Introduction to Process Modeling in Membrane Separation Process

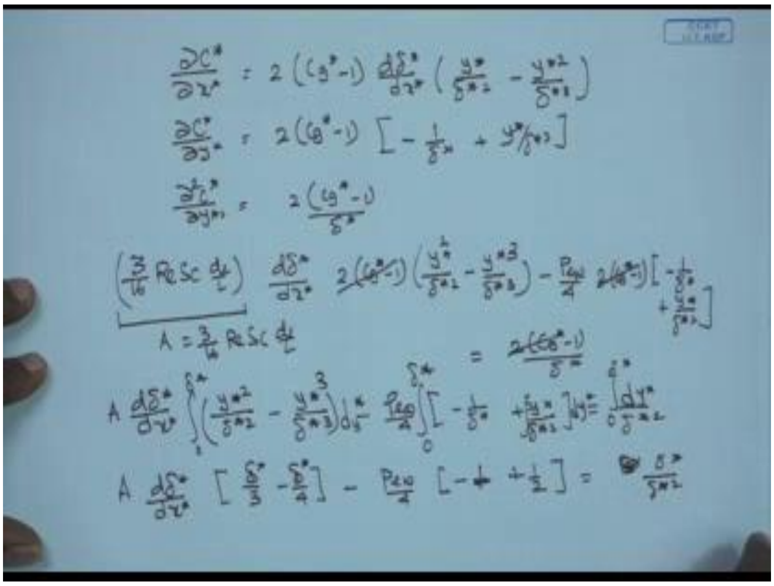
Prof. Sirshendu De
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture -16
Design of Membrane Module

Welcome to this session. Now we are basically looking into the modeling of gel layer power controlling filtration in a rectangular channel with two dimensional model with growing developing mass transfer boundary layer.

Now as you have seen earlier, we have already determined the concentration profile.

(Refer Slide Time: 00:39)



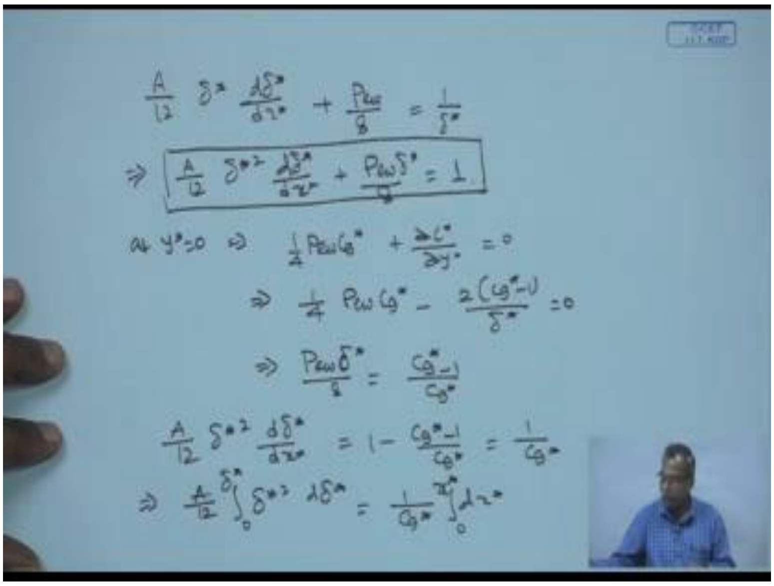
And now we will be evaluating the various derivatives of the concentration profile. So, del c star del x star will be nothing, but 2 cg star minus 1 d del star dx star y star by del star square minus y star square by del star cube. And I am just omitting a step and writing the final expression and del c star del y star will be 2 cg star minus 1 minus 1 by del star plus y star by del star square and del square c star del y star square will be 2 cg star minus 1 over del star.

