

Adiabatic Two-Phase Flow and Flow Boiling in Microchannel
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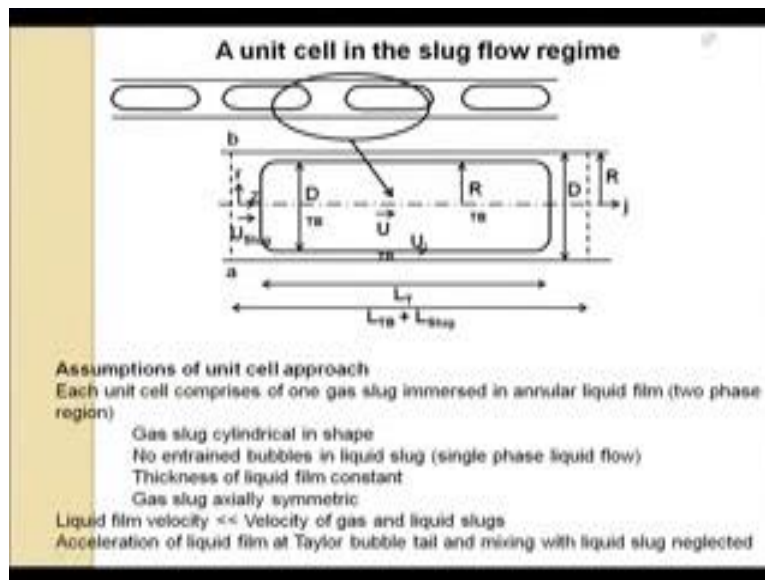
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Flow Pattern based Modelling – Slug Flow Model

Hello everybody. So, we continue with our discussions on the flow pattern based modelling approach, situated started in the last class. In the last class after discussing the reflux model sorry after discussing the homogeneous flow model, then we went over to discuss the reflux model, what the reflux model basically does? It incorporates some particular constraints, which have been derived considering the distribution of the two phases in the conduit.

So, therefore, it incorporates different constraints in the equations if the flow is bubbly or if the flow is slug and based on those particular constraints it tries to predict the hydrodynamics of the different flow patterns. Now this was definitely good approach, but more specifically if 1 considers the exact distribution of the 2 phases and then tries to modulate from the basic mass momentum equations, then it is expected to give much better representation or much realistic representation of the physical phenomena. In this class, we will be discussing one such analysis for the slug flow pattern, keeping in view that the slug flow pattern, is the most widely occurring flow pattern in micro channels and as we have already discussed.

Now, in micro channels also there have been several attempts, to model the slug flow pattern and the usual approach of modelling, is the unit cell approach what is the approach? Let us see now.

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In this approach, we assume that the flow is completely or perfectly intermittent or periodic, where the entire flow passage can be divided into unit cells and each unit cell comprises of a liquid slug and a gas plug or a gas Taylor bubble. So, therefore, how can we imagine this? We can imagine the entire flow passage to comprise of a large number of unit cells and each unit cell comprising of one gas slug, which is immersed in a liquid film and a liquid slug in narrow passages. We find that the liquid slugs are almost pure or they comprise of single phase liquid flow. So, naturally they can be modelled as single phase dynamics and the Taylor bubble region is the two phase region, where the gas plug and the liquid film which flows in the annular passage between the Taylor bubble and the pipe wall. It flows counter current to the Taylor bubble.

So, therefore, this can be modelled by approximating the flowing Taylor bubble region to be or to resemble the annular flow. So, therefore, the other assumptions which we make here is that, the thickness of the liquid film is constant which is quite a logical expression, considering them because the thickness of the liquid film. It becomes asymmetric, it is usually wider in the lower portion as compared to the upper portion and this occurs primarily due to the effect of gravity. Now since in micro channels there is a decreasing effect of gravity and increasing effect of surface tension. So, therefore, the assumption of axially symmetric gas slugs and constant thickness of the liquid film are justified and naturally the other assumption

is the liquid film velocity, is much less as compared to the velocity of the gas as well as the liquid slugs and in addition, we find that in this particular case.

Therefore, the usual step of modelling is we try to find out the pressure gradient in the Taylor bubble region, we try to find out the pressure gradient in the liquid slug region and in addition, we consider another particular pressure gradient which occurs because the counter current flowing liquid film comes and meets the upward flowing or the forward flowing liquid slug as a result of which, some mixing and some turbulence results at the end of the Taylor bubble, which also should be contributing to the pressure drop of the entire slug unit, but this usually, it is neglected under most of the circumstances.

