Heat Transfer
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Lecture - 16
Fundamentals of Convection

We are going to start with another mode of Heat Transfer namely convection, which is almost omnipresent when where is a contact between a solid and a fluid and the solid temperature and fluid temperature are different. Now, in addition we may have the fluid, have a velocity with which it flows over the solid. So, in that case the heat transfer, the major heat transfer the maximum amount of heat transfer is going to take place through convection. So, convection requires the presence of a medium and in most of the cases there would be an imposed flow of the fluid over the solid.

So, convection is characterized by the presence of a moving fluid stream, in contact with a solid. In some special cases, there maybe this motion of the fluid can be induced by a density difference by a, by a bind force without the presence of an external agency that drives the fluid over the solid surface, which is known as natural convection or free convection. So, an object which is or a hot object which is placed in, let us say water it is going to lose its heat by convection and unless there is a flow imposed flow of the, of the of the water stream, water. Then it is going to be natural or free convection, where the liquid near the solid, it is temperature will increase, it is density will decrease and due to the bind force that hot liquid will rise up along the solid and to be replaced by cold water from the surrounding.

So, a natural current would therefore, set in which is known as natural convection. So, we will treat natural convection separately towards the end of the, end of our discussion on convective heat transfer. But right now we are going to mostly concentrate on situations in which there is an imposed motion of the fluid in contact with the solid, the applications of convective heat transfer or the occurrence of convective heat transfer is everywhere in industrial processes. So, wherever you have to heat up a fluid which is entering a reactor or you would like to cool a stream of liquid before you discharge it to somewhere, it requires the exchange of heat in between two fluid streams. It is also very important to ensure that the plant the chemical plant is going to operate at its highest efficiency, if you can regenerate some of the heat which would otherwise be lost.

So, convection plays an important role in how you can effectively design and equipment, which is going to have the maximum amount of convective heat transfer between a solid and the fluid or between two fluids streams separated by a solid barrier. These considerations play an integral role in evaluating, in determining the efficiency of the process. So, the heat exchangers that we would again consider towards, on in this course the heat exchangers, the design operating principle of heat exchangers they rely heavily on convective heat transfer.

So, it is important that we understand and learn convection, but in order to have a formal mechanism to study convective heat transfer one has to start with the simplest possible case. Where, we have a solid plate which is at a higher temperature, which is at a high temperature, in relative to the fluid which, with which it is in contact. And as I said we are going to concentrate mostly on forced convective heat transfer; that means, the fluid which is in contact with the fluid with the solid is moving with a certain velocity.

The whenever a fluid comes in contact with a solid which is stationary, then there is going to be a change in the hydrodynamic pattern of the fluid which is flowing over heat. So, will discuss that and the concept of boundary layers would be relevant in describing the convective heat transfer, that is expected out of the solid plate, out of the flat solid plate. So, we have chosen flat solid plate because it is the simplest possible geometry that you can think of. Any change any curvature in the solid with which the liquid is in contact will give rise to additional complexity, which is slightly more complex. So, starting point would be we would like to see how the heat transfer from a solid plate would take place when it is in contact with water. Let say of the fluid is water, which is very common which we see almost every day.

So, what is going to happen to it? So, which start with the fundamental concepts of boundary layer, some of you are most of you already are aware of the concept of boundary layer, when we when we considered the momentum transfer part of it.

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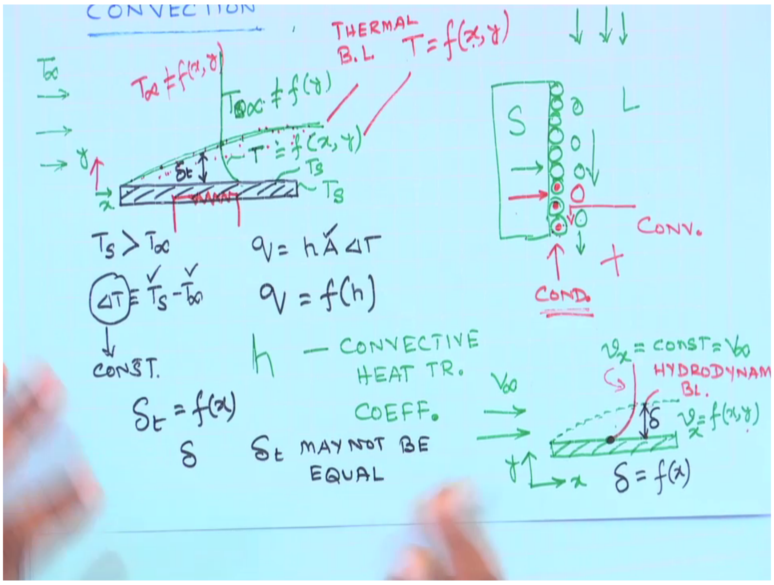