Now, you substitute all these derivative in the governing equation and see what you get. 3 by 16 Reynolds Schmidt d by L d delta star d x star raise on y star should have been multiplied with that. So, we will be putting into that, 2 cg star minus 1 y star square by del star square whether a y, so it will be multiplied with this minus y star cube by del star cube minus Pw by 4 del c star del y star at 2 cg star minus 1 minus 1 by del star plus y star by del star square is equal to del square c del y star square. So, it will be 2 cg star minus 1 del star. So, 2 cg star minus 1 will be cancelling out from all the sides. And what will be getting lets have this is A. A is equal to 3 by 16 Reynolds Schmidt d by L. So, A d delta star d x star y star square by del star square minus y star cube del star cube minus P w by 4 minus 1 by del star plus y star by del star square is equal to 1 over delta star. So, you multiply both sides by dy star.

And integrate across the boundary layer thickness 0 to delta star, 0 delta star and see what we get. So, these will be you will be obtain that e A d delta star d x star. Since delta star is full function of x star, the derivative will be full function of x star and it will be out of the integral sign. And if you really do that it will be 1 by 3 minus 1 by 4 minus Pw. So, there will be delta star there. So, delta star by 3 delta star by 4 minus P e w by 4 minus half minus 1 and plus half, y star square by 2. So, del star del star will be cancelling out. So, this will be the 1 and it will be delta star so this will be 0 to delta star. So, you will be having a delta star there, delta star there.

Now and since there will be square there will be a 1 delta star on the other on the right hand side. Now after this, what will be getting is A delta star A by 12 delta star d delta star d x star plus Pew by 8 is equal to 1 over delta star. And ultimately you will be getting A by 12 delta star square d delta star d x star plus Pew del star by 8 is equal to 1.

(Refer Slide Time: 04:52)

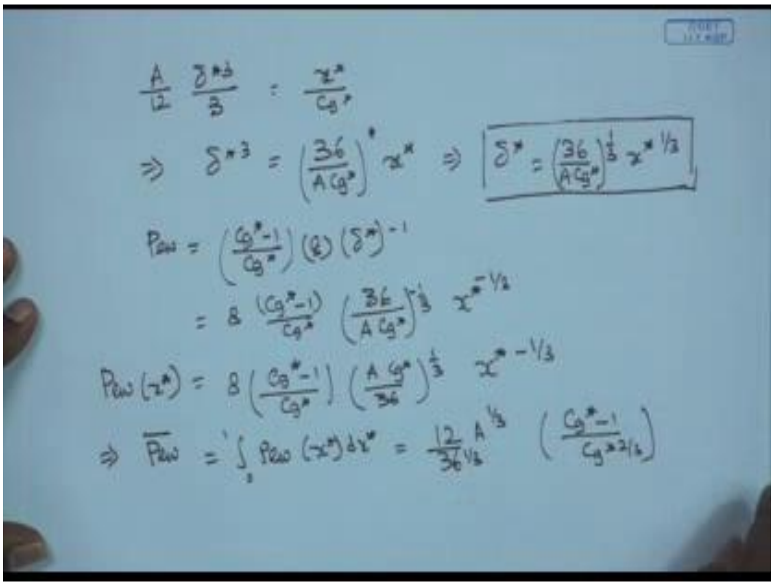


Now, let us look into the, so this will be the form of the governing equation that one would getting after this. Let us look into the boundary condition at y star is equal to 0. So, you have already seen that Pew Cg star by 4 plus del c star del y star will be equal to 0. So, we have the expression of del c star del y star and put it there. So, this becomes 1 by 4 Pew Cg star minus 2 Cg star minus 1 by del star will be equal to 0. And ultimately it will be getting Pew del star by 8 is equal to Cg star by minus 1 divided by Cg star.

So, once you put this Pew delta star by 8 here and finally, you will be getting A by 12 delta star square d del star d x star is equal to 1 minus Cg star minus 1 by Cg star and Cg star will be cancelling out. We will be getting 1 over Cg star. So, these can be integrated over length. So, A by 12 delta star square d delta star, from 0 to delta star and 1 by Cg star d x star, from 0 to x star. That will be giving me and expression of delta star is a function of x star.

