Computational Fluid Dynamics

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Module No#06

Dealing with complexities of physics of flow

Lecture No.#34

We now have the set of equations that we want to solve to represent almost any realistic case, whether it isflow with heat transfer, without heat transfer, with mass transfer, with chemical reactions or all the things put together ah.We have a set of equations and we have seen that all these equations formingthat generic scalar transport equation.Now the question is, are we now equipped to deal withany realindustrial problem.

As far as the equations are concerned, we have already highlighted the fact that,we can only deal with geometries which can be fit into the Cartesian coordinate system. Or may be stretching it a bit, we can do withcylindrical polar coordinate system or even spherical polar coordinate system.But we cannot havewe cannot yet deal with physical system which has partlyCartesian coordinate, for example, a rectangle duct and partly spherical example, rectangle duct containing a ball around which you have flow going through.

So that kind of physical complexity is something thatwe cannot yet deal with but, can we deal with corresponding equations which represents which represent all thisreacting flow type of situation.On the face of it one would say yes but, it is not yet quite…We are not in a position to say yescompletely. Because most real world flows are turbulent flows and turbulent flows are very special they have special characteristics which makes it extremely difficult tomake use of the kind of techniques that we have evolved,for example, for the C F D techniques.Ah to be directlyapplied to thecalculation of turbulent reacting flows for example, or even turbulentnon reacting flows or even isothermal turbulent flows.

And what we mean by this is that, the very nature of turbulent flow isis such that turbulent flow is not one-dimensional or two-dimensional and it is not steady. In the in theusual sense ofsteady one-dimensional or two-dimensional flow that we can expect for a lamina flow,we cannot expect in a turbulent flow. It is inherently unsteady and inherently three dimensional.

So that means that, even when we are dealing withwith for example, flow fully developed flow, steady flow througha long pipe, cylindrical pipewith constant pressure gradient, pressure differencebetween the inletand outlet. So that we have essentially constant boundary conditions, even then if you were to puta probe which would measure the instantaneous local velocity at a particular point within that pipe, we would find that the velocity would be fluctuating with respect to time. And if you had two probes which are close to each other and then if you were to recordthe instantaneous velocitiesmeasured by each probe you would find that even though they should be under one-dimensional flow condition, they should be recording the samevelocity. We find that the instantaneous velocities will be different.

Soit is in that sense, the flow is not two-dimensional or one-dimensional. There'llbe fluctuation if you go from one time to another time, there'll be variation if you go from one point to another point, even in the direction in which there is supposed to be no mean flow, no flow.So it is this aspect that the flow is inherentlyunsteady and three-dimensional that makes the computation difficult. But, you can say we have the equations for the three-dimensional flowand we have the equations for transient flows.

So in theory we can actually solve thetransient three-dimensional form of navierstokes equations and get a solution. So that is possible andin theory one can therefore, say that we can deal with turbulent flow. But, what is special about the turbulent flow fluctuations is that, the fluctuations are very rapid in time and that means that, if you want to look at a steady flow througha pipe with Reynolds numberof say 50000 or 100,000 which we expectto be in fully turbulent flow. We cannotsay that the flow is fully developed, so we will consider the variation only in the radial direction.

Because turbulence is three-dimensional, we have to havea three-dimensional we have to solve for not only the axial momentum but, also for the radial momentum and the tangential momentum equations, along with the continuity equations. So it becomes a three-dimensional problem and because the flow is transient we have to deal with the transient form of it.

So the dimension of time also comes into picture and because the fluctuations are veryrapid you need to solve with a very small delta t. And because there's equally rapid fluctuation in the spatial dimension, you need to have very small delta r and since turbulence is three-dimensional,that means it can exhibit fluctuations even in the axial direction even though it is supposed to be fully developed, so you can…You need to have small delta z, small deltar, small delta theta and small delta t. And you need to do for a large number oftimes in order to get something likesteady flow condition.So this makes it computationally veryexpensive. It is not that it can't be done, it has been done. And this approach to calculating turbulent flows by directly solving the transient three-dimensional form ofnavierstokes equation is called direct numerical simulation of turbulence or the d n s of turbulence flows.

So thatthat approach is feasible only when we havesuper computers at our disposal forweeks and months and even then when the Reynolds numbers is not very high. So whenthe Reynolds number increases the computational time required to geta turbulent flow calculation is supposed to go asReynoldsnumberto the 9/4. So that means that, if you are looking at a Reynolds number10000 and Reynolds number10 to the power5,then the computationaleffort wouldgo as 10 raise to the power9 by 4, so it is more than 100. So, if it takes one day to get a Reynolds numberflow simulation at 10000, it will take 100 days simulation to get Reynolds number simulation at 10 to power 5. And even 10 to power 5 is not really industrially a large Reynolds number if you are looking at flows in ducts in alarge industrial concern.

Typically the Reynolds number may be the order of 10 to the power of 6so that would be even more. So from that point of view, approachingcalculation of turbulent flows directly by the solution of the transientthree-dimensional form ofthe equations which is what we can do now,is not does not make a reasonable proportionproposition.Andone have to one has tocome up with a more efficientway of calculating it, because although it is three-dimensional and transient we as engineers or scientists we probably do not need the full variation right down to the smallesttime step and smallestspaceincrement.

What we may be interested in, isthe overall behaviors if you are looking at flow over a car then what is the drag coefficient that we want, that we get for this particular profile. We are not interested in how the drag coefficient changes with every millisecond of the time.We would like to get only the overall behavior and we would like to find out what where specifically the flow is not goodfrom the point of view ofdrag coefficient and where we can make changes.

We are not looking at variation of a velocityat every millimeter or every fraction of a millimeter with every fraction of millisecond time. So, that is the kind of information you would be generating from a fully transient three-dimensional flow calculation flow of turbulent flowwith very small delta t, delta x and all that. And most of that information is useless, because we are interested only in the average quantities.

So that kind of solutioneven though we can get in principle is not a good thing and there are also difficulties associated withC F D type of approach that we have discussed here ah, where we are usingfirst order or second order accurateapproximationsfor thefor the derivatives. Usually these brings these kind ofapproximations, low order of accuracy of approximationsbrings in artificial effects ofdissipation and so on, which will damp out or which may damp out or orfurther accentuatethe dissipation that is present in a natural turbulent flow through a natural mechanism.

So, one has to have highly accurate numerical schemes todo the discretization of the governing equations, in order to get accurateestimates ofof the velocity fluctuations. So on the whole, the direct numerical simulation ofturbulence is a proposition which is not acceptable for an ordinary engineer or a scientist.They have those kind of simulations have their place they set the bench marks by which we can validate simple models simpler models they set the bench marks and they give the insight into how theflow is behaving and thereby we can use that to derive and developmodels which arelower order models which are more amenable to computation ok.

Sothat is usually thethat, in that sense D N S is very useful but, not inthe regular computation of turbulent flows. So realizing the nature of turbulent flowwhich is, that it is inherently three-dimensional and fluctuating.Reynolds has proposedin 1800 itself, a decomposition ofvelocity at a particular point into a time average quantity and an instantaneous quantity.

So using this decomposition he has come up withan averaging approach so that for the averaging the navierstokes equations, which has a property that if you are looking at time average behavior of a particular flow, then in theory you could solve this time average equations in which the variables should be the time average quantities.That is you are no long no longer saying that my the solution will not give you the velocity at every millisecond,it will give you the time average quantityvalue of the velocity at that particular, may be what you'll see in anot what you see in an instantaneous snapshot,with a high resolutionof the flow. But, something that istaken with a long time exposure thatthat kind of picture and the advantage of this time averaged approach is that if you are looking at the time averaged equations will exhibit thethree dimensionality or two dimensionality or one dimensionality or the steady state nature of the equations.

Therefore if you are looking at fully developed steady flow steady turbulent flow through a pipe then using the time averaged equation you need to solve for the steady form steady one-dimensional form of the corresponding equation. You do not need to study; you do not need to calculate the full three-dimensional transient equation. So there's a great advantage that ispossibleAh when you go by the time averaged equation.

