Computational Fluid Dynamics

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Module No. # 02

Governing equations

Lecture No. # 5

Topics

Kinematics of deformation in fluid flow

Stress vs strain rate relation

Derivation of the Navier - Stokes equations

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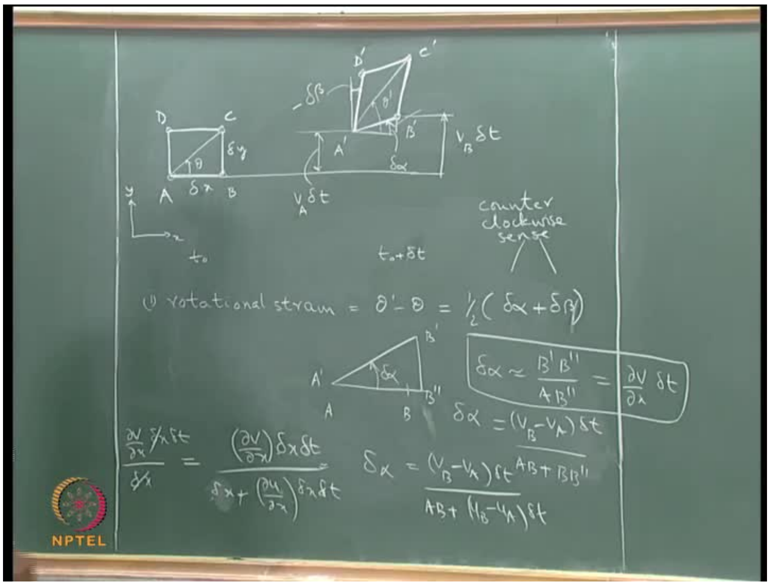

Next, now try to convert the, let us try to examine the kinetics of fluid motion and see what kind of standards are happening as a fluid flows from t naught to t naught plus delta t and as exemplified by the relative motion of the points A B C D which represents four fluid particles, which in a interval of time of delta t have move to A prime, B prime, C prime, D prime.

And we had said that this change of relative position from A B C D to a general quadrilateral is a result of, rotational, rotational strain, shear strain and extensional strain will quantify these things and see how we can express in terms of velocity gradients in solve.

Let us consider rotational strain. What you mean by rotational strain is that diagonal of this quadrilateral initially is theta with respect to the horizontal. Now, it has become theta prime, and one can say that the rotational strain is theta prime minus theta is in indication of the rotational strain.

So, now, what is theta prime? We can say that if the bottom side has moved with respect to the horizontal through rotated through an angle delta alpha, and if the side A D has rotated in the clockwise direction by an element beta delta beta. Now, we always measure the angles in the counter clockwise direction, so, this delta beta which is actually in the clockwise direction.

This is in the clockwise direction. So, we should be putting it as minus delta beta. We can say that if there is an shift of, if delta alpha is positive in this direction, then theta prime would have gone a, in, in the counter clockwise direction.

And if delta beta positive delta beta, that is, if this side we have to come like this, that would also contribute to theta prime. So, we can see that theta prime minus theta is equal to half of delta alpha plus delta beta, that is, the shifting of the diagonal coming from the rotation of this side A D and this side A B, and we note that initially delta alpha is 0 and delta beta is 0. So, any change from 0 value of delta alpha to the new value as a reason because of the relative movement. So, we can say that rotational strain is theta prime minus theta it is given by this. Here, we measure these things both the angles in counter clockwise sense. We understand from this thing when, theta, delta beta is negative, like this one, we have identified this as negative.

The fact that side A D has rotated in this direction would reduce the rotational strain. In fact, it would make theta prime less than what it is because of the rotation in this direction. So, all the way we have put it as delta beta. In this thing, it is reduced because we have got minus delta beta. So, we can say that, in general, when delta alpha and delta beta are measured in the counter clockwise sense, then the rotational strain is given by theta prime minus theta which is half of delta alpha plus delta beta.

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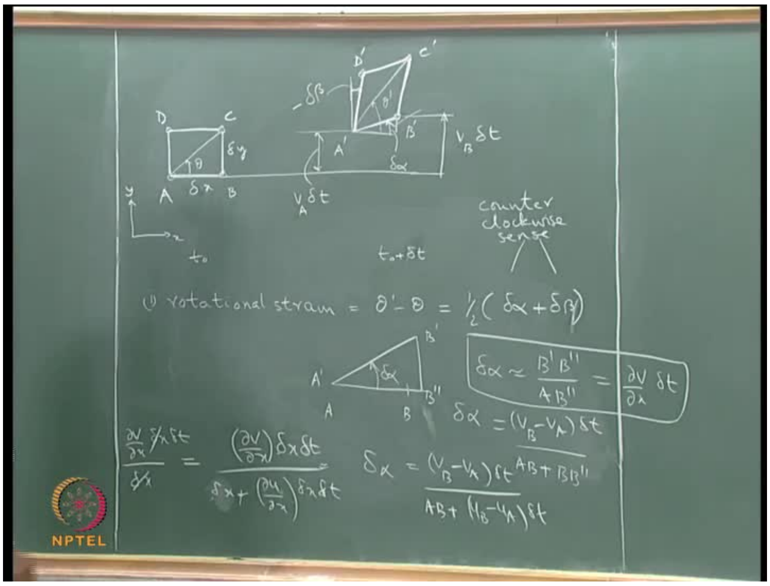

Now, what is delta alpha? So, let us magnify this bit here. This is originally this is A B, and now, this is A prime B prime. So, we have brought A prime and A to B the same thing because we are only looking at rotation, and this is our delta alpha. So, if you draw perpendicular here and call this as B double prime, then delta alpha is roughly equal to B prime B double prime divided by A B double prime. So, that is this distance divided by this distance, that is, A B prime is given roughly by this.

Now, B prime, this B prime B double prime is this distance here, this distance, and one can say that this height is equal to V A times delta t - where V A is the vertical velocity of point A - vertical velocity component - and this distance here is V B times delta t. Solve them, because particle b has a vertical velocity component V B which is greater than the vertical velocity component of point A. This has gone a higher vertical distance. That is why this B prime is relatively at a high position compared to A prime.

So, the difference between V B and V A has actually led to relative movement of B prime B prime in this. So, we can write this one as V B minus V A times delta t by A B prime, A B double prime, and what is A B double prime? A B double prime is nothing but the original distance that was there between these two which is A B. We have, let us say that this is point B; so, this is the original distance, this much plus this B B double prime, and what is B B double prime? It is the distance that this has B has travelled in the horizontal direction relative to A. So, we can write this as, we can write delta alpha as V B minus V A times delta t divided by A B plus U B minus U A times delta t.

So, now, we can write this thing as dou v by dou x times delta x delta t. This is the gradient of velocity; (Refer Slide Time: 08:17) vertical velocity in the x direction. So, that is times delta x will give us V B minus V A times delta t, and this is A B plus d u by d x times delta x delta t. So, and this is itself is delta x; A B is original delta x, and we can write this roughly as dou V by dou x times delta x delta t divided by delta x neglecting this thing in comparison with this because both are delta x is and this is a small time delta t which cancels out here, and finally, we get delta alpha to be equal to dou V by dou x times delta.

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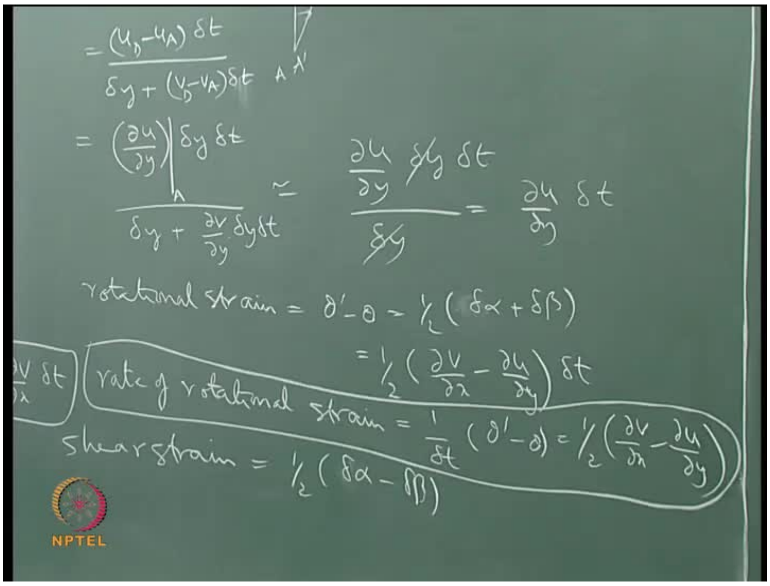

So, we can say that, and similarly, if you consider delta beta, if you want to get an expression for this, we are looking at A D which is the original side, and relative to this, this is our D prime and this is A prime. So, we can say that this is our delta beta minus delta beta. We draw a perpendicular to this, which is we call as d double prime and this is our D here, and we can write delta beta equal to D prime D double prime by A D plus D double prime.