So, this will becomes A by 12 delta star by delta star cube by 3 is equal to x star Cg star. And we will be getting delta star cube is equal to 36 ACg star to the multiplied by x star and one will be getting an expression of delta star as 36 by ACg star to the power 1 upon 3 x star to the power 1 upon 3.

(Refer Slide Time: 07:14)



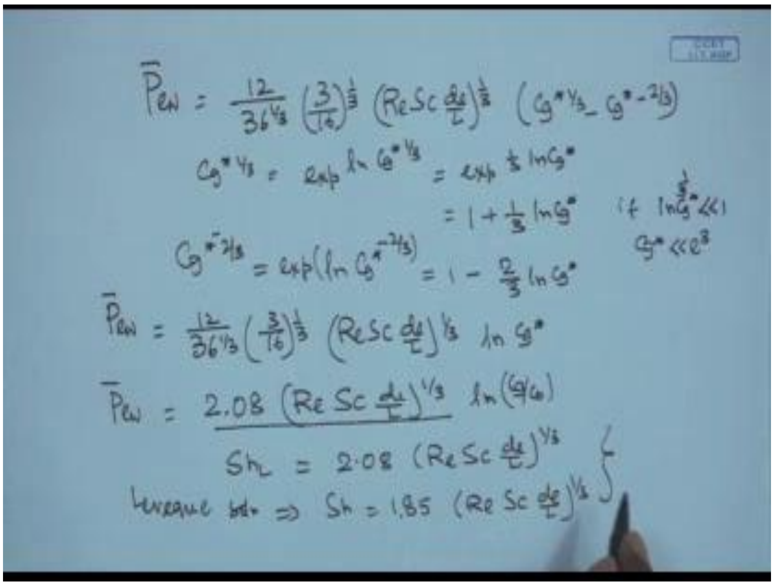
That is how the delta star, the mass transfer boundary layer as a function of x star and if you remember the functional variation of mass transfer boundary layer thickness in case of osmotic pressure control filtration is also del star is a function of x to the power 1 upon 3.

So, next will be looking into the permeate expression of the permeate flux. Pew will be, if you remember Pew is nothing, but Cg star minus 1 divided by Cg star multiplied by 8 into delta star. So, we are going to put the expression of del star there. So, 8 Cg star minus 1 divided by Cg star and del star 36 by A Cg star to the power 1 upon 3 x star to the power 1 upon 3. So, in fact if you remember the expression it was Pew delta star by 8 is equal to Cg star minus 1 by Cg star. So, we have put A. So, what is a pressure value of Pew? Pew will be nothing, but 8 by delta Cg star minus 1 by Cg star delta star will be the denominator. So, these will be delta star will be the denominator.

So, there will be a minus 1 upon 3 and there will be minus 1 upon 3 here. So, you will be getting 8 Cg star minus 1 divided by Cg star A over Cg star divided by 36 to the power 1 upon 3 and x star to the power minus 1 upon 3. This is how parameter flux non dimension parameter flux is varying as a function of x star. We can integrate it out. If you can integrated out, the length of h parameter flux will be given as Pew x star d x star 0 to 1 and integration all this will be constant and the integration of these will be giving you a value of 1.5.

So, 1.5 should be a multiplied by 12, 1.5 should be multiplied by 8 and it will return value of 12. So, 12 by 36 to the power 1 upon 3 A to the power 1 upon 3 and Cg star to the power 1 upon 3, this will be Cg star minus 1 divided by Cg star to the power 2 by 3, 1 minus 1 upon 3 is 2 by 3. So, we have almost close to the expression of length permeate flux.

(Refer Slide Time: 10:34)



So, Pew bar is equal to 12 divided by 36 to the power 1 upon 3, A is 3 by 16 1 by 3 Reynolds Schmidt de by L to the power 1 upon 3 and Cg star minus 1 divided by Cg star to the 2 by 3. So, it will be Cg star to the power 1 upon 3 minus Cg star to the power minus 2 by 3.