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A unit cell in the slug flow regime

A more generalised approach

$$\left(\frac{dP}{dz}\right)_f = \frac{1}{L_{TB} + L_{SLUG}} \left[\left(\frac{dP}{dz}\right)_{TB} L_{TB} + \left(\frac{dP}{dz}\right)_{SLUG} L_{SLUG} + \Delta P_{DRAIN} \right] \quad Re_{TB} = 2 Re_L R_{TB} (U_{TB} - U_f) / \mu_G$$

$$\left(\frac{dP}{dz}\right)_{SLUG} = f_{SLUG} \frac{\rho_L U_f^2}{2D} \quad f_{SLUG} = \begin{cases} 64/Re_{SLUG} & Re_{SLUG} < 2,100 \\ 0.316/Re_{SLUG}^{0.25} & Re_{SLUG} > 2,100 \end{cases}$$

$$\left(\frac{dP}{dz}\right)_{TB} = f_{TB} \frac{\rho_L^2 (U_{TB} - U_f)^2}{4R_c} \quad \text{Drainage of liquid film into liquid the bubble tail}$$

Dukler and Hubbard (1975) $\Delta P_D / s = \rho_L \left[1 - \frac{R_c^2}{R_c^2} \right] \frac{(U_{TB} - U_f)(U_f)}{2}$

So, therefore, with these particular assumptions we start the modelling as I have already mentioned. So, as discussed the frictional pressure gradient.

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The image shows handwritten mathematical derivations on a blue background. The top equation is:

$$\left(\frac{-dp}{dz}\right)_{TB} = \frac{1}{L_{LS} + L_{TB}} \left[\left(\frac{-dp}{dz}\right)_{TB} L_{TB} + \left(\frac{-dp}{dz}\right)_{LS} L_{LS} + \Delta P_{c/s} \right]$$

The middle equation is:

$$\left(\frac{-dp}{dz}\right)_{LS} = \left[\frac{f_{LS} \rho_L U^2}{2D} \right] = f_{LS} \cdot f_f(Re_{LS}) = f_f \left[\frac{\rho_L U D}{\mu_L} \right]$$

The bottom equation is:

$$\frac{\Delta P_{c/s}}{L_{LS} + L_{TB}} \approx 0$$

I would again like to remind you that the pressure gradient it is predominantly frictional for micro channels and is very less effect of orientations since gravity is not very important in this in this particular case. So, therefore, the frictional pressure gradient, this is equal to, this should comprise of the pressure gradient in the Taylor bubble region.

This should comprise of the pressure gradient in the liquid slug region and it should also comprise of the pressure gradient due to the drainage of the liquid film into the liquid slug at the Taylor bubble and each pressure gradient, it should be multiplied with the relative lengths of the individual Taylor plugs and the Taylor slugs is in it. So, this is going to give you, this particular complete expression is going to give you the total pressure gradient deflectional pressure gradient which occurs over unit cell, rather it occurs from here to here which occurs over the unit cell of the slug flow pattern.

Now, let us see how to E value, how we go about evaluating the individual pressure gradient terms naturally start with the simplest the frictional pressure gradient in the liquid slug. Now since just only liquid is flowing in this particular case, so, naturally this can be evaluated from near single phase flow. Hydro dynamics, which gives the expression of frictional pressure gradient as $\rho L U^2$ square by $2 D$ where naturally f_{LS} , it is a function of the Reynolds number in the liquid slug,

which is again a function of the Reynolds number, which can be defined as $\frac{\rho L J}{\mu D}$ by μL is in it.

So, therefore, in sorry I made a mistake, this should have been actually $J T P$ square because remember one thing, in the liquid slug region the liquid is flowing over the entire cross section and it is flowing at the total mixture velocity in this particular case. So, therefore, in the slide I have represented it as J , but J in this case means the 2 phase volumetric plugs, more specifically I can refer to it as $J T P$.

So, therefore, we find that the frictional pressure gradient in the liquid slug is, it can be obtained from single phase hydro dynamics. There the unknown f_L the friction factor in the liquid slug can be expressed in terms of the Reynolds number, which is expressed in this particular we and we already know from or single phase knowledge of single phase hydro dynamics that f_L is, goes to 64 be $\frac{16}{Re_L}$ for laminar flow and the Blasius type equation for the turbulent flow provided, we have to remember this equation is applicable, provided we are considering the Darcy friction factor. Right now again.