So that is why the standard way of approaching turbulentflows is to do the time averaging of the governing equations and thereby derive the time averaged equations.But in the process as with any kind of averaging process we lose out some information and one has to give this additional information to theto theequations, to the time averaged equations, so that the corresponding physics of the fluctuations is properly accounted for.

So all this together constitutes the subject which is known as turbulence modeling andturbulence modeling isisan ever evolving field and it is being evolving for the pastmore than 100 years. Andwe have now a plethoraof models to deal with turbulent flows and whereas, in lamina flows for example, lamina isothermal flows, we have only four equations for a three-dimensional flow, namely the continuity equation and the three linear momentum equations.

In turbulent flow we may haveadditional 7 equations. So we may have to solve 11 equations to represent the time average behavior of a three-dimensional turbulent flow. But, the advantage of thismodeling is that all these equations fit into a genericscalar transfer equation, which we know how to solve andthis kind of advance has made it possible to extend our ability to calculateflows to even turbulent flow situations and one can readily do these days turbulent reacting flowsusing C F D, even in complicated geometry.

But there'san attendant loss ofaccuracy in makingthis calculations because the moment we do time averaging, the moment we do some sort of modeling of the fluctuations, we are introducing uncertainty. And therefore, for turbulent flows the equations are no longer fundamental. They are they do not representwholly correctly the conservation of for example, the linear momentum. We have a modified form of the a equation which is valid to set an assumptions and we have another form of the same time averagelinear momentum equations which brings in other assumptions which are supposed to be better than the previous assumption.

So in that sense you have a hierarchy of modelswhich are progressively more accurate and which are also progressively more difficult to compute and at some point we have to make a compromise between the accuracy that we want and theeffort that we want to get a generalC F D solution.

So this is the kind ofinterplay and the choice that comes up in turbulent flow. We will briefly look at the overall picture and approach to turbulence flow modeling,we will do, we will go up tothe two equation model which is considered as themodel industries standard model for turbulent flows and which has proved to be very robust, even if it is not accurate it is proved to be very robustin giving reasonable solutions formany turbulent flows.So once we are armed with a two equation model for turbulence then we can say yes we can doturbulent flow calculations of a of anon isothermal reacting flow in a complicated geometry.

So we will be building towards thisturbulence flow and how to deal with turbulent flow and how to deal with turbulent flow within the context ofcomputational fluid dynamics.So let us start withwith recollection of our generic scalar transport equation, we have said that doubydou t bydou phi plus dou by dou x kdou by dou x kof say some gamma phiplus s phi.

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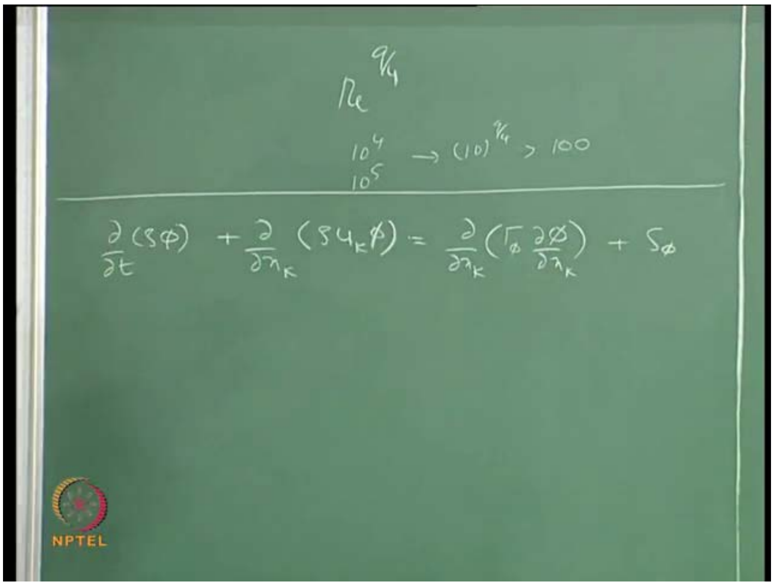

As usual a repeated index here meanssummation overthe 3components and here we have the phi is a scalar and it usually represents the specific quantity for unit mass of that particularthing, for example, enthalpy or linear momentum in which case phi is nothing but v, that is the velocity and for continuity we get phi equal to 1, so that we getthe continuity equation. By substituting for different expressions for the effect the diffusivity and the source term, we can make use of this equation to represent the continuity equation, the momentum equations,the energy equation, reacting flow equations for the species conservational equation.

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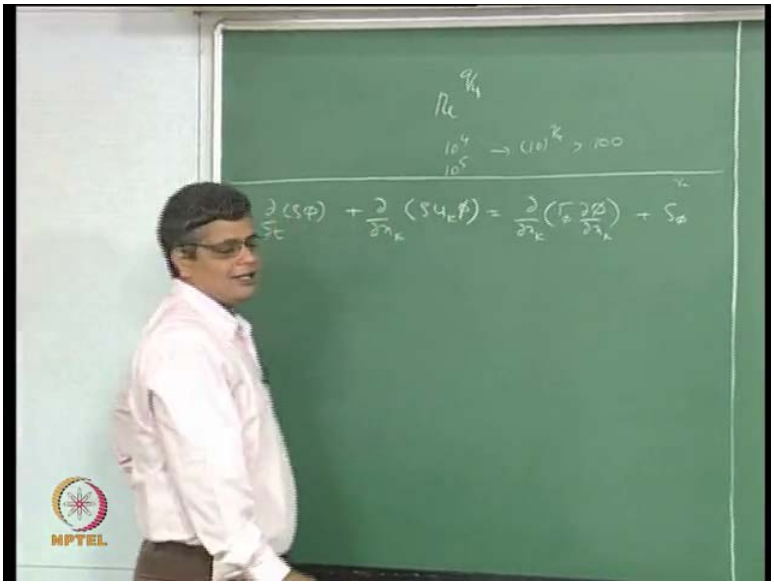

We know that in a species conservational equation the source term is the rate of generation r i.Andthis is thatdiffusivityd i j which we have said isa fairly complicated things for a multi component diffusionto be done exactly and for a linear momentum equation this becomes the pressure gradient and this becomes viscosity andforheat conduction, for the energy equation, this may be someinternalheat sources if there are any and this becomes the thermal conductivity. And for the case ofeven radiatingheat transfer we can represent that as we can add some minus del dot q r, where q ris the radiative heat flux.

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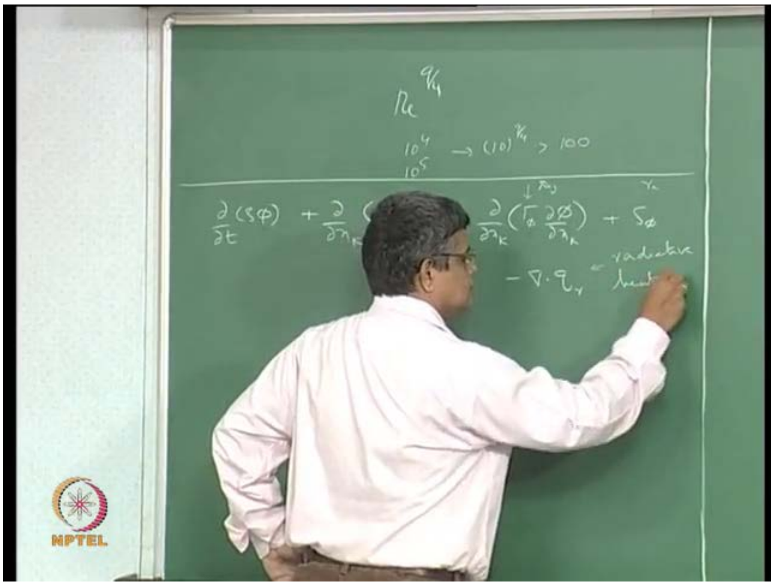

Ok soq r hereis the radiative heat flux and the evaluation of q r is also done using anumber of models using number of different approaches, so the evaluation of radiative heat transfer in C F D isis a major topic of it is own and it is not as simple as the corresponding conductiveheat flux which we can write as minus k gradient of temperature.