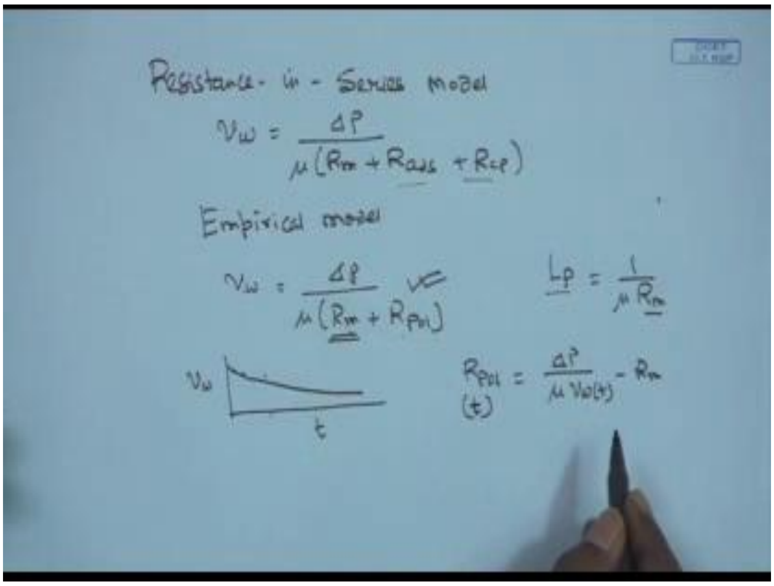
Now for the limiting case under some, so this will be the expression of Pew under certain case Cg star we can write Cg star to power 1 upon 3 is equal to exponential ln Cg star to the power 1 upon 3. So, that gives me exponential 1 by 3 ln Cg star and if this becomes less than 1 then we can have an exponential expansion. So, neglecting the higher order terms one third ln Cg star. If Cg star ln Cg star to the power 1 upon 3 is much less than 1 so that means, Cg star should be much less than e to the power 3. So, that will be around 20 that will be around 20.

So, Cg similarly Cg star to the power 2 by 3 can be written as exponential minus 2 by 3 can be written as exponential ln Cg star to the power minus 2 by 3 and will be having 1 minus 2 by 3 ln Cg star. So, Pew bar will nothing, but 12 by 36 to the power 1 upon 3 2 by 16 to the power 1 upon 3 Reynolds Schmidt de by L to the power 1 upon 3 and this minus this or this minus this 1 minus 1 it will be cancelling out one third minusplus two-third, so it will be 1 only. This becomes ln Cg star; this is Cg by C naught.

Now if we really calculate this numerical value the numerical coefficient this turns out to be 2.08 Reynolds Schmidt de by L to the power 1 upon 3 ln Cg by C naught. So, what will be this? This will be the Sherwood number or length average mass transfer coefficient is 2.08 into Reynolds Schmidt de by L. And if you remember to the power 1 upon 3 for the laminar flow, if you remember for the Leveque solution, we have Sh is equal to 1.85 Reynolds Schmidt de by L to the power 1 upon 3.So, whenever will be considering a developing mass transfer boundary layer, there will be around 12 percent enhancement of permeate flux one can expect out of the system. So, that completes modeling of gel layer control filtration.

In the next we will be moving over to the module design. So, we have looked into the designs of the osmotic pressure control filtration, we have looked into details of the gel layer control filtration then one more modularized, I think I should touch over that will be the resistance and series model.

(Refer Slide Time: 14:03)



So, in case of resistance in series model, so this is nothing, but a phenomenological equation. We assumed that permeate flux is equal to driving force divided by all the resistances that we have put into the series, absorption resistance plus concentration polarization resistance. So, all these resistance are put into series, but this is internally and empirical model. So, what is done here? This is generally we take request to the empirical models like resistance in series models, in case where there will be have a complex real fluid for example, like you know effluent or a juice or a very real life effluent which will be contracting thousands of components.