Let me tell you that for most of the cases, we neglect the pressure gradient due to the drainage portion, but in case someone wants to do a very accurate modelling, then this should be consider and the person is advised to refer to the paper of Dukler and Hubbard who have derived this particular expression, considering the counter current motion and relative velocity between the film as well as the liquid slug.

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Handwritten notes on a blue background:

$\left(\frac{-dp}{dz}\right)_{fTB} \ll \ll \left(\frac{-dp}{dz}\right)_{fLS}$

Macro system - $\left(\frac{-dp}{dz}\right)_{unit\ cell} \approx \left(\frac{-dp}{dz}\right)_{fLS} (1-\alpha)$

2 Mini channels

$\left(\frac{-dp}{dz}\right)_{fTB}$ significant for ① Mini channels
② Mini channels for refrigerant flow

Well now we come to the modelling of the Taylor bubble portion. Now what is the Taylor bubble portion? What do we find that? Firstly, I would like to mention that for macro systems usually we notice that the frictional pressure gradient in the Taylor bubble region, it can be neglected because it is much smaller as compared the frictional pressure gradient. It is usually much less as compared to the frictional pressure gradient in the liquid slug region. So, therefore, very frequently for macro systems, what is the conventional way?

Or conventional approach is that the frictional pressure gradient for the entire unit cell for the 2 phase slug flow, it is often approximated as the frictional pressure gradient in the liquid slug. It is better written as approximated into 1 minus Alpha assuming that Alpha is the void fraction and the entire gas flows as the Taylor bubble and the liquid slugs are completely unedited. So, naturally that portion and also we neglect the liquid in the film as if we assume it to be negligible as compared to the liquid in the slug.

So, therefore, we assume that the entire 1 minus Alpha portion of the liquid flows as slug and accordingly the pressure gradient of the unit cell is approximated as 1 minus Alpha times minus dp/dz , A film f L S.

Now, all the this is, fines are macro systems as well as for mini channels, but the pressure gradient in the liquid rather in the Taylor bubble region, it becomes

significant for micro channels and also for mini channels for refrigerant flow right for these two cases, it becomes almost it becomes quite important for micro channels. I also for mini channels for refrigerant flow because in this case we cannot completely neglect the gas velocity and density as compared to the liquid velocity and density. So, therefore, for micro channels and mini channels during refrigerant flow, this particular term has to be considered otherwise we can neglect this. Now in case we have to consider this term, then let us discuss how this term can be evaluated.

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$$\left(-\frac{dp}{dz}\right)_{TB} = f_{21} \frac{\rho_g^2 (U_g - U_{21})^2}{4 R_{TB}}$$

Interfacial friction factor f_{21} Relative velocity $(U_g - U_{21})$

$$f_2 = f_1(Re_{TB})$$

$$Re_{TB} = Re_g = \frac{\rho_g R_{TB} (U_g - U_{21})}{\mu_g}$$

$U_g = U_{TB}$

Now, we know minus dp/dz frictional Taylor bubble, this can be written down just like we are written down for the liquid slug region in the same way, we can write down for the Taylor bubble region. In the Taylor bubble region then this is sorry this is f at the interface between the bubble and the liquid film $\rho_g U_g^2 (U_g - U_{21})^2 / 4 R_{TB}$ up to diameter of the Taylor bubble. So, we have expressed the frictional pressure gradient arising due to the friction between the Taylor bubble and the liquid film. So, naturally it is the function of interfacial friction factor and it is the function of the relative velocity between the bubble and the interface.

Now, in this particular equation you find that U_{21} is an unknown. So, next we try to find out or other, we next we discussed how to find out U_{21} , but before that I would

like to mention that, just like single phase dynamics as well as the hydro dynamics of the liquid slug region, in this case also F I it is nothing, but a function of R E T B where we can write down R E T B which is nothing, but equal to the gas phase Reynolds number that is equal to, in the same way R T B into U T B minus U I divided by Mu G, right and remember one thing, I think it should not be using both the symbols, it will confuse you further. Remember one thing since the entire gas flows as a Taylor bubble. So, in this particular case U G equals to U G B. So, even if I use both the symbols, remember one thing it is the velocity of the Taylor bubble and the gas phase velocity the inset velocity inside the conduit, they are identical right.