The same way as what we have written here, and this distance is because A and D have different horizontal velocities; so, because of the difference in horizontal velocity, A has moved to A prime and D has moved D prime, and if they have the same velocity, then D prime would also be, at the same, in the same x velocity x position as D, but because of the horizontal velocities being different, this now a gap between the 2. So, we can write this as U D minus U A times delta t divided by delta y plus and this D prime D prime is again related to V D minus V A times delta y times delta t.

So, we can write using the same argument du by dy times delta y delta t; du by dy because particles D and A are separated by vertical distance; so, they variation is with respect to y here. So, we can say that dou u by dou y at A times delta y times delta t is this divided by delta y plus dou v by dou y times delta y delta t. Again, we neglect this as small compared to this; so, we can write this roughly as dou u by dou y times delta y delta t divided by dou y delta y. So, this this cancels out and we get dou u by dou y times delta t, and this is for minus delta b.

Now, we can say that the rotational strain which is theta prime minus theta, which is equal to half of delta alpha plus delta beta here, is now becoming half of dv by dx minus du by dy times delta t. So, the rate of rotational strain is this much strain has happened in a time of delta t. So, if we divide by delta t, we get this. So, this is 1 by delta t of theta prime minus theta. So, that is equal to half of dv by dx minus du by dy. So, the rate of rotation strain is expressible in terms of, velocity gradients, velocity gradients of, of a certain combination.

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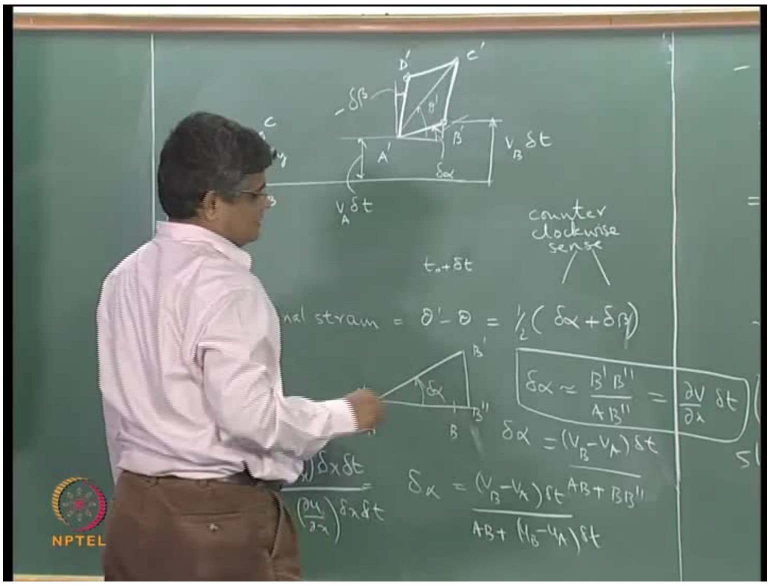

Now, let us consider the shear strain. We have said that this is equal to half of delta alpha minus delta beta, because delta beta supposed to be going in the counter clockwise direction and minus delta beta is coming in this direction.

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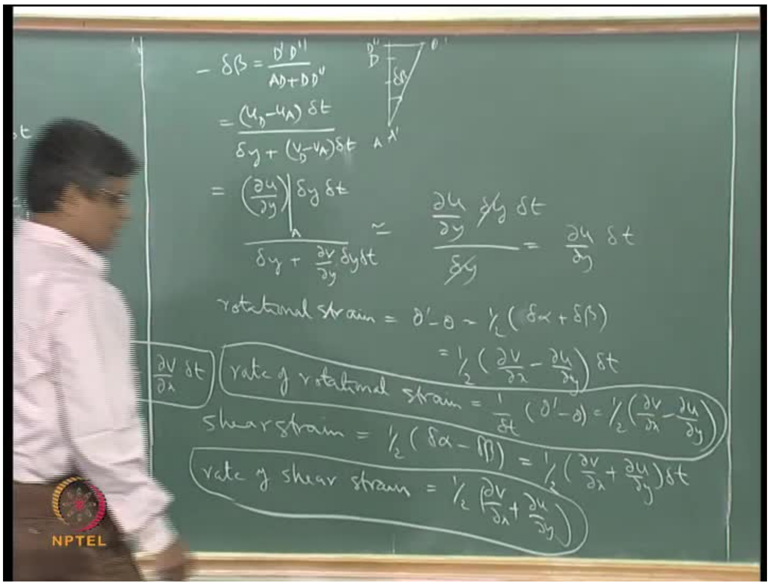

So, this is we already have evaluated this each term here, and this is equal to half of d v by d x plus d u by d y times delta t. Therefore, rate of shear strain is half of dou v by dou x plus dou u by dou y.

Again, we see our shear strain rate being expressed in terms of dou v by dou x and dou u by dou y, and this is a term that we recognize as this strain rate term which appears in the Newton’s law of viscosity - where tau is equal to mu times du by dy. You see that d u by d y term coming here. So, that is coming in the shear strain rate and also in the rotational strain rate in this way, in this particular way.

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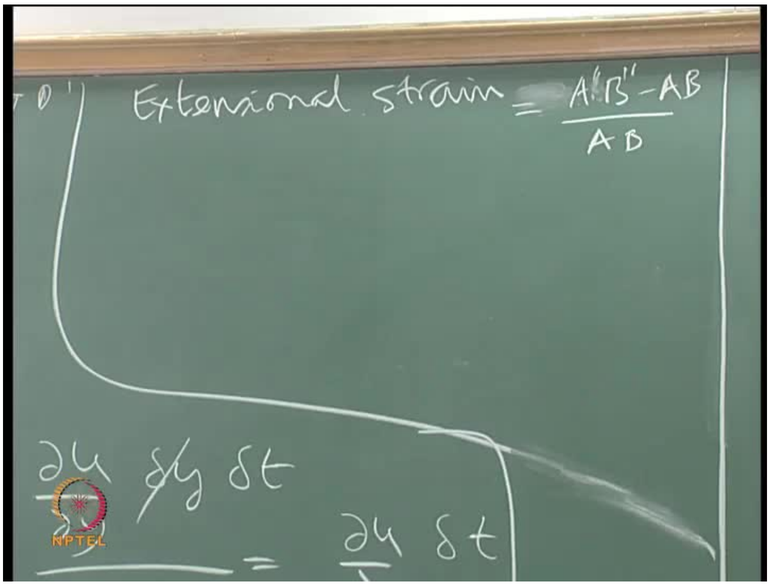

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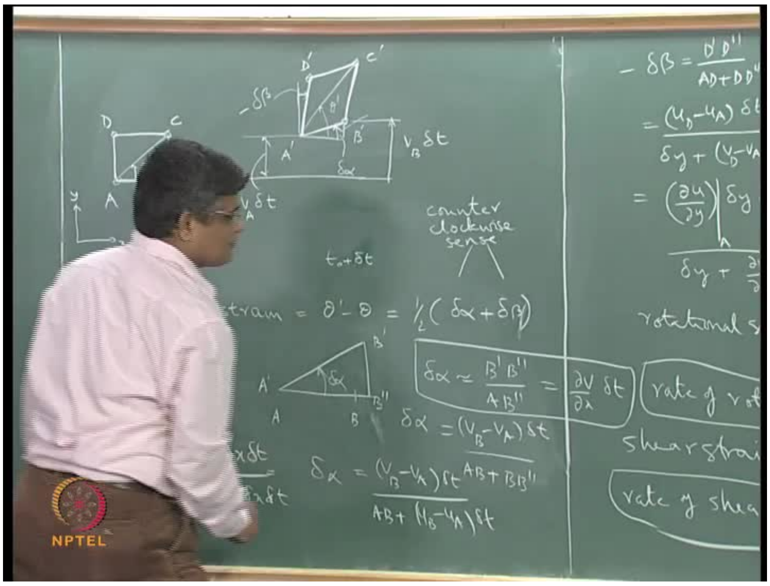

Now, we have the last one is the extensional strain. Let us see if we can do it here. This we said is the change in length divided by the total length. So, this is A prime B prime minus A B divided by A B, and we should be specifically considering this A prime B double prime which is horizontal velocities. So, this this particular thing is we have already evaluated as this A B double prime here, is A B plus B B prime and B B double prime and bottom double prime was what we said as u B minus u A times delta t.

