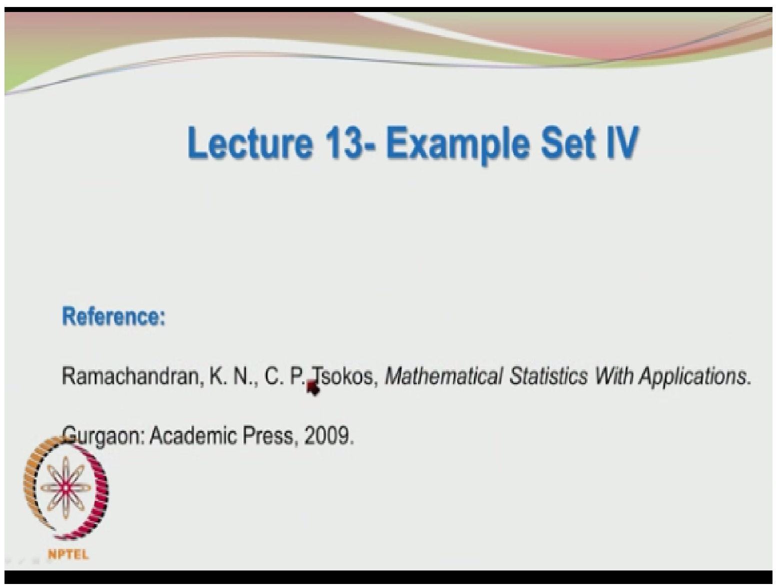
Statistics for Experimentalists
Prof. Kannan. A
Department of Chemical Engineering
Indian Institute of Technology – Madras

Lecture – 14
Example Set – IV Part A

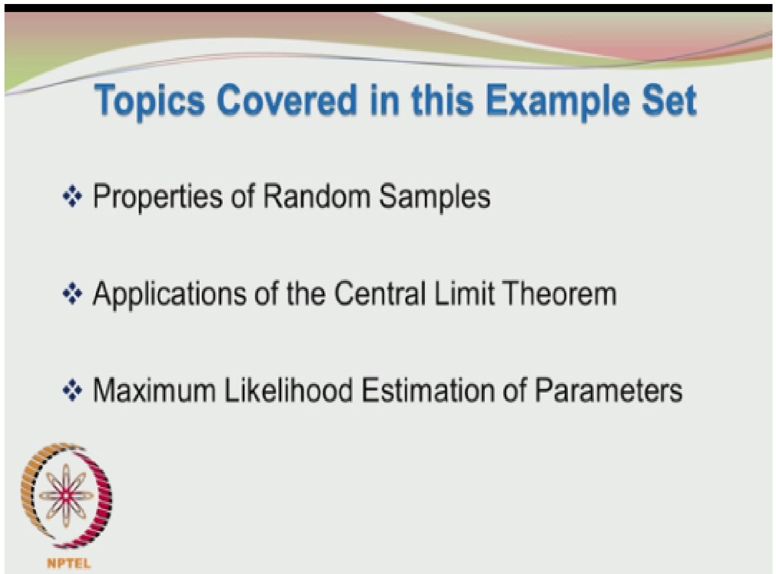
Hello, welcome back in today's lecture, we will be solving the few problems. The reference I used for solving one of problems is the book written by Ramachandran and Tsokos, mathematical statistics with applications, academic press, published in 2009. It has an interesting set of both examples and problems. So, the topics covered in this example set are properties of random samples, applications of the central limit theorem and maximum likelihood estimation of the parameters and also the method of moments.

