Introduction to Process Modeling in Membrane Separation Process
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Lecture – 17
Design of Membrane Module (Contd.)

Good morning everyone. So, we will be start as we have seen in the last class that we were looking into the modeling of membrane modules. We have looked into the very simple version of the module modeling. We concentrate the constant permit flux that is coming out from the wall or from the membrane surface. Initially, we have concentrate the flow through a rectangular geometry, which will be nothing but simulating the spiral wound module and similarly we have looked into the tubular module and although we have not derived the detailed equations from the tubular module. But this could have been done in the exactly the same way we have done from the overall material balance and solute balance and the pressure drop calculations in a rectangular channel.

So, modeling of membrane module indicates that we have to calculate the axial pressure drop that will be developing inside the membrane module. This axial pressure drop calculation is very very important in almost in almost all chemical engineering applications, whether it will be a (Refer Time: 01:26), whether it will be distillation column or it is absorption column, whether it is absorption column. Because once you know the pressure drop or flow through a pipe, so you whenever you know that pressure drop then you will be able to know the rating of the pump by which the fluid has to be transported. So, that the design of the pump or the in case of the in case of the liquid or design of the compressor in case of gas flow can be can be calculated easily, if you know the pressure drop in across the equipment.

So, the selection of the equipment now driving agent for example, pump and compressor is very, very essential in you to know the knowledge to know the knowledge of the pressure drop in the process stream. So, that is why the pressure drop calculations are always very important in any chemical engineering applications and so in our application as well in the membrane module.

So, now, we will be doing step by step we will be adding step by step complication to the whole theoretical modeling as we have done earlier, next analysis we will be doing that the permeate flux is not constant it will be proportional to the trans membrane pressure drop neglecting the osmotic pressure. So, you will be in the next analysis we will be doing the I have negligible osmotic pressure case, where permeate flux is proportional to the trans membrane pressure drop. In the third analysis and final analysis, we will be considering the axis number three where osmotic pressure is not at all negligible and we will be doing a real and modeling of the membrane module.

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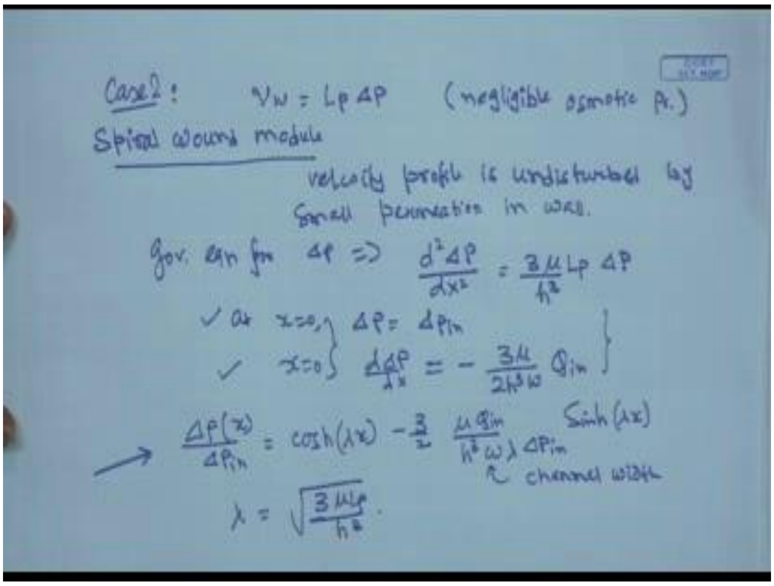

So, in these case, case 2 will be considering the permeate flux is proportional to trans membrane pressure drop, we have negligible osmotic pressure of the solution. So, first we will talk about the spiral wound module. So, as we have done earlier that the governing equation of trans membrane pressure drop will be remaining same that we will be assuming that the velocity profile remain is undisturbed by small permeation in the wall.

So, the pressure drop profile the governing equation for trans membrane pressure drop or axial pressure drop for delta P remains same and this becomes we have seen in the last class that d square del P d x square is equal to 3 mu h cube times v w, but in this case v w will be nothing, but L P del p, where L P is the membrane permeability. The boundary condition that we have already one was the pressure drop at the inlet at x is equal to 0 these delta P is known to us. And also the flow rate is known to us at the inlet condition and that will be at x equal to 0, d delta P d x is equal to minus three mu divided by two h cube w q in.

