

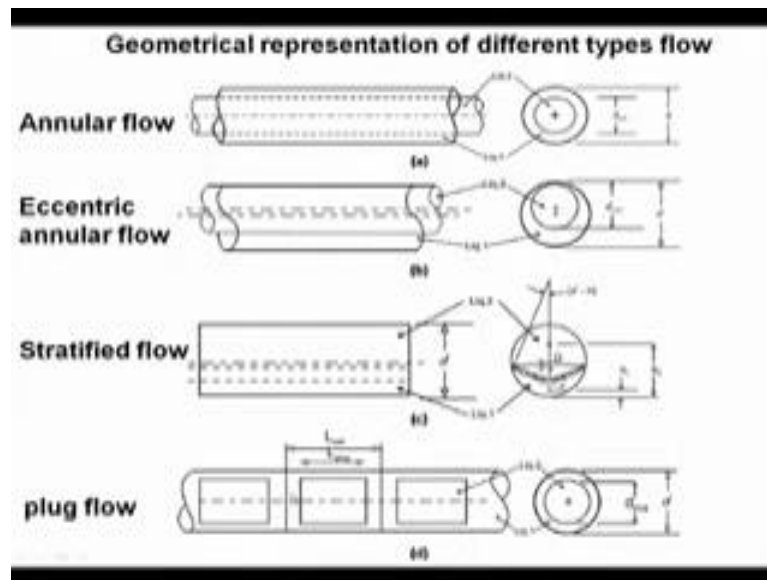
Adiabatic Two-Phase Flow and Flow Boiling in Microchannel
Prof. Gargi Das
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 08
Pattern Transition from Energy Minimization Principle

Well to continue with our discussions, what we were discussing; we were discussing the development of an analytical model developed from basic physics in order to predict the flow patterns in a liquid flow systems under reduced dimensions from the principle of minimization of total system energy; where the system energy was postulated to comprise of kinetic energy, potential energy and surface energy of the 2 phases and between the liquid solid interfaces. Now what we have assumed we have assumed that kinetic with these is this is particularly applicable for surface energy dominant phenomena.

Naturally kinetic energy induced instability like (Refer Time: 1:03) instability etcetera, they are not applicable in this particular case and the other thing which we had assumed was, quite common which we usually assume like fully developed steady state and smooth interface etcetera. And we are try to do it for suffice tension dominant situations, therefore we have confined our analysis for, low to moderate velocities of the 2 liquids and in this particular case, the analysis was validated to with the experiments performed in the multiphase flow laboratory, with water and toluene as the test rids. Now since we are trying to confine our attention on low to moderate velocities. Naturally this particular analysis has been developed for the plug flow pattern, where we have assumed that we have neglected.

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The nose region of the liquid plugs and we have assumed rectangular liquid plugs; in this particular case organic plugs. Since water is the wetting phase, organic plugs traversing through the continuous water media and there are water slugs in between the toluene slugs and there is also water fill in stabilizing the toluene slugs.

This is the idealize plug flow model that we have assumed and the geometric parameters which will be adopting for the analysis also given here, we have taken up a units cell; for each unit cell comprises of organic plug and one particular equals plug such that the 2 combines and then gives us a total length of a liquid slug and the diameter of the plug has been taken as d plug and it is flowing in a conduit of diameter d , on the other hand we find that for the higher velocity of the other phase; the organic phase we have annular flow pattern.

We have assumed both situations, we have developed the model for a concentric core as well as for an eccentric core. In the eccentric core we have assumed that, a very small very; low thickness of liquid fill is flowing on the top valve such that there almost touching, but yet there is going to be some particular interfacial shift between the organic and the water phase throughout. Now these 2 configurations why we have developed them, for very low dimensions as we have seen the flow is perfectly axis symmetric annular flow with the diameter of the core taken as d core and as have mention for all cases the diameter of the pipe is d and length of the pipe is l . Now as we keep on

increasing the dimension of the conduit we find that the tendency of eccentricity or the core appears to be eccentric phase.