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$$f'_{w} = \begin{cases} 64/Re_{int} & Re_{int} < 2,100 \\ 0.316/Re_{int}^{0.25} & Re_{int} > 2,100 \end{cases}$$

Momentum Eqn for laminar flow

$$-\frac{dP}{dz} + \frac{u}{r} \frac{d}{dr} \left(r \frac{du}{dr} \right) = 0$$

General Solution

$$u = -\frac{1}{4\mu} \left(-\frac{dP}{dz} \right) r^2 + B \ln r + E$$

Since liquid film in most cases laminar but gas flow turbulent, a simpler semi analytical solution by first deriving solution for liquid laminar film

$$u_L(r) = \frac{1}{4\mu_L} \left(-\frac{dP}{dz} \right)_{int} \left[R^2 - r^2 - (R^2 - R_{TB}^2) \frac{\ln(R/r)}{\ln(R/R_{TB})} \right] + U_I \frac{\ln(R/r)}{\ln(R/R_{TB})}$$

Similarly for the gas phase

$$\left(-\frac{dP}{dz} \right)_{int} = \frac{\rho_G (U_G - U_I)^2}{4R_{TB}}$$

Where f'_I can be related to Re_G

$$Re_G = 2_{\rho G} R_{TB} (U_I)$$

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From the idealised flow field, approximating bubble as cylinder,

$$\alpha = \frac{L_g R_c^2}{(L_g + L_{\text{liquid}}) R_c^2}$$

Garimella et al. (2002) modelled intermittent regime pressure drop in minichannels when liquid film is laminar and the gas core is turbulent


$$\frac{L_{\text{gas}}}{L_{\text{gas}} + L_{\text{liq}}} = [0.7228 + 0.4692 \exp(-0.9604 D_c)] \frac{J_g}{J_g + J_l}$$

$$\frac{(L_{\text{gas}} + L_{\text{liq}})}{D_c} = \frac{1}{2.4369} \text{Re}_{\text{liq}}^{0.7601} \quad \text{Where } D_c \text{ is in millimeters}$$

$$\frac{R_{\text{liq}}}{R_c} = 0.9$$

Dukler and Hubbard (1975) $\Delta P_f / s = \rho_l \left[1 - \frac{R_{\text{liq}}^2}{R^2} \right] \frac{(U_{\text{liq}} - \bar{U}_l)(U_{\text{liq}} - \bar{U}_l)}{2}$

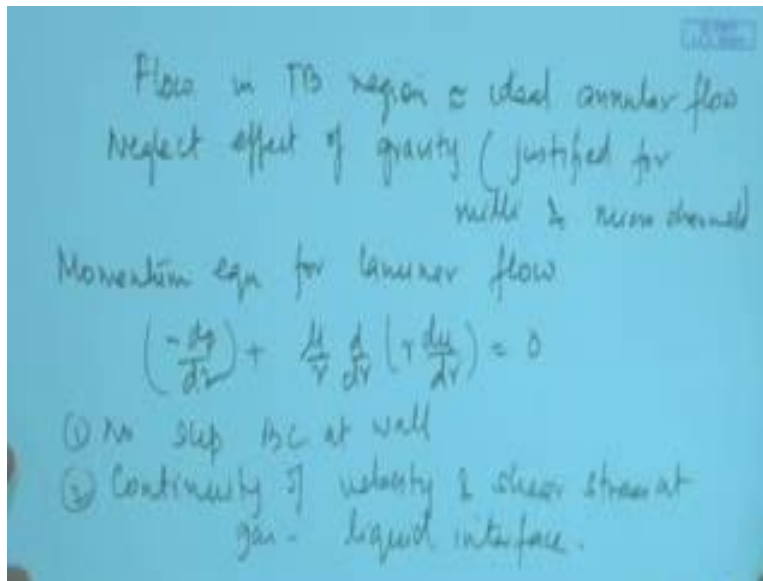
Where $\bar{U}_l = U_l / 2$ is the mean liquid velocity



So, therefore, f_i can be determined from as a function of R E T B again, thus the same equations are applicable in this particular case as well that, the other same type of equations are applicable and R T B, it is nothing, but the various of the Taylor bubble portion.