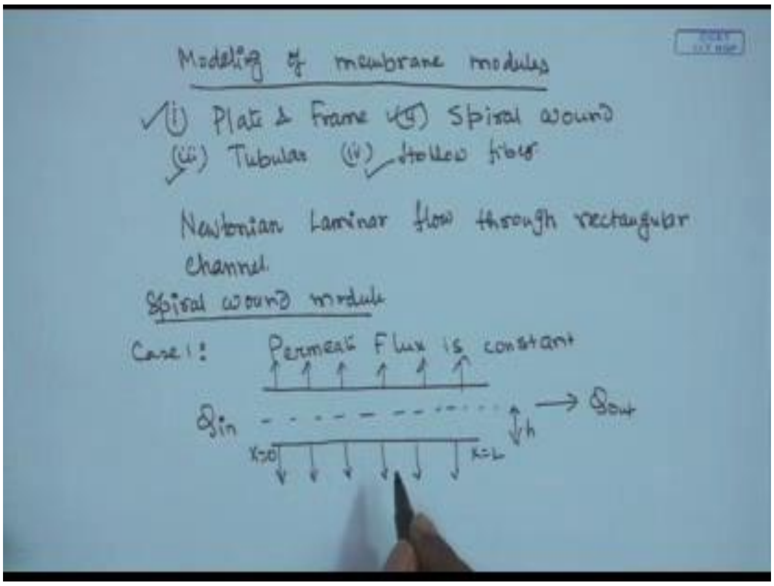
As we have discussed earlier, it is difficult to identify these components. Not only that, it is difficult to identify the concentration as well as the property. So, it is better to form to measure the permeate flux and every time point and evaluate this resistances mean there are ways to measure this resistances and then you, for example, if you would like to have a two resistance model that is membrane resistance and polarization resistance. So, it is very easy to find the membrane resistance because we can know we have already seen we can independently find out the membrane permeability where from the permeability data, we can get the membrane resistance.

And from the permeate flux data, if you we measure a, if you conduct an experiment and record the permeate flux data then from at every time point, we can calculate the what is the polarization resistance because we know the operating pressure, we know the membrane resistance we have the experimental flux data. So, we can calculate the, what is the polarization resistance every time point. So, that will be nothing, but mu Vw function of time minus Rm. So, will be getting the R polarization where polarization layer of time point and then we will be divide, you know getting these values of this the polarization resistance for different operating pressure and operating time.

Then we evaluate a relationship between the polarization resistances with the operating conditions. Once that is done, then we can get we can put back the that expression into this model and this can be used as a this will be validated for a small range operating conditions then this can be used as the, in the predictive more for different operating conditions for the same complex feed. So, absolutely complex feeds where there is very difficult to understand or estimate the properties, streams, concentrate components their concentration etcetera. Then one can take request to the resistance and series model.

So, we have looked into the various types of models and then we will be moving into the modeling of membrane modules.

(Refer Slide Time: 16:57)



So, what are the modules? So, as we discussed earlier, the membrane modules are basically the housing of the membrane and all these membrane step operations are occurring under high pressure. So, there is a housing of the membrane that is very very required and membrane housings are really complicated. There are some known well known modules. This membrane housing is known as module, the well-known modules plate and frame plate and flame.

Secondly the spiral wound, third one is tubular, fourth one is hollow fiber. We just discuss about them, hidden frames are basically the module were there will be the plates will be held between the frames and membrane will be held to the plates by the frames and across the plate there will be two surfaces and one can put two membranes membrane in the circular disk or may be rectangular in shape. Such plates and frame arrangements can be kept one after another in the module and can increase the surface area that is number one.