So, let us say this is a solid, which we have, I have said it is a the temperature of T s and this stream of fluid is approaching the solid with a temperature of T infinity and let us assume that T s is greater than T infinity. So, the fluid is going to come in contact with the solid, extract heat by the convection process from the solid and as a result of which the temperature of the solid, temperature of the liquid in the vicinity of the solid will rise.

So, this increase in the temperature of the fluid very close to the solid plate is something that we need to model in order to get an idea of what is the heat transfer coefficient. It was the principle equation, the relation that we are going to follow, that we would we are going to use extensively in describing convective heat transfer is Newton's law of cooling. With simple tenses at the amount of heat, which is lost from the solid in this case, is can be written as q equals h A delta T. Where h is the convective heat transfer coefficient, A is the area in contact and delta T is the temperature difference between the solid and that of the that of the liquid, which is at a point far from the solid.

So, when I look at this one this the amount of heat lost from the solid would simply be equal to h times A delta T, where delta T is defined as T S minus T infinity and if there is an mechanism, if. Let us say there is a heater which maintains, which maintains the temperature of this, of the solid at constant at T S then T S is a constant the temperature with which the liquid is coming in contact or approaching the solid that is also a constant. So, delta T is simply going to be a constant and if delta T is a constant the area which is in contact with the fluid that is also a constant. So, essentially the heat q that is there is lost from the solid is going to be a function of h.

So, how can we manipulate h, the convective heat transfer coefficient heat transfer coefficient that is or how do we can evaluate the convective heat transfer coefficient that becomes the study of convective heat transfer. So, how do I relate h with other parameters, now what are the parameters on which h would depend on. So, if you think heuristically, let us see the fluid if it is moving at a higher velocity, all of us realize that when you are outside and a cold wind blows on a winter day, you feel more cold as compared to the to the to the case where the air velocity has significantly reduced.

So, the temperature in both cases will remain the same; however, the velocity will be able to extract, because of the velocity the cold air would be able to extract more energy from your body and their by you would feel you would feel cooler. So, h the convective heat transfer efficient is going to be a strong function of velocity, convective heat transfer coefficient would also depend on the thermo physical properties of the fluid which is flowing over the solid.

So, what are the thermo physical properties on which it would depend on, since we talk about flow, the two properties which come to our mind automatically, one is what is its viscosity, that is an important that that will play an important role in what is going to be its density. So, rho and mu would come into any expression of h, specially when we concentrate only on the momentum transfer part of it; that means, only fluid flow part of it.

Now, this heat is being taken up by the fluid by as sensible heat. So, the temperature of the fluid in contact with the solid will start to rise and whenever you are going to have an increase in temperature of the fluid the capacity that heat, the thermal capacity of the fluid must be taken into account. And the one of the parameters, one of the properties which define the thermal capacity of the fluid in terms of extracting heat from the solid has to be c p the specific heat.

So, c p is going to play a major role and how is heat going to transfer between the solid and the liquid. So, the molecules of the fluid which are flowing over the solid, when they come in contact with the solid due to the no slip condition. Which I am sure you are aware of from your fluid mechanic study; that means, the molecules of the fluid, molecules of the moving fluid, which are in contact with the solid they do not move so, they become static. So, there would be a decrease in velocity as we approach the solid and on the solid the velocity of the fluid would be equal to 0, which is the no slip condition.

