

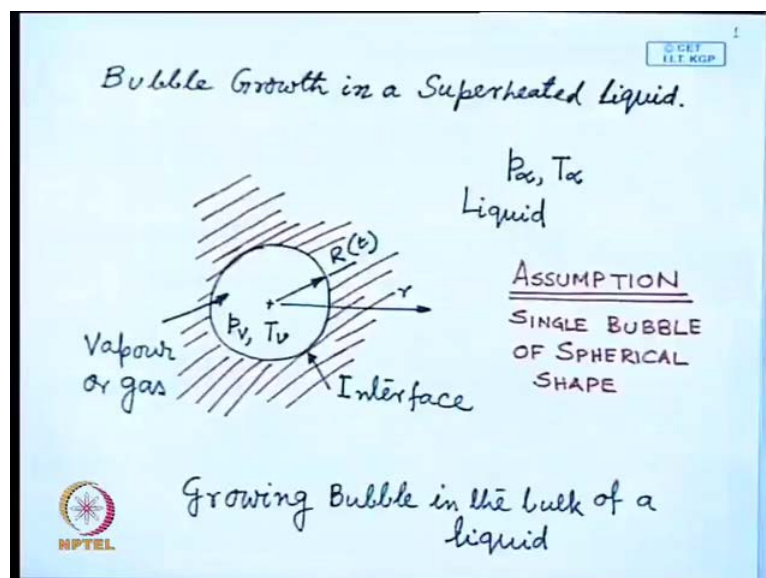
**Multiphase Flow**  
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**Indian Institute of Technology, Kharagpur**

**Lecture No. # 30**  
**Bubble Growth**

Good morning everybody. Yesterday we had a brief introduction to volumetric transfer, and in that I mentioned that in the process of boiling there will be nucleation due to that bubble will grow come up on the heated surface, then that bubble will grow and ultimately it will depart from the heated surface, so you can see that bubble growth is a part and parcel of the process of boiling, it is a very important step of boiling transfer, so today we like to devote on bubble growth, again some preliminary introduction was given regarding bubble growth in last day's class.

So, I mean to say that the though I am discussing the growth of bubble in connection with boiling heat transfer, bubble growth is a process which is also encountered in other industrial situations other day to day life situations or other natural phenomenon, like in case of boiling you can see it, in case of flashing you can observe the growth of bubble, even during degassing of liquid you can see the generation of bubble and its growth, so my discussion will be to some extent targeted towards boiling the transfer, but in general it can be applicable for other processes of bubble growth which I have just mentioned.

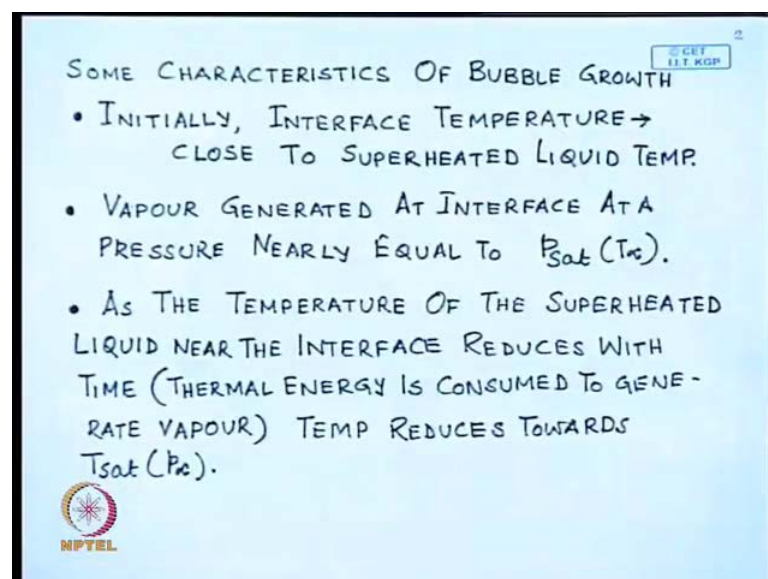
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Now, on the growth of bubble the pioneering work was done by Lord Rayleigh, and here you can see a simplified version of bubble growth. So, what has been done in a very large bulk of fluid a spherical bubble has been assumed, so in actual practice there could be a large number of bubbles, and the bubbles shape need not be always spherical; the bubble cannot be always in the bulk of the fluid, suppose for boiling it will be on the heated surface, but to simplify the process, here the bubble has been shown that it is in the bulk of the fluid and only a single bubble is existing. So, assumption I have mentioned here that a single bubble of spherical shape, inside the bubble there could be vapor; there could be a permanent gas or there could be a mixture of vapor and permanent gas, so the gas and gas or vapor which is there inside the bubble, that will have a different or a separate pressure and temperature denoted by  $P_v$  and  $T_v$ .

Now, this pressure and temperature in the liquid that will change or that will vary as we move away from the bubble, and this variation is a variation both in space and time, so at a distance large from the bubble which mathematical one can tell that at infinity, the pressure is  $p_\infty$  and temperature is  $T_\infty$  for the liquid. And then there is an interface of the bubble which is spherical in shape denoted by  $R(t)$  where  $r$  is the bubble radius, again it is the function of time that is why it is denoted by  $R(t)$ . And as the due to the growth or shrinkage of the bubble there will be change of motion or change of fluid velocity, so conveniently we can take a spherical symmetric coordinate that is what has been shown by the small  $r$  that is the coordinate system.

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SOME CHARACTERISTICS OF BUBBLE GROWTH

- INITIALLY, INTERFACE TEMPERATURE → CLOSE TO SUPERHEATED LIQUID TEMP.
- VAPOUR GENERATED AT INTERFACE AT A PRESSURE NEARLY EQUAL TO  $P_{sat}(T_w)$ .
- AS THE TEMPERATURE OF THE SUPERHEATED LIQUID NEAR THE INTERFACE REDUCES WITH TIME (THERMAL ENERGY IS CONSUMED TO GENERATE VAPOUR) TEMP REDUCES TOWARDS  $T_{sat}(P_w)$ .

NPTEL

So, with this we can make some sort of idealization and discuss the basic series some characteristics of bubble growth, initially interface temperature close to the superheated liquid temperature, particularly in case of boiling what we are assuming that the liquid that is at the constant temperature initially, and if a bubble has to grow then the temperature inside the bubble that should be lower than the liquid temperature this is from second law of thermodynamics, so initially we have started with a hot mass of liquid which has got a temperature which is equal to  $t_{\infty}$ , now inside it if a bubble has to appear the temperature of the bubble cannot be more than the temperature of the liquid rather it would be less than the temperature of the liquid, so that is why this liquid is turned as the superheated liquid.