We have developed models for or other energy patience for both of them such that, we can see that under fourth condition, the eccentric annular flow becomes more stable than the concentric annular flow or in other words eccentric annular has got lower energy content as compared to the annular flow pattern, but in both the cases we have assumed smooth interfaces between the 2 liquids.

Now, as we know that the conduit diameter mode, then there is the tendency of stratification; what happens and the that condition the organic phase it is starts touching the upper wall and then, we find that the entire conduit is divided into 2 areas where the upper portion occupied by the lighter phase, which in liquid liquid phases is usually the organic phase and the lower portion it is occupied by the equals phase, where this particular equals phase or other for our case it is usually the equals phase which occupies here. Now since we are working with small diameters systems.

Therefore, we have assumed that, the interface separating the stratified flow it will be a curve rather than a plane interface and we assumed that this particular curve it is a circular one, and the analysis for developing the equations for stratified flow have been taken from the analysis of browner at all and that can be found out from the reference of the people which have all ready referred. This particular case we find that, the height of the centre of gravities H_2 and the height of the centre of gravity of the water portion is H_1 and the that for the toluene portion is H_2 and we find that the entire geometry it is represented or it can be represented by 2 particular angles 50 and 5 star.

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Annular flow

- Negligible energy dissipation due to wall friction (valid for low viscous liquids)
- No wall liquid entrained in the core

$$\Delta PE_A = \frac{g\alpha d}{(u_2 + u_1)^2} \left(\frac{\rho_2}{\rho_1} - 1 \right)$$
$$\Delta KE_A = \frac{\rho_2 \beta^2}{\rho_1 \alpha} + \frac{(1 - \beta)^2}{1 - \alpha} - 1$$
$$\Delta SE_A = \frac{8\sigma_{12}}{d\rho_1(u_2 + u_1)^2} \sqrt{\alpha}$$

Kannan Aadithya, Ray Subhabrata, Das Gargi, Liquid-Liquid Flow Patterns in Reduced Dimension Based on Energy Minimization Approach, *AIChE*, (2016), 62(1), 287-294

Therefore, as I was telling the annular flow that are 2 things that we have assumed, but both these assumptions there were extremely valid, as we have observed from our experiments. The number one is negligible energy dissipation due to wall friction, which is valid definitely for low viscous liquids, again I would like to mention that this analysis is applicable for low viscous liquid liquid flows and the other thing is no amount of wall liquid is entrained in the core which is also a very reasonable assumption as for as your small diameter conduits are concerned; accordingly the expressions for the change in potential energy with respect to the reference substance, which is water flowing at the mixture velocity has been developed. Now this analysis it can very well be done by considering the geometry I am not going into the details of this analysis; similarly we have performed the analysis for eccentric core annular flow.

As is a evident that the kinetic energy and the surface energy terms are going to remain unchanged for eccentric core, the only thing which changes is going to the potential energy term why because thus a centre of gravity appears to shift in this particular case

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Eccentric core annular flow

$$PE_{EA} = \frac{g\alpha d}{(u_2 + u_1)^2} \left(\frac{\rho_2}{\rho_1} - 1 \right) (2 - \sqrt{\alpha})$$

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Accordingly the potential energy term it should be actual delta PE there is a typo here. This particular term this has been only developed and the surface energy terms, from the anti kinetic energy terms, from the concentric core has been adopted the three have been added up to give us the total energy term in this particular case

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Stratified flow
- curved (circular arc) liquid-liquid interface

$$PE_S = \frac{(1-\alpha)g\rho_1 h_1 + \alpha g\rho_2 h_2}{2} \frac{dz}{(u_2 + u_1)^2}$$

$$KE_S = \frac{\rho_2 \beta^2}{\rho_1 \alpha} + \frac{(1-\beta)^2}{1-\alpha} - 1$$

$$SE_S = \frac{(\pi - \phi_2)\sigma_2 + \phi_2\sigma_1 + \sigma_{12}(\pi - \phi^*) \frac{\sin(\phi_2)}{\sin(\phi^*)} - \sigma_1}{\frac{d\rho_1(u_2 + u_1)^2}{8}}$$

Similarly for the stratified flow case also, as you can see we have developed the potential energy, kinetic energy and the surface energy terms in this particular case. We find that

this surface energy their governed by the 2 angles as have mention 50 and 5 star and they define alpha and the other geometric parameters.