So for conductionq c is given by this. But, radiative heat transfer cannot be written as simply as that andso that is usually brought in as an additional flux term in this particular form.So the point that we are making is that we have these kind ofgeneric scalar transport equation which can represent allthedifferent conservational equations that we have derived so far.

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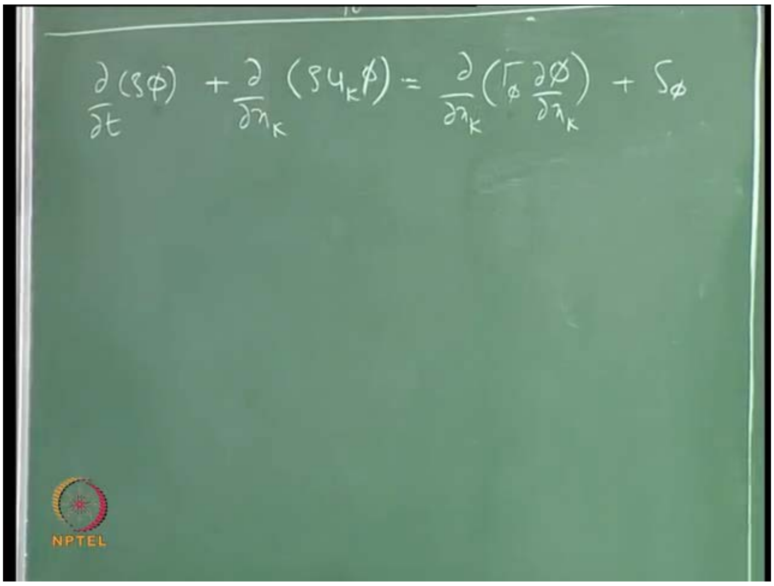

In this form these conservation equations are valid only from lamina flow. In turbulent flow, unless we make sure thatthe calculation procedure,honorsthe special featuresof turbulent flow, thiskind of method cannot beused with the conventionalC F D approach.

Now,what we mean by turbulent flow ithas turbulent flow hassome special features.It isfluctuatingfluctuatingand it is fluctuations are very rapid. This isan inherent feature and typically these are so rapid that these are of not of any real interest. In the sense that typically they are so rapid that they pass over within the blink of the eye and we do not really want to know why it isso rapidly changing and how it is so rapidly changing.These things are present.

The second feature about these fluctuationsis that although they are fluctuating and therefore, represent some sort of instability, these are well contained fluctuationsandthat is why we do not really need to knowso much about this.

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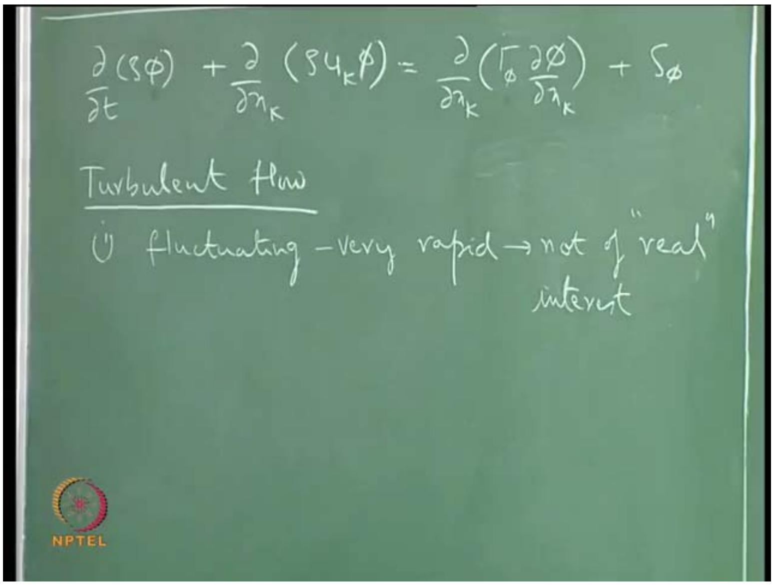

Well contained in the sense thatalthough theythey seem to be theflow seems to beunstable and so on it is notinstability which needs towhich needsthe perditiondistraction. But, it is definitely these fluctuations do lead to chaos.

Ok so there is achaotic behaviorof the flow properties. When we say chaotic behavior chaosimmediately tells us that it is unpredictable and that brings into the philosophicalquestion as to how we can geta chaotic unpredictable behavior from deterministic equations. So that isa strand of thought which we will not pursue but, by this chaotic behavior means that given a particular value for example, of the velocity at this particular instant here. If the flow is turbulent you cannot predictwitha great deal of accuracy what it is going to be at the next instant of time.

And similarly, because the fluctuations are not only with respect to time but, also with respect to space,if the flow is laminar then if this is the velocity here and if this is the velocity here and this is the velocity here,I can saythe velocity here is going to be somewhere in between this.But in turbulent flow, if I know the instantaneous values at two points here like this, then I will not be able to give the velocity at anintimate pointto agreat deal of accuracy at that particular point, because, within the two points the velocity is going to change.

So in that sense there isthe loss of predictability there’s an uncertain element in thisturbulent flow but, since the overall fluctuations are well contained one can still be confident about the overall behavior. So there is the inherent feature of turbulent flow, it is very rapid fluctuations which are not of real real world interest but, whichthrow in a bit of chaoticnature of the flow properties but, on the whole they are well contained fluctuations. Therefore, it is sufficient to knowessentially thethe steady statethe average value of thisflow behave flow parameters and it is notnecessary to know all the full details of of this so that is one comfort we can take when we are dealing with real world problems.

And the other thing which we have already mentioned about this is that turbulent flow is neverunsteady and neversteady and never 1-d or 2-d. It is always 3-d. Even when you have the typical assumption of fully developedone-dimensional flow, fully developed flow through a very long pipe, with constant properties and constant pressure gradient and so on. Even then when we..in the normal case when we don't expect variation in the flow direction in a turbulent flow there will be variation in the flow direction.So this means that we have to solve alwayswe need to solve the full3-d transient NS equations the navierstokes equations and all its extensions to represent the real behavior.

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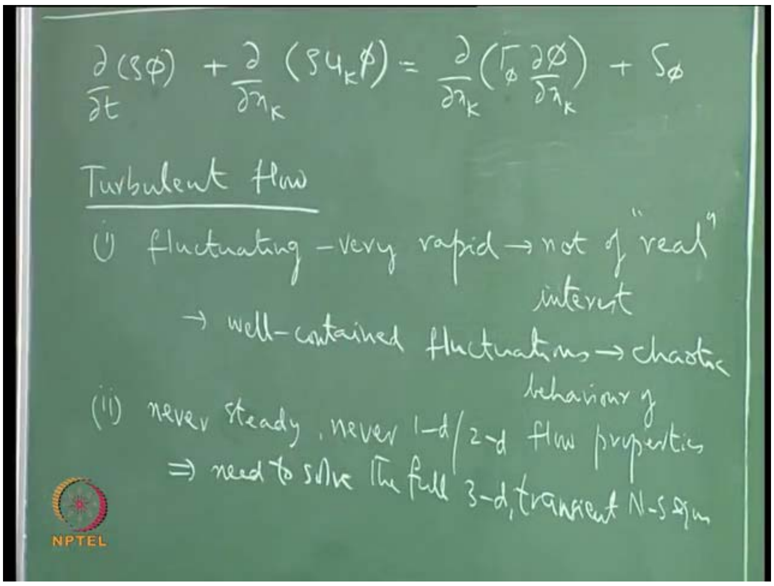

The other feature aboutturbulent flow:it is highly diffusivediffusive. In the sense that if you put a drop of ink in turbulent flow it very rapidly mixes. If the flow is laminar is you put a drop of ink and the flow is going in this direction, you can trace the thing, you can see how the ink is goingalong. But, if it is turbulent flow the moment you put it gets dispersed so quickly thatthere's hardly any trace of it so this highly diffusive nature is alsorelated to the structured chaotic behavior, structured madness in inthe fluctuations that we see.