So, initially the interface temperature bubble interface temperature is closed to this super heated liquid temperature, but inside the bubble the temperature could be slightly lower, vapor generated at interface at a pressure nearly equal to  $p_{\text{sat}}$  corresponding to  $t_{\infty}$ , so how the bubble is growing the bubble can grow as the pressure equalization takes place, or the bubble can grow due to more evaporation, and evaporation will take place only in the interface only at the interface, so the pressure at the interface is given by this particular condition that  $p_{\text{sat}} t_{\infty}$  is it is very close to this value, as the temperature of the superheated liquid near the interface reduces with time, why it will reduce with time? Because there is heat transfer. So, if the heat transfer is there from the surrounding liquid to the bubble so the temperature of the surrounding liquid will reduce.

So, thermal energy is consumed to generate vapor that is why there will be falling temperature, the temperature reduces towards  $e_{\text{sat}}$  corresponding to  $p_{\infty}$ , so this is another condition. Then what about the pressure, pressure inside the bubble is high initial, initially the pressure inside the bubble is high and gradually it will drop gradually it will become lower, why it will gradually become lower? See initially when a bubble generates then its radius is small then gradually it is growing up so its radius will grow.

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• PRESSURE INSIDE THE BUBBLE IS HIGH INITIALLY; DROPS GRADUALLY, AS RADIUS INCREASES & CAPILLARY PRESSURE DIFFERENCE DECREASES

• TEMP. & PRESSURE RANGES DURING GROWTH PERIOD ARE AS FOLLOWS

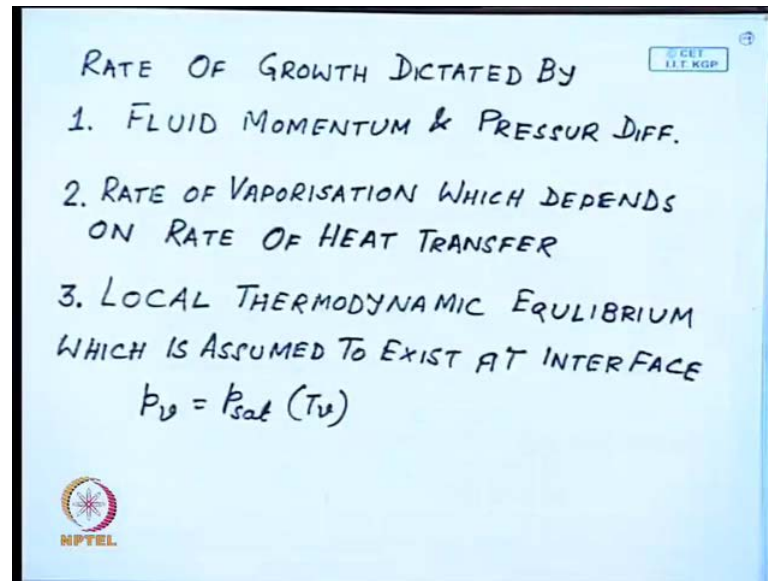
$$P_{\alpha} \leq P_b \leq P_{sat}(T_{\alpha})$$
$$T_{sat}(P_{\alpha}) \leq T_b \leq T_{\alpha}$$

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Now, we know that if there is a curved interface then across the interface there is a pressure difference from young Laplace equation, and this pressure difference is balanced by this substantial force, now as the bubble grows then this components become smaller and smaller the difference of pressure between the inside of the bubble and outside liquid that becomes smaller.

So, there will be drop in pressure as the bubble grows, temperature and pressure range during the growth period are as follows, so from this discussion we can see that during the growth period we had the pressure values within this range; and we have the temperature values within this range.

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So, these are the pressure and temperature ranges during the growth period, now these three are very important points how the bubble growth take places, the rate of bubble growth is dictated by fluid momentum and pressure difference, that is first thing. Second thing rate of vaporization which depends on the rate of heat transfer it will also depend on the rate of vaporization, and rate of vaporization will depend on the rate of a transform, and then at the interface local thermodynamic equilibrium is assume to exit.

So, thermodynamic equilibrium will not be there over the entire bulk of the fluid considering both the vapor and the liquid, but thermodynamic equilibrium it is assume to it gives on at least at the interface, so at the interface if the vapor pressure is given by  $P_v$  so that should be equal to  $P_z$  corresponding to  $P_v$ , this I had briefly mentioned in our last class that there are two limiting cases of bubble growth, so first one is inertia controlled, so if I see what are the characteristics of inertia controlled bubble growth then we can find that it is the initial stage of bubble growth, then pressure has its maximum value, so when the bubble is very small it is growing and it is controlled by the inertia of the bubble.

So, pressure has been maximum value, so it is obvious as the bubble will grow the pressure inside the bubble that will reduce, then  $P_{sat}$  that is given by  $p_{sat}$  corresponding to  $T_{infinity}$  and  $T_v$  that is approximately equal to  $t_{infinity}$ , these are more or less the condition at the initial stage of bubble growth, then growth rates dedicated by

momentum transfer not by rate of heat transfer, so at this stage we can assume that the growth rate that is dedicated by momentum transfer not by heat transfer, and due to this the growth rate will be faster. So, these are the characteristics of your inertia controlled bubble growth which is at the initial stage of bubble formation and growth.

Now, if I consider inertia controlled growth the situation is like this, this shows part of the bubble, so this bubble is growing, so as it grows so it will boost the liquid **it will boost the liquid** in the radically outward direction, at any point  $r$  and at any point the instant of time  $t$  the velocity  $t$  is given by  $u r t$ , so obviously it is a it is in a radial direction, then very simple continuity equation can be written, so this is  $2 \pi r$  square d **sorry**  $4 \pi r$  square  $dR dt$  which is the rate of expansion of the bubble that is equal to your if we take some sort of a surface anywhere so equating the mass flow rate and considering that the velocity is  $u$  of  $r t$  at a radial position  $r$  we will get the first equation.