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Slug Flow
- Axisymmetric cylindrical plugs

$$\Delta PE_p = \frac{g\alpha d}{(u_2 + u_1)^2} \times \left(\frac{\rho_2}{\rho_1} - 1 \right)$$

$$\Delta KE_p = \frac{\rho_2 \beta^2}{\rho_1 \alpha} + \frac{(1 - \beta)^2}{1 - \alpha} - 1$$

$$\Delta SE_p = \frac{4\sigma_{12}}{d^2 \rho_1 (u_2 + u_1)^2 x_2} \times (d_{plug}^2 + 2d_{plug} L_{plug})$$

$$\frac{L_{plug}}{d} = (1.3 + 0.15u_2 + 0.06u_1)(10d)^{0.44}$$

Well we have also done it for slug flow as have mentioned axis symmetric cylindrical plugs and this particular case we find that there is one more unknown which is the length of the plug. This is the only parameter which has been taken from a correlation which has been developed from experimental data and we have used not only our experimental data, but also data from literature to develop this particular correlation, which gives us the plug length with respect to the diameter of the conduit, in terms of the velocities of the 2 fluids as well as the diameter of the conduit for the diameter is in millimeters.

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Non-dimensional expressions for total energy

$$\Delta TE_a = \frac{1}{Fr_m} \left(\frac{\rho_2}{\rho_1} - 1 \right) \alpha + \frac{\rho_2 \beta^2}{\rho_1 \alpha} + \frac{(1-\beta)^2}{1-\alpha} - 1 + \frac{8\sqrt{\alpha}}{We_m} \left\{ 1 - \alpha \left(1 - \frac{\rho_2}{\rho_1} \right) \right\}$$

$$\Delta TE_{ea} = \frac{1}{Fr_m} \left(\frac{\rho_2}{\rho_1} - 1 \right) \alpha (2 - \sqrt{\alpha}) + \frac{\rho_2 \beta^2}{\rho_1 \alpha} + \frac{(1-\beta)^2}{1-\alpha} - 1 + \frac{8\sqrt{\alpha}}{We_m} \left\{ 1 - \alpha \left(1 - \frac{\rho_2}{\rho_1} \right) \right\}$$

$$\Delta TE_s = \left(\frac{2(1-\alpha)h}{d} \right) \frac{1}{Fr_m} + \left(\frac{2ah}{d} \right) \frac{1}{Fr_m} \frac{\rho_2}{\rho_1} - \frac{1}{Fr_m} + \frac{\rho_2 \beta^2}{\rho_1 \alpha} + \frac{(1-\beta)^2}{1-\alpha} - 1 + \frac{8(\pi-\phi)}{Bo_1 Fr_m} \frac{\rho_2}{\rho_1} + \frac{8\phi}{Bo_2 Fr_m}$$

$$+ \frac{8}{We_m} \left(1 - \frac{\rho_2}{\rho_1} \right) \frac{(\pi-\phi) \sin(\phi)}{\pi \sin(\phi)} - \frac{8}{We_m}$$

$$\Delta TE_p = \frac{1}{Fr_m} \left(\frac{\rho_2}{\rho_1} - 1 \right) \alpha + \frac{\rho_2 \beta^2}{\rho_1 \alpha} + \frac{(1-\beta)^2}{1-\alpha} - 1 + \frac{4}{We_m} \left(\frac{\alpha d}{L_{plug}} + \frac{\alpha d}{d_{plug}} \right)$$

Now, what we have done, we have non dimensionless the total energy and we have expressed it in terms of non dimensional groups. If you now see observe the non dimensional groups, what do we find; we find that the same dimensional less numbers are appearing in the total energy expressions, of annular, eccentric annular, stratified and the plug flow pattern. What are the groups that are appearing, at first appearance you see it is the front number and the webber number of the mixture, apart from this, other important groups and naturally.