So thisthis is closely related to the fact that we havelots of this eddies in turbulent flow. So this eddies are like, almost likemini whirlpools they mix rapidly in that particular thing. And you have not only eddies like this, you have eddies like this you have eddies like this you have small eddies,you have large eddies and then you haveeven larger eddies.

So there is if one were able to visualize eddies for example, going from time domain into the frequency domain, that is usual mathematically way of looking ateddies and spectra and fluctuations.Then one would find a continual range of size it is not an eddy of a specific size but, eddies of several sizes and a continuous range of sizes will be present. And it is this eddies that are characteristic of turbulent flow which cause a lot ofdiffusionand this diffusion that actually affects the way that the parameter is going to change. This is the equation which governs the change of the parameter phi here.And there's a diffusive component here and turbulence by it is very diffusive nature directly place in onto this diffusivity.

So this diffusivity which is essentially done by molecular type ofmechanisms in lamina flow is now aided by gross fluctuations, these eddies which are much much larger thanthe molecular distances and (( ))ok. So these eddies act like mini molecules mini atoms and they interact with each other and then therefore, they cause they give riseto the turbulent transport properties likediffusivity of heat and momentum andmass and all that. So theydirectly play into these equations.

That's why it is important to resolve this diffusive nature and quantifying in the form of additional diffusivity is coming into this because, once this is disturbed once this is no longerdue to molecular motions alone, then the whole way that thethe scalar phi evolves will be changed. So it is important to know this diffusivity in turbulent flow. And the final character that is of interest to us fromthis is that this additional diffusivityis a strong functionfor example, ofReynolds number.

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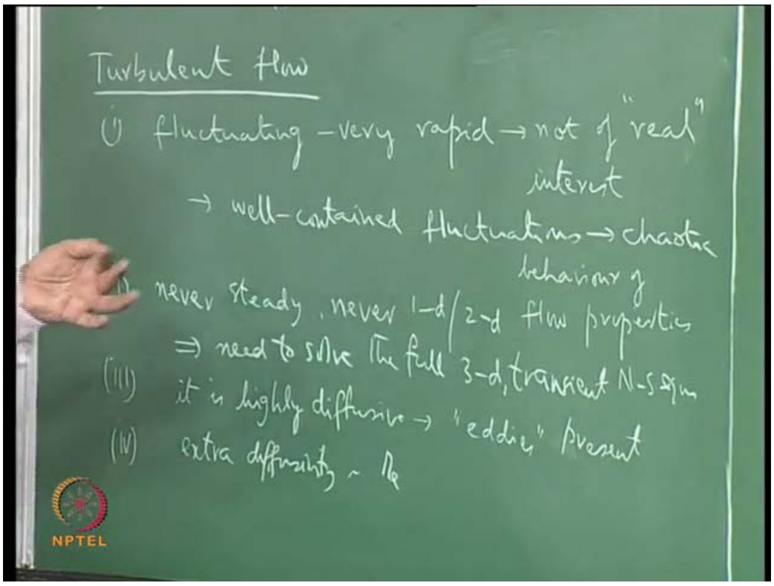

In laminar flow, we know that the nozzle number or the heat transfer coefficient does not change with Reynolds number but, not in turbulent flow.So we can havethis additional diffusivity or for example, the heat transfer coefficient or friction factor in turbulent flow divided by friction factor in laminar flow, is not a constant parameter and it can be a factor of 10, it can be a factor of 100, it can be a factor of 1000, depending on what kind of Reynolds numbers you have that is in simple flows.

So what that means is that if your friction factor is increased by an unknown amount and by large amount, unless you know what that increase in the friction factor is, you will not be able to size yourpump properly. The pump that you say requires so much of horse power say one horse power based on laminar flow calculations may actually berequiring 10 horse power or 100 horse power ok.

Similarly if you havesizedthe heat exchanger and the totalheat exchanger is3 meters length because of the higher diffusivity of aturbulent flow, actually the real size may beonly $1/10^{th}$ of that or $1/3^{rd}$of that ok.So you may be grossly over sizing or under sizing your process equipment dealing withthe fluid flow and associated heat and mass transfer, if you do not take correct account of the extra diffusivity that is coming into picture. And because it is not only extra but, how much extra depends significantly on the flow parameters like Reynolds number and the flow geometry,it's necessary to take proper account of this and then evaluate this. And that is why we need to have the set of equations which willwhich willaccurately and quantitatively describethese special features related to the fluctuating properties of turbulent flow. We cannot simply just neglect these fluctuating properties because these are too rapid and they are contained and all that.

When you look at the overall effect of this in terms of diffusivity and in terms of the overall transport coefficients these fluctuations have a very large role to play and it is necessary to resolve these things and quantify these,so that we can get the proper evolution ofof the scalarin this. So and that is the purpose ofof turbulence modeling.

These fluctuations which are not of real interestfrom their own sake in their own sake will have to be properly accounted for in the in the equation form. Andtaking into account, this particular nature and also this particular nature, we derive thetime averaged form of the governing equation which enables us to bring in the information of these fluctuations through certain extra terms.

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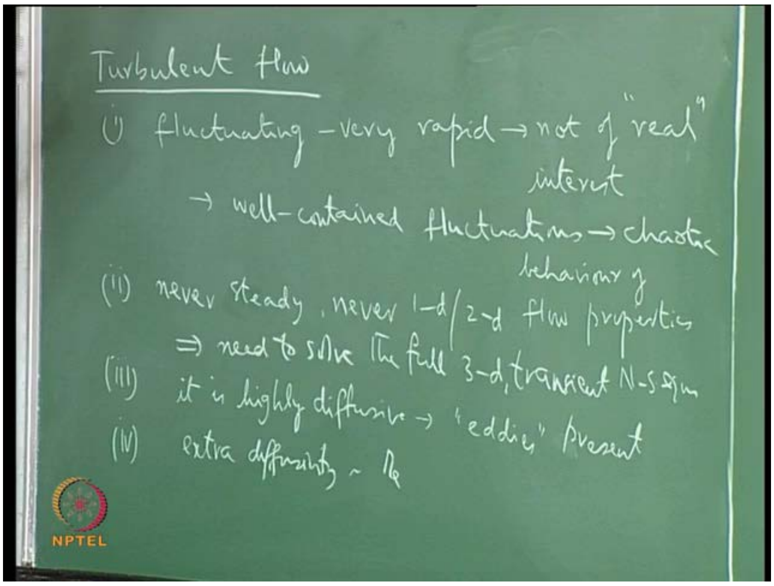

So how do we take account of this extra diffusivity that is coming from turbulence.Although we are talking about increased heat coefficient andfriction factor and so on we should realize that when you look at the actual equations that we havestarted out with these are equations which arewhich are valid at every point and in the context of C F D these are valid at every grid point.

So the turbulence that we have the turbulence diffusivity that we have is going to change from location to location because it is a function of the velocity and we know that in general case the velocity is the function of space and time.So in a transient three-dimensional flow, the additional diffusivity that is coming fromthefrom the turbulence is going to change from location to location, even in a fully developed flow through a straight pipe, it would change on a time average basis, it would change with radial distance.So it is not constant additional causefactor,multiplicative factor that we candirectly put in the equations. So we have to have a mechanism whereby we can predict the local enhancement of turbulence diffusivity that is appearing in the equations.