(Refer Slide Time: 00:21)



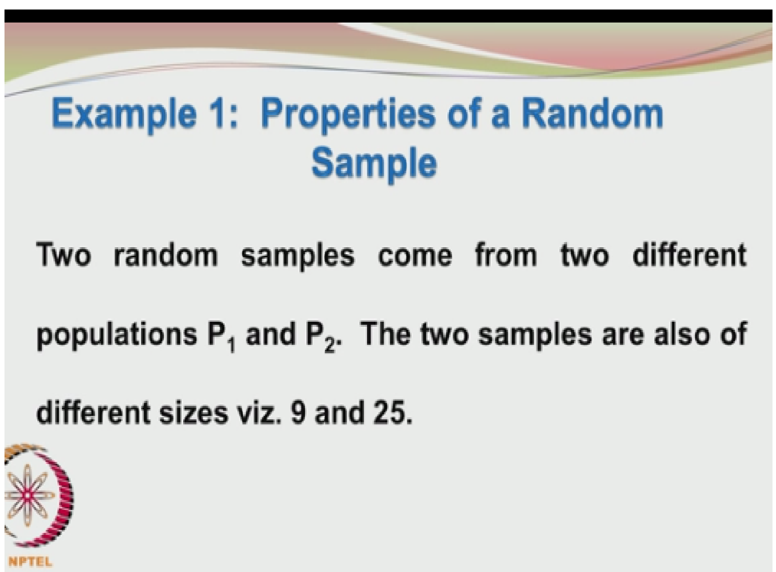
The first example is 2 random samples come from 2 different populations P1 and P2. The 2 samples are also of different sizes 9 and 25. If the 2 samples distributions however are to have the same standard deviation, what should be the ratio of their respective population standard deviations? So, we are asked to find the ratio of the population standard deviations such that the 2 unequal size to samples have the same standard deviation.

(Refer Slide Time: 00:41)



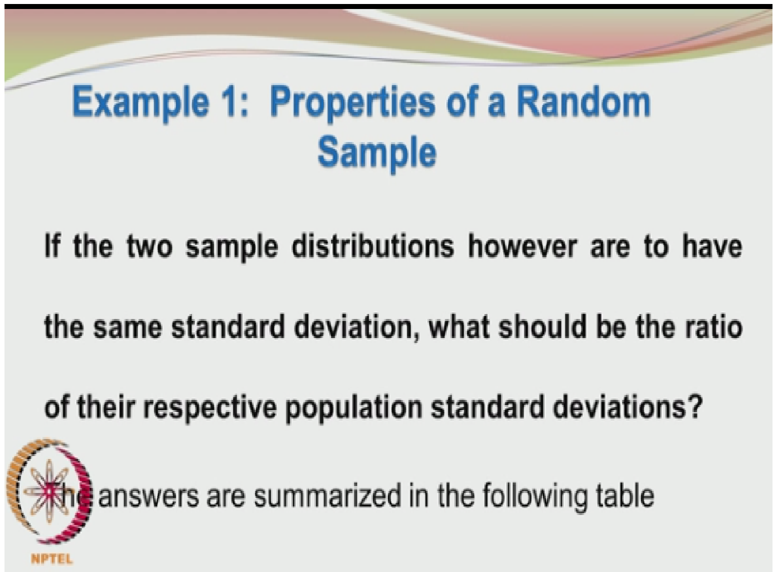
So, depending upon the size of the sample you can have different sampling distributions. You also know that the sampling distributions of the mean are centered around the population parameter mu itself, but have a lesser spread given by sigma square/n, there sigma square is the variance of the population from which the random sample was taken n is the size of the sample taken. So, using this information we can do the following.

(Refer Slide Time: 00:55)