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15/11/20

β
 α

$\frac{L_{plug}}{d} = 10 - 20$

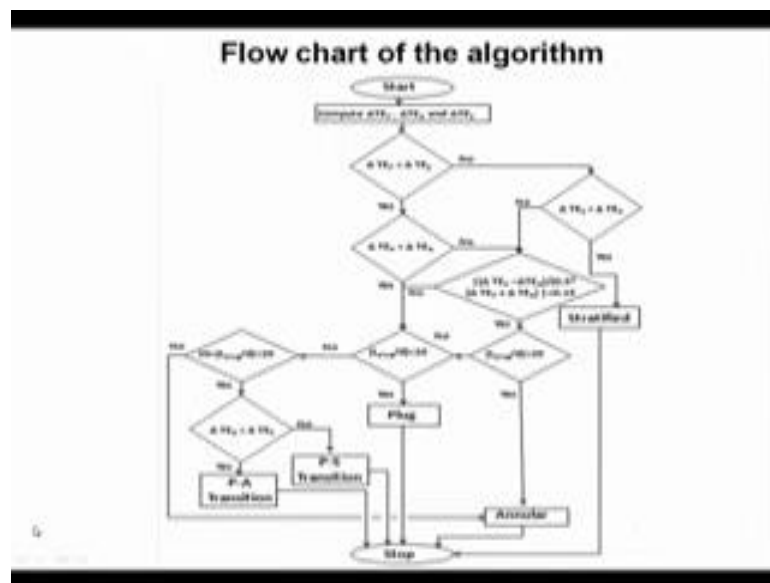
Onset of asymmetry \Rightarrow
Comparable total energies of
EA & CA flows
- $Bo \approx 0.1$

Onset of stratification - $Bo \approx 10$

Range of minichannels $0.1 \leq Bo \leq 10$

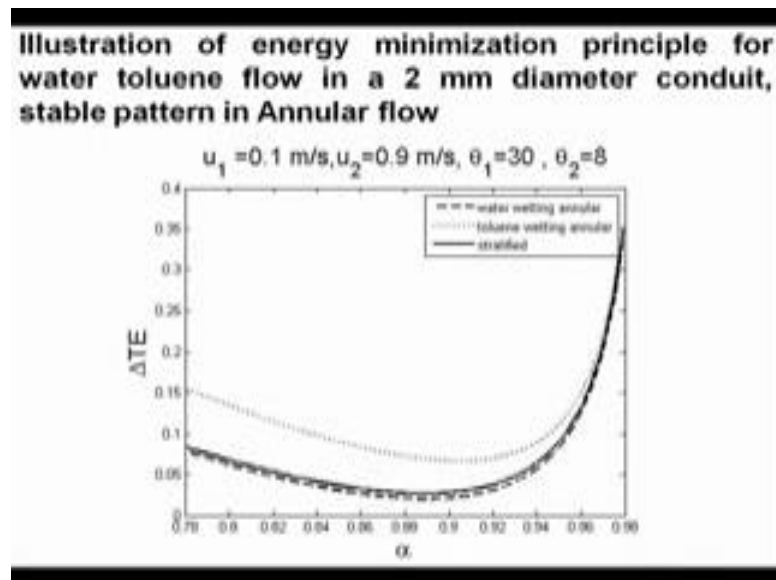
In let volumetric fraction of the 2 phases, incitu fraction of the 2 phases that's it; I do not find anything else, only in the stratified flow expression; we have one number of phase 1 and phase 2 which is quite expected. Therefore, what we find that the same dimensional less numbers are appearing in all of them. what we do we first plot, rather we first develop this particular expressions then for a given alpha for given flow conditions in a particular pipe diameter, for 2 particular fluid pears so that we know all the physical properties of the fluid, we know the for any particular alpha we can compute the total energies for different distributions.