So there are twomethods for this two approaches for this one is of course, go to thetime dependent navier stokes equation and thensolve them in the direct numerical simulation of turbulence, which we have said is very timecomputer intensive, computational effort intensive. So the other more practical approach is to try to smooth out these very rapid fluctuations and then use some simpler approximate but, fairly accurate models to represent thecontribution coming from these fluctuations. So this is what is known as the time average approach and we havethe Reynolds time average approach is the most common for essentially incompressible fluids.

So we look at this Reynoldstime averaging and the concept and we will ultimately derive the RANSE equations, Reynolds AveragedNavierStokes Equations which will form the basis for the C F D of turbulent flows in the general case.We look at the RANSE equation approach which is the more practical approach to turbulent flow calculations.

We arelookingat Reynolds averagenavierstokes equations. This will be in terms of time averaged quantities which enable us tosolve only thetime average steady state equation for a steady turbulent flow. Time average simplifiedtwo-dimensional equations for a two-dimensional flow andso on.For example when we consider turbulent flowin a straight pipe under fully developed conditions, then we will be solving a single equation which is the Reynolds averaged axial momentum equation to resolve theturbulent velocity profile. So that is advantage that isobtained in this.

Now what is this Reynolds averaged equation. We need to understand first of all what is meant by time averaging. If you look at turbulent flow and if you plot any velocity at a particular point x 0 as a function of time, u isthe velocity component andt isvarying in this thing.

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Andtypically what we will haveis a variationquite unexpected at a particular point x not y not z not as a function of time. So we are not doing anything except recording the velocity at a particular point and this recording may be over a second and within that second you may have these kind offluctuations and although I have drawn this large kind of humps like this. The variation is not even as structured as this and if you were to take a small section here and then broaden it up,if you take for example, a 100 milliseconds in this and then plot u versustime in that 100 milliseconds, they will also see further squiggles. So the rapidity of the fluctuations that may be encountered is so, pointed that I cannot draw it fully usinga chalk piece like this.

So in in this context we say that this is the instantaneous velocity. If you take a particular time instant here,t not then this velocity hereis the instantaneous velocity. So thisis u ofx byx 0 t 0 but, we can define also a time average velocity at t 0 by taking a time interval say capitalT, such that thiswhole thing is T,this is a time period and we average the u over this time period centered around t 0, so thisdistance ist by 2 so this is t 0 minus T by 2 this is t 0 plus T by 2and the time averagevelocitydenoted by u barat x 0 t 0is written as 1 by t ofu d t betweent0 minus T by 2 tot 0 plus T by 2. So this is that average velocitythat weobtain at t 0.Now what is this T herethis is the time period it is a time window over which we arewe are averaging and this time window is such that it is sufficiently large to smooth out the turbulent like fluctuations that wehave here.

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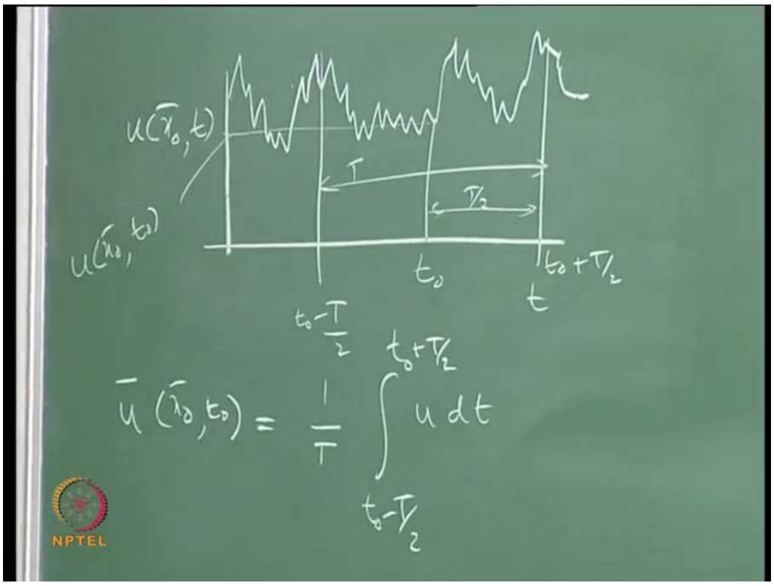

But, it is not so large so wide, that it will suppress inherenttime dependence of the system itself.For example, if you are looking atthetemperature at a particular spot for a metrological application for weather related application , then the temperature is going to change from instant to instant every second it is going to change but, what we are interested in is what is average daily temperature what is the average temperature at say 10’o clockin the timeinterval of between8 and 12’o clockandhow does this time average quantity, how does this temperature vary on a weekly basis, on a monthly basis on a seasonal basis, on a decadal basis and even on a century basis, in thethese days when we are interested in the global warming andsituations like that.

So we are interested not in the second by second variation of the temperature but, we are averaging over say 1 hour and then gettinga value of the temperature at that location and we take this as theaverage temperature represented on that particular day and then we do the same thing next day, next day and these are the time averagedtemperatures recorded at that particular location on a day by day basis.So this is that kind of time averagevelocity and using this we can write theinstantaneous velocityat x 0 t 0as asum of two quantities, that is thetime averagedquantity plus a fluctuating component u primeat x 0 t 0.

Now what are these things?Obviously this is this is theu at x not t not hereand averagevelocitymay bethis one so this value hereis theaverage velocityover which is averaged over this thing.And the difference what is left out of u 0 afterthe averaging so that is this much here,this intervalso this interval is u prime of x 0, t 0.So the total velocity u at x 0,the instantaneous velocity is decomposed into the average velocity plus a fluctuating velocity.

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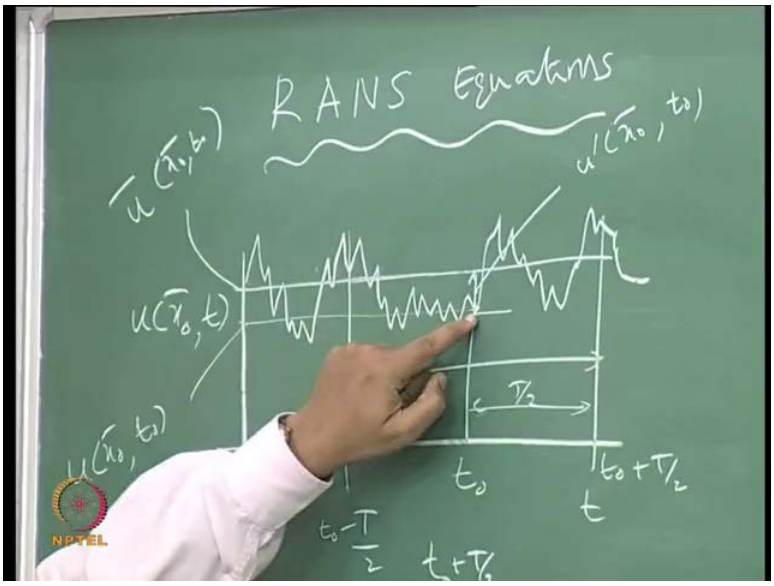

And in this particular case we can see that the fluctuating velocity component is less is negative.Ok. And in general over this time period oft not minus capital t by two and t not plus capital t by 2,we will find that the fluctuating component will be both negative and positive and when averaged over the entire interval that we are looking at thethe time average quantity will be 0, thefluctuating component will be 0.So that is u primed t between t 0 minus T by 2 to t 0 plus T by 2 is 0. Sometimes its negative, sometimes its positive.

So for example, herethis is u prime and that ispositive and this is negative.So if the averaging is done correctly then it'll be such that the time average of the fluctuating component over that interval averaging interval will be 0 andwe can decompose the instantaneousvelocity intotime average velocity and theinstantaneous velocity.