So, since both the boundary conditions specify on the same boundary and located at x equal to 0 this is a typical quasi boundary condition and we have already seen earlier that equalization of these two boundary conditions in the previous class. So, now, this is a simple a first standard in second ordinary differential equations two boundary conditions are specified. So, this problem can be solved quite easily and I am just writing the solution directly computing a couple of steps in between. So, kindly do the derivation by yourself. So, delta P x divided by delta P in is equal to cos hyperbolic lambda x minus 3 by 2 mu q i h cube this q in w lambda delta P in sin hyperbole lambda x, where w is nothing but the channel width and the parameter lambda is nothing but root over 3 mu L P divide by h cube. So, this is the expression of pressure drop at any a distance x in the module or in the channel.

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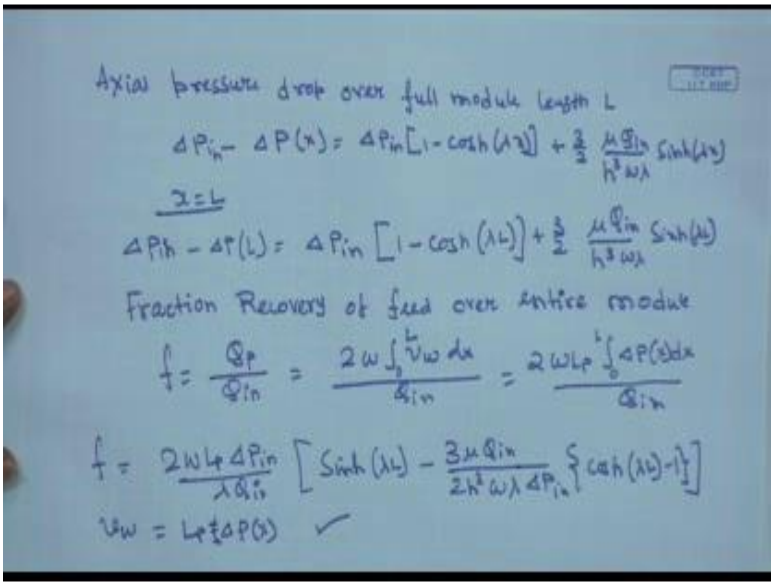

Now the pressure axial pressure drop over the full-length module length L becomes delta

P i minus. So, it becomes delta P in minus delta P x is equal to delta P i 1 minus cos hyperbolic lambda x plus 3 by 2 mu q in divide by h cube w lambda sin hyperbolic lambda x and for point x is equal to L that will give you the axial pressure drop in the across the channel. So, delta P in minus delta P L is equal to delta P in this will be in 1 minus cos hyperbolic lambda L plus 3 by 2 mu q in h cube w lambda sin hyperbolic lambda l. Now, one can get an expression of fractional recovery of feed a fractional recovery of feed over the entire model becomes f is equal to the total flow rate in the permeate stream divide by total flow rate going into the system q in, this will be nothing but two w v w d x over the entire length L divide by q in. So, this will be nothing but 2 w L P 0 to L delta P x d x and divided by q in and we know the profile of delta P x.

So, this can be inserted here and this can be integrated it out and the final expression becomes 2 w L P delta P in divided by lambda q in sin hyperbolic lambda L minus 3 mu q in divide by 2 h cube w lambda delta P in cos hyperbolic lambda L minus 1. So, we will be getting an expression of fractional feed recovery. So, we can also know we can calculate the profile of permeate flux, because v w is equal to nothing but L P del P as a function of x. So, we can we can put the expression of delta P x and can get the permeate flux as a function of x.