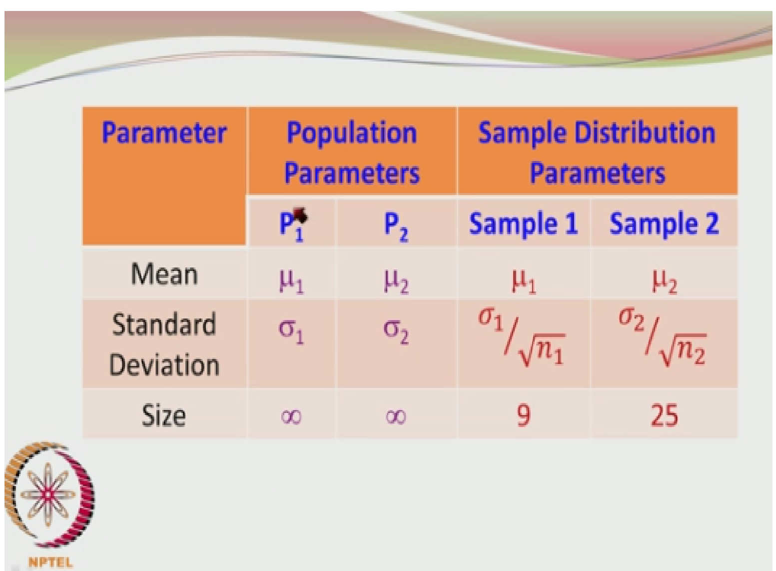
I have given a table here. In this table, you can see the population parameters listed, P1, P2, mu1, mu2, the 2 population means sigma1, sigma2, the 2 population standard deviations and of course the population would hypothetically comprise of infinite size or very, very large size and when you go to the sample, again the sample probable distribution of the means will have a mean value of mu1 and mu2 for sample1 and sample2 corresponding to the 2 populations from which they were taken.

(Refer Slide Time: 01:10)



Standard deviations sigma1/root n1, sigma2/root n2 and what should be the ratio of the sigma1/sigma2 such that these 2 are equal. So, the question is very simple. So, sigma1/root n1 = sigma2/root n2 and then, we have sigma1/root n1, root n1 would be root of 9, so that is not difficult to get sigma1/3, sigma2/n2. What is n2? 25. Again that is easy to get, root of 25 was 5. So, you have sigma2/sigma1 is 1.67.

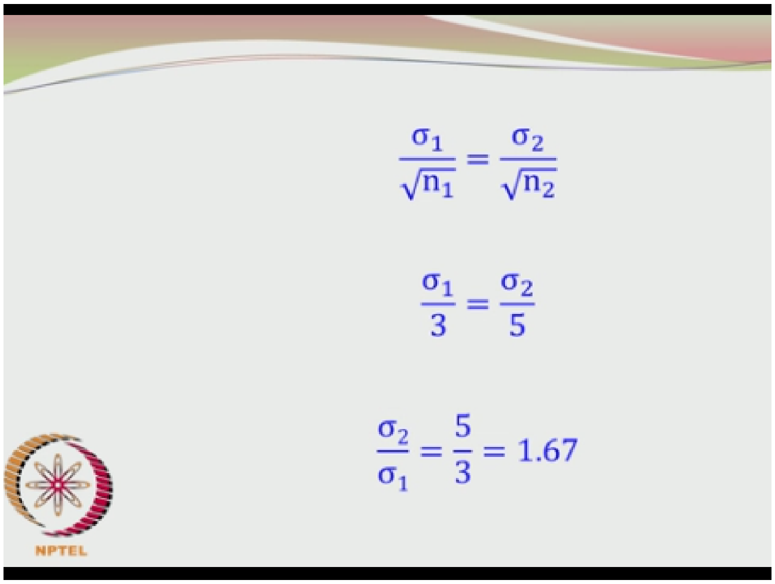
(Refer Slide Time: 02:41)



Parameter	Population Parameters		Sample Distribution Parameters	
	P_1	P_2	Sample 1	Sample 2
Mean	μ_1	μ_2	μ_1	μ_2
Standard Deviation	σ_1	σ_2	$\sigma_1/\sqrt{n_1}$	$\sigma_2/\sqrt{n_2}$
Size	∞	∞	9	25