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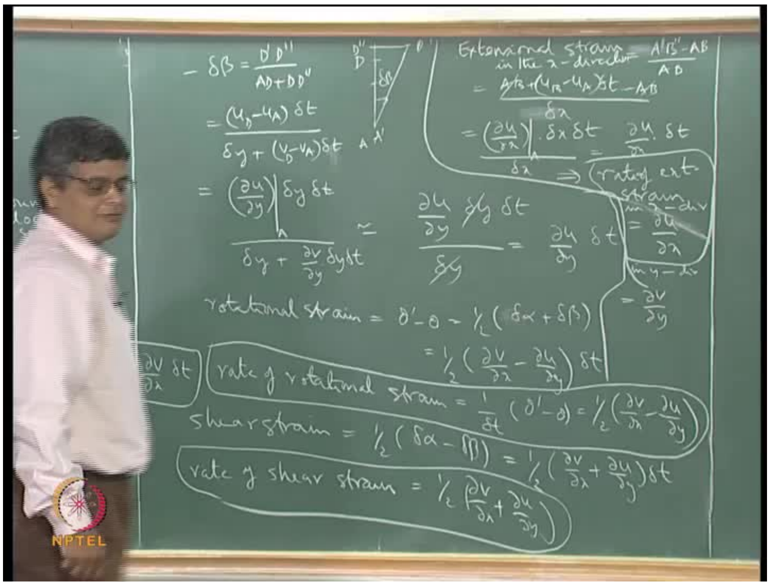

So, this particular thing is A B plus u B minus u A times delta t minus A B by A B. This and this cancels out, and this A B is nothing but delta x; so, we can write this as dou u by dou x at A times delta x times delta t divided by delta x. So this gives us dou u by dou x times delta t. So, the rate of extensional strain is this divided by delta t, so, that is dou u by dou x.

So, this is the rate of extensional strain in the x direction, and similarly, we can show that the rate of extensional strain in the y direction, and in y direction, this will be given by dou v by dou y. So, from this point of view, from this, what we can see is that the different strain rates, the rate of extensional strain, the rate of rotational strain and the rate of shear strain are all expressible in terms of the velocity gradients; velocity gradients which are of different kinds. In this particular equation dou u by dou x dou v by dou y here and this dou v by dou x minus dou u by dou y. These are in the same direction like the normal strains, and these are shear strains; these are shear gradients and this is minus here and plus here.

So, let us sense the various combinations of velocity gradients described the rates of strain that a fluid particle will be suffering as it is going through the flow, and our idea is to follow the analogy of solid mechanics, in which, we relate the stress to the strain in a linear way. We want to express a relation - linear relation - between stress and the strain rate.

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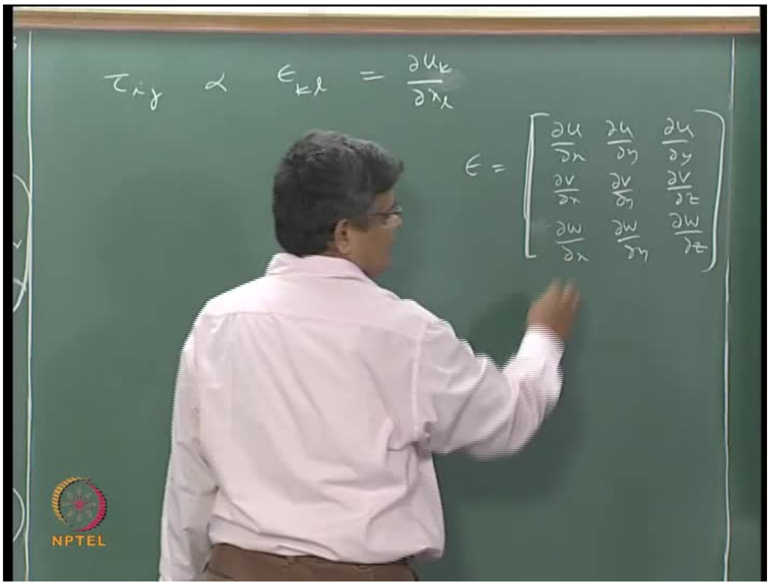

So, we would like to say that tau i j is proportional to epsilon k l - where this epsilon k l, this stress is, we know is tensor, and we make up a strain rate tensor deformation rate tensor which is nothing but dou u k by dou x l. So, the, the particular component of this strain rate tensor with index k l is given by U k component of the velocity and x l component of the direction. So, if you were to write this out, then this will be three components there and then three more.

These are all the different, different velocity gradients, and different components of shear and strain and shear rotation and extension are different combinations of these velocity gradients. So, we put everything here together and we make a linear relation between the shear stress, the viscous stress and the strain rate as a general expression.

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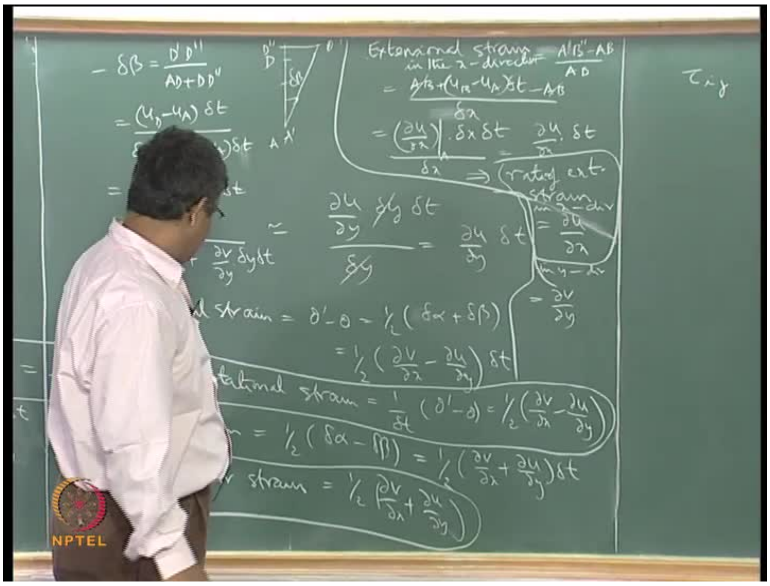

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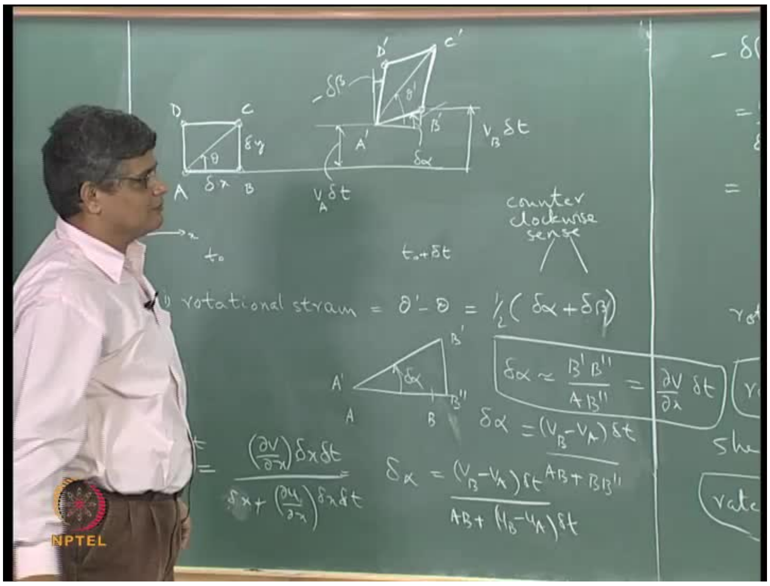

So, we would like to capture what we are saying is that the strain that the fluid element is suffering over a particular time delta t can be at attributed to some stresses acting on these phase, such that, the resultant will be distortion in this particular way.

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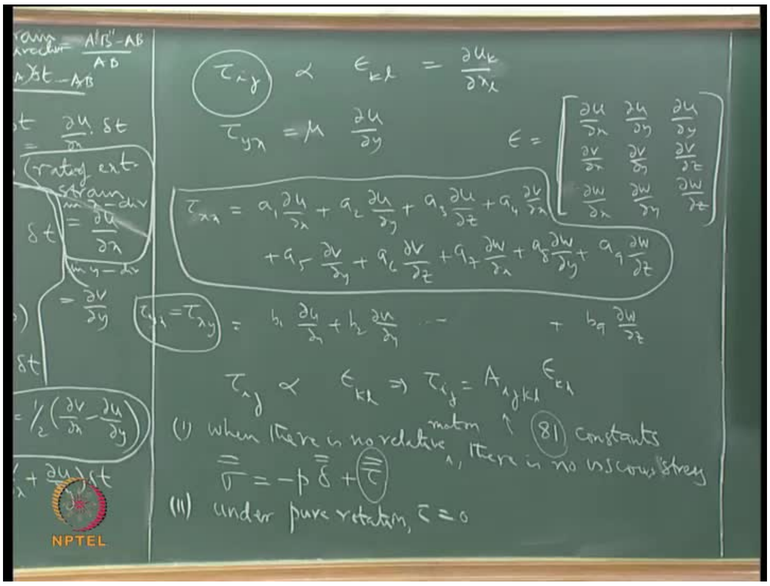

And because these are all because of various components of the deformation like shear or rotation and extension are linear combinations of these velocity gradients. We can seek a general relation between stress and a strain rate tensor which is defined like this. So, we would like to say that a generalization of the Newton's law of viscosity where shear is equal to is proportional to the velocity gradient.

Can now be express in a general frame work as a stress component is proportional to the velocity gradient component in this particular way. Now, what will be the proportionality constant? If you are looking at a single component tau y x, then if you want to say that this is proportional to a single component like dou u by dou y, then you can say that this equal to mu dau u by dau y.