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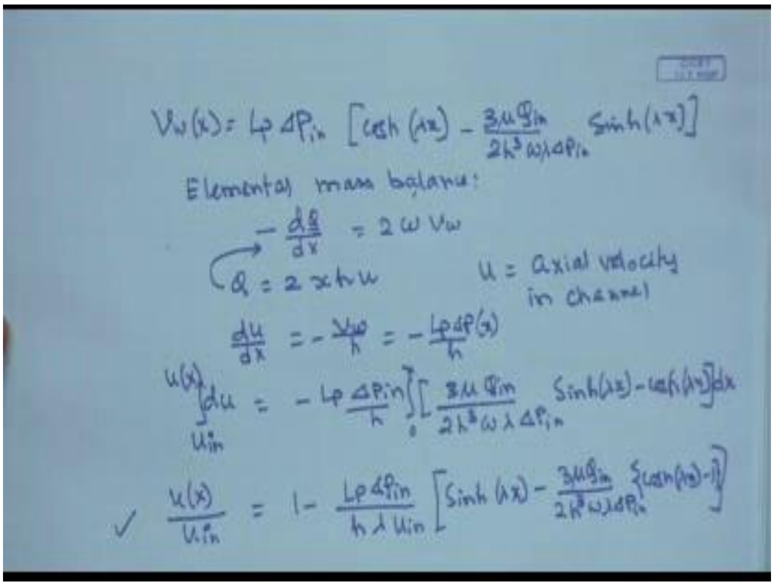

If you really do that the expression of permeate flux becomes L P delta P in cos hyperbolic lambda x minus 3 mu q in divide by 2 h cube w lambda del P in sin hyperbolic lambda x. Now that profile of v can be obtained the profile of cross the retuned velocity in the channel that can also be obtained from this expression. So, if you remember that the governing equation of q from the elemental mass balance, that we have already done in the last case in the previous class. So, elemental mass balance is. So, basically mass balance over an element in the in the channel will give you minus d q d x is equal to 2 w v w and in this case we have to put q is equal to 2 x h times u, where u is the velocity. u is the axial velocity in the channel h is the height and 2 x is the length. So, that will be the cross sectional area. So, you are a going to get d q d x. So, now, will be getting and v w in this case will be L P del p. So, d u you just substitute it over here.

So, we will be getting d u d x is equal to minus v w over h and you know the expression of v w, v w is nothing but minus L P del P divided by h, but del P will be a function of x. We have already calculated how profile of pressure drop varies along the x. So, we will be substituting that over here and then we can we can integrate at integrate over u. So, if you really do that, so it will be nothing but minus L P by h del P in by h and then we integrate over the you put the expression of delta P and integrate over 0 to x and there will be from u in to u at in x location will be giving you these expression 3 mu q in divided by 2 h cube w lambda delta P in sin hyperbolic lambda x minus cos hyperbolic lambda x d x. If we can if you if you really carry out the integration the final expression will be nothing but u x divided by u in will be equal to 1 minus L P delta P in divided by h lambda u in sin hyperbolic lambda x minus 3 mu q in divided by 2 h cube w lambda delta P in cos h hyperbolic lambda x minus 1.

So, this will be complicated equation and if you really evaluate this will be a positive quantity. So, a 1 minus some positive terms, so it will be decreasing. So, as you go along the length the velocity of the in the feed channel will decrease. Why the velocity will there in the feed channel will decrease; simply, because we are extracting some amount of material or the in the in the boundary of both the walls. So, its velocity has to go down and this is the expression of velocity in the flow channel as a function of (Refer Time: 14:26) module length.

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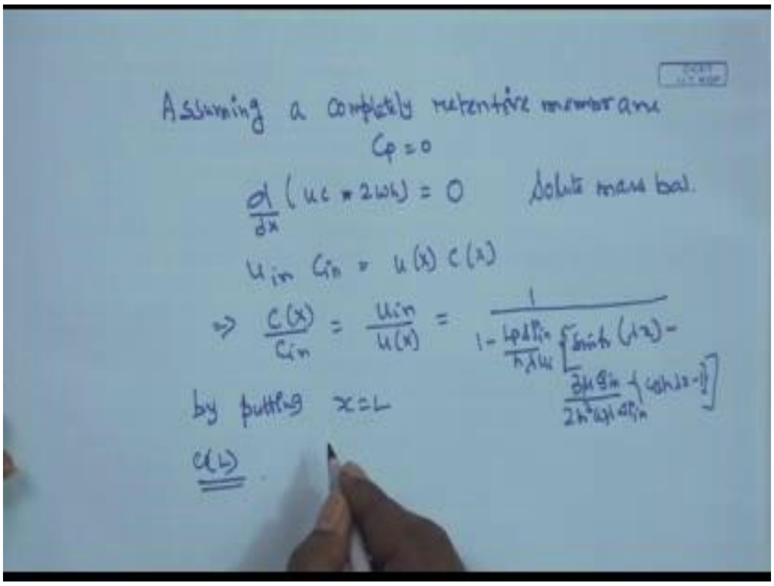