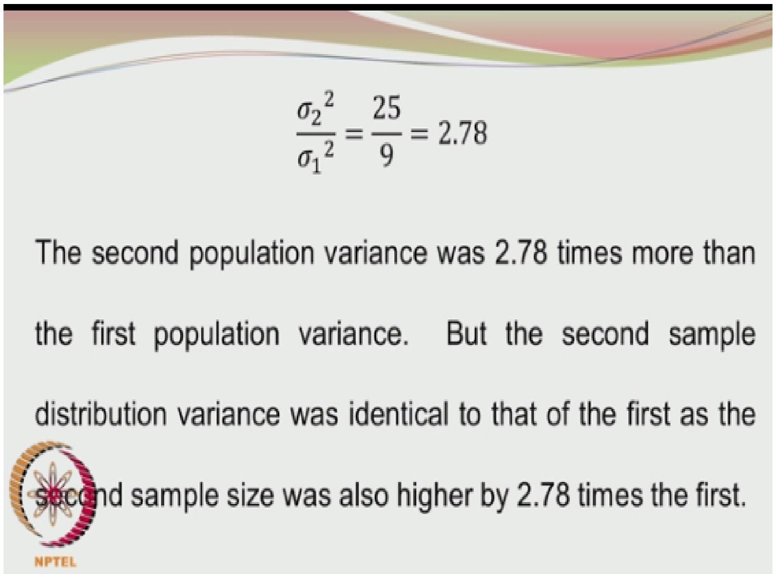
And sigma2 square/sigma1 square, the ratio of the 2 population variances would be 25/9, which is 2.78 rather than doing the mental mathematics, let us do with the calculator 25/9 that is 2.777 so on. So, you can truncate it to 2.78. So, the second population variance was 2.78 times more than the first population variance. But the second sample distribution was identical to that of the first as the second sample size was also higher by 2.78 times the first.

(Refer Slide Time: 03:50)



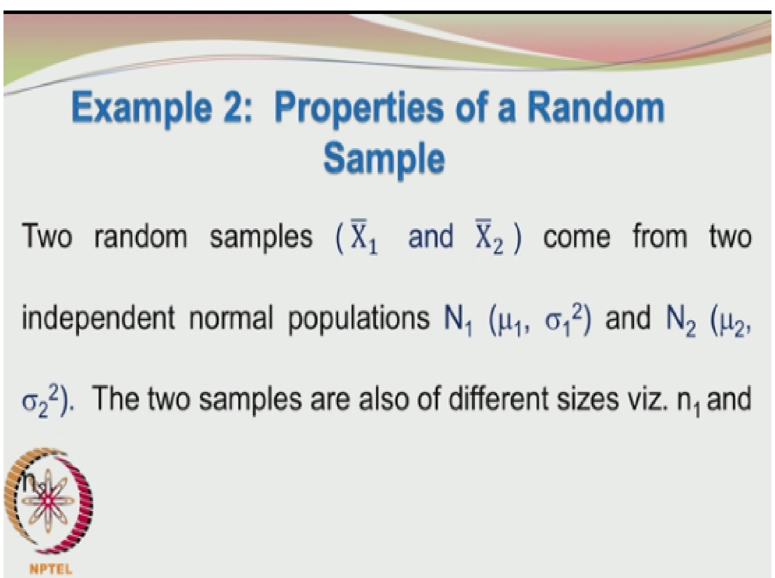
So, when you normalize the variances of the 2 different populations by the sample sizes taken. In this case, we were equal because we sample size taken from the second population was higher than the first. So this sort of balanced out the higher variance of the second population, okay. Let us go to the next example again the simple example you have 2 random samples, X1 bar and X2 bar, they come from 2 independent normal populations N1, mu1, sigma1 square and N2 mu2 sigma2 square.

(Refer Slide Time: 04:18)