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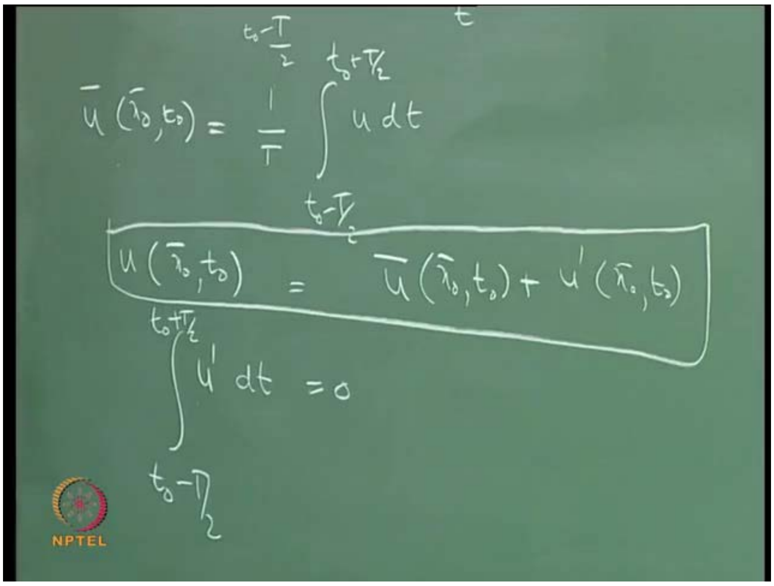

Typically in turbulent flows, this instantaneous velocity is not large andone couldone could represent root mean square velocity, root mean square quantity root mean square quantity so that is u prime square. If you take n different samples you take the u i square u i prime square you square each of them sum it and then divide by n, so that gives you the root mean square value of this. Androot mean square means thatthis issquare root of this.

And we can say that typically u r m sby u average at a particular pointis between0.5 to0.1 or even less it is of this particular order.So the fluctuation is not like 50 percent that one would record will be only of the order of five to ten percent or even two percent and so on but, these small fluctuations are big enough to cause a tenfold increase in the friction factor or even a 100 fold increase in the friction factor andheat transfer coefficients and mass transfer coefficients.And that 10 fold or 50 fold or4fold depends on the particular point of time and point of space that we’re considering in a particular flow and that is what is of interest here.

So now, what we canusing thisdecomposition of velocity into an average velocity and a fluctuating velocity. We cando this for all all quantities so we can write u as u barplus uprime, v as v barplus v primeand w as w barplus w prime and other quantitythat we have is t is t barplus t prime.

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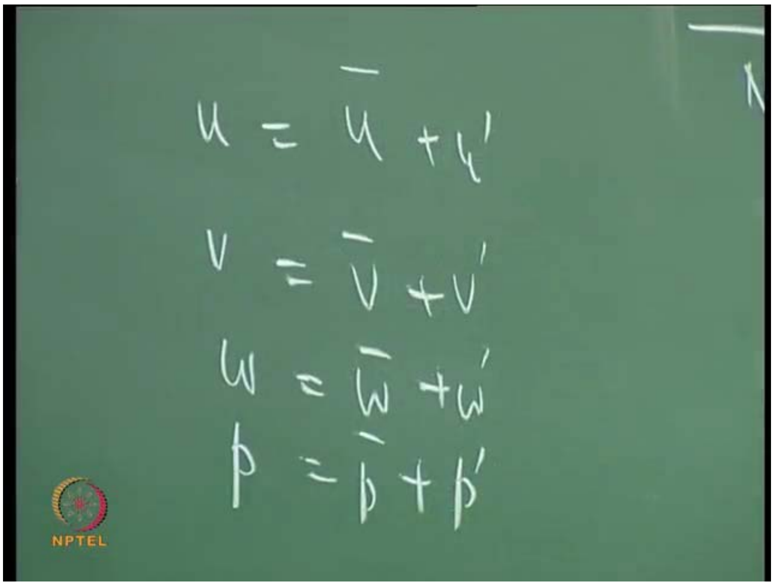

And in each case the quantity isintegrated over a particular time period and capital time periodis much much greater thanthe slowest perturbationassociated with turbulent flow,associated with turbulenceand it is much much less thanthe typicaltime scale,time periodof transient phenomena of interestok. (No audio from 47:53 to 48:03)So there is an upper limit and lower limit.

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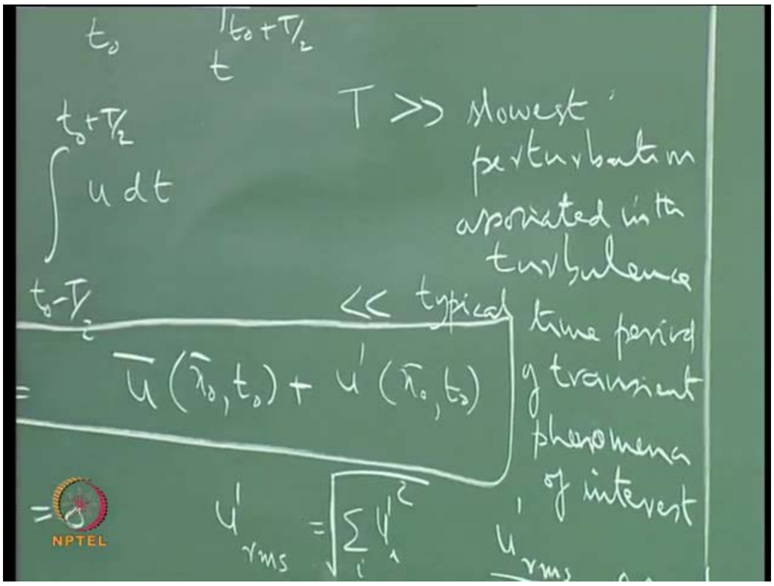

Sothe time period has to be much smaller than the transient phenomena that we want to resolve using ourequations. For example, in case of the temperature variation if we are interested in thein thedaily temperature variation or weekly temperature variation, then we cannot average the temperature over aover a week or a month.Obviously it has to be the averaging period has to be much less than the daily variation or the monthly variation.If you are interested in the monthly variation the time period of averaging can be may be over a day or may be a couple of days and if it is a daily variation that we are interested in then it has to be differently of the order of one hour or half an hour like that.

So depending on that we fixan upper limit to the t to the time period such thatit we can still resolve the transient phenomena associated with the flow thatwe we are interested incapturing but, it must also be much much greater than the slowest perturbation associated with turbulent flow. And in this particular casethis kind of thing so it must be much bigger than this,so that we can smooth out this these variations. So that the smooth roads if you were to plot u bar as now a function of time, although it is varying like this that u barmay be varying only slowly.

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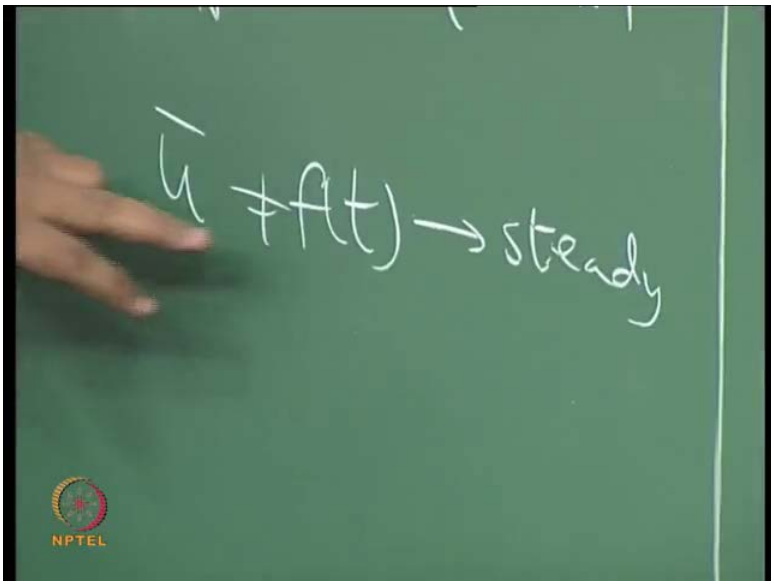

Ok so the the kind of variation that you are seeing here the rapid variation will disappear from u bar and in the case where u bar is not a function of time, then we say it is a steady turbulent flow, it is turbulent flow so we are expecting these kind of rapid fluctuations but, it is also steady flow because your average quantity is not a function of time. So its steady turbulent flow andAh if you have maintained a constant pressure gradient in a across a pipe of a givendiameter straight pipeand if the Reynolds number is out of 50000 then we can expect the pipe to reacha steady flow condition where the time average velocity at a particular radial location will not vary with time ok.So although instantaneous velocity will change the time average quantity will will not vary but, if you were measuring the velocity at a different radial location then definitely we canexpect it.