From the first equation we will get the expression of velocity at any position  $r$ , so you can see the velocity will decrease as  $r$  increases that is quite obvious, that the bubble is going so near the bubble the velocity will be larger and as we moved away and away from the bubble the velocity will reduce, the kinetic energy of the liquid surrounding the bubble that can

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INERTIA CONTROLLED GROWTH:

RADIAL FIELD OF VELOCITY →

$$4\pi R^2 \frac{dR}{dt} = 4\pi r^2 u(r,t)$$

$$u = \frac{dR}{dt} \left(\frac{R}{r}\right)^2 \dots (2)$$

THE KE OF LIQUID SURROUNDING THE BUBBLE

$$(KE)_L = \frac{\rho_L}{2} \int_R^\infty u^2 dV$$

$$2\pi \rho_L \left(\frac{dR}{dt}\right)^2 R^3 \dots (3)$$

NPTEL

now be calculated very easily, and you can see the kinetic energy of liquid taking the liquid density as  $\rho_l$  so we can get this particular expression where integration is to be

taken from  $r$  to infinity. So, within  $r$  from small  $r$  equal to 0 to capital  $R$  there is only vapor or gas, so only beyond small  $r$  is equal to capital  $R$  we have got the liquid and that liquid is expanded up to infinity, so that is how your the limits of integral can be explained, and then this is a very simple integration and you can get the result like this.

Now, what the bubble is doing? The bubble is growing, so if it grows then what it is what it has to do now if it grows it is pushing the liquid in the outward direction, so it is doing some work on the liquid, now if it is pushing the liquid in the outward direction so it is doing some work which is  $w$ , where that  $w$  is going.

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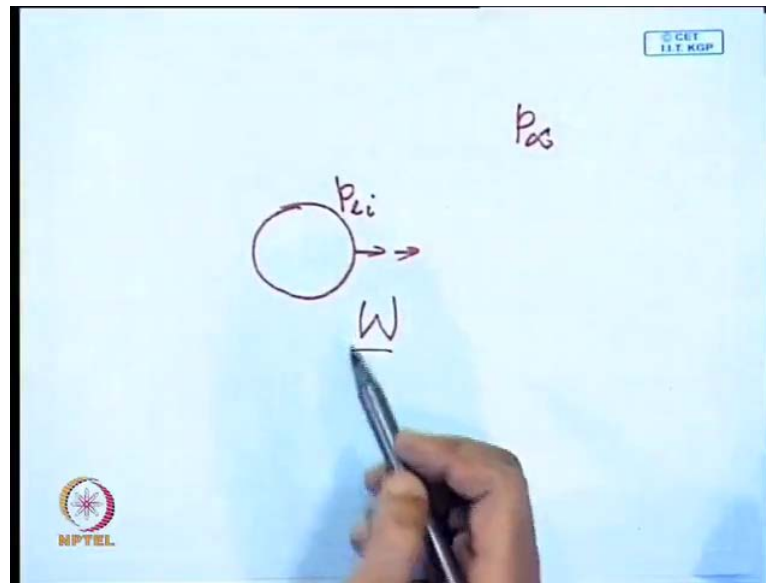
NET WORK DONE AGAINST THE SURROUNDING LIQUID, AS BUBBLE GROWS  $R=0$  TO  $R=R$

$$W_L = \int_0^R P_{Li} \cdot 4\pi R^2 dR - \frac{4}{3}\pi P_{\infty} R^3 \dots (4)$$

- $P_{Li} \rightarrow$  INTERFACE PRESSURE
- $\frac{4}{3}\pi P_{\infty} R^3 \rightarrow$  WORK DONE AGAINST AMBIENT PRESSURE, DURING VOLUME CHANGE OF BUBBLE.

This  $w$  is going to the liquid and increasing the kinetic energy of the liquid, so we can now equate calculate what is work done and equate it with the change in kinetic energy that is what we are going to do in the next stage. So, network done against the surrounding liquid as bubble grows to the bubble grows from  $r$  equal to 0 to  $r$  is equal to  $r$ , so this is your network done.

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How can we get an expression of the network, you see again if we consider this particular example; or this particular figure this is pressure at the liquid interface, and the surrounding pressure initially it was  $p$  infinity, now this  $P L i$  this is not constant  $p$  infinity was constant  $p$  infinity is the surrounding in both pressure initially and at infinity also now the pressure is equal to  $p$  infinity, but  $P L i$  that is the interfacial pressure and that changes with time, so considering this we can get an expression of work done, so in this expression  $P L i$  is the interface pressure, then there is another term in the hand side of the equation four third  $p_i p$  infinity  $r^3$  that is work done against the  $p$  ambient pressure during volume change of the bubble.

During the volume changing of the bubble so not only the interface pressure is changing but there is a constant ambient pressure, so due to the this extra term will come. Work done and change in kinetic energy can be equated now, the way I have explained that the bubble is expanding so it is doing some work, and that work is getting stored inside the liquid as kinetic energy so this two things can be equated, so if we equate them we will get equation 5, and then what we do you see that we have got  $1 P L I$ , what is that? That is the interface pressure, now interface pressure as I have told that it will change, it will change with radius that means it will change with the time also, and we had already 1 variable pressure  $P v$  that is the pressure inside the bubble, so let us try to relate between  $P v$  and  $P L I$ , and that can be done very easily with the help of your Laplace equation.



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WORK DONE & CHANGE IN KE CAN BE EQUATED

$$2\pi\rho_l\left(\frac{dR}{dt}\right)^2R^3 = \int_0^R \rho_{li}(4\pi R^2)dR - \frac{4}{3}\pi P_v R^3 \quad \dots (5)$$

APPLYING YOUNG-LAPLACE EQN.  
ONE GETS THE INTERFACIAL PRESSURE

$$P_{Li} = P_v - \frac{2\sigma}{R} \dots (6)$$

$\sigma \rightarrow$  INTERFACIAL TENSION OF LIQUID  
& DIFFERENTIATION OF THE EQN. GIVES  $\rightarrow$

$P_v$  and  $P_{Li}$  can be related introducing surface tension of the liquid by this particular equation, now if we replace  $P_{Li}$  and then we differentiate the equation once again we will get the equation given in 7 and this is known as Rayleigh's equation of bubble growth, this is the very well known equation and you see here we are getting a non-linear differential equation for the bubble radius capital  $R$  with respect to  $t$ , where we have got this variables, what are these variables? The property variables like density of liquid surface tension of the liquid and some process variable  $1$  is  $p_{\infty}$ , this  $p_{\infty}$  is the pressure in the liquid field at some distance from the bubble, and another is  $P_v$  that is your vapor pressure within the bubble, now you see for all particle purposes one can assume  $\rho_l$  and  $\sigma$  to be constant, so we are taking a liquid for which there is no variation approval and  $\sigma$ ,  $p_{\infty}$  is the imposed pressure field so either one depending on the situation.