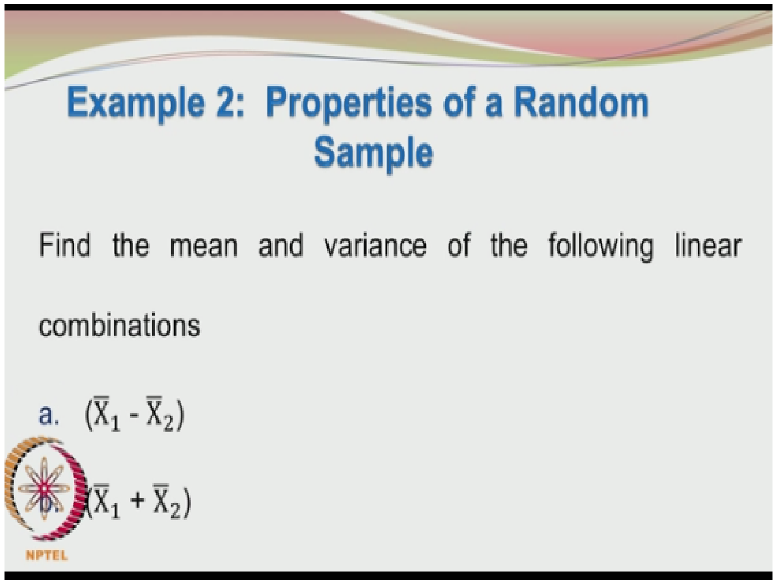
The 2 samples are also of different sizes namely n1 and n2. So find the mean and variance of the following linear combinations X1 bar - X2 bar, then X1 bar + X2 bar. Very nicely, the problem statement gives us all we require. It says that the 2 parent populations are normal and they are also independent of one another. So, when you take a random sample out of these 2 populations, we have to get the random sample means that is easy.

(Refer Slide Time: 05:36)



So, you will have X1 + X2 + so on to Xn divided by n and again X2 would be from the second population. Again, you add up all the attributes or values of the random sample elements and then divided by that particular sample size. So, you will get sample mean 1 and then you will also get sample mean 2. The important result is, suppose you take random variables X1, X2, they come from independent normally distributed populations.

(Refer Slide Time: 06:02)

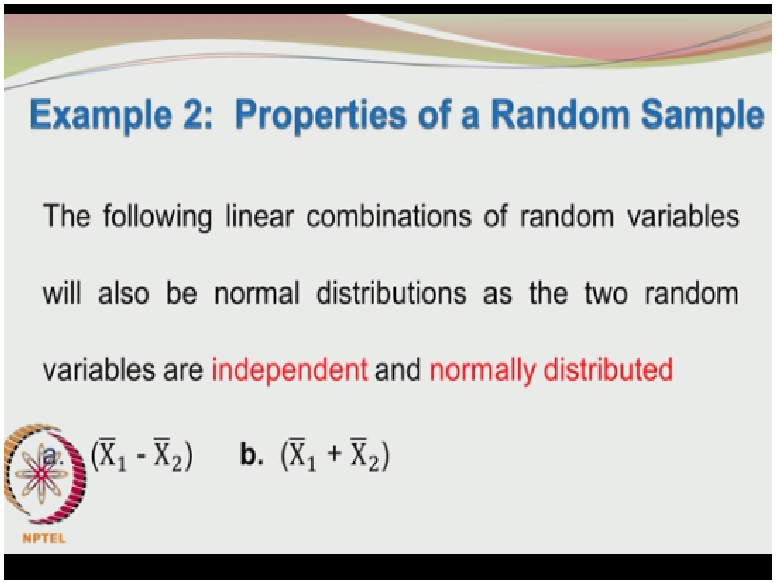


Then, a linear combination of X1 and X2 would also be a normal distribution that is an important result. Now, we are having X1 bar and X2 bar. X1 bar in turn is formed by taking the elements of the first sample adding all the attributes of those sample elements divide it by the sample size. Similarly, do for the second random sample. So, now you are going to combine these 2. So, rather than thinking of them as X, all the elements divided by n1.

Then, all the elements of the second random sample divided by n2. You think of X1 bar and X2 bar as random variables themselves and they are coming from 2 independent populations. So, the distributions of X1 and X2 are independent of each other and if you think on these lines, it is easier to proceed further. So, now you have to find mean and variance of the 2 linear combinations.