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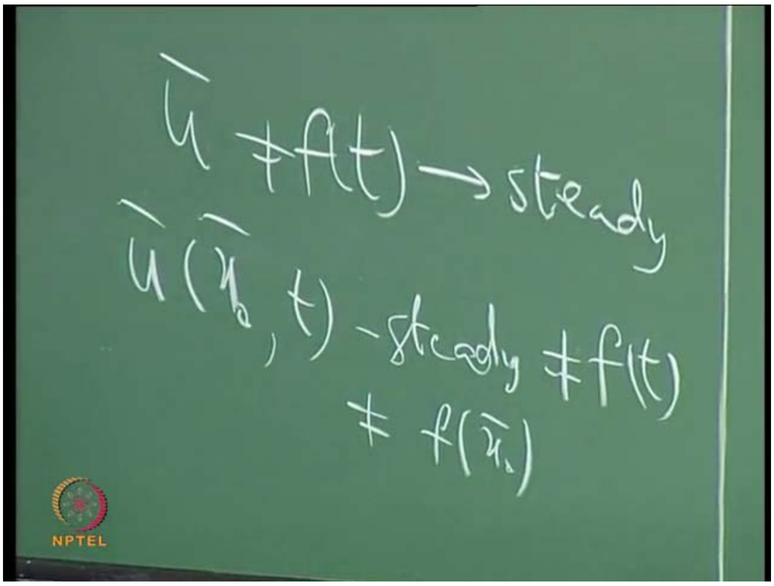

So ifu bar as a function ofgeneralized spatial coordinate x 0and t,so this will be steadyif it is not a function of timeand it will be homogenous if it also not a function ofx 0. But, any way that isthat ispoint which we do not stress upon but, what you want towhat you want to stress upon is that even under steady conditions, u bar the time averaged quantity can change as a function ofthe spatial location.

So given this definition we can show certainsimple mathematical operations for example, we canshow that, what we are interested in is not just inthe velocities themselves in our conservation equations, we have the gradients ofof the various quantities that are coming into picture and so, in order to use these things in our original navierstokes equations and then bring out the time averaged equations, we should not only be able to write aboutu as a function of u bar plus u prime but, also about dou u by dou x and dou p by dou y and all those things.

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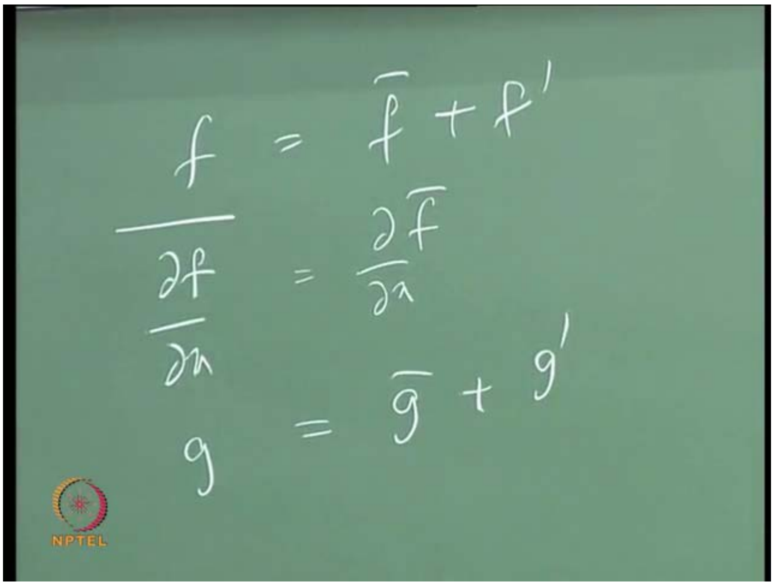

So if f is the quantitywhich is decomposed intof barplus f prime. We can showthat dou f by dou x,the time average of dou f by dou x isdou by dou x of f bar and ah, similarly, ifg isanother quantity which is decomposed into g bar plus g prime, then we can show thatthe time average of f bar plus f plus gis equal tof bar plus g bar and the time average of a scalar quantity times f is equal to,the scalar quantity times f bar obviously the scalar is not a flow parameter it is not fluctuating. And we can also show thatthe product of f g where f and g are two parameters of the flow and therefore, are exhibiting turbulent flow fluctuations arethe product of f bar and g bar so that is the product of the time averaged quantities plusthe time average of the fluctuating components.

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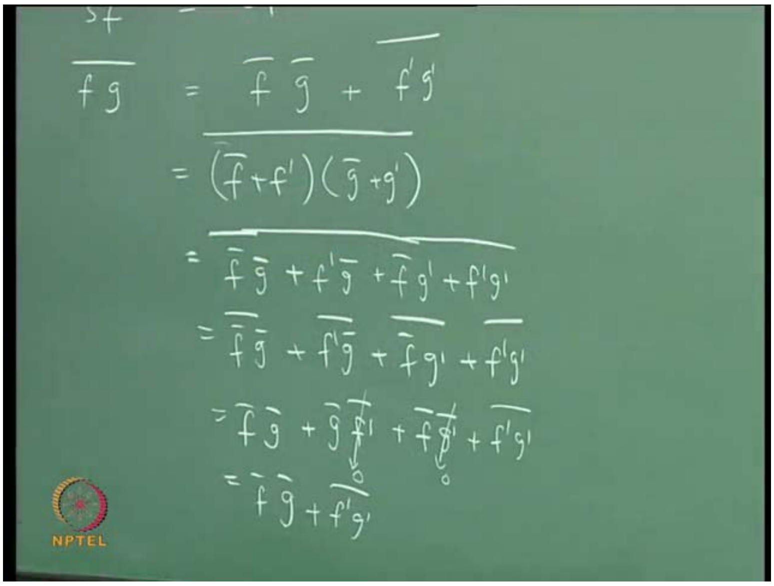

So we can justdo showdemonstrate it like this. We can show f asf bar plus f prime andandg as g bar plus g primeand we take the time average of this whole thing and then we can write this astime average of f bar g barplus time.We can expand this so that we have f bar prime g bar plus f bar g prime, plus f prime g prime bar, this isthis product and we can apply this rule and then we canwrite this as f bar g barprime,plus f prime g barprimeg bar bar,plus f barg prime barplus f primeg prime bar.

And because these are constant within the time period capital T they are not function of time so this will be nothing but, f bar g barand here g bar is not a function oftime within that time periodin which we are doing the averaging so this will come out of the time integral so this will be g bar times f prime bar.And similarly, this f prime f bar will come out, so this will be f bar g prime barplus f prime g prime bar. And so this is equal to f bar g bar and this is 0 because we have said the integral of the timecomponent fluctuating component is 0 and this is 0 and we have f prime g prime bar.

Now the question is, is this going to be 0.So it depends on the independence the relative dependence of f and g, so is f and g are truly independent then we expect this thing to be close to 0 or 0 if they are entirely independent. But, if the two dependon each other if there is some sort of relation between the two, then this f prime g prime bar can bepositive it can be negative also and we can takesimple example to illustratethis point.