So, at the interface, at the solid liquid interface there would be solid in one side and static molecules of liquid on the other side. So, if I draw it, if this is my solid part of it, if this is the solid and this is the liquid. Then even if the liquid is flowing some the molecules of the liquid, which are in contact of with the solid due to no slip condition, they have a 0 velocity. Now, when they have a 0 velocity, so the heat is going to get transferred from the solid to this static liquid molecules by conduction, because, if you remember conduction does not require the conduction is prevalent when there is no motion of the molecules in contact, in contact.

So, the mechanism by which heat gets transferred from the solid to the liquid molecules, solid to the static liquid molecules is by conduction, but the, but the molecules out of this, molecules beyond this static layer they have a velocity. As they have a velocity now, the heat transfer between two liquid molecules one which is static and one which is moving the heat transfer here is going to be by convection. So, conduction and convection both will exist in order to have convective heat transfer from the solid to the liquid.

So, we they we realize we understand that an important part here that the, you can never have convection without conduction. So, you have to have conduction through the layer, through the static layer of the fluid molecules which are clinging to the side of the solid and thereby having gaining heat by from the solid by conduction. In on the other hand on the other side they are exposed to mobile liquid molecules and these, the interaction between the static liquid and the mobile liquid the heat transfer is going to be by convection. So, when we say that a convection from the solid takes place, we need to understand that the convection is going to be preceded by conduction so the layer of the immobile molecules. So, we can never have convection without conduction.

So, let us now concentrate on what is going to happen to the liquid or to the fluid which is coming in contract with the solid. So, the molecules which are over here, there going to get heated and, but the molecule at this point they do not know that a hot plate exists.

So, as I move to some move distance, the molecules at a slightly higher depth if this I call as my depth, at a slightly higher depth would know that there is a hot plate which exists. And the extent of the influence of the solid plate would propagate more and more into the liquid as I move in the direction, in the x direction. So, this is my x direction and this is my y direction. So, for small values of x, the effect of the plate in terms of a change in temperature is going to be limited to a point very close to the surface, as I move further with x we depth the penetration of the temperature so on, will be more and more into the liquid.

So, if I approximately join them together, I am going to get a layer which more or less demarcates the range where you would expect a change in temperature. So, the temperature over here is simply going to be T S, the temperature sorry t infinity the temperature over here is T S. So, in between this point and this point there is going to be shark change in temperature from T infinity to T S and if you go beyond this, beyond this point the temperature everywhere is going to be equal to T infinity. So, the temperature profile probably would look something like this, vertical because T infinity is not a function of y for the region beyond this, this, this, this imaginary layer and here T the temperature is going to be a function of both x and y. So, inside this layer, I would write it clearly again T is a function both of x and y.

So, further you are in terms of x, the temperature is going to be more further you go away from the solid the temperature will reduce. So, therefore, t is a function both of x and y, but in here t infinity is not a function of either x or y. So, this line imaginary, line which demarcates the change in temperature field and a constant temperature field due to the effect of convective heat transfer from a solid to a flowing fluid is known as the boundary layer, thermal boundary layer. You are aware of what is a hydrodynamic boundary layer, which is defined in the same way so if I, just to recap some of these fluid mechanics part of it which is going to be required for us study of convective heat transfer. If this is my plate, where I have flow, which lets say it is the velocity is V infinity then there would be an imaginary layer like this there, would be a layer like this in which the velocity this is my x and this is y.

So, in here the velocity is going to be a function of both x and y, out here the velocity this is a x component of velocity v x is going to be a constant and for a flat plate this should be equal to V infinity. So, for a so this layer which is in here, the velocity varies from 0 due to no slip condition at the solid it increases asymptotically and then becomes a constant. So, this is a v x over here which is a constant, but in here the velocity is a function of both x and y. So, all the effects of convective or flow is, is limited is confined within this layer and most of this layers are, if this layers which are called boundary layers, why they called boundary layers because they demarcate between two different types of flow.

When we considered the flow over a flat plate, whose temperature is equal to the temperature of the fluid, no heat transfer is taking place, but the only thing that is happening is that due to no slip condition, on the solid the velocity is 0. And as you move away from the solid the effect of the solid will be felt lesser and lesser as you move away from the solid and after a certain point, the velocity or the moving fluid will not realize that there exists a stagnant solid plate below it.