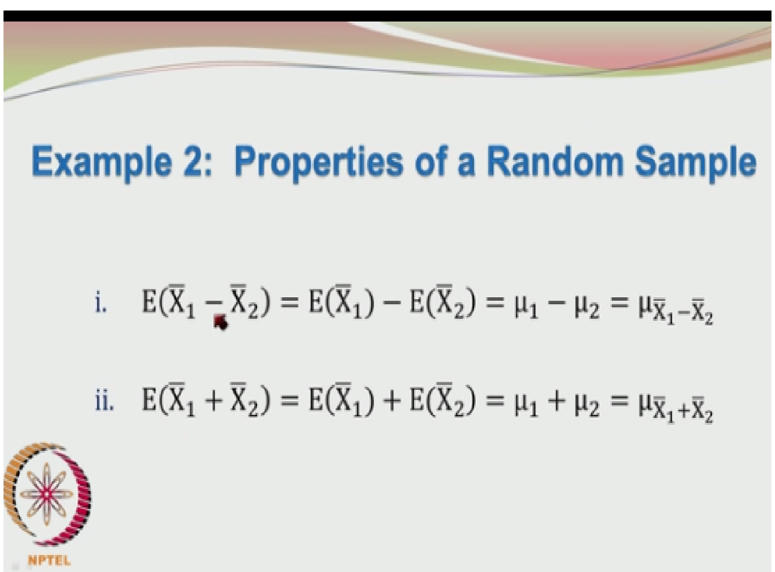
Why I gave this example as we encounter such crises very frequently even when different kinds of problems, okay. So, the following linear combinations of random variables will also be normal distributions as the 2 random variables are independent and normally distributed. So, these would also be normal distributions. So, this would be one normal distribution. This would be another normal distribution.

(Refer Slide Time: 08:54)



What are the mean and variances of such normal distributions for the 2 cases? So, the expected value for X1 bar - X2 bar would be expected value of X1 bar – expected value of X2 bar that would be mu1 - mu2 and that is represented as mu of X1 bar – X2 bar, mu of the probability distribution formed by X1 bar – X2 bar. Again, you have expected value of X1 bar + X2 bar that would be expected value of X1 bar + expected value of X2 bar that = mu1 + mu2, which is represented by mu of X1 bar + X2 bar.

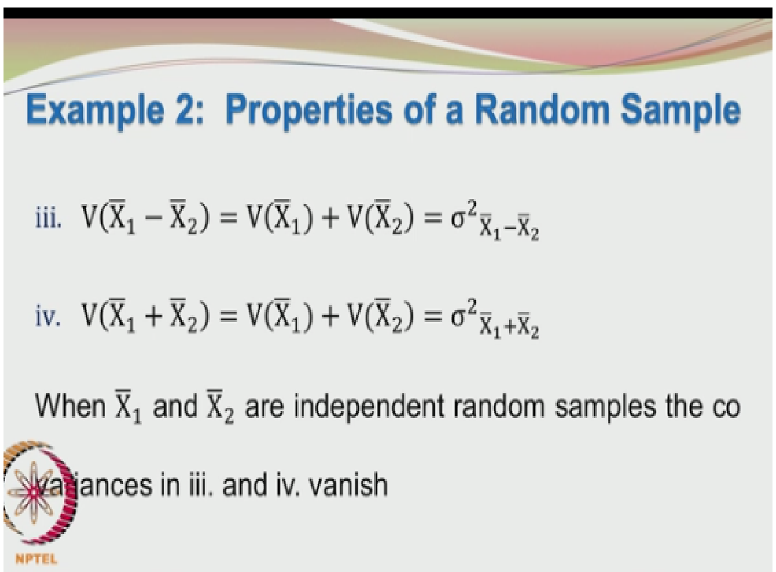
(Refer Slide Time: 09:28)



So, the linear combinations of the probability distribution of X1 and X2 bar would also result in a normal distribution which is centered at mu1 – mu2. Well, you can ask what will happen if mu1 > mu2, no problem, it is a positive value. If mu1 < mu2, it is a negative value. So what, let the resulting probability distribution be centered on a negative value, there is no harm in that. So, again if you look at expected value of X1 bar + X2 bar that would be E of X1 + E of X2, which is mu1 + mu2.