Then we can really calculate the concentration in the channel as well. So, assuming a completely retentive membrane, that means permeate concentration is 0. So, will be having d of d d x of u C times 2 w h will be equal to 0. So, d is basically the solute mass balance over the differential length of the channel. So, therefore, u at inlet C at inlet should be is equal to u at any x location C at any x location. So, therefore, one can get an expression of C at x, C in is equal to u in divide by u x and it will be inverse of that. So, will be getting the profile of that 1 minus L P del P in divided by h lambda u i sin hyperbolic lambda x minus 3 nu q in divided by 2 h cube w lambda delta P in cos hyperbolic lambda x minus 1. So, therefore, the concentration in the feed channel will be increasing as a function of x, because the denominator will be decreasing as the as the length of the channel.

So, inverse of that, so therefore, the concentration will be increasing in the feed channel and at the outlet. If the concentration can be obtain by putting x is equal to l, we can get concentration at the outlet of the module. We have already got the expression of pressure drop and from that we can get the rating of the pump through whatever is required what is or kilo whatever pump is required to pump the fluid to overcome this pressure drop in the module. So, that gives the rough idea about the design of the module whenever we are talking about a rectangular channel, where in the permeate flux is proportional to the trans membrane pressure drop.

So, next will be carrying out an analysis for we will be pulling out the results in the tubular module we will do the exactly the same thing and the finally, we will be I will be writing that the governing equations and the final solution of the tubular module. So, you just do the exact the derivations following the all the in the steps exactly the same way in the rectangular channel it will be getting the results in the tubular module.

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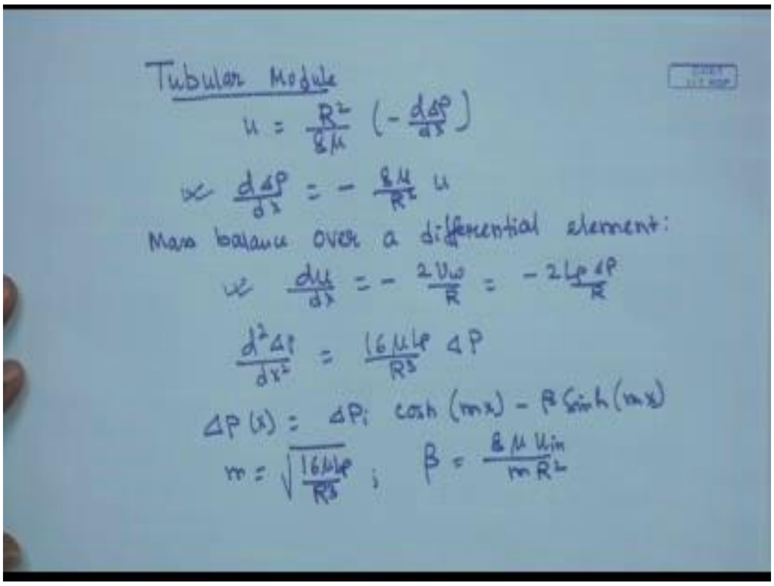

So, in case of tubular module the velocity is given as R square the across section average velocity is given as u is equal to R square by 8 mu minus d delta P d x and we will be getting the governing equation of d delta P d x is equal to 8 mu by R square times u. If you do a differential mass balance, mass balance over a differential element will give us d u d x is equal to minus 2 v w by R you can put minus 2 L P del P divided by r. Similarly, one will be getting an expression of you just you can differentiate it once again and get d u d x. Substitute it over here combining these two equations one will be getting the governing equation of d square delta P d x square is equal to 16 mu L P over R cube times delta p.

The solution of this equation will be straight forward and the solution will be delta P i cos hyperbolic m x minus beta sin hyperbolic m x and m becomes a parameter, which is nothing but sixteen mu L P over R cube R is the radius of the tube and beta is the parameter 8 mu u in divided by m R square. So, one can calculate the axial pressure drop as a function of channel length a tube length.