So, the layer up to which the effect of the solid is filled by the moving fluid is known as the hydrodynamic boundary layer and of course, the motion of the fluid molecules sleeping past one another near the solid plate, the property of relevance is; obviously, viscosity. So, the viscosity is the one which transfers the presence, the effect of the presence of the plate into greater depths of the fluid. So, viscosity in rho they play a very important role in defining what is the hydrodynamic boundary layer thickness, which are going to, which are generally very small of the order of a few milli meters for normal sized plates.

So, within this few milli meters, the effect of viscosity is important, outside of this boundary layer the viscous effects are unimportant and the entire flow can be treated as inviscid flow. A flow we where in which the viscosity can be assumed to be equal to 0 because there is no momentum transfer in a direction perpendicular to the flow and therefore, it is inviscid in nature. However, inside the thin layer close to the solid surface the effect of viscosity cannot be neglected.

So, we have viscous flow inside the boundary layer, hydrodynamic boundary layer and in inviscid flow outside of the boundary layer. The, from your fluid mechanics you probably also remember that in inviscid flow can be express, can be explain by Euler’s equation. In Euler’s equation is the one which is the simplified form of Navier stokes equation, which Euler’s equation is for inviscid fluid. Where viscosity can be set equal to

0 in Navier stokes equation, in order to obtain the Euler's equation and we get Bernoulli's equation starting from Euler's question, but that is a separate story.

So, inside the boundary, inside the hydrodynamic boundary layer the flow is viscous. So, the viscous transport of momentum as well as the convictive transport of momentum, both will have to taken, you will have to be taken into account. Similar to hydrodynamic boundary layer what we are discussing right now is thermal boundary layer. So, thermal boundary layer is the region in which the temperature changes, temperature varies with y from a value equal to T S, all the way to the constant value of T infinity.

Here I have the velocity changing from 0 to V infinity, here the temperature changes from T S which is the temperature of the substrate to infinity, beyond this point the velocity is constant, beyond this point the temperature is constant. So, there is a similarity between the thermal boundary layer, boundary layer and this is known as the hydrodynamic boundary layer. So, in most in most of the cases, the thickness of the hydrodynamic boundary layer which is generally denoted by delta and the thermal boundary layer which is denoted by delta T they are not equal.

So, in one hand you have delta T and over here you have delta. So, this delta the thickness of the boundary layer; obviously, as you can see from this figure is a function of x, here delta T is also a function of x; however, delta and delta T may not be equal.

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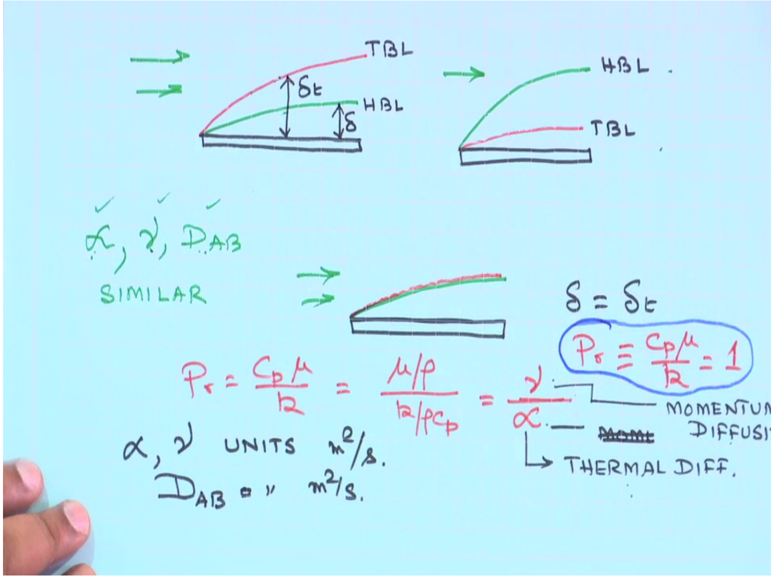

So, we make it situation in which, if this is the solid plate; you have the thermal boundary layer and you have the hydrodynamic boundary layer. Or in you may have situations in which the thermal boundary layer would be below and the hydrodynamic boundary layer would be above.