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The flow chart is as shown you can follow the flow chart and use it for predicting the serial flow pattern in this particular case.

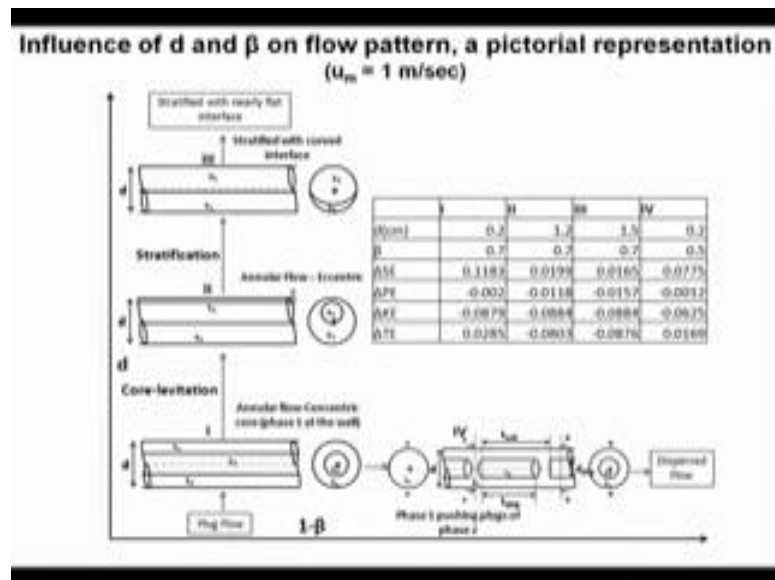
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And after developing the flow chart what we do, we have plotted the total energy term as a function of alpha and here what do we find very interestingly you will find that for higher alpha values, we find that water wetting annular flow is applicable. Since in this particular case we have just plotted from 0.782, 0.98 we find that, the water wetting annular flow is the has the minimum amount of energy and interestingly we find that the difference between stratified and water wetting annular flow it decreases as we shift to lower alphas, we have not plotted beyond it.

I will be showing it in a another particular slide and I would also like to mention that, we have developed this rather discuss this model assuming the tube to be hydrophilic or in other words we are since we are going to validate it for toluene water flow in a glass capillary, so therefore it is hydrophilic tube, and you can very well see for this particular situation we have got the water wetting annular flow as a more stable and you can very well see that the toluene wetting annular flow is much unstable. We will see a reverse case later on in one of the slides where we will work with the hydrophobic tube.

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Therefore, once the model was developed the algorithm was developed we could use the model and then after that we wanted to extract some more extra information from this. For that, what we did we try to plot the influence of diameter and the water fraction, because we found out from these particular expressions if you observe, we found the diameter is the most important parameter for defining the total energies, so that explains why; when previous researches had used hydraulic diameter to classify the channels more or less accurate results were obtained.

After developing the energy equations, now let me tell you certain things. What were the postulations that we used to decide the stable flow pattern? Firstly, the energy expressions were derived; The pattern which had the lowest energy, for example, in this particular case it was a water wetting annular flow pattern was considered as the stable flow pattern provided it had energy substantially less than the other flow patterns, if the energy defects between 2 patterns was less than 7.5 percent we assumed that, either of the flow pattern can be exist. Naturally it must be a transition.

The other thing which we had considered was for the plug flow patterns it is quite evident, that the plugs cannot become indefinitely long and they cannot occupy the entire channel. Therefore, there must be some particular limiting length for the plug flow pattern to exist. Based on our experiments and data from literature, we have postulated that l plug by d it should be it should not exceed 10 to 20. So, therefore, this was the only