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$R \frac{d^2R}{dt^2} + \frac{3}{2} \left( \frac{dR}{dt} \right)^2 = \frac{1}{\rho_L} \left( p_v - p_\infty - \frac{2\sigma}{R} \right) \dots (7)$

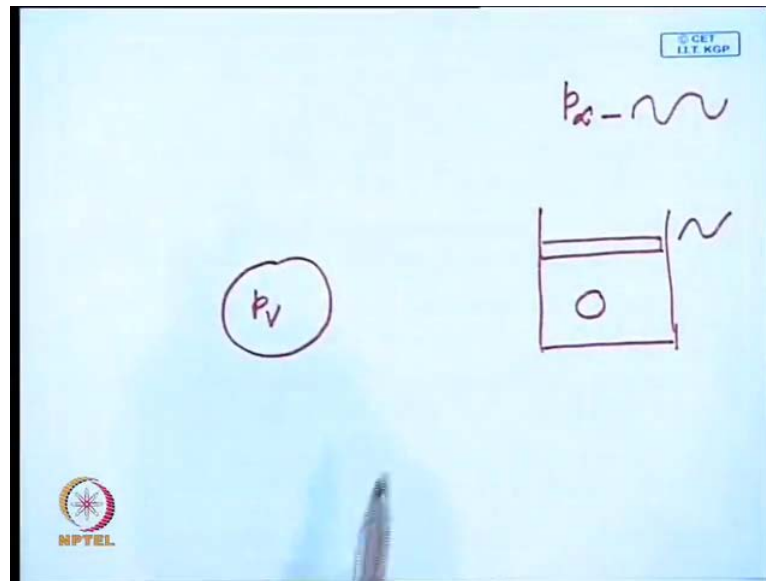
KNOWN AS RAYLEIGH'S EQN. OF BUBBLE GROWTH

- CAN BE DERIVED FROM MOMENTUM BALANCE
- VISCIOUS INTERACTION HAS BEEN NEGLECTED

NPTEL

So either one can take it as constant or the way you are imposing the pressure field you can give mathematical expression for this,  $P_v$  is the vapor pressure which is related to the temperature, and the relationship will come from the equation of state. So, you see the situation I will come back to the situation once again, but this situation is like this that you have got a bubble inside this is  $P_v$  which is a variable pressure, here we have got  $p_\infty$  which is the liquid pressure it has to be known, either it is to be constant or let us say that we are considering a situation where we have got a bubble and this piston is having some sort of a sinusoidal motion, that means the  $p_\infty$  that is a sinusoidal function.

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So, for all these cases we can we should be able to at least theoretically it should be able to solve the rally equation, so if I solve the rally equation, then I will get how  $r$  is changing with time, that means how the bubble growth is taking place, 2, 3 points are to be noted from the rally's equation, first thing how I have derived the equation I have determined what is the work done by the bubble due to the expansion process, and that work done is going through increase the kinetic energy of the fluid, so I have equated this, this is one approach, another approach could be that I could have taken from the very beginning in momentum equation, or I could have solved the Nervure's tropes equation along with proper simplification, because this is a simple case.

So, along with proper simplification I could have solved Nervure's tropes equation and then also I could have got the similar result what I am getting now, then another thing I have done that here I have neglected the viscous interaction, so assuming that the velocity are small and the liquid viscosity does not play that much of a role I have neglected discuss interaction, but somebody can include this, in fact there are analysis after rally it will could I mean people try to include the effect of viscosity and see what will be the result, so that that has also been done. Now, you see inertia controlled growth  $P_v$  is minus  $p_\infty$  that is what the initial stage is much larger compared to  $2\sigma$  by  $r$ , that means the pressure change is due to inertia is much larger compared to the pressure deferent imposed by the capillary.

So, neglecting the last term the last term can be neglected, and we can induce or we can take the help of celosias equation, then  $P_v$  minus  $p$  infinity that can be written by the equation written in 8, so equation 8 gives a relationship between  $P_v$  and  $p$  infinity, then again we can make another assumption that at the initial stage  $T_v$  is almost equal to  $t$  infinity, so if I assume  $T_v$  is almost equal to  $t$  infinity what happens that  $T_v$  no longer depends on time it becomes a constant.

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FOR INERTIA CONTROLLED GROWTH

$$P_v - P_\infty \gg \frac{2\sigma}{R}$$

NEGLECTING THE LAST TERM & APPLYING  
CLAUSIUS CLAPYRON EQN.

$$R (P_v - P_\infty) = \frac{P_v h_{LV} [T_v - T_{sat}(P_\infty)]}{T_{sat}(P_\infty)} \dots (8)$$

TAKING  $T_v \cong T_\infty$  FOR INERTIA CONTROLLED  
GROWTH, EQN. 7 CAN BE SOLVED AS:

$$R(t) = \left\{ \frac{2}{3} \left( \frac{[T_\infty - T_{sat}(P_\infty)]}{T_{sat}(P_\infty)} \right) \frac{h_{LV} P_v}{P_\infty} \right\}^{1/2} \dots (9)$$

NPTEL

So, for inertia control growth then we can get some sort of a relationship by this simplification, without this simplification it cannot be solved that non difference non-linear differential equation, now it can be solved and we can get  $r$   $t$  that means the radius as a function of  $t$  as this particular equation, let us look into this equation bit carefully, so variation of bubble radius with time in inertia controlled growth is given by the equation 9, and this equation it satisfies the initial condition  $r$  is equal to 0 at  $t$  is equal to 0 you see.

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VARIATION OF BUBBLE RADIUS WITH TIME  
IN INERTIA CONTROLLED GROWTH IS  
GIVEN BY.

$$R(t) = \left\{ \frac{2}{3} \left( \frac{[T_\infty - T_{\text{sat}}(p_\infty)]}{T_{\text{sat}}(p_\infty)} \right) \frac{h_{\text{lv}} R_\infty}{c_p} \right\}^{1/2} t \dots (9)$$