So, let us call this is hydrodynamic boundary layer, in this is the thermal boundary layer so here it is going to be TBL and HBL on for some very special cases you discuss them in detail later on these two would coincide. So, when these two coincide here I have for hydrodynamic boundary layer, this is delta thermal boundary layer thickness at any location is given by is denoted by delta t. So, in this case delta would be equal to delta t. So, when we would see that this special condition only appears when Prandtl number which is defined as C p mu by k to be equal to 1 mu k.

Now, why would that happened, let us let us expand this a little bit little bit more, Prandtl number is C p mu by k. I can write it as dividing both the numerator and the denominator by the rho, the density I can write it as k by rho C p and if you remember your fluid mechanics this mu by rho is known as the kinematic viscosity and this k by rho c p which you have just seen is denoted by alpha. So, this is the, known as the momentum sorry this is known as the momentum diffusivity and this alpha is known as the thermal diffusivity. Both have units both alpha and nu have units of meter square per second, which is the same as the diffusion coefficient that you probably have heard of which is the diffusion coefficient of A and B this also have units of meter square per second.

So, another important observation which would not probably with related to heat transfer, but you are going to come across this a many times is that conceptually alpha nu and D AB are all similar all will have units of meter square per second. In one case this refers to momentum transfer, this refers to heat transfer and this refers to mass transfer. So, conceptually there is not much difference between these three transport processes, heat, mass and momentum transfer and they in there at some point of time would be the base on which the unified treatment of heat mass and momentum transfer can be undertaken. That you are going to study in a separate course which is transport phenomena that looks at the fundamentals of all these transport processes.

But coming back to convective heat transfer or coming back to the nature, to the growth of these boundary layers, what we see is that the value of Prandtl number which is mu by rho they simply tell us about momentum diffusivity and thermal diffusivity. So, the growth of these layers their strongly dependent the TBL, the Thermal Boundary Layer would strongly depend on the thermal diffusivity. How fast, how easily the temperature front is getting into the moving fluid and the for the case of hydrodynamic boundary layer its alpha which is defining, how the thermal how the hydrodynamic boundary layer is growing.

So, when numerically these two are equal both the HBL and the TBL would grow together, would grow at the same rate therefore, the value of Prandtl number equal to unity which appears only when momentum diffusivity and thermal diffusivity are equal. This is a special case which would let us, which would let us take the value of the thickness of these two layers to be identical. So, this is the background which we are going to, which we are going to utilize in the in deriving some of the equations of convective heat transfer.

So, we understand here that conduction and convection both exist for the case of convection, but conduction can be a standalone process, which do not required the presence of convection. Conduction mostly, conduction when it happens inside a solid that is no question of any movement of the molecules, no net movement of the molecules and therefore, conduction is specified by 0 velocity or no velocity. Whereas, convection you have to have a velocity which could be imposed, that is the that occurs in most of the industrial processes or it could be without the imposition of a velocity it is there because of the presence of a velocity.

Because, in the presence of a temperature gradient which induces a difference in buoyancy a change in the value of the density are so therefore, have buoyant force would make the liquid, make the fluid in make the fluid in contact with the solid, hot solid rise and that is what is known as a natural convection.

Before we take this, slightly further and do of mathematical treatment of convection. Next another concept which I would like to introduce, a new I will follow it up with the in the next class with further details. In many of these cases the equations can be obtained, the governing equations can be obtained if you assume small control volume through the faces of which heat mass and momentum can enter the control volume. So, I can assume in a, in a free space a cuboid of side, size del x, del y and del z. So, through so; obviously, this cuboid will have 6 faces and truth is 6 faces the, let us say the mass is allowed to come and heat is allowed to come as well. So, I am going to first write or draw this cuboid and try to identify mentally, what are the process, what are the ways by which, let us see heat can enter into this control volume.

So, I am going to write the physics of flow into the control volume and the associated energy which comes into the control volume. So, this difference equation when I write and when I divide all sides by del x, del y, del z what I do is I convert the difference equation, which is a statement of the physics of the situation to a differential equation. In this differential equation can then be integrated with appropriate boundary conditions to obtain either the velocity profile or the temperature profile.