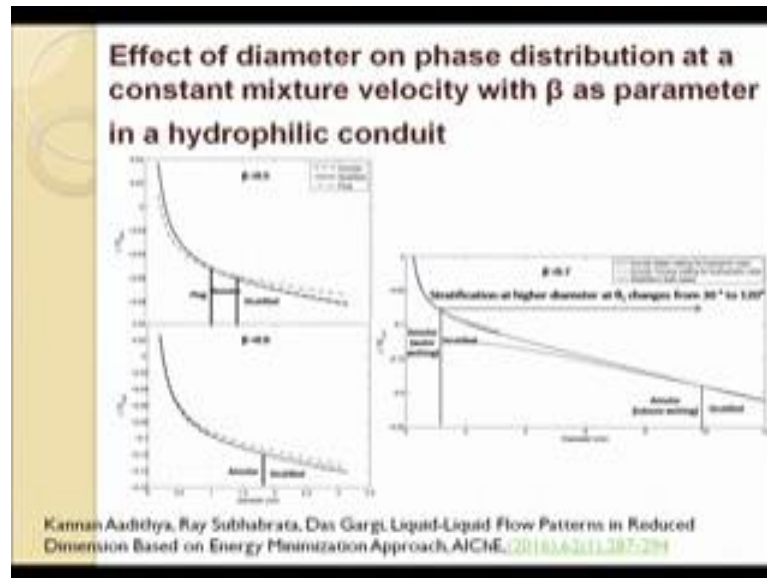
thing which has assumed we have also referred to this also. Therefore, the length of the plug should not exceed 20 times the diameter.

Based on these 3 postulations, Firstly; the minimum energy. Secondly, the energy difference between the most stable pattern and the next higher stable pattern should not be or rather should be greater than 7.5 percent; otherwise it lies in a transition and if it is plug flow, then a check on the plug length. Based on this we had developed the stable flow pattern or other we have developed a flow pattern map, which is then the paper I have not shown it here because I wanted to show some other interesting aspects, which have come out from the model for what do we see.

We find out that, for low diameters definitely and low one minus beta, plug flow pattern is more stable; is quite obvious we find that when the water fraction is higher and that the same time your diameter is lower than on the that case we get the plug flow pattern. And as we decrease the water fraction we gradually shift to the annular flow pattern and again your plug flow pattern here we had plug flow pattern here and from the plug flow pattern we find that as we decrease the water fraction you get annular flow; as we increase the diameter we get annular flow. Therefore, there is a coupled effect of decreasing beta and increasing diameter to attain the annular flow pattern. Now if we keep on increasing diameter, keeping the water fraction same; we find that gradually, the concentric core it becomes eccentric and there again if you increase the diameter further t it becomes stratified, and as we increase the diameter; the interface it shifts from a curved interface to a flat interface. Therefore, this particular graph gives you an idea of the influence of diameter and the volume fraction of the 2 phases in a particular fashion, just to understand how diameter and beta they complement each other in deciding the flow pattern.

We have also noted down the surface energy, potential energy kinetic energy and also the total energy of the 4 situations which have been depicted here just to highlight the dominance of surface energy in deciding the plug flow pattern and even if the surface energy is not so very high for larger by dimensions.

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After this the diameter effects and diameter is so very importantly found. So, therefore, the diameter effect has further been plotted or other the total energy change of the 2 phase system has been plotted as a function of diameter for, 2 different in let toluene fractions; one is 0.5 other is 0.9, if you are deserve the curves minutely what do we observe we find that for beta equals to 0.5 it is very evident that, for low diameter situations. What is stable plug flow pattern is stable, then for a very low diameter range we find that the stratified flow beyond the particular diameter we find that this its stratified flow which is stable and I think here the annular flow is stable for some particular small diameter range the annular flow is stable, and beyond that, we find that the stratified flow is stable, but in this particular case initially we find that is the stratified flow has comparable total energy with the eccentric core annular flow; this is what is expected.

Therefore, we find that for low to moderate water fractions and for small diameters we get only plug flow pattern and beyond the particular critical diameter which in our particular case in our particular case of water to (Refer Time: 19:39) we find is about 1.5 centimeters, we find that stratification axis for both beta equals to 0.5 as well as for beta equals to 0.9. So, therefore, what do we find; we find that when we are operating at moderate water fractions, initially the plug flow pattern is stable, then gradually annular on stratified they compete with each other and finally, the stratified flow pattern becomes more stable.