IT SATISFIES INITIAL CONDITION  
 $R=0 \quad t=0 \quad \dots (10)$

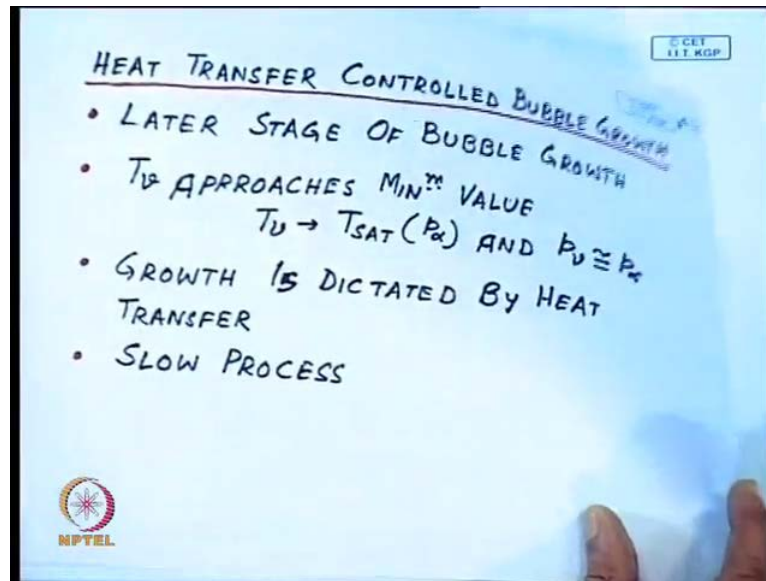
RADIUS INCREASES WITH TIME DURING  
THIS PHASE

NPTEL

So,  $r$  is a function is direct function of  $t$ , so if  $t$  is equal to 0 then  $r$  is equal to 0 that also satisfies the initial condition, then radius in previous with time during this phase and if this increase is linear increase, and let us see whether it is I mean conforming to physics or not, see how it is increasing if the difference between  $t$  infinity into saturated that is more than the increase will also be more, and what we can see that if  $h_{\text{lv}}$  is more  $h_{\text{lv}}$  is the latent heat of vaporization, so if latent heat is more then also this increase is more.

Next if we come to the heat transfer controlled bubble growth, so what are the characteristic features of this, so it occurs at march later stage of bubble growth, as inertia controlled growth takes place at the initial stage the heat transfer controlled bubble growth that takes place at a march later stage, now  $T_v$  approaches the maximum value,  $T_v$  approaches  $T_{\text{sat}}$  corresponding to  $p$  infinity, and  $P_v$  is almost equal to  $p$  infinity so this is what is the condition when heat transfer controlled growth takes place, growth is dictated by heat transfer, so basically then we have to consider the energy equation, so far we have not considered energy equation we have considered only the moment of equation continue equation or force balance and continuity equation.

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So, that we have to **sorry**, and then this is your slow process as heat transfer process is slow process it is a diffusion process so we have to understand that this phase of bubble growth is relatively slow is a growth of bubble. So, again if we consider that the flow field and temperature field are more or less radial only there is radial variation and of course, there is variation with time then we will have the following equation of heat transfer between the liquid and the bubble, so you see this is your simplified version of the energy equation in a spherical co ordinate system.

In this spherical co ordinate system we have considered only  $r$ , and  $\alpha_l$  that is the. What is  $\alpha_l$ ? That is the thermal diffusivity of the liquid, then in this equation  $u$  is coming which is the bubble velocity, and bubble velocity already we have derived.

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HEAT TRANSFER BETWEEN LIQUID & BUBBLE

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial r} = \frac{\alpha_l}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) \dots (11)$$

ONE CAN USE FOR VELOCITY

$$u = \frac{dR}{dt} \left( \frac{R}{r} \right)^2 \dots (12)$$

INITIAL & BOUNDARY CONDITIONS ARE

$$\left. \begin{aligned} T(r, 0) &= T_\infty \\ T(R, t) &= T_{sat}(p) \\ T(\infty, t) &= T_\infty \end{aligned} \right\} (13)$$

NPTEL

So, this is given by equation 12 that we can substitute in equation 11, and then we can consider whatever the initial and boundary condition, what is the initial condition? Initial condition is that initially the entire field of liquid is having a uniform temperature  $T_\infty$  at  $t = 0$ , and then  $T(R, t) = T_{sat}$ , that means at whatever time it may be at the interface there will be saturation temperature corresponding to the pressure of the bubble, and at the infinity at all the time the temperature is  $T_\infty$ , so these are the 3 conditions first one is the initial condition, and then the second and third those are the boundary conditions. Additionally we have to consider another energy consumption equation that is at the interface, so at the interface whatever conduction of heat will be that will be consumed for the generation of vapor, so you see this is the very simple energy conservation equation at the interface, now energy equation can now be solved by both approximate and exact methods.

Now, obviously this solution are not very easy to do, and exact solution particularly is very involved, so what has been done an approximate solution has been given here, and this approximate solution assumes that it is for large Jacob number, Jacob number has been defined later on, so if it is solve for a large Jacob number then variation of  $r$  with  $t$  we are getting by this equation 15, where  $C r$  that is a function of Jacob number and the Jacob number is defined Jacob number is non dimensional number it is defined by equation 16, so what is Jacob number? Jacob number actually is a relationship between the change of energy or between the energy contained by a fluid for I mean stored energy

it is kind of a stored energy and it is given by the  $C_p$  specific heat of the fluid corresponding to that the energy which is needed for phase transfer, so the ratio of these two energy that is your Jacob energy **sorry** Jacob number.

Now, physical significance is that that when they are there is an extra amount of evaporation for the growth of the bubble then there will be a change in temperature of the surrounding liquid.

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ENERGY CONSERVATION AT THE INTERFACE -

$$k_L \frac{\partial T}{\partial r}(R, t) = \rho_v h_{LV} \frac{dR}{dt} \dots (14)$$

ENERG. EQN. CAN BE SOLVED BOTH APPROXIMATELY & EXACTLY.

APPROXIMATE SOL<sup>N</sup>. FOR LARGE "JACOB NO" ( $J_a$ )

$$R(t) = 2 C_R \sqrt{\alpha_L t} \dots (15)$$

$$C_R = \sqrt{\frac{3}{\pi}} J_a \quad J_a = \frac{\rho_L C_{PL} (T_K - T_{sat})}{\rho_v h_{LV}} \dots (16)$$