This is the case where you have a single non-zero shear stress and a single non-zero velocity gradient, but in the general case, we do not have a single non-zero shear stress and we do not have only one of this being non-zero. So, in the general case, all these can be non-zero, and if you are expressing a general relation, one particular component of the shear stress may be expressed as a linear combination of all of them. For example, if you, if you want to put a linearity relation between the 2, we should be able to say that, for example, tau I will just put as x x 1 component is equal to some constant a 1 times dou u by dou x plus a 2 times dou u by dou y plus a 3 times dou u by dou z plus a 4 times dou v by dou x plus a 5 times dou v by dou y plus a 6 times dou v by dou z plus a 7 times dou w by dou x plus a 8 times dou w by dou y plus a 9 times dou w by dou z.

This I would say is a general expression for this component being linearly proportional to all possible combinations of this strain rate stress tensor components, and we should be similarly writing tau x y is equal to b 1 times dou u by dou x plus b 2 times dou u by dou y so on plus b 9 times dou w by dou z and so on like this for all the 9 components of this stress tensor.

So, if you want to express a relation, a general relation between tau i j as being proportional to epsilon k l, then we have to express this as tau i j equal to some a i j k l times epsilon k l - where this is a matrix with 81 constants, and we can see where the 81 constants coming. For example, the first element of this has 9 constants a 1 to a 9 and the second element has from b 1 to b 9, that is, 9 more components.

So, generalize the expression between shear stress and the strain rate here will involve 81 constants, not a single constant view. So, this is a requirement in case we want to make it the most general component, the general relation, and how do we get these 81 constants? These are properties of the fluid just as in the simplest case mu is a property of a fluid. This is a property of fluids and we to determine all these 81 constants empirically, only then we can claim that we have a relation like this.

That is difficult and it is also not necessary. It can be reduced to much simpler level by noting certain properties of the stress tensor the strain rate tensor. For example, we have all the nine components of this stress tensor that the stress tensor is symmetric. If you apply the principle of angular momentum conservation, then we can show that the stress tensor is symmetric, and therefore, instead of having nine independent components in this, we will have only six components.

So, this tau x y is actually is also equal to tau y x. So, instead of having, 6, 9 equations like this, we will have only six equations. So, that reduces the number of Equations here. Instead of 81 constants, we should be having few number of constants, and the other thing is that while stress tensor is symmetric, the strain rate tensor here is not necessarily symmetric.

But we can make certain assumptions. We can make the assumption that under hydro static conditions, that is, when there is no directive motion, when, when there is no relative motion, then the stress part is equal to 0. See if you were to, there is no viscous stress, so that means that we can break up the stress that we have been using, in, in this into two components - 1 is minus p, that is, the pressure the hydro static pressure times delta which is chronicle delta plus this tau, and this tau is the viscous tensor which is equal to 0 when there is no relative motion. So, under pure hydro static conditions, the stress tensor, that is, appear in the sigma consist only of the three normal stresses normal compressive stresses in the three directions and that this is equal to 0.

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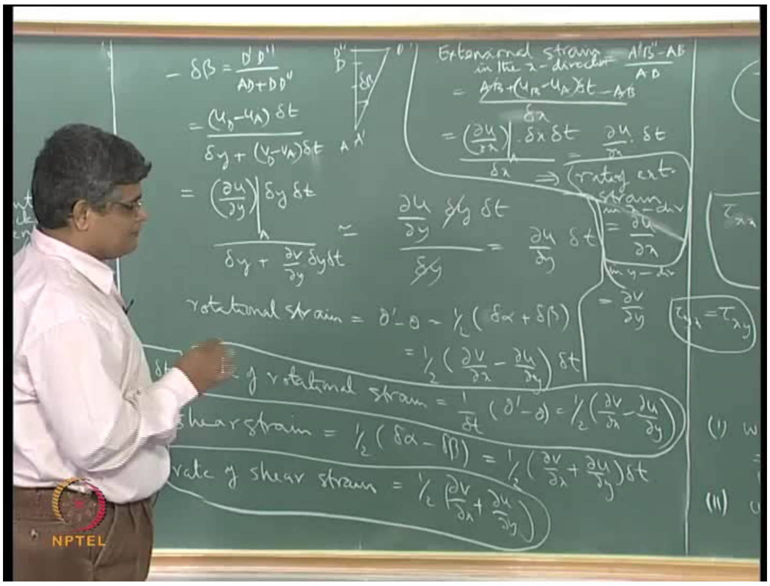

And we also make the second assumption that under pure rotation, the stress is equal to 0. So, that means that when we have only rotational strain, then there is no stress coming out of that. So, even though the velocity gradients are non-zero in this particular case, the strain rate there is no stress coming from that.

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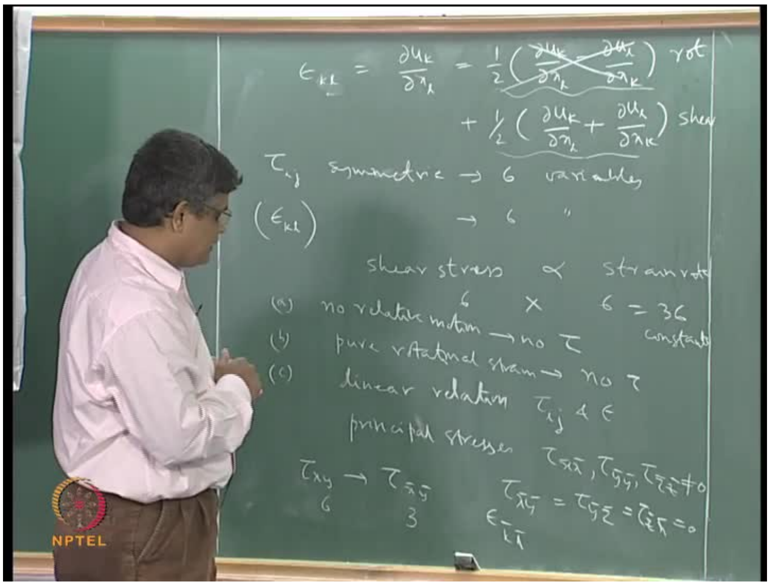

And what this implies is that relations - linear relations - among these things with a negative sign here are not allowed. Again, write epsilon i j epsilon k l as dou u k by dou x l. Then we can also write this as half of dou u k by dou x l minus dou u l by dou x k plus half of dou u k by dou x l plus dou u l by dou x k. If you do it, what we are writing here is that this is the typical rotational stain rate part, and this is shear rate, shear strain rate of a part.

Now, if you say that pure rotational strain rate does not come in to picture, does not cause this stress, then this particular combination is will not come in to picture and we have only a combination like this that comes in to picture, and this is symmetric; so that means that if you interchange l with k, you get the same thing, whereas here, because of minus sign, you do not get you. If this, this becomes, this is anti-symmetric; this is the symmetric component.

So, we can say that only the symmetric component the strain rate tensor will come in to the linear relation between stress and strain rate, and this allows us to say that just as the shear stress component has a symmetric. Therefore, you have six variables in this. Even the epsilon k l component which is defined now in this, in this way, will also have 6 strain rate components in this.

So that the relation between shear stress as being proportional to the strain rate will involve only six components and six components here, so that it will have only 36 constants.

By making the assumption that rotational strain - pure rotational strain - does not cause any shear stress. We can make the whole relation between shear stress and shear rate being among symmetric tensors so that we have only 36 constants, but even 36 constants is quite tedious to find and it is very it is very difficult to find experimentally, and this we would like to reduce it to much smaller level and we make the third observation. So, first observation is that no relative motion, no shear stress; the second is pure rotation, pure rotational strain no tau and the third is that a linear relation between tau and epsilon, and which when is between two symmetric tensors will mean that. Now, this is in general coordinate frame of i and j.

So, any tensor can be converted into a principles coordinate frame, in which, out of the 3, out of only tau x x tau y y and tau z z in the new coordinate frame x bar y bar z bar are non-zero and tau x bar y bar equal to tau y bar z bar equal to tau z bar x bar are 0. So, this is the principle stress directions. Instead of having x y z, we can do a transformation. For example, using the Eigen values and in to a new coordinate frame x bar y bar z bar, in which, all the half normal stresses, the shear stresses will be 0. So, and only non, only normal stresses are non-zero. So, in such a case, the, instead of having six stresses here, will have only three stresses; so, only three non-zero stresses, and so, from tau x y, if we go to tau x bar y bar, this has six components and this has three components, and similarly, the strain rate tensor are here.

This, in this, in the form of general x y z. Even that can be converted in to the the principle strains and that will also have only three components: epsilon k bar l bar in that. So, a general relation that we are seeking now is not in terms of among six variables and six variables here. It is in terms of three variables here and three nonzero variables here. So, underlying this, is the, is the requirement that the relation between the shear stress and strain rate that we are seeking here is in variant to coordinate frame rotation.