Just keeping everything constant, the mixture velocity etcetera we just shift from rather we go to a higher toluene fraction. Very interesting what do you find in this particular case, in this particular case we find that; for low diameters in this case plug is not stable, but the annular flow pattern is stable; annular flow pattern continuous to be stable, till the critical diameter which is around 1.75 centimeters in this case, after that it is the stratified flow which become stable.

Therefore, from here what do we understand, we it is very evident that the stratified flow exists for some particular critical diameter; for this particular case of water toluene flow, we have observed the critical diameter to be 15 centimeters quite evidently the diameter will be a function of phase physical properties. It is a weak function of the inn let volumetric fraction and below critical diameter we find that, the plug and annular they compete to be the energy more energetically more privileged flow pattern for a low to moderate water fractions, the plug flow pattern is more stable and for higher water fractions the annular flow pattern is much more stable.

And the other interesting thing which we have observed, we have done it for 3 betas and in this case we have plotted it for a hydrophilic tube or other we have considered class when the contact angle is 30 degrees and we have also done it for a hydrophobic tube where the class was coated with xylene and the contact angle with water was observed to be 120 degrees, in this particular case we find that, when it became hydrophobic the tendency of forming a stable water film decreased and therefore, in this particular case if you observe properly we find that the annular flow with toluene wetting that is the more stable as compared to annular flow with water wetting case. And interestingly we find that in this particular case since water has a very less tendency to wet the pipe wall, the onset of stratification is shifted to a much higher diameter.

Therefore, these were the interesting things that we could observe from this particular diameter rather from this particular analysis; another important thing is from here what. What are the things that we could get, we could predict the stable flow pattern that was first next we could use the analysis to predict the stable flow pattern both for hydrophobic as well as for hydrophilic cases. We could also predict the zone of transition, and very interestingly from this particular case we could also predict the domain of micro channel and the domain of macro system. For example, for our case we

find that for hydrophilic case we found that the onset of stratification, started beyond centimeters sorry it started beyond 1.5 centimeters.

Therefore, in this particular case what we postulated was that, definitely the transition from micro to mini should occur; when the distribution shifts from symmetric to asymmetric distribution. Therefore, we assumed that for micro system concentric core annular should have the minimum energy. As the energy of concentric and eccentric core started becoming comparable or the difference was less than 10 percent and above when the eccentric core became more stable as compared to concentric we assumed that it is the onset of mini channels. Therefore, the transition from micro to mini was postulated as occurring when eccentric core became more stable as compared to concentric core and the transition from mini to large conventional channels was postulated when stratification came into picture.

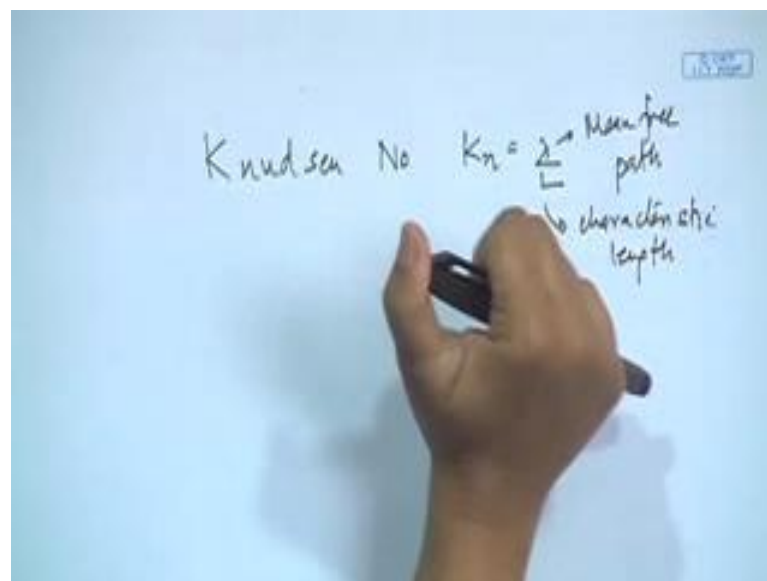
Therefore, based on this we found out the critical diameters and since it is a common convention as we have seen in the last lecture that usually the domains are expressed in terms of bond number. Accordingly we also expressed in terms of bond number and we found out that, the onset of asymmetry; which is marked by comparable energies, comparable total energies of eccentric annular and concentric annular flows, this occurs for a bond number close to 0.1 and the onset of stratification that occurs for a bond number close to 10.