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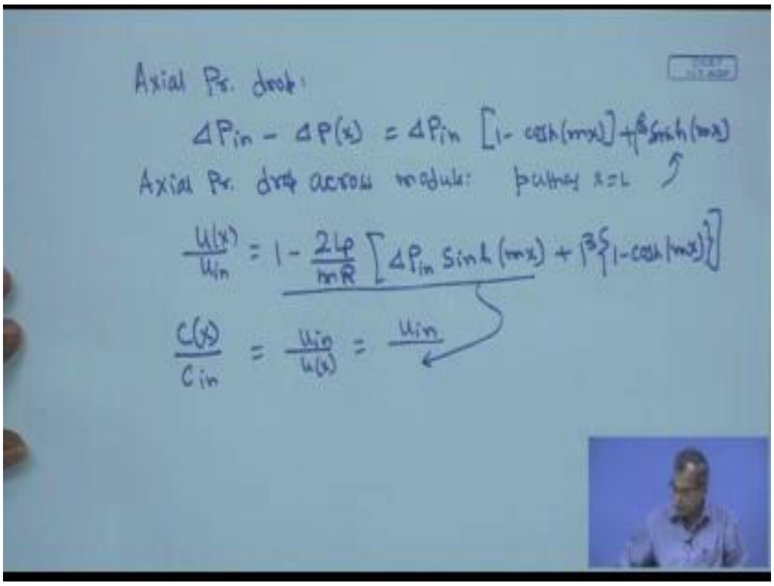

The axial pressure drop becomes at any point any location x delta P in minus delta P x is equal to delta P in 1 minus cos hyperbolic mx plus beta sin hyperbolic mx. One can get the axial pressure drop across the module can be obtained by putting x is equal to L in the above expression. Similarly, the other parameters can be calculated u over u i u in is equal to 1 minus 2 L P divided by m into R delta P in sin hyperbolic m x plus beta 1 minus cos hyperbolic m x. You can get an expression of feed concentration retuned concentrations is a function of C in is equal to u in by u x it will be just you know reverse of this expression.

So, this will be u in and the denominator will be putting up this expression over here and one can get so. The axial pressure drop where one will be getting the idea is that the axial pressure drop will be varying as a function of x and whenever will be calculating the permeate flux v w L P delta P minus delta pi. So, delta P is no longer a constant at every x location the delta P will be varying as a function of x and that will be taken care of.

One will be taking the axial pressure drop across the module the permeate flux profile, across the module the velocity profile, across the module the concentration profile in the retuned stream. So, once that is done now let us look into a more and the most realistic case of the module when we are not neglecting the osmotic pressure difference.

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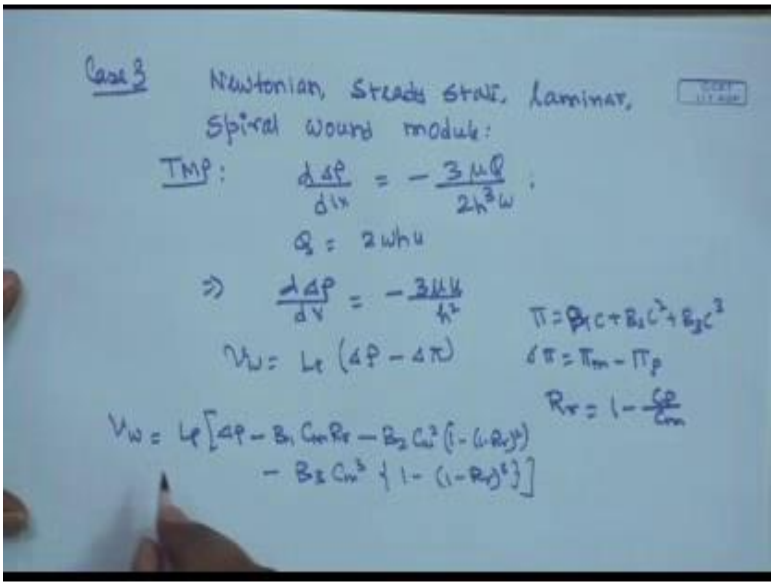

So, we will be looking for case number 3 now while we are talking about a Newtonian, fluid Newtonian, steady state, laminar and spiral wound module. So, the governing equation of trans membrane pressure drop remains the same. So, it becomes d delta P d x is equal to minus 3 mu q divided by 2 h cube w and by putting q is equal to 2 w h u this can be expressed in terms of velocity. So, d delta P dx becomes now minus 3 mu u divided by h square.