If you rotate from x y z to x bar y bar z bar, then you can get into the principle coordinate frame here and also for the shear rate and this rotation would not affect the cost ends that are involved in the linear relation.

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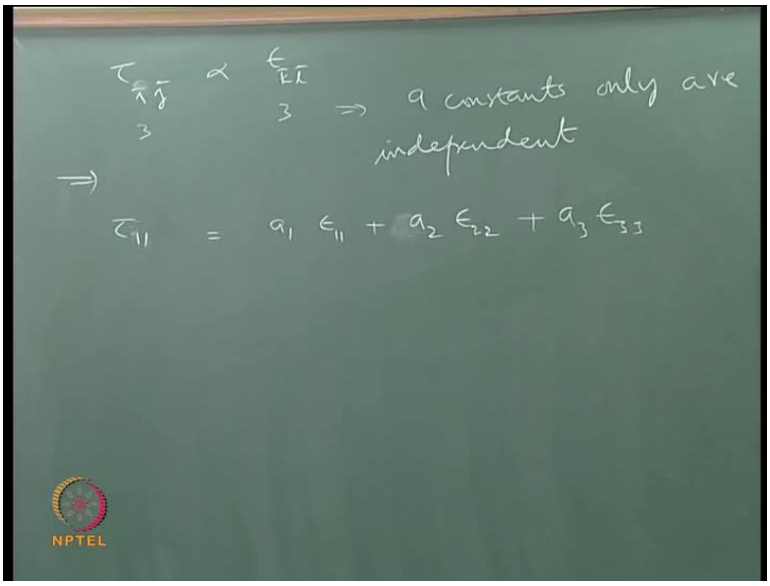

So, that kind of mathematical invariants means that the relation between stress and strain rate involving 36 constants should actually have only three times 3 9 constants. Now, if you see the same relation in terms of tau i bar j bar as being proportional to k bar l bar, then this has three components and this has three components, so that is three constants must be there.

So, only nine independent constants exist. If we want the principle that the relation - the linear relation - is invariant to coordinate transformation, which is usually a requirement for any constitutive relation, so these nine constants, for example, can be if you were looking at tau x x is expressed in terms of a 1 epsilon 11 plus epsilon a 2 epsilon 22, let we put also as 11.

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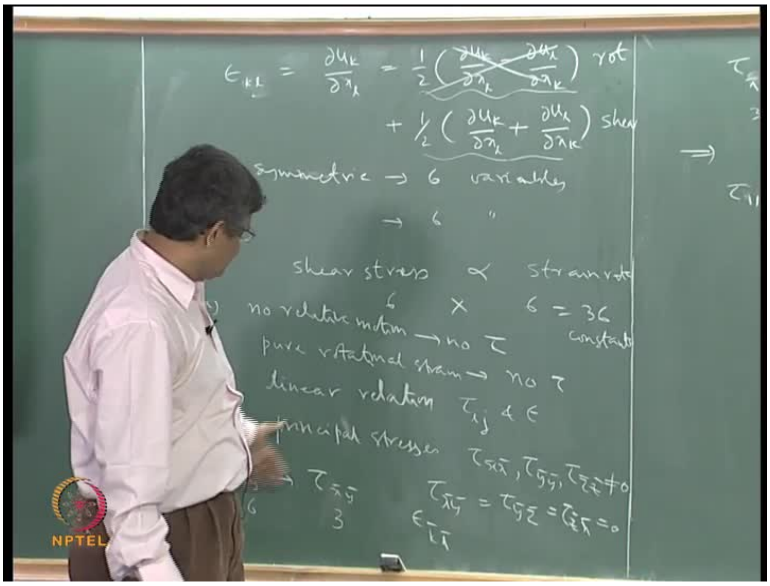

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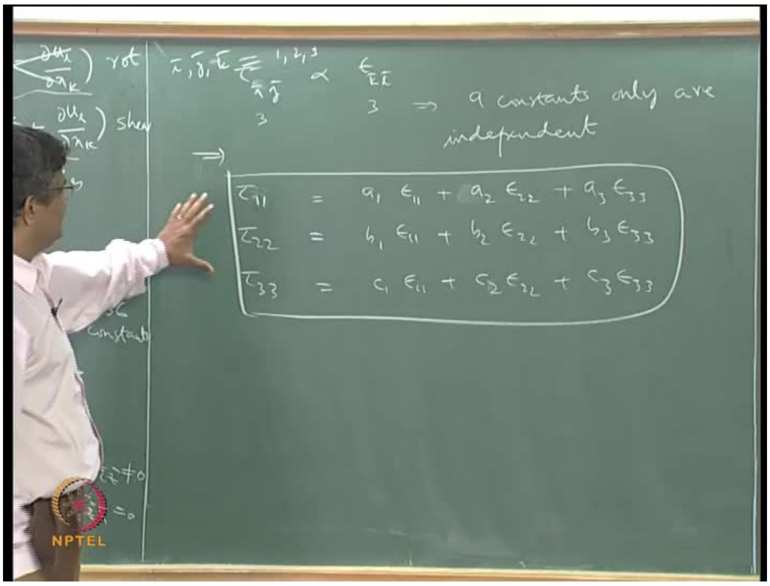

We should also realize that if you are seeking a linear relation between these stress tensors, then it is necessary that the rotation that we give to the stress tensor, that is, the principle axis for the stress tensor and the strain rate tensor must coincide. So, the 1 2 3 directions of the i j k, if we say that these are 1 2 3 directions, these are three principle directions, they must be the same for this stress as well as the strain rate tensor.

So, we can say that tau 1 1 is now a is a linear function of these three non-zero strain rates. So, and the general expression will be that this is proportional to with a proportionally constant of a 1 and a 2 and a 3, and similarly, tau 2 2 is b 1 epsilon 1 1 plus b 2 epsilon 2 2 plus b 3 epsilon 3 3 and tau 3 3 is c 1 epsilon 1 1 plus c 2 epsilon 2 2 plus c 3 epsilon 3 3 here. We have three principle stresses being proportional to the three principle strain rates involving nine constants - three for a, three for b, three for c.

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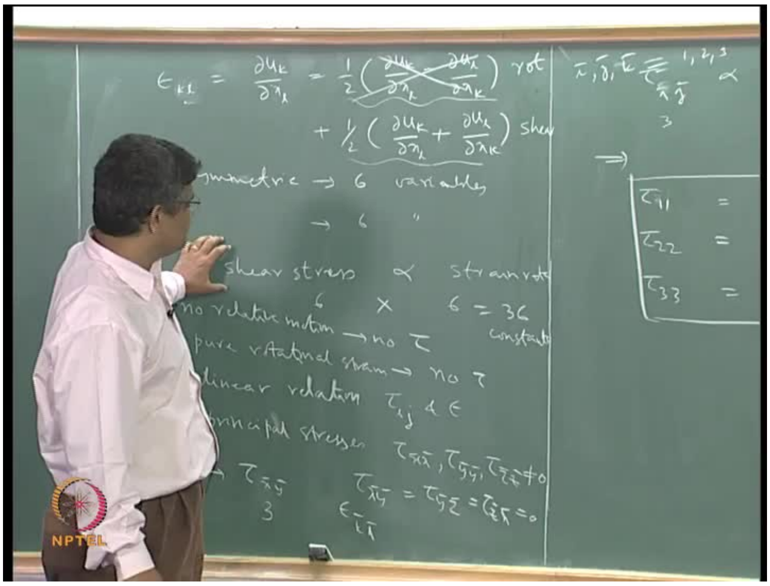

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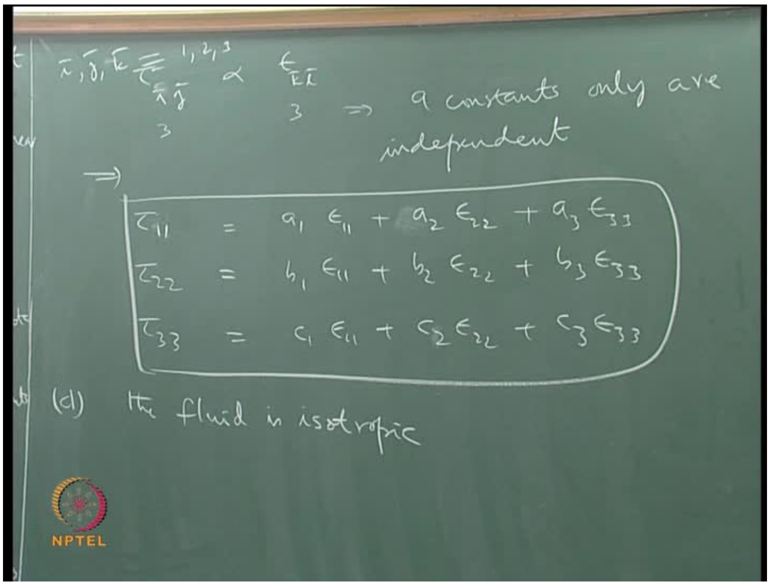

So, this is what we mean by a coordinate transformation involving two symmetric tensors being mathematically invariant, in which, there are only 9 independent constants that are permissible. Now, at this point, we introduce the forth condition that we have the fluid is isotropic. When we say an isotropic fluid, then it is fluid which gives us same stress versus strain relation or strain rate relation, no matter in which direction, the, that is relation is same in all directions. We can take an example of a paper to illustrate this. Let us take an extreme view. This is a paper and I tear it along. So, if I tear it, then I am applying a shear stress because of which is tearing and this is corresponding deformation and you can see that it is tearing like this. If the paper is truly isotropic, I should get the same sort of tearing ability in this direction also.