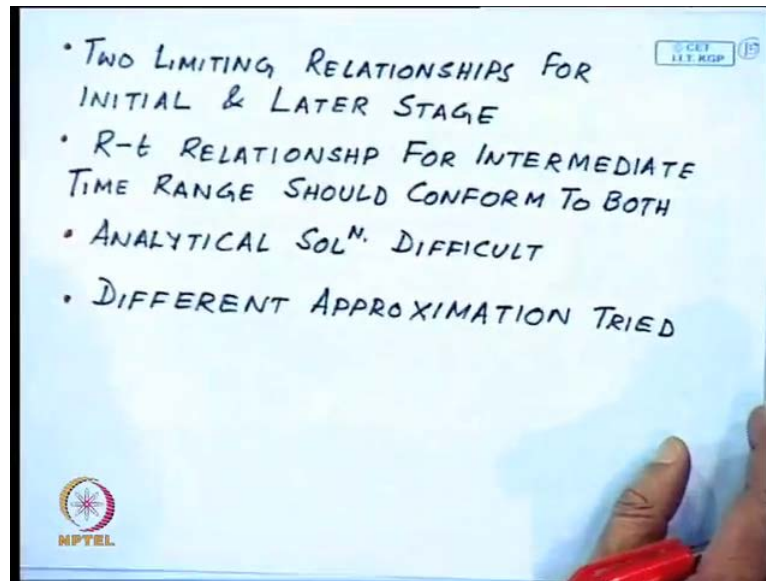
RADIUS ∝ SQRT. OF TIME

NPTEL

So, how these two things are related, how the mass of vapor and the temperature change of the liquid I related so that is given by or that is indicated by your Jacob number, and here you see that which is very important the radius is proportional to the square root of time, so at the initial stage we have got radius is proportional to time, here we are getting radius is proportional to the square root of time, so obviously the initial stage if growth is much faster compared to the final very final stage of growth.

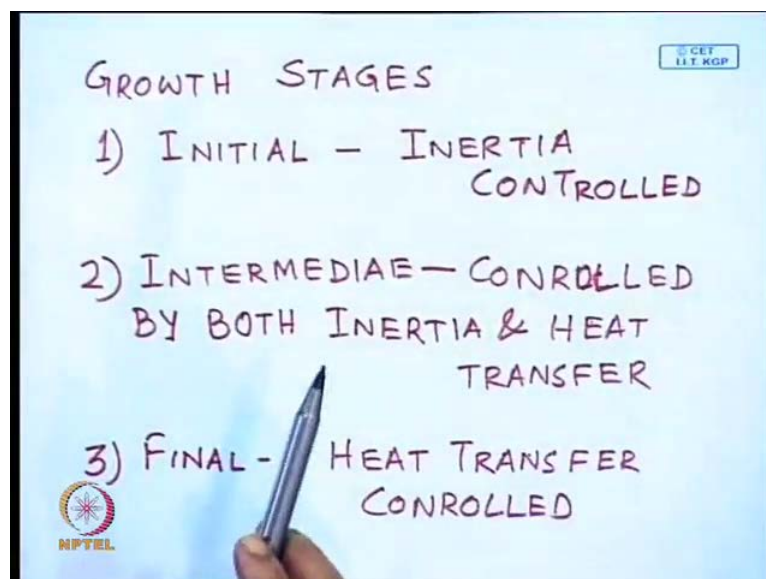


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Now, you see that we have got two Limiting relationship, one is for the initial stage of growth and another is for the final stage of growth, so obviously there is something in between, and how did I get this get the relationship if you recap later little bit, how did you get the relationships? The relationships what we have get what we have obtained we have started from the rally's equation for the initial growth of the let me elaborate little bit and then we will come to the this slide that growth stages initial that is inertia control then we can write final that is heat transfer control, then obviously there should be some intermediate.

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And logically this intermediate stage should be controlled both by inertia and heat transfer, now let us see how we have done this analysis, so first we have got the two Limiting relationship for initial and later stage, for the initial stage if we see what the initial stage we have used rally's equation, and in that equation it is only a force balance equation that means we have considered a force we have to consider continuity equation, and along with that we have to consider momentum equation, though the momentum equation has been derived in a different manner from the thermo dynamic first law of thermo dynamic point of view we have defined the momentum equation, but it is your the continuity equation and momentum equation, and in that momentum equation basically these are mechanical energies so that is why the momentum equation we could derive from the approach of first law of thermo dynamics.

Now in the later stage what we have done? We have solve the energy equation, and in the energy equation we had a an expression of velocity that means we had to also consider the continuity equation, from the continuity equation this velocity equation came, so what the later stage we had to solve the energy equation and continuity equation, now intermediate stage will be controlled by both inertia and heat transfer, so there we have to solve all the 3 equations continuity equation momentum equation and energy equation, and obviously it will be an involved exercise, it is not impossible it can be done but the way we have got the analytical solution we cannot get the analytical solution for the intermediate stage  $p$   $g$ , so what has been done by people? People have done some sort of approximation analysis, if somebody does some approximation analysis.

So, what will be the guide line for this approximate analysis is that the approximate analysis for the intermediate zone should match with the solution of the initial stage, and it should also match with the solution of the final stage, there should be a smooth variation the physical process does not know that it has got 3 stages, it will smoothly go from one phase to another phase, or it will smoothly have a transition from one stage to another stage, only we engineers for our own benefit or for our own convenience what we do we define that it is first stage second stage third stage like that, so obviously what people have done mechanically they have tried to find out mathematical functions which will also relate the first stage of the growth and the last stage of the growth, once such equation is this.

So, if you write down this equation it is  $r$  plus that is related to  $t$  plus, so far we have dealt with dimensional variables now we are dealing with non dimensional variable, so  $r$  plus is the non dimensional temperature and  $t$  plus is the non dimensional time, to non dimensionalize this terms again two parameters namely  $a$  and  $b$  has been introduced, so  $r$  plus is equal to  $r a$  by  $b$  square, and  $t$  plus is equal to  $t a$  square by  $b$  square, where  $a$  is given by equation 19, so it is given by a number of physical parameters, and  $b$  again  $b$  includes the thermal diffusivity of the liquid and the Jacob number, the Jacob number has already been defined so this is your Jacob number.

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$$R^+ = \frac{2}{3} \left[ (t^+ + 1)^{3/2} - (t^+)^{3/2} - 1 \right] \dots (17)$$

$$R^+ = \frac{RA}{B^2} \quad t^+ = \frac{tA^2}{B^2} \quad \dots (18)$$

$$A = \left\{ \frac{2 [T_k - T_{sat}(p_k)] h_{lv}}{3 \rho_c T_{sat}(p_k)} \right\}^{1/2} \dots (19)$$