In case of spiral wound module, a sandwich of membrane support system will be spiral wound across a central rod and there is a very complicated special arrangement and then this whole thing can be you know (Refer Time: 18:42) to the internal rod and then it will be done. In case of tubular module, the module is in the form of a tube where the internal surface of the tube is basically a specially wound by a membrane. So, is called a tubular module. And in case of a hollow fiber modules there are fine hollow fibers which will be having internal diameter four hundred to five hundred microns or even slightly higher and this hollow fibers will be packed in a bigger fiber and one can get large number of fibers packed in a bigger you know diameter pipe. And then one can realize a huge surface area even in small volume. What is the diameter of the filtration area? What is the filtration area in these hollow fibers?

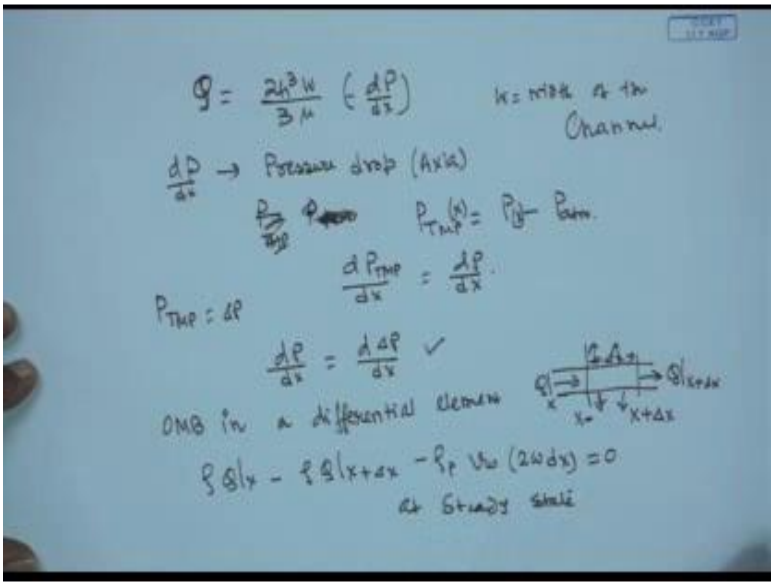
If the diameter is d filtration area is pie d l if n numbers are packed then we will be having a total surface area if total filtration area is n pi d l. So, all though the diameter is less in number, but for example, 400 micron, 500 micron, 600 micron in that range, but the number is very high. Let us say around 10000, 15000. If you really pack such huge number of fibers in a bigger tube then we can realize, you know one meter square, two meter square area in a small volume. So, therefore, the idea of every module is to get the maximum filtration area surface or surface area in a small volume so on, so that the design of the module would be very very compact.

Now there are two types of analysis are required in order to design these modules, one is the flow through a rectangular channel and a flow through a tube. If you consider a flow through rectangular channel that will be closely you know resemble case one for the plate and frame and case two the spiral module; that means if you open up the spiral module it will be like a flow through a flow or a flat sheet. Now, for the flow through a tube will be resembling to the tubular module and the hollow fiber module.

Now let us consider the Newtonian fluid. Newtonian laminar flow through rectangular channel and will be again being a very simplified case in the beginning then we will build up the case and going to the more realistic case or complicated case at the end. So, first case will be the, doing analysis for the spiral wound module. So, it is basically flow through a rectangular channel where the membrane is present on the both top and bottom surface of the channel and case one will be considering the permeate flux is constant. Let us consider a membrane channel where this is the middle of the channel of half height h this Q in total volumetric fluorite inside towards going inside the channel and this is Q out that is going out of the module this x equal to L 0 and this x equal to L length. L is the length of the channel and will be considering a constant permeate flux in the module. So, all these are constants.

As we have discussed earlier that velocity in the, the retented velocity in the flow channel will be in the order of meter per second and permeate velocity will be in the order of 10 to the power of minus 5 to 10 to the power of minus 6 meter per second. So, there is a at least 5 or 6 order or magnitude difference between the permeate flux and the retended fluid. So, therefore, we will be assuming the laminar profile that is a parabolic velocity profile that is existing inside the channel will be remaining undistorted by the permission at the wall because it will be at least 5 to 6 magnitude less than the bulk flow.

