your surface tension becomes important under that circumstance, we find that g D square rho L minus rho G by sigma. This particular group which I had written down here (Refer Slide Time: 52:49), this particular group naturally sigma becomes very large.

So, therefore, the inverse of this which is nothing but known as the Eotvos number this becomes small. So, therefore, surface tension dominant condition your surface tension dominant (( )) happens, when your Eotvos number value is less than 3.37 under this condition the bubble does not raise at all, is it clear. So, when inertia is dominant, Froude number is important. When viscosity is dominant, one type of particular number is important, this number we do not use much instead of this number we use the inverse viscosity number. And when surface tension is important, the bubble does not raise at all.

If the surface tension becomes so important that the Eotvos number becomes less than 3.37 or less than 4, then the bubble does not raise at all. And when more than one force are important; then naturally when inertia and viscous forces are important under that particular conditions, I have already given you the expressions here (Refer Slide Time: 53:58). When inertia and viscosity sorry your viscosity and viscous your inertia is important, this is the expression.

When inertia and surface tension is important, this is the expression and when everything is important then, usually this is the expression that we use. But, normally we do not use this analytical expressions, we prefer to use the graphs which I have already shown you; these graphs using this graphs if we know the Eotvos number, we know the liquid property group value, they are very easily found out, we can find out u infinity.

And once we find out u infinity for the particular condition considering the forces which are important under that particular condition, we can find out your alpha; and once you can find out alpha, we can find out the pressure drop and other things. But, remember one thing the u infinity that you have found out in this particular case u infinity is fine it is the velocity of the single bubble in a stationary liquid; but using this u infinity, the u bubbles which you are trying to find out, that u bubble it is not a function of u infinity only, it is a function of j as well.

So, therefore, it is also inference by certain other parameters and it needs further modifications for accurate estimation. So, tomorrow we are going to do the additional modifications, which we shall be introducing in order to get a modified expression of the bubble raise velocity for the slug flow pattern in vertical tubes, thank you very much.