$$B = \left( \frac{12 \alpha_l}{\pi} \right)^{1/2} Ja \quad \dots (20)$$

$$Ja = \frac{\rho_l c_{pl} [T_k - T_{sat}(p_k)]}{\rho_v h_{lv}} \dots (21)$$

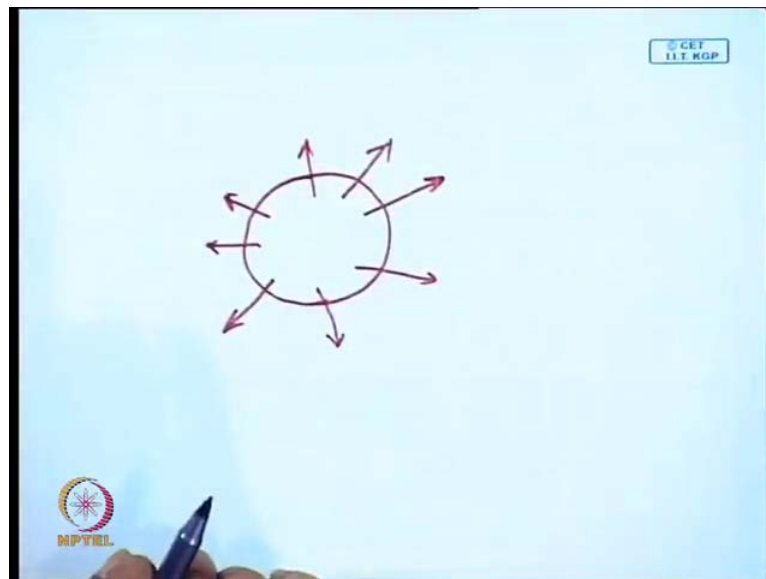
Now, you see the relationship given in 17 we have to take it carefully, now what I have shown the derivation for initial stage of growth and final stage of growth though this have been these equations have been derived with some approximation, these equations can co relate with the or can explain the experimental result also very well, where as the equation which is given in 17 it has been derived with mechanical, so it is comparison with experiment is not very good, so unless we have got some other numerical mean if you do not adopt any numerical means then only this type of equation are to be used otherwise I mean we do not recommend the use of this equation.

If somebody has got a better means of finding out the change of  $r$  with time during intermediate phase, somebody if somebody can solve the entire Nernst's tropes equation along with the energy equation and continuity equation then he should do it,

otherwise if quickly we have to guess the value of  $r$  how it is changing with time then probably the equation of the form which is given in 17 that can be used, it would be a good exercise that if you take this equation and try to check it with the limiting value of bubble growth what the initial and final stage, for that what he have to do? For that initial stage is that have to take a very small value of  $t$ , and for the final value final stage you have to take the large value of  $t$  that means time.

And you see how these the equation are telling with the earlier equations separate equation given for the initial growth stage and final growth stage, so this is an exercise which you should do. Now, let me discuss a few other things, the condition what I have or the this situation which I have describe is an isolated bubble in the bulk of the liquid which grows bigger and due to the how the pressure and temperature changes so that that is what we have done, now generally this problem or the situation which I have described is much more relevant for bubble during some sort of flashing or gravitation.

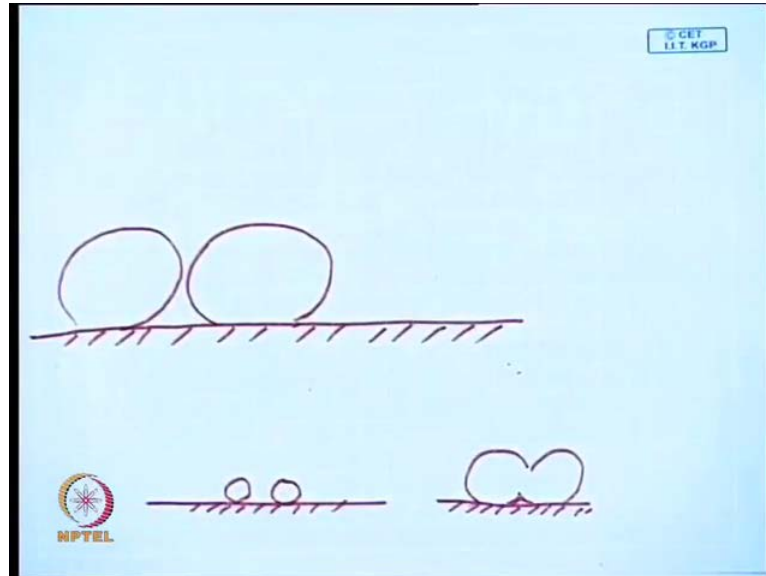
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It is not that way very relevant for the boiling process, but as we told that in this few classes we will concentrate on boiling, so let us see how it can be related to the process of boiling, now in the process of boiling what will happen that we have got a heated surface and on the heated surface we will have growth of bubble, **so on the heated surface we will have the growth of bubble.** Now, let us see what is the difference between the earlier process at the process, the difference first difference is that it is not

an infinity bulk of liquid it is attached with the surface, at the top the bulk of the liquid may extend up to infinity.

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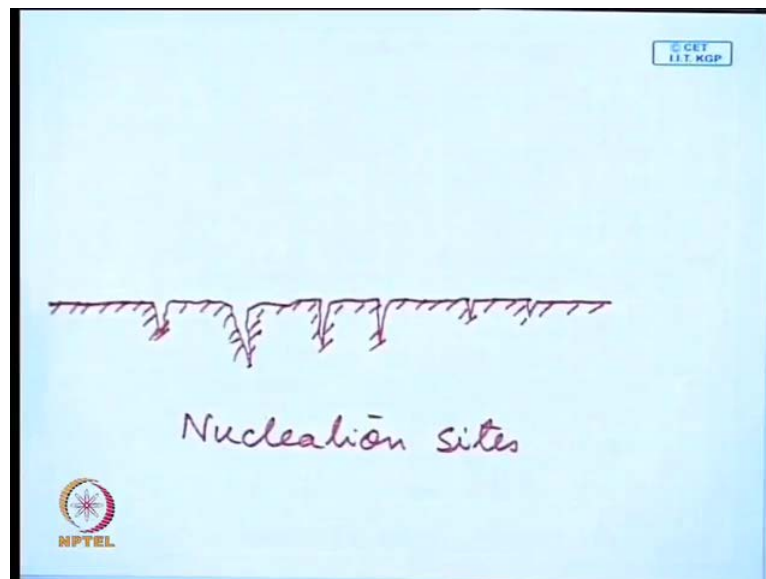
But one side its growth restricted by the solid wall, second thing is that it is not completely spherical, it can have different shapes and it is not completely spherical, third thing which of course, we have neglected in the previous case and in this case also what the time being we will neglect, but it is important that in an on an on a particle surface when boiling takes place there is not a single bubble where could be number of bubbles, so initially let us say there are two small bubbles, but as they grow they can quails, so this type of processes are very difficult but very typical in case of boiling, and it is obviously the type of analytical approach we are taking by that analytical approach we will not be able to model them or predict this kind of bubble margin.