(Refer Slide Time: 23:03)



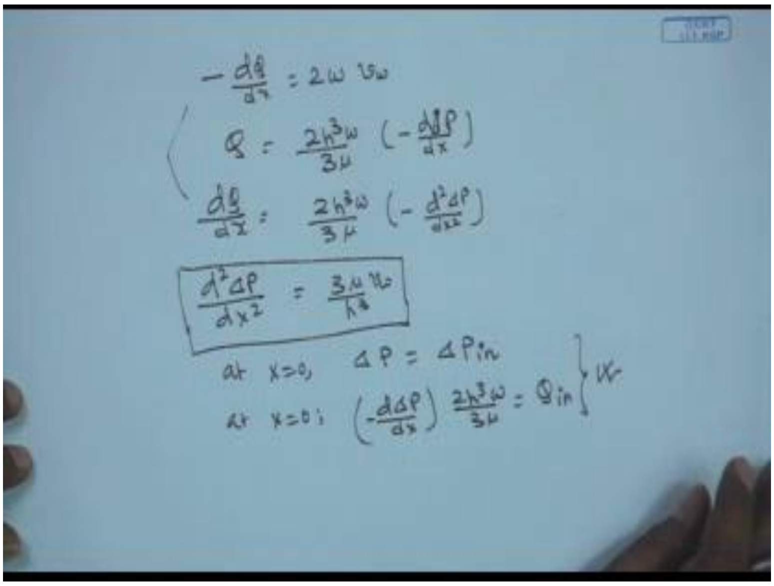
If you really do that then will be, we can write down the famous law of volumetric flow rate under laminar condition through the channel that will be 2h cube w by 3 mu times d minus d P d x. And h is half height; mu is the viscosity of the solution. w is the width of the channel.

Now you can see that what is P? P is the pressure these this dP dx is actual pressure drop.

So, dP dx is the actual pressure drop. And let us see what is dP dx is basically this P can be written as P upstream sorry P trans membrane pressure drop. P trans membrane pressure drop is nothing, but P TMP is nothing, but P in the feed side minus P in the permeate side. So, that is the atmospheric pressure. So, this P is similar same as this P. So, d P, P trans membrane pressure drop d x will be nothing, but dP dx because P atmospheric is constant. So, what is dP dx is saying, is to same as the d trans membrane pressure drop dx. So, we assume dP dx d del so, this same as delta P by dx and P trans membrane pressure drop is nothing, but delta P. So, dP dx will be treated as d delta P dx. So, this will be any point of time, so this will be absolute pressure at any point of time.

Once we identified this, we can write down the solute you know total overall material balance. Overall material balance in a differential area, in a differential element of the channel whose there is a channel and this is we are talking about cross section located at x and another cross section which is located at x plus delta x ok. So, total material going into the system, total material going out of the system and total material going out of the wall because membranes are there. So, rho this is Q at x, this is the flow rate volumetric flow Q at x plus delta x. So, rho times Q at x minus rho times Q at x plus delta x it is going out minus rho permeate Vw two W dx is these differential area. W is the width in this normal plain to the paper and this delta x. So, 2W dx, there will be two surfaces that is why two ways appearing is equal to 0 at steady state.

(Refer Slide Time: 26:14)