Now, we come to discussing how to find out the interfacial velocity, once we can find out this particular velocity, we can very easily calculate the frictional pressure gradient in the Taylor bubble region now. In order to find out U_I , what do we do? We start from the basic momentum balance equation for the liquid film and the gas core. So, therefore, we first start by considering the Taylor bubble to be more or less approximated by the annular flow pattern.

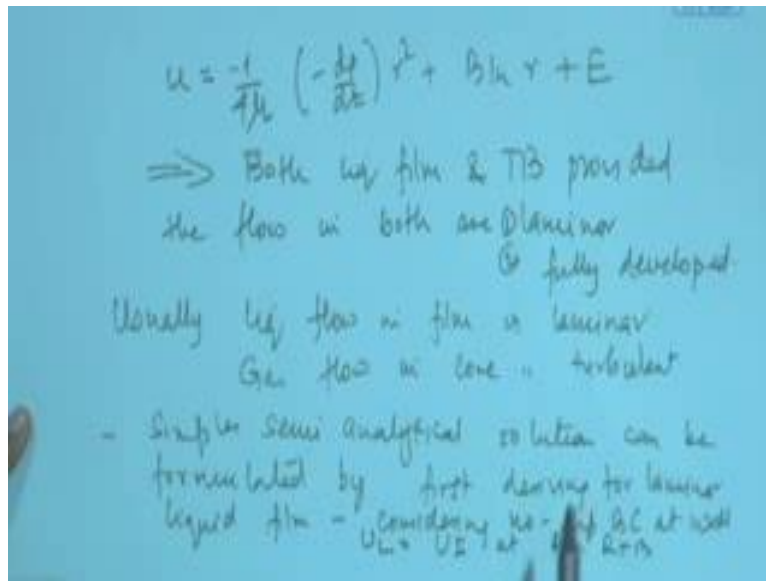
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So, we assume that flow in the Taylor bubble region as ideal annular flow region as ideal annular flow and we neglect effect of gravity. This is definitely justified for mille and micro channels. So, naturally if we can neglect the effect of gravity then the momentum equation for laminar flow of any particular flow, it can be written down as minus dp/dz plus μ/r d/dr in cylindrical cornets. This can definitely be written down provided; we have neglected the effect of gravity for any laminar flow. We this is applicable provided we what I mean to say is this is equally applicable to the Taylor bubble region as well as in the liquid film region, only the boundary conditions, where solving this equation are going to be different for the 2 cases.

Now, the general solution can be derived for both phases in laminar flow, how to derive it, we apply the no slip boundary condition at the wall and continuity this is 1 and this is 2 continuity of velocity and shear stress at gas liquid interface, based on these 2 assumptions we find that the general solution to this particular equation which are also mentioned here. So, therefore, the general solution is as I have given.

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This also would like to write down. So, that you can follow it better the general solution can be given as minus dp/dz , r square plus $B \ln r$ plus E . Now this is highly applicable for both liquid film and Taylor bubble provided the flow in both are laminar 1 and 2, fully developed. Then we can, you can use this and we using this, we can find out U and then using the boundary conditions as I have mentioned the no slip boundary condition at the wall and the continuity of velocity and shear stress at the gas liquid interface we can solve it and we can find out $U I$.

But here we need to remember one particular point, that is for most of the cases, we find that the usually the liquid film although it can be laminar, but the gas slug or the gas flow is usually turbulent. So, that is the case in what do we do, how do we proceed in that case, for that particular case as simple semi analytical solution, it can be formulated by first deriving the equation or the solution for laminar liquid film flow.

Usually liquid flow in film is laminar that gas flow in core is turbulent. So, for such a situation for do we do, we find out a simpler semi analytical solution for this particular case. It can be formulated how, by first deriving for the laminar liquid film by first deriving for laminar liquid film considering no slip boundary condition at wall and U_L equals to $U I$ at R equals to $R T B$ can be now do this.

So, therefore, if for the case of laminar liquid flow in the film and turbulently gas flow in the core, the simpler and semi analytical solution can be formulated by first deriving the expression for laminar liquid film and the boundary conditions for deriving expressions is mostly boundary condition at the wall and the liquid film velocity is going to the interfacial velocity at R equals to R T B

Now, for such a case we find that the liquid velocity profile is obtained by the expression which is given here, now 1, we have derived this then we go for deriving or further gas phase in the Taylor bubble region.