The permeate flux u w is now no longer proportional to delta p, now you have L del P minus del pi and what is in terms of when you express phi is equal to B 1 C plus B 2 C square plus B 3 C cube and we replace. So, delta pi is nothing, but pi m minus pi P. So, pi evaluated and membrane surface concentrations C m and phi P is phi evaluated and membrane at permeate concentration C P. So, then we will replace R r in we will be replacing C P in terms of R r and C m. So, by defining R r is equal to 1 minus C P by C m and we will be getting an expression of v w in terms of C m only L P del P minus B 1

C m R r plus minus B 3 C m square 1 minus 1 minus R r square minus B 3 C m cube 1 minus 1 minus R r cube. So, this is the expression of permeate flux now if there.

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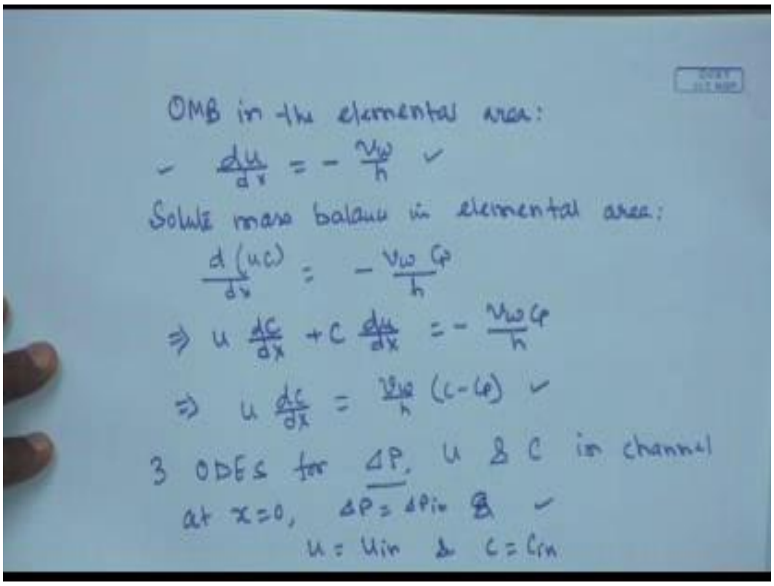

Now we will do a solute mass balance in the differential area if we do that will and if you doing overall material balance then we will be doing a solute material solute balance. So, doing an overall material balance in the channel in the elemental area exactly like the previous case we will be getting d u d x is equal to minus v w by h. So, this will be the governing equation of u in the channel and v w is nothing, but L P del P minus del pi. So, what is the then you do a solute mass balance in elemental area, in this case, these becomes d u C, d x becomes minus v w C P divided by h is the upper w channel C P is the concentrated permeate concentration and you just open it up the left hand side it becomes u d C d x plus C d u d x is equal to minus v w C P over h.

Then we can replace d u d x from this equation where and ultimately you will be getting the governing equation of d C d C u governing equation of concentration. So, these become v w by h C minus C p. So, this will be by the governing equation of they will be governing equation of trans membrane pressure drop or axial pressure drop this will be the governing equation for permeate flux, these will be the governing equation for velocity, these will be the governing equation for concentration in the feed channel.

So, I will be having three ordinary differential equations for the accounting trans membrane pressure drop velocity cos velocity in the retuned channel as well as the concentration in the retuned channel. So, three ordinary differential equations for delta P u and C in channel or module and they will be having the conditions we know the boundary conditions on delta P at x is equal to 0. So, at x equal to 0 we have 2 boundary conditions on delta P delta P is equal to delta P in and another one what that related that q in d delta P d x that will be known to us at x equal to zero, u is equal to u in and at x equal to 0, C is equal to C in. So, these will be coupled, but C n is still not known.

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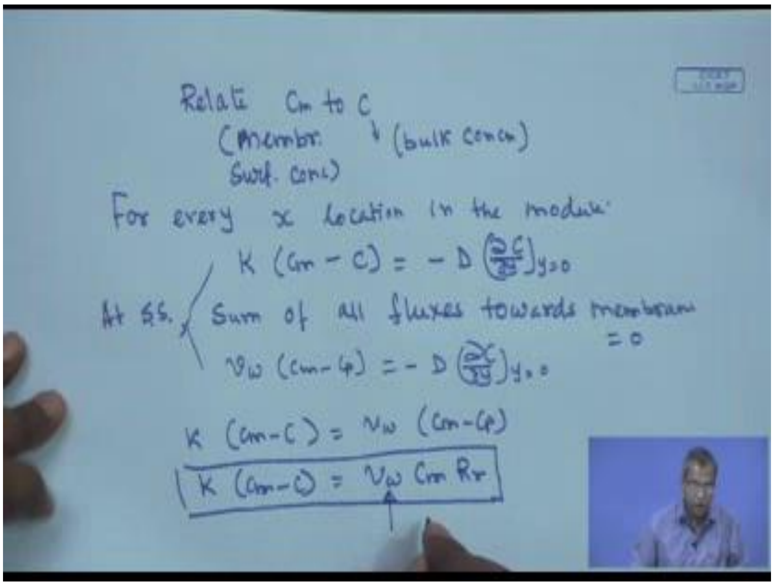