Accordingly what we postulated was that the range or the domain of mini channels that occurs between rather that can that can occur where I would like to mention that this is the I should write this is the bond number of the 2 phase mixture. So, therefore, we find that the range of mini channels is 0.1 so this is the range of mini channels that we that we obtained in terms of the 2 phase bond number and it will be interesting to note that if you compare this particular value with the table that I had shown yesterday you will find that, all the transition values which were given by the past researches they lie within this particular domain. Therefore, this model could predict a large number of things and I think we rather we have not tried it, but this should be quite applicable for your gas liquid systems slightly intent flow patterns and also it can be used for other flow patterns as well.

Now before end this particular session I would also like to mention just a basic difference between 2 phase flow and single phase flow. In 2 phase flow we found that it was very it that the physics of flow was very different in mini or micro channels as compared to your macro systems or conventional systems therefore, it was essential to define the range or the domain of micro systems and mini systems in single phase flow if you find we find that suppose it is just water flowing through a micro system usually we it is noted that the equations for laminar and tabulator applicable for that case as well and the critical Reynolds numbers is also closed to 2100 2000 in this particular case.

Therefore, the equations which have been developed for conventional systems can be applied for your micro systems as well. On the contrary for single phase gas flow we find that, when we go for micro systems, we find that the no slip condition is no longer applicable and there is the slip along with that; there is a temperature jump also and therefore, the range of micro channel needs to be defined for only gas flow and this particular range it is defined in terms of Knudsen number.

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Which is defined as Kn and it is equal to λ by L , where λ is the mean free path and L is the characteristic length of the conduit. So, we find that based on this value of Knudsen number, the micro effects in gas flows appear.

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Knudsen Number and it's Significance

Knudsen number (K_n) is a dimensionless number defined as the ratio of the molecular mean free path length to a characteristic dimension (can be the dia of the tube)

$$K_n = \frac{\lambda}{L}$$

where, λ = Mean free path
 L = Characteristic dimension

$K_n < 10^{-3}$	<ul style="list-style-type: none">> Continuum flow> Modelled by Navier Stokes eqn> No slip
$10^{-3} < K_n < 10^{-1}$	<ul style="list-style-type: none">> Slip flow> Navier Stokes eqn applicable, provided velocity slip and temperature jump accounted
$10^{-1} < K_n < 10$	<ul style="list-style-type: none">> Transition flow> Navier Stokes eqn no longer valid> Intermolecular collision not yet negligible
$K_n > 10$	<ul style="list-style-type: none">> Free molecular flow> Intermolecular collision negligible compared to the collisions between gas molecules and wall

And as a result the normal equations for macro systems cannot be used, for example; we find that for Knudsen number less than 10 to the power minus 3, we find that the continuum flow is applicable. It can be modeled by Navier stokes equation and no slip no slip condition is also applicable, but as we go for higher and higher Knudsen number, we find that the flow the no slip conditions is no longer applicable and there is a velocity slip between the wall as well as the gas, and the Navier stokes equation all though it is applicable, but we have to incorporate velocity slip and temperature jump there.

Then again if we go for still higher Knudsen number, we find that then Navier stokes equation is no longer applicable because under that condition, wall fluid particle interaction becomes much more important as compared to intermolecular collision and then for still higher Reynolds number we find that, it becomes a free molecular flow and the single phase equations are no longer applicable. Therefore, this was just to give an idea about how this phase flow is much more different than single phase flows and we should keep it in mind 5 modeling on dealing with any particular situation of 2 phase flow.

Thank you very much