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For the gas in the TB region

$$\left(-\frac{du}{dr}\right)_{RTB} = \frac{1}{2} \left(-\frac{dp}{dz}\right)$$

Momentum balance eqn on gas core gives

$$\mu_L \left(\frac{du}{dr}\right)_{RTB} = \frac{\rho_{TB}}{2} \left(-\frac{dp}{dz}\right)_{RTB}$$

$$U_I = \frac{1}{4\mu_L} \left(-\frac{dp}{dz}\right)_{RTB} (R^2 - R_m^2)$$

$$U_I, U_L, \alpha \Rightarrow \left(-\frac{dp}{dz}\right)_I$$

So, for the gas in the Taylor bubble region what we do; we can write it down the momentum balance equation as the Taylor bubble region, This is again interfacial friction in this particular case, yeah this is the interfacial friction f_i , the way I have told for the gas phase in the Taylor bubble region I am extremely sorry, the momentum balance equation. The momentum balance equation on the gas core gives $\mu_L \frac{du}{dr}$ at R equals to R T B, they should be R T B by 2 minus $\frac{dp}{dz}$ frictional for the Taylor bubble portion; you can write it down in this particular way and then from there, we can find out the interfacial velocity from this particular expression and expression I have shown, we can find out the interfacial velocity as U_I is equal to $\frac{1}{4\mu_L}$ the pressure gradient in the Taylor bubble region in this particular form. So, therefore, what do we have now?

We have the expression for the frictional pressure gradient in the Taylor bubble region which I had mentioned this is that, frictional pressure gradient in the Taylor bubble region, here f_i can be derived from this particular expression for U_G sorry, for U_I we have got this particular expression right. So, from these 3 equations, we can solve for the 3 unknown which we have. So, if we know J_G if we know J_L , if we know α , then from here we can find out $-\frac{dp}{dz}$ frictional for the entire unit, slap the only thing which we do not know in this particular case, which we need to find out is the relative length of the liquid slug and the Taylor bubble.

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$$\alpha = \frac{L_{TB}}{L_{TB} + L_S} \frac{L_{TB}}{R^2}$$

$$\frac{L_{TB}}{R} = 0.9$$

$$\frac{L_S}{L_{TB}} = 1 - \frac{L_{TB}}{L_{TB} + L_S}$$

$$\frac{d}{Dz} = f(L_S)$$

Now, this is often derived by considering the expression of α the void fraction. Usually we know α , it is equal to L_{TB} by L_{TB} plus L_S into R_{TB} square by R square, and several experimental investigation have shown that usually R_{TB} by R it is close to 0 point 9. So, therefore, this can easily be substituted in the expression of α and from there one can very well estimate the relative length of the liquid Taylor bubble and the relative length of the liquid slug is nothing, but 1 minus the relative length of the Taylor bubble as is quite evident right.

So, therefore, this can also be derived and once this has been derived then we know all the terms here we can substitute the terms and find out the friction pressure gradient for unit cell in the slug flow pattern and this gives us the frictional pressure gradient for the entire slug flow pattern. Here I would also like to mention

that, even if you do not consider the value of Alpha from experiments, we can find out the value of Alpha from several correlation which are available from the several correlation also available from which even without knowing Alpha we can find out the relative lengths and the other things which I would also like to mention is that.

If you if we can either assume this $R T B$ by $R \rho$ equals to point 9 or there are several correlations available for the liquid film thickness in terms of the hydraulic diameter as the function of capillary number such correlations can also be adopted and they can also be used to find out $R T B$ by R square from which the relative length of the Taylor bubble and the liquid slug region can be obtained. So, therefore, this is one particular model, which we use for the slug flow pattern and I also like to mention that since I had expressed that, the when we observed the flow pattern in a micro channel, we find that several flow patterns exist at the same time. So, a better model will be if we consider we develop the pressure gradient for the Taylor bubble region for the liquid slug region for single phase gas flow for the annular flow region and we also know the relative time of existence of the different patterns and if you wet each particular pressure gradient with the time of existence of that particular distribution we would be getting much more accurate expression of pressure gradient, but in order to do that, we have to know the time of existence of the different flow patterns and again we have to depend on experiments for this particular evaluation.

So, with this I come to an end to on the analytical models for the hydrodynamics of gas liquid and vapour liquid flows in micro channels and at the end in the last lecture I would be dealing with some flow boiling aspects in micro and mille channels and that will be the end of my lecture series on adiabatic two phase flow and flow boiling in micro channels.

Thank you very much.