So, this kind of approach where are small cuboid is assumed in the flow space, the difference equation written convert it to differential equation and then solved is known as the shell momentum balance or shell heat balance or in the case of mass transfer it is known as the shell species balance. So, the first thing that one should do in order to derive all these equations or all these concepts is define a shell. So, let us try to define a shell and identify through the faces what is going to come in terms of energy into the space, into the space that I have defined over here.

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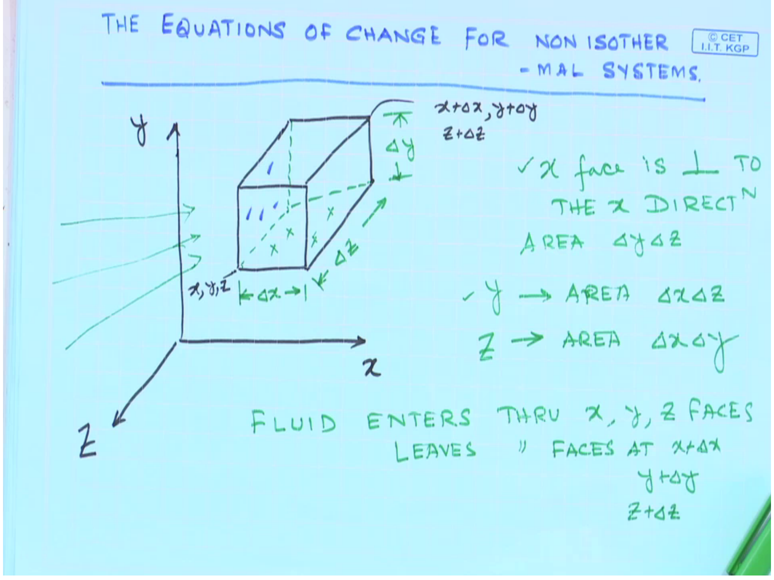

THE EQUATIONS OF CHANGE FOR NON ISOTHER-MAL SYSTEMS.

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✓ x face is $\perp$ TO THE x DIRECT^N AREA $\Delta y \Delta z$

✓ $y \rightarrow$ AREA $\Delta x \Delta z$

$z \rightarrow$ AREA $\Delta x \Delta y$

FLUID ENTERS THRU x, y, z FACES

LEAVES " FACES AT $x+\Delta x$, $y+\Delta y$, $z+\Delta z$

So, let us just draw, so this is my coordinate system and I am going to draw the shell. So, this is the one which I have, let us say this is my x, this is my y and this is the z. So, as you can clearly see this is delta x, this is delta z and this is delta y, this is the space which I have defined and I have flow of a liquid in all possible directions which approach in this.

So, this point is, the coordinate of this point is x, y and z and coordinate of this point is x plus del x, y plus del y and z plus del z. So, I have 6 faces in this case, the face which is perpendicular to the x direction; that means, this face, which you do not see, this face is known as the x face. So, the x face is perpendicular to the x direction, its area as you can clearly see the area is going to be del y times del z, del y del z. Similarly, y face has will have area, y face is the face below the bottom face which is this one, you y face will have area of del x, del z and this z face which is the one which is see over here, this z face will have an area of del x, del y.

So, what we would assume is that the fluid is going to enter through x, y. So, the fluid enters through x, y and z faces and leaves through faces at x plus del x, y plus del y and z plus delta z. So, this is what the fundamentals of shell moment, shell heat balance is. So, you have this shell of dimensions del x, del y del z which is situated in a flowing, flowing in a in a flow field and the, and the flow is going to come, it would entered through all these faces in a win and leave through the other side. That means, they what vary it can come through x it leaves at x plus del x, y and y plus del y, z and z plus del z.

It is a three dimensional flow so, there is going to be components of velocity as v x, v y and v z and the temperature is going to be a function of x, y and z as well. Whenever liquid enters into the control volume, it will carry with it some amount of energy and when it leaves it is going to carry some amount of energy. So, some amount of energy is coming into the control volume, some of it is going to live in the control volume, the coming to that of the energy to the control volume is through three faces, it is going to leave through the other three faces.