So, when I am taking the linear combination of independent random variables which are normally distributed, I am going to get a resulting probability distribution which is also normally distributed and having the mean at the some of the means of the 2 probability distributions, I am adding. So, this is again quite straight forward. Let us look at the variance, the variance is quite interesting.

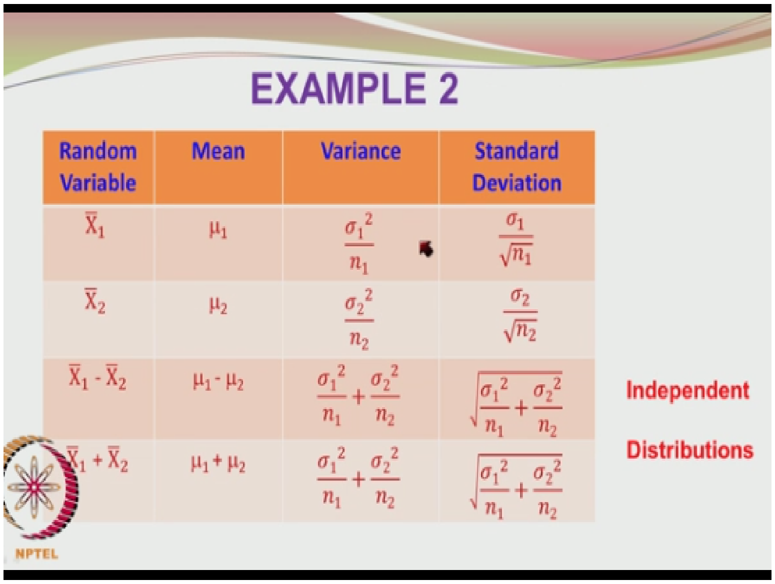
(Refer Slide Time: 11:30)



The expected value was sign dependent depending upon what was a sign used here, but when you look at the variance, variance of X1 bar – X2 bar is variance of X1 bar + variance of X2 bar. Variance of X1 bar + X2 bar is variance of X1 bar + variance of X2 bar. So, the negative sign or positive sign does not matter. The negative sign or positive sign would really matter when you look at the covariance and here in the first case, it will be – covariance of X1 bar and X2 bar.

Here, it will be + of covariance of X1 bar, X2 bar, but the covariance will vanish because X1 bar and X2 bar are independent. So, we simply have variance of X1 bar + variance of X2 bar in both the cases. So, summarizing the results from this example, we have the random variable X1, having a mu1 as mean and sigma1 square/n1 as variance, so the standard deviation would be sigma1/root n1.

(Refer Slide Time: 12:24)



Random Variable	Mean	Variance	Standard Deviation
$\bar{X}_1$	μ_1	$\frac{\sigma_1^2}{n_1}$	$\frac{\sigma_1}{\sqrt{n_1}}$
$\bar{X}_2$	μ_2	$\frac{\sigma_2^2}{n_2}$	$\frac{\sigma_2}{\sqrt{n_2}}$
$\bar{X}_1 - \bar{X}_2$	$\mu_1 - \mu_2$	$\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$	$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
$\bar{X}_1 + \bar{X}_2$	$\mu_1 + \mu_2$	$\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$	$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

X2 bar for the second case, again I think it is better if I sort of go back a little bit. What is X2 bar? This is the random sample taken from a second population. The second population is normally distributed. So, you take the elements of size n2, then add the attributes or values of these elements divided by n2, you will get X2 bar and similarly, you can take many such random samples from the second population and each one would have a different average value.