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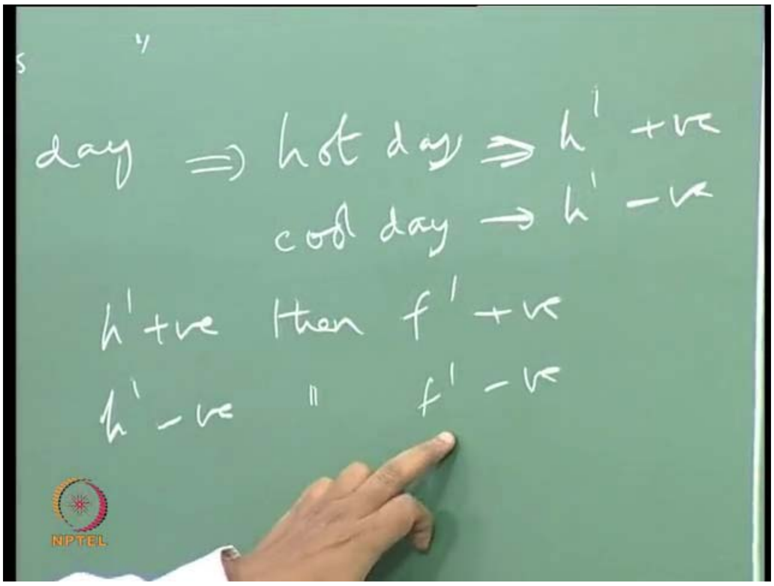

For example, if f represents thenumber oficecreamsthat are sold in the eveningonon the beach, on the sea shore. And if g represents the number ofsay coffeecups that are soldon thebeach each dayand if hrepresentsthe temperature of the day. Then we can see if you plot the f and g and h every day they are going tovary, they are going to show a typical variation, because you would not be selling exactly the same number of icecreams.

In a case where you have plenty icecreams tosell and plenty customers tofor these things, but, if you were to look at the time averaged quantity, thenthere'll be some value expected valuefor a particular season and there's going to be a daily variation but, if one were to look atf g bar f prime g prime bar, then this isif you were to look at f prime h prime this means thath is, on a particularly hot day h prime is positive. Hot day means,in place h prime is positive and relativelycool dayis h prime is negative and f prime positivemeans, more icecreams sold.

By general common interest, we would if it is a hot day relatively hot day, we want icecream and if it's relatively cool day, we would probably like coffee. So what that means is that if you were to take the daily product of the instantaneous of that particular fluctuating component of that particular day andthe temperature thing.Thenwe can say that, when h prime increases, h prime is positive thenf prime is also positive and h primeis negative, so that is when it is a cool day, there'll be lesser than the usual number oficecreams sold, thenf prime will also be negative.So if you now take the product of f prime and h prime it is going to bepositive, when h is h prime is positive and it is also going to be positive, when h prime is negative because f prime is also going to be negative.

So when you now take the time average of this when you average it out you'll see that thiswhole thing will be positive and greater than 0this will be greaterthan 0. And similarly, if you were to look at g prime h prime, again by the same token, on a hot day so that is when h prime is positive,g prime is going to be negative, because we will have fewer number ofcoffee cupssold on that particular day.So that means that h primepositive means,g primewill be negativeand h primenegative means,g primeis positive.

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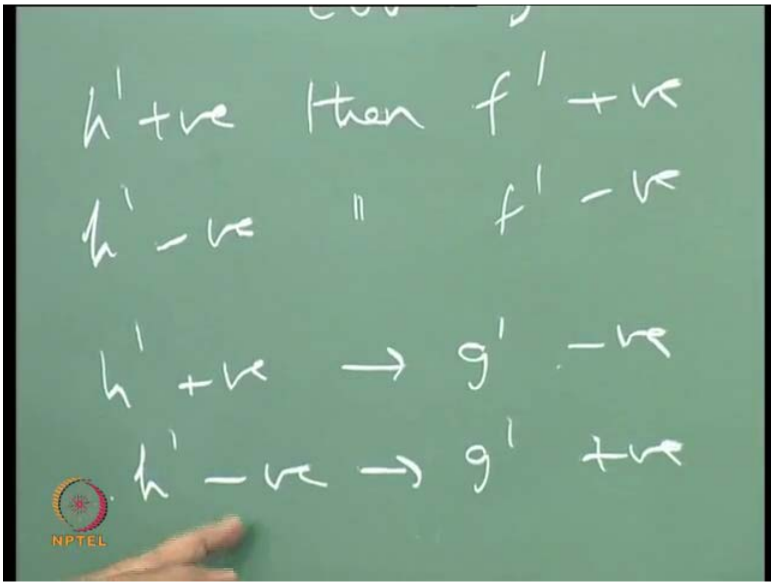

So, if you now look atproduct of h prime and g prime, it is going to be negative when h prime is positive and it is again going to be negative when h prime isnegative because g prime is going to be positive.So in this case there is a certain relation between the fluctuation in the daily temperature and the fluctuations in the coffee cups and the fluctuation is such that on a average basis this is going to be non zeroand its going to be negative.

If you nowthink of a different quantity likeg h i as thenumber of visitorsto the museum,number of visitors to the museumor number of visitors, number of workersreporting toreportingat a factory. Even this i is going to change from day to dayso there’s going to be i negative and i positive, i prime will be positive and i prime will be negativegiving us anaverage value ok.

But if you now look at variation ofi primeand h prime together, co variation of these things, then we won’t go to we won’tstart or stop going to work because it is a hot day or a cool day relatively cool day or hot day. Because we may stop going to work because we are ill or because there’s somesports match that is going on or there is an interesting movie or some visitors have come something like that but, it is not we do not sayor today is a bit hot let me not go to work.

Soand so because of that there is no linkage between whether it is relatively hotday or cold day in terms of whether or not I’ll go to worklike this. So it may be that even on a hot day, there may be more number of workers reporting towork or there may be fewer number. So there’s no strong correlation between i prime and h prime and in such a case this is going to be close to 0.

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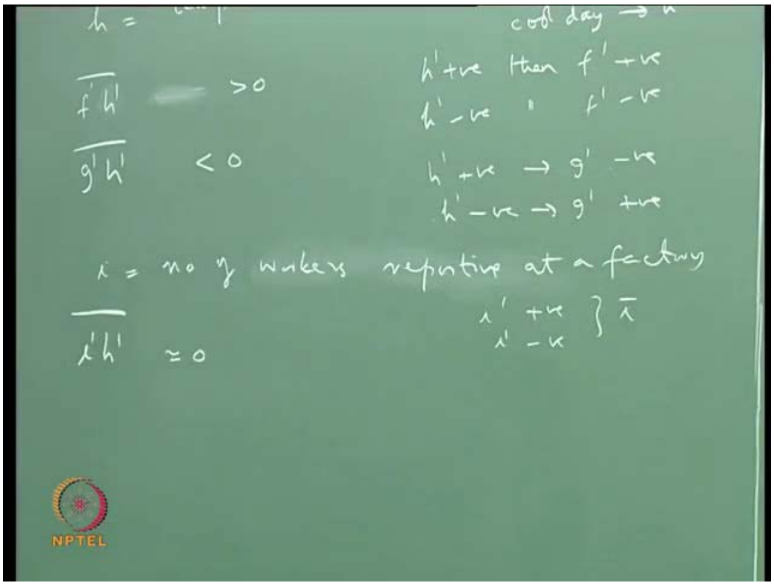

And in the case where there is absolutely no correlation between the two, then we can expect these two to bethe inter relation the co relation between these two to be exactly equal to 0.So from that point of view, when we come back to the product ofthe time average of the product of the two varying quantities f g here, it is obvious that the mean quantities will have a role to play in this. But, the fluctuating quantities the time average is the fluctuating quantities is not necessarily 0 it is 0 only when f and g are independent.

Now in our navierstokes equations we will havethis kind of term,we will have this kind of term,we will have this kind of term and we will have this kind of term. Making use of these simple relations, we can startlooking at time averaging of the navierstokes equations and then derive the time averaged form of the navierstokes equations in which the time average quantities that is u bar,v bar, w barand t barwill appear as the primary variables which we would like tosolve for.so…