So, probably a good numerical approach should be taken for dealing the this type of problem, but let us come back when boiling occurs what type of phenomenon we can observe, a normal surface by our naked eye may appear very smooth, and we can take lot of precaution we have we can take lot of precaution to make this surface very smooth that means good machining polishing etcetera can do, but if we see it under some magnification we will find that there are number of cracks, (( )), groves, micro groves etcetera we can find, so these will be the nature of the actual surface, now you can see or you well imagine appreciate that it is very difficult to characterize this surface in a very

definitive term, because the cracks (( )) will have different geometries shape size and distribution.

And it will depend on a very large number of parameters, those parameters are the material which I am using, the type of machining process surface finishing process I have adopted, the suppose I am doing boiling studies on it so the type of oxidation it has gone through, during boiling heating is there so there could be oxidation, and also during boiling there would be fouling and contamination, so the type of fouling and contamination this surface as experienced, so depending on all these we will have the shape and size of the cracks (( )) like this, now all his cracks or grooves what it will do before the liquid is poured over it, so in this cracks (( )) says we will have some permanent gas, this permanent gas is nothing but air, for the time being we can think that it is air, it would be other permanent gas also so it is air, but as these are very small.

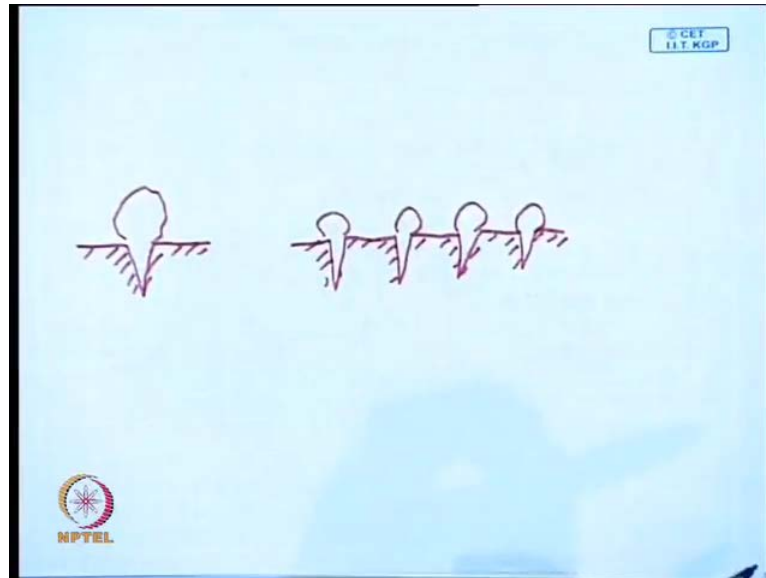
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So, when it is filled up with surface is in contact with the liquid, so the air that will come out of this that will stay inside the cracks itself, and they has got I mean they have got a very premier role very important role as far as the boiling process is concerned, as far as the bubble generation is concerned, now these are called nucleation sights, for boiling heat transfer these are known as nucleation sights, and they will give raise to the evolution of bubble, now again it has to be told it has to be noted that this nucleation sights all of them are not operate together, what I say what it means that a particular

nucleation sight will be operative only at a particular temperature, not that at every temperature I mean at each and every temperature we will find all the nucleation sight working.

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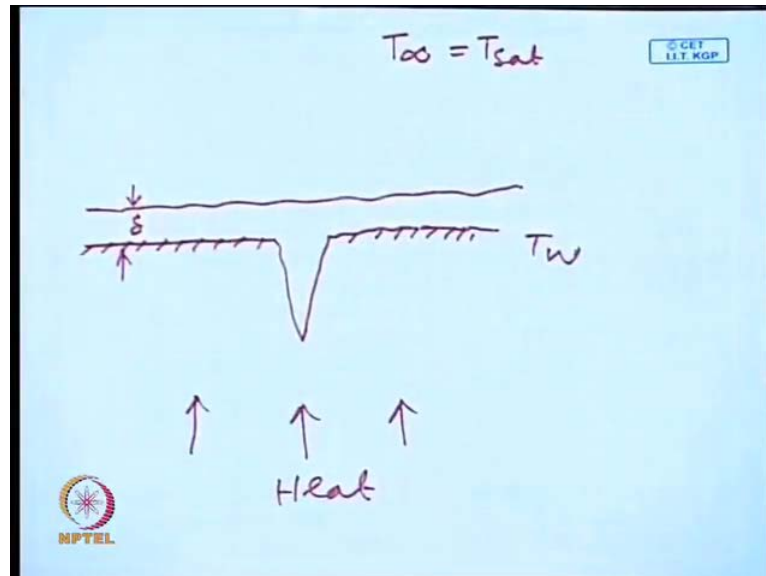


I will draw your attention into a common day to day phenomenon, suppose you are boiling water on a pan just observe what is happening, initially the water is at room temperature then slowly the temperature of the water increases, how it is increasing? Because slowly the temperature of the bottom movable of the pan that increases, and then you will find that we will have bubble generation at one point, after sometime we will find that in number of points are h generating the bubble, that means with time as the surface temperature has increased we are having more number of nucleation size, so as I have told that the nucleation size that will depend on the temperature also.

Not only it will depend how the material has been prepared, so it will depend also on the temperature of the surface, or in other words the nucleation size which are there already in the surface they will get activated more number of nucleation sights will get activated as the temperature increases, so this is another feature, the third thing which we can see that what is the role of the nucleation sights, so let us consider a particular nucleation sight, so it will have some as I told some permanent gas like air, and e to l from the name it suggest that it will it is helping in the nucleation of bubble, so nucleation of bubble

how it is helping that it will give 1 or 2 molecule or it may be a few molecule of the permanent gas to form as the bubble nucleus.

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So, if we consider this surface this is at a wall, and heat is coming from this side, one feature I will find that though the bulk of the liquid is at  $t_\infty$  here I will find a boundary layer  $\delta$ , so this is a heated boundary layer whose temperature is more than the  $t_\infty$ , and  $t_\infty$  if I call it  $t_{sat}$  then in the boundary layer the temperature of the liquid is more than the saturation temperature, so this is one very important saying you consider that though near the wall we will have liquid at a temperature more than the saturation temperature boiling will not take place, so to boiling to take place we need some sort of active nucleation size, so from the active nucleation size now the boiling will take place.

So, here a bubble will first generate or nucleate then it will grow probably that growth stage we can correlate with the bubble growth which we have studied so far, and that is what we are going to do in our next class. If there is any question please.