And I can see that here it is coming out very neatly, whereas, here it is not coming out like that. See it is coming out in a different direction. So, this is an example, of a, of a, of an anisotropic material, in which, a stress produces different strain depending in which direction supplied. If you have a co-screened newspaper, like may be a cheap newspaper, then if it is rolled in a certain way, then that changes the structure of the material, and in such a case, it is, it is normally easier to tear it along the grain boundaries rather than across the grain boundaries. So, if you are the material that you are talking about is constituted in such a way that it has some intrinsic fault lines intrinsic strength pattern and intrinsic orientation of changes in all that.

In such a case, you can expect forces applied in certain directions to produce certain strains and appeared in a different direction to produce different amount of strains. So, such a material is not isotropic material, but if it is a truly isotropic material, like a fine grant paper will be high quality paper, if you try to tear it, it tears uniformly in all directions, and this one may be a cheap quality paper may not tear in all in the same way in all directions.

So, we are talking about the fluid being isotropic so that it exhibits a same relation for strain verses strain rate verses stress, and if you have a fluid which is for example of polymeric fluid with long chain molecules, which are oriented in a particular direction, then in such a case you can except because of the orientation of chains of the polymeric molecules, you could get different strain verses stress relation in different directions. So, in such a case, such a fluid contain long chain long chain poly polymeric molecules may be an anisotropic it may not be isotropic, but typical fluids like air and water, which has, which are not polymeric, and will, which are, which contain very simple molecules like that without any preferred orientation, exhibit the condition of isotropic.

So, under isotropic conditions, if you were to change the orientation of the stress, and then it will look, if you were to look at the resulting strain rate along with these principle axis directions, then you find that that would be invariant.

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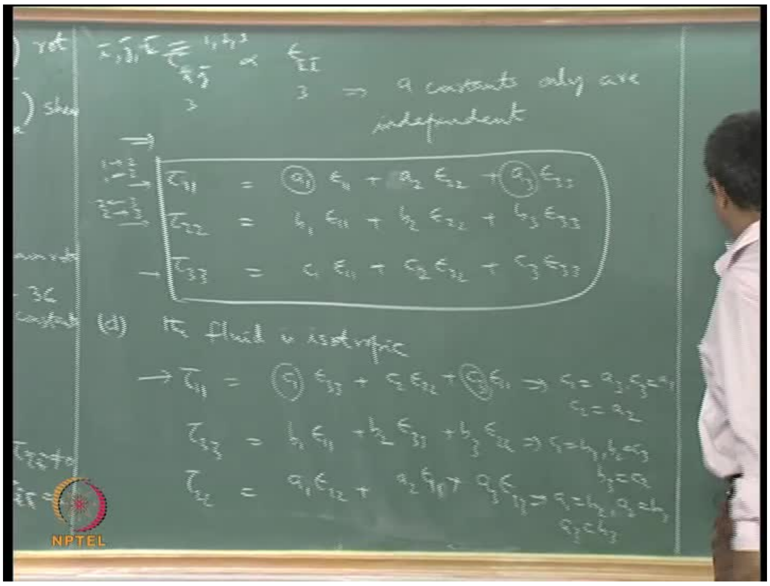

So, under those conditions, for example, if you were to substitute between these two, instead of 1, we make it 3, and instead of 3, we make it 1. So, if you do that here, then this relation gives us tau 1 1. We are making 3 to 1 and 1 to 3.

It is c 1 times epsilon 3 3 plus c 2 times epsilon 2 2. This 1 1 has become 3 3 plus c 3 times epsilon 1 1. Now, if you were to compare this relation with this relation, because this is a stress applied in this. So, now, you can see that 3 3 is coming here, and these two to be the same c 1 must be equal to a 3 and c 3 here must be equal to a 1. So, this means that c 1 is equal to a 3 and c 3 is equal to a 1 and c 2 is equal to a 2.

Now, let us do the shift between 2 and 3 2 to 3 and 3 to 2 in this equation. So, this will give us tau 3 3 equal to b 1 epsilon 1 1 - 1 is unchanged - plus b 2 epsilon 3 3 plus b 3 epsilon 2 2. Now, you compare this tau 3 3 with what you are getting here. This means that c 1 must be equal to b 1 and b 2 is equal to c c 3 and b 3 is equal to c 2.

Now, let us do in this change between 1 to 2 and 2 to 1. So, this will give us tau 2 2 is equal to a 1 epsilon 2 2 plus a 2 epsilon 3 1 1 plus a 3 epsilon 3 3. If you compare this with this, you get a 1 equal to b 2 and a 2 equal to b 1, and finally, a 3 equal to b 3. Now, if we compare this c 1 is equal to a 3, so we can just summarize this, and a 3 is already equal to b 3 here.

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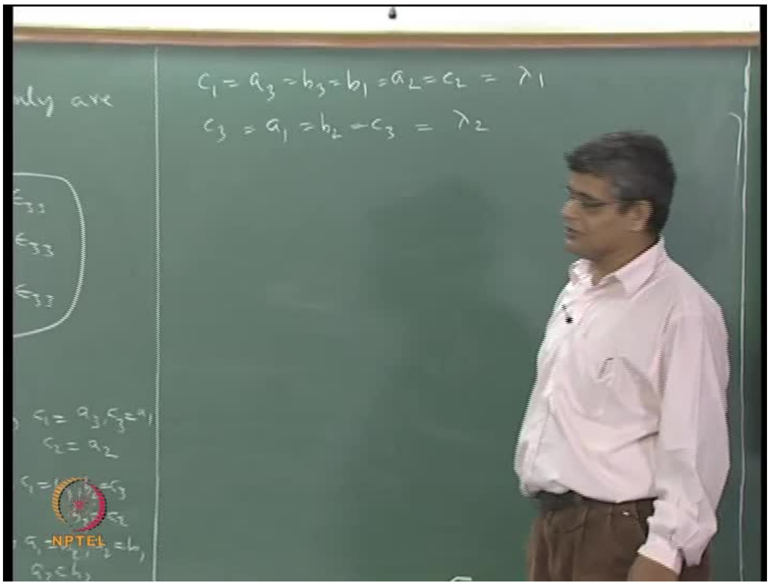

And c 1 is already equal to b 1 here and b 1 is equal to a 2 and b 3 equal to c 2. So, let us call this as sum lambda 1 and we have c 1 a 3, and let us see. Let us look at c 2. Now, let us see where is c 3; c 3 is equal to a 1 is not appearing here and a 1 is equal to b 2; b 2 is equal to c 3 and c 3 is equal to a 1. So, this is, that is, it, so we have lambda 2. So, out of these nine coefficients here, six of them are all equal and you call that lambda 1 and the other three are all equal again with respect to and you can call this as lambda 2.

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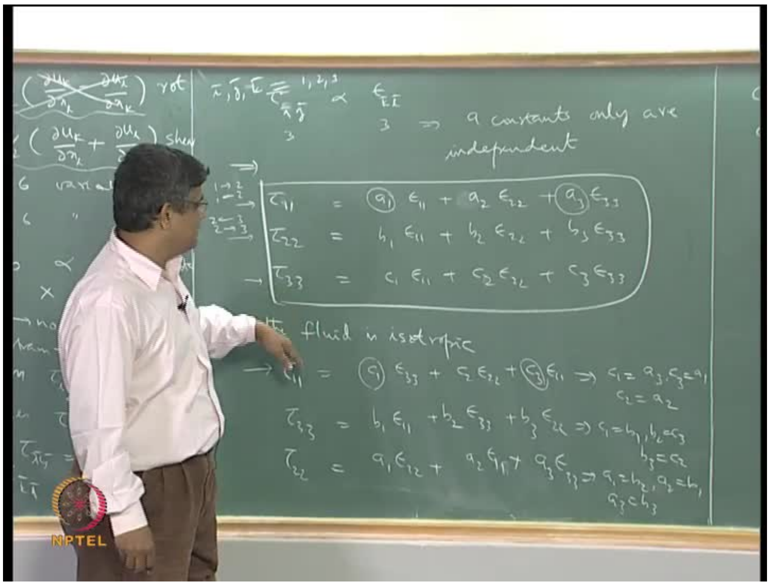

So, that means that once you apply the condition of isotropy, the nine independent constants become only two independent constants. So, if you want to have a relation between principle stresses and principle strain rates which is linear and which exhibits a condition of isotropy, which is a property of the fluid, then instead of having nine constants, we can have only two independent constants.