So, for that what we are going to do next is we do we relate the bulk concentration with the membrane surface concentration. So, we have to relate C m to c. So, this is bulk concentration this is the membrane surface concentration and C is the bulk concentration, how these are related these are related through the mass transfer coefficient. So, for every x in the module for every x location in the module are we have this relation holds good K C m minus C is equal to minus d del C del y at y equal to 0. So, this is the definition of mass transfer coefficient that is that is valid at every location and at steady state we have seen earlier also sum of all fluxes towards membrane equal to 0. So, if we do that this becomes v w C m minus C P is equal to minus d del C del y at y equal to 0.

Therefore, we can equate this two and we will be getting K C m minus C is equal to v w C m minus C P this can be replaced v w C m minus C P can be replaced in terms of real retention this becomes v w C m R r. Now, we will be getting these algebraic equations to be solving at every step. So, then we will be what we will be doing we will be putting the expression of permeate flux here as L P to delta P minus delta pi and we will be expressing delta pi in terms of C n and let us see what we get.

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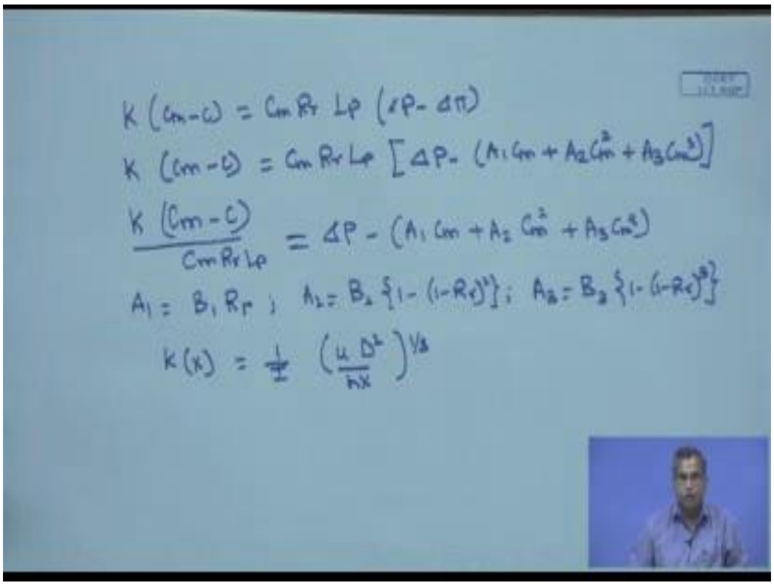

So, by doing that one will get K C m minus C v C m R r and v w will be L P del P minus delta pi K C m minus C is equal to C m R r L P and this will be delta P minus A 1 C m plus A 2 C m square plus A 3 C m cube or K C m minus C divided by C m R r L P is equal to delta P minus A 1 C m plus A 2 C m square plus A 3 C m cube and the Constance a one or related to original osmotic coefficient and real intention. So, A 2 is equal to B 2 1 minus 1 minus R r square of that then A 3 is equal to B 3 1 minus 1 minus R r cube of that. So, now, we have to find out what is the expression of these algebraic expression has to be solved also K is the function of x as we have seen earlier that K of x is equal to 1 over I u d square by h x to the power 1 upon 3. If you remember the mass transfer coefficient analysis for the flow through a rectangular channel, what I will be doing now I will be stopping here in this class.

In the next class I will be looking of mass transfer coefficient of function of x and writing the complete solution complete expression of the algebraic equation that will be that has been solved at every step of three governing equations of delta P u and C. Then we will be seeing how the module design can be done in a very complicated and realistic case.

Thank you very much.