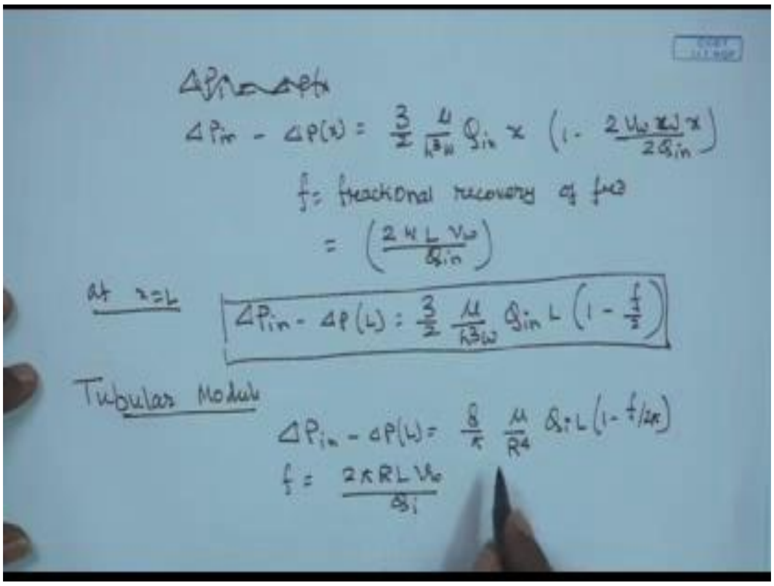


Now what it will be giving you? This will be giving you minus dQ by dx is equal to 2W Vw. So, this equation will give you governing equation of how Q or you know volumetric flow rate is varying as a function of x in a membrane module in a actual membrane module. Now we can, we have written that Q is equal to 2 h cube W over 3 mu minus d dP dx will be delta P d x. So, just differentiate it, once more time with respect to x will be giving you dQ dx is equal to 2 h cube W by 3 mu minus d square delta P by dx square.

So, we equate this two and will be getting the governing equation of pressure drop across the channel is equal to d x 3 mu Vw by h cube. This gives you the governing equation of delta P across the channel. This is a second ordinary differential equation and you need to have, you need to specify two boundary conditions to solve this problem. One boundary condition at x is equal to 0. We know the delta P inlet, delta P is equal to delta P in that is known to us. And at x is equal to 0, we know the volumetric flow rate that is going into the system. So, d delta P dx is known to us because d delta P dx will be known to us minus two h cube W 3 mu is equal to Q in. So, these two conditions are known to us. So, if you remember that these two conditions are specified on the same boundary, so this is known as quasi boundary conditions.

So, using these two conditions, this equation can be solved and once this equation will be solved you will be a getting a solution of pressure drop across the channel. If you really solve this equation, this is straight forward everything is constant to the right hand side substituting the boundary conditions.

(Refer Slide Time: 28:23)



Ultimately you will be getting a solution delta P i is equal to delta P x. delta P i in minus delta P at any x location is equal to 3 by 2 mu h cube w Q in x 1 minus 2 Vw x Vw w times x divided by 2 Q in. So, why you put in this form? Because that will be giving you what is Vw wx because 2 Vw wx will be giving you, what is the total recovery. So, f is the fractional recovery of the feed in the filtrate or in permeate fractional recovery of feed. So, that will be nothing, but two w at x is equal to L times Vw divided by Q in. That will gives you total recovery of the feed that is coming into permeate. So, we have to maximize these. So, it is basically at x is equal to L will be at end of the module, delta P in minus delta P L is equal to 3 by 2 mu h cube w Q in L 1 minus f by 2. So, this is the pressure drop across the channel and then this according to this pressure drop, one can select a pump, so this much pressure drop in the module will be sustained by the pump.

Similarly, the same expression we can do the similar type of analysis in case of a flow through a tube. For that tubular module, we can get the expression of the pressure drop and the expression of the pressure drop will be similarly delta P in minus delta P L is equal to 8 over pi mu by R to the power of 4 where R is the radius of the module as Q i times L 1 minus f by 2 pi where f is the fractional recovery. It is 2 pi R L Vw divided by Qi. So, total surface area is 2 pi R L and that will be multiplied by Vw permeate flux that will be the total amount of flow rate that is obtained in permeate divided by total permeate flow rate going into the system that is the fractional recovery.

So, I stop this class. In the next class what I will be doing, I will be going for one more step of complication that is will be considering that osmotic pressure is negligible and Vw is equal to LP del P; that means, permeate flux is proportional to delta P in the, then I will be considering another case the most realistic case, that is the Vw is equal to the LP into delta P minus delta pi and then will be looking into the how to do the module design in that case.

Thank you very much.