So, there could be as a result of this process and net amount of energy, which is added to the control volume this net amount of energy could also reduce. So, in that case I am going to simply use a minus sign, but let us assume that some amount of energy is added to the control volume, now this control volume can also do some work or some work maybe done on it ok.

So, that is a possible since we are considering, taking into considerations all possibilities we should also considered that the sees, that the control volume can do some work or some work can be done on it. So, the amount net amount of heat which you add and the amount of work that this system does or it is being done on the system, this sum total of this must be equal to the time rate of change of internal and kinetic energy of the system.

So, if you recall first law of thermodynamics what I have stated. So, far in terms of the control volume is nothing, but the statement of first law of thermodynamics for an open system. Where all effects are considered, the energy which comes with the flowing fluid, it will have a thermal energy component; it will also come with a velocity. So, there will be some internal energy component sorry, there will be some kinetic energy component and an internal energy component, kinetic energy is because of the velocity of the fluid stream, internal energy is because of its temperature, whatever be its temperature.

So, we would like to write all those terms containing kinetic energy and internal energy through all the 6 faces, there going to give me the net heat being net energy, both kinetic and internal being added to the control volume. We will also have to take into account whether the control volume does any work or some work is being done on the control volume, that is going to be another component of the difference equation. As a result of this the total energy content, internal and kinetic, total energy content of the control volume will change with time.

So, we are not restricting our self to steady state, we also allow the energy in the total energy can change which time inside the control volume. So, when I express in that terms, what I am stating is nothing, but the first law of thermodynamics, where all effects are considered in from this difference, difference equation when we think about, when we when we take all the appropriate terms into account for example, let see work done on the system or by the system. So, what are the forces against to which what can be done, one obviously, is a body force for example, a gravity which acts on the entire volume of the control volume.

So, gravity is a body force which for against which the control volume may do some work, the other forces are not everywhere, not acting everywhere on the control volume, but acting on the surfaces, So surface forces will also have to be taken into account. So, what are the, for most what is the most common surface force expression, so the pressure force is acting on the control volume. So, these are two examples of the forces which can operate on the control volume, one is a body force the other is a surface force. So, we have to identify the most common body forces and surface forces plugged into the equation that I have just described, in then try to see mathematically; what is the end result of it.

So, the end result of it should give us an equation which is an energy equation, which should have embedded into it both conduction, convection the work done and as a result of which the time rate of change of temperature of the control volume. So, that would give me the complete energy equation, which I will be able to use for a specific application, cancelling the terms which are not relevant for the problem that we are dealing with and get to a simplified form of equation. The same way we have obtain the equation of, of conduction in the case of conductive heat transfer. So, and stop here today, but what I just brief summary, I have discussed about conduction, the thermal boundary layer the velocity boundary layer and the concept of shell heat balance.

And while describing shell heat balance I have use the concept of first law of thermodynamics for an open system and the shell that I have defined of size del x, del y, del z which is fixed in space and through all the faces the fluid is coming with some internal energy and some kinetic energy. And some work is being done on the system or by the system and the work is being done either against gravity, which is the example of a body force or some pressure forces which are acting only on the surfaces of the define control volume. When a sum all of them together what I get easy total, the time rate of change of internal and kinetic energy is of the control volume of the shell that we have to just define.

So, we will write down the terms in the next class and see how beautiful is this concept would result in a compact differential equation for temperature and this for the special; that means, x, y, z variation of temperature, as well as the temporal variation of temperature, that means how the temperature changes with time. So, my aim is to obtain an expression of t as a function of x, y, z and time that is what I would like to get out of this energy equation. And once I have that, then I am going to transform this equation for the case of convective heat transfer to see whether or not I get a compact expression for h, the convective heat transfer coefficient the engineering parameter of interest that engineers would like to find out before designing in equipments.

So, starting from fundamentals through some, little bit of mathematics I would like to get a differential equation, from the differential equation and expression of each which can be used by practicing engineers. So, that is the whole change which I would like to cover in the, in some of the future classes.