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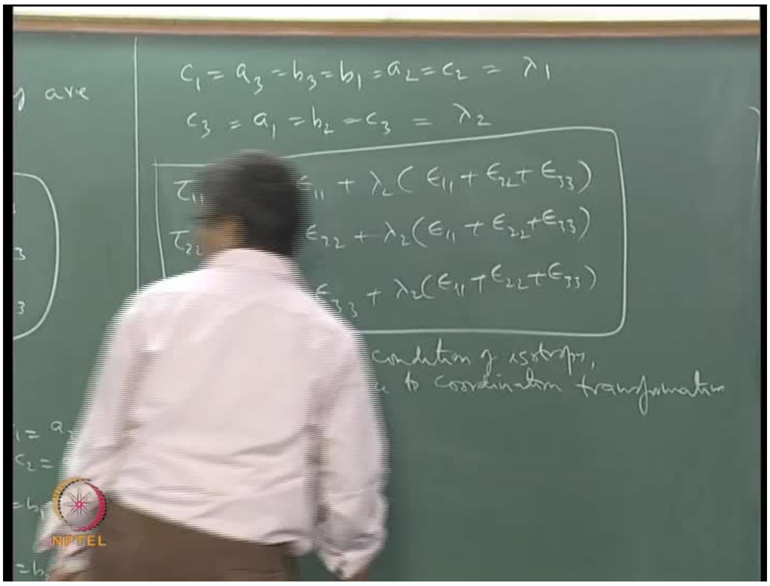

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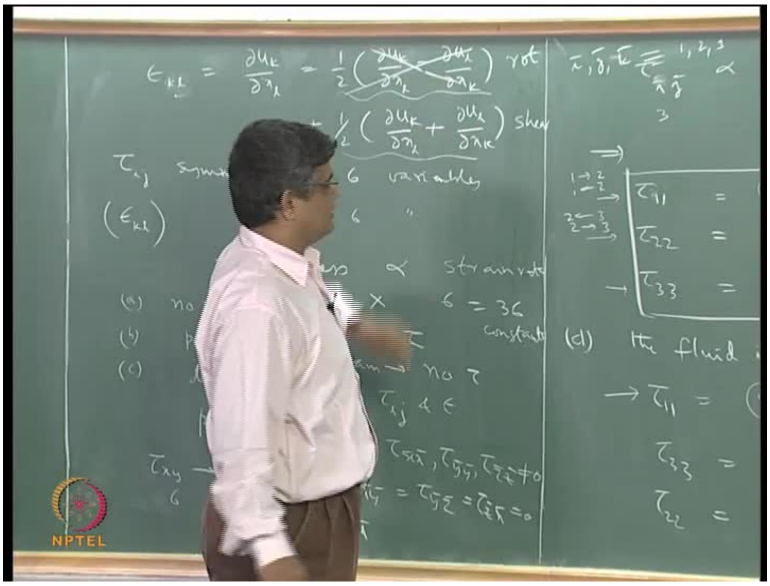

Using this, for example, you can have tau 1 1 equal to lambda 1 times epsilon 1 1 plus lambda 2 times epsilon 1 1 plus epsilon 2 2 plus epsilon 3 3 tau 2 2 is equal to lambda 1 times epsilon 2 2 plus lambda 2 times epsilon 1 1 plus epsilon 2 2 plus epsilon 3 3 and tau 3 3 is equal to lambda 1 times epsilon 3 3 plus lambda 2 times epsilon 1 1 plus epsilon 2 2 plus epsilon 3 3.

So, this expression, these relations here embed a linear relation between stress and a strain rate, in which, we are not forcing all the stress components to be the same, and a linear relation which involves only two constants lambda 1 and lambda 2. So, in that sense, this is a reformulation of this. In the form of two independent constants - lambda 1 and lambda 2, which obeys the condition of isotropy, so this is obeys the condition of isotropy and invariance to coordinate transformation as well as symmetry of the strain rate in the sense that it only allows, this kind of, this kind of relation, not this kind of relation, which is anti-symmetric part, in which, the anti-symmetric part does not come into picture.

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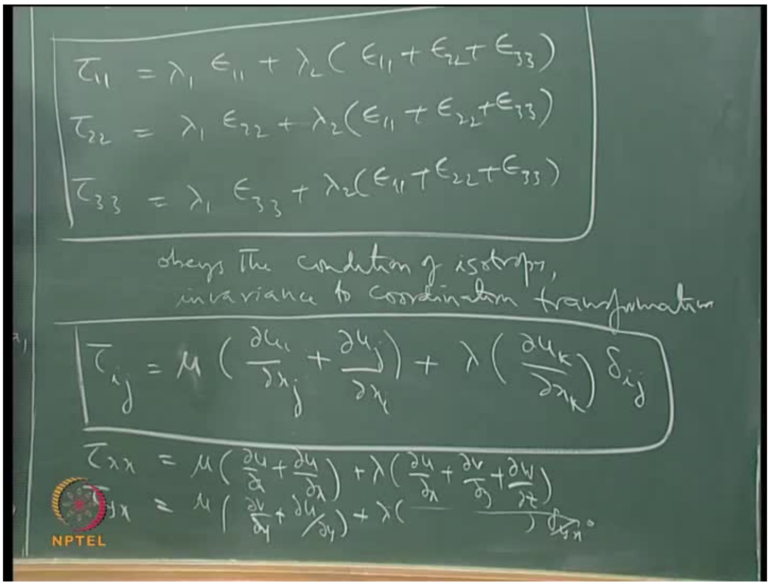

So, that this is a kind of relation between which is generic, which involves all the three-dimensional strain rates and three-dimensional stresses and this can be expressed in the general coordinates as tau i j is equal to half of d u i by d x j plus d u j by d x i plus times lambda 1 mu times this plus lambda times dou u k by dou x k.

So, this is a general relation for the shear stress and the strain rates involving two constants, mu which is our familiar dynamic viscosity, which we call as dynamic viscosity and lambda which is called the second coefficient of viscosity and you can see that this is a term with repeated index. So, this is sum of all the three normal strain rates which is same as what we have here and this is a chronicle delta.

So, this relation is a is a general three-dimensional relation between linear relation between stress and the strains rate involving two independent constants which obeys the condition of isotropy and mathematical invariants coordinate transformation.

So, using this, for example, we can now write tau x x as u times i is now x and j is also x. So, this will be dou u by dou x plus dou u by dou x plus lambda times dou u by dou x plus dou v by dou y plus dou w by dou z times delta. When i and j are the same, then this is equal to 1, so it gives us like this, and tau y x is mu times, so i is equal to y and j is equal to x or we can say i is equal to 2 and j is equal to 1. So, this will be dou v by dou x plus dou u by dou y plus lambda times this whole thing times delta y x is 0. So, this comes out as only this thing. So, this term will not be there.

So, we can using this expression, we can write down the individual components in terms of the velocity gradients and in terms of two coefficients - u and mu and lambda. So, this gives us the additional relations that we are seeking the constitutive relations and we can write the momentum balance in equation in this way.

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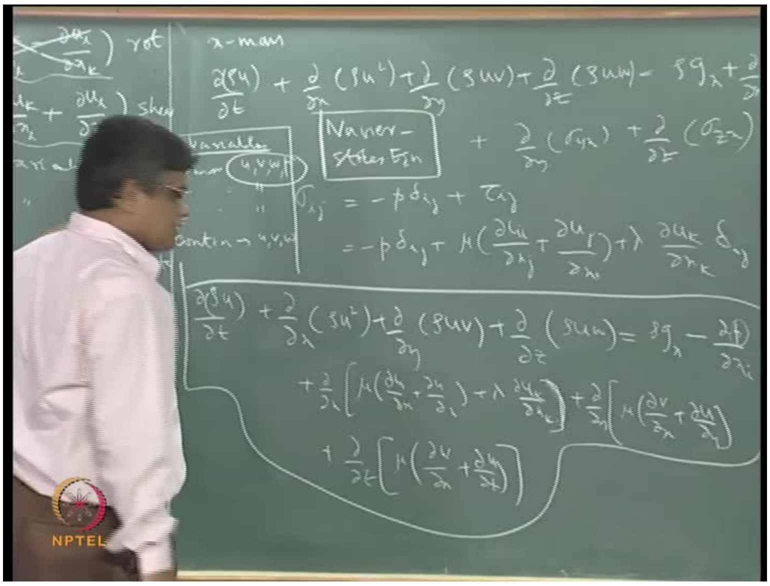

We have the x momentum balance written as dou rho u by dt plus dou rho u square by dx plus dou rho u v by dy plus dou rho u w by dt minus rho g x plus dou sigma x x by dx plus dou sigma y x by dou y plus dou sigma z x plus dou t. This is how we have written, but in the course of trying to express this sigma in such of coordinate, in such of constitute relation, we have put sigma i j to be minus p delta i j plus tau i j and we have tau got an expression for tau i j in terms of just like this. So, this is equal to minus p delta i j plus mu times dou u i by dou x j plus dou u j by dou x i plus lambda times dou u k by dou x k times lambda z.

So, this is what we have to substitute for each of the sigma x x and y x and z x, and using this, we can write the momentum balance equation the x momentum as rho g x, and here we have dou by dou x of this is appearing only in the sigma x x; sigma y x this will be 0 and sigma z x will be 0. So, only dou by dou x of minus p will come here. So, that this is minus dou p by dou x I, that is, this term.

And this particular term will come into all these things dou by dou x of sigma x x tau x x is written here plus dou by dou y of sigma y x is mu dou v by dou x plus dou u by dou y plus dou by dou z of mu times sigma z x will be mu times dou w by dou x plus dou u by dou z.

So, this is the x momentum balance equation. We can also making use of this expression. Write down the momentum balance equation in y direction z direction, and what we observed from this is that this equation here contains only one new variable P and of course, it has u. So, mu and lambda are material properties and rho is also material property. So, in the x momentum equation, if you consider the variables in the x momentum equation, we have u v w and p; u is here; v is here; w is here and p is here; rho and g and all are specified here. (Refer Slide Time: 1:01:10)

In the y momentum equation also will have the same four quantities; z momentum equation will also have the same four quantities, and in the continuity equation, you will have u v w; so, that means that the total number of variables is just for and we have four equations - x momentum equation, y momentum equation, z momentum equation and the continuity equation. So, with this modeling, with this constitute modeling for the stresses that are arising out of fluid motion. We are able to come up with an overall scheme of equations, in which, there are four equations and four variables. These together are called navier stokes equations. These are the equations which govern the fluid flow which determine the fluid flow but only subject to the condition that rotational strain does not give rest to stress.

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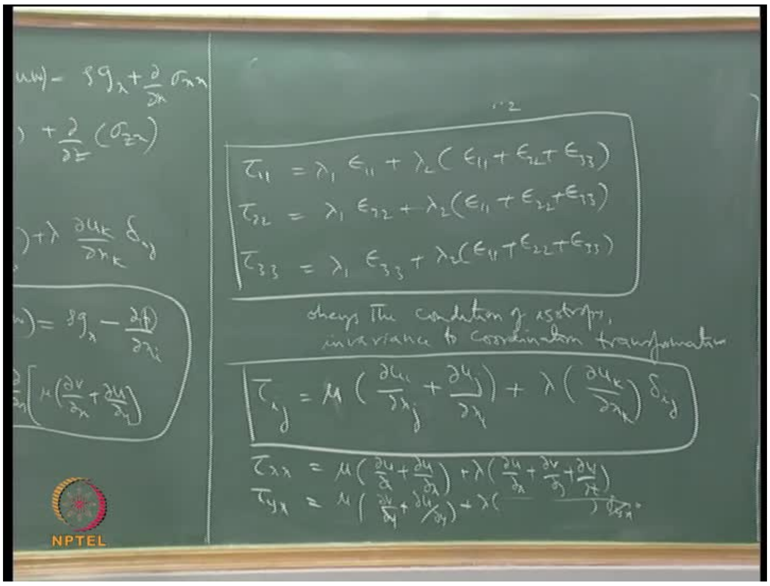

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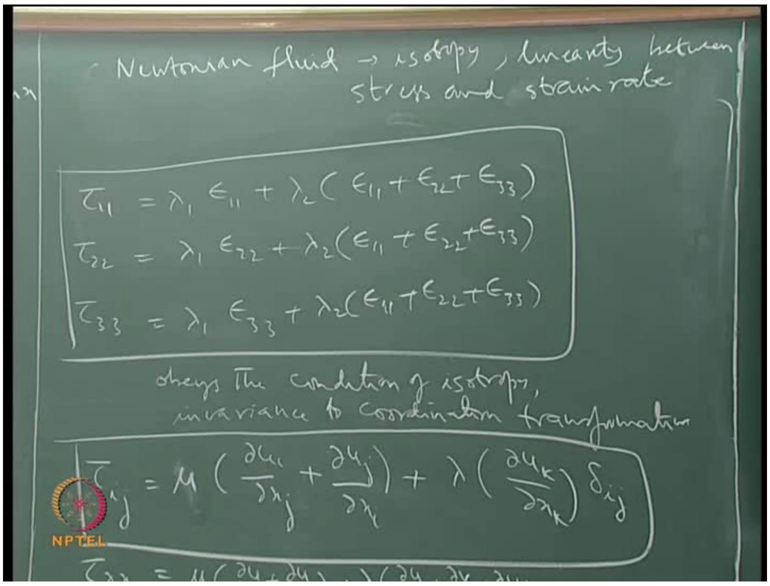

And that the fluid that we are considering is isotropic and that the relation between stress and strain rate is linear involving certain constants. So, and that kind of fluid, which obeys these conditions of isotropy and linearity between stress and shear stress shear rate is called a Newtonian fluid. So, a Newtonian fluid is one which obeys isotropic condition and linearity between stress and the strain rate and which also does not cause any stress under pure rotations strain.

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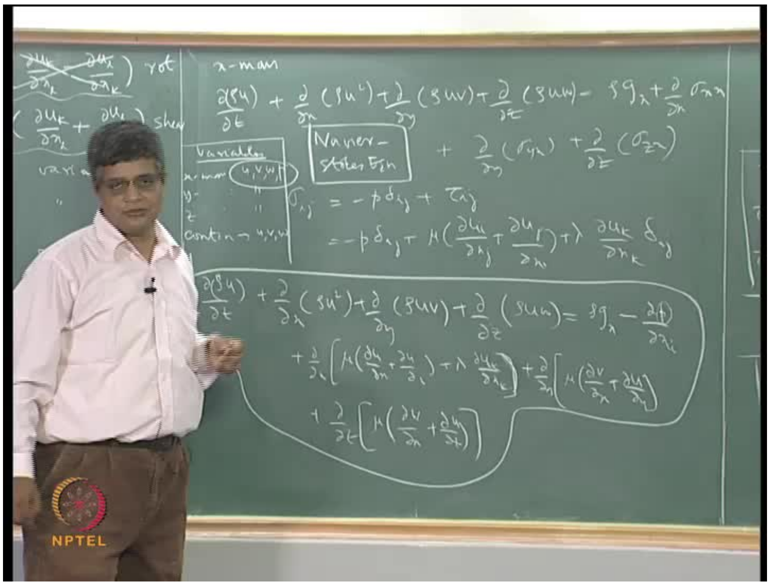

So, for a Newtonian fluid, the governing equations are navier stocks equations and we have seen the example of the x momentum equation. In the next lecture, we will write down the full set of equations which are valid for the navier stokes equations, and common fluids like air and water are Newtonian fluids, but there are many other common fluids, like for example, the blood which flows through our bodies and may be sugar solutions syrups which are, non, not Newtonian fluids. So, for such thing, for such fluid, this relation is no longer valid and the navier stocks equation are also not valid. You will have to have different kind of constitute relation which expresses the shear verses strain relation.

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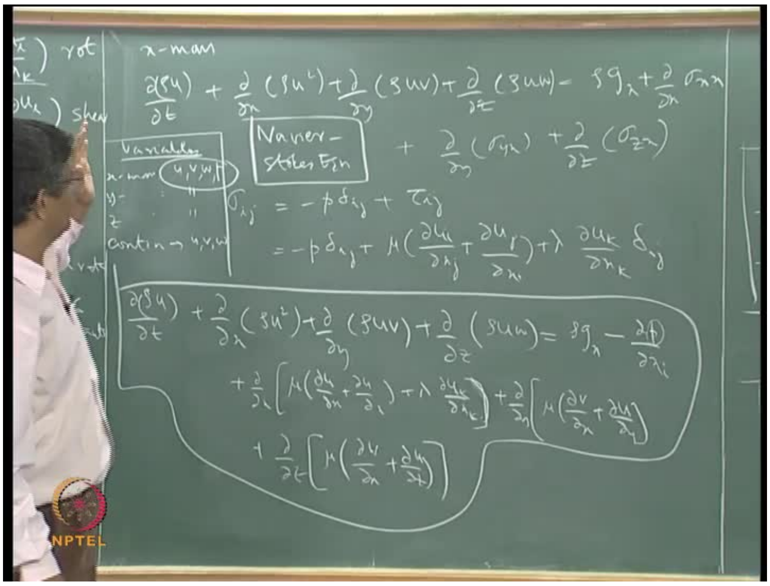

And there are power-law fluids, Bingham plastic models and then dilatant fluids and viscous elastic fluids which have even more complicated constitute relation between a stress and strain rate. These are different types of constitute equations which finally go into determination of the stress, which is appearing in the momentum equation as one of the external forces. So, if you have a description of the stress which is arising out of fluid motion in terms of computed variables, then we can have ultimately a situation where we have equal number of knows and un-knows and the problem becomes mathematically close problem, and for that kind of close problem for Neutronian fluid is the set of navier stokes equations.

So, that is what we have today.