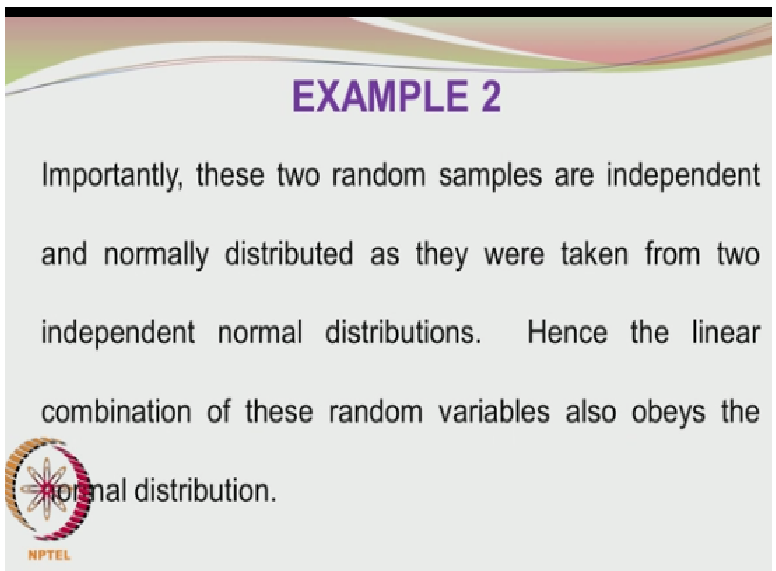
So, they will form a distribution of the sample means. This distribution of the sample means would be normal with mean at mu2 and variance at sigma2 square/n2. What is mu2? It is not only the mean of the sampling distribution, but it is also the mean of the parent population from where the random sample was taken and sigma2 square again is the variance of the second parent population.

And the variance of the probability distribution of the sample means taken from the second population will be smaller and it will be given by sigma2 square/n2. So, the standard deviations of course would be sigma1/root n1 for the first case, sigma2/root n2 for the second case and the X1 bar – X2 bar, a linear combination of the 2 random variables would have a mean of mu1 – mu2, we saw that it is sign dependent.

We just saw it a couple of slides back and the variance would be variance of X1 bar which is sigma1 square/n1 + variance of X2 bar which is sigma2 square/n2 and so, they are added up and we take the standard deviation, it would be square root of sigma1 square/n1 + sigma2 square/n2. When you take X1 bar + X2 bar as the linear combination of the 2 random variables.

The 2 random sample means they will be distributed around mu1 + mu2 at the center and having a variance or spread given by sigma1 square/n1 + sigma2 square/n2. The square root of that would be sigma1 square/n1 + sigma2 square/n2. This applies for independent distributions. When X1 bar and X2 bar are independent of each other, then this results I have shown here would apply, okay.

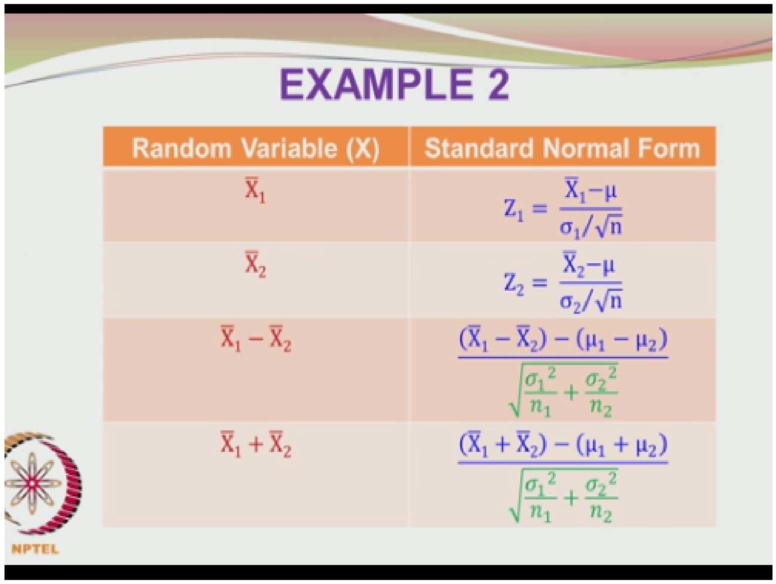
(Refer Slide Time: 15:45)



So importantly, I would like to re-emphasize, the 2 random samples are independent and normally distributed as they were taken from 2 independent normal destructions. Hence the linear combination of these random variables also obeys the normal distribution. Since it obeys the normal distribution, we can express this in the standard form so that we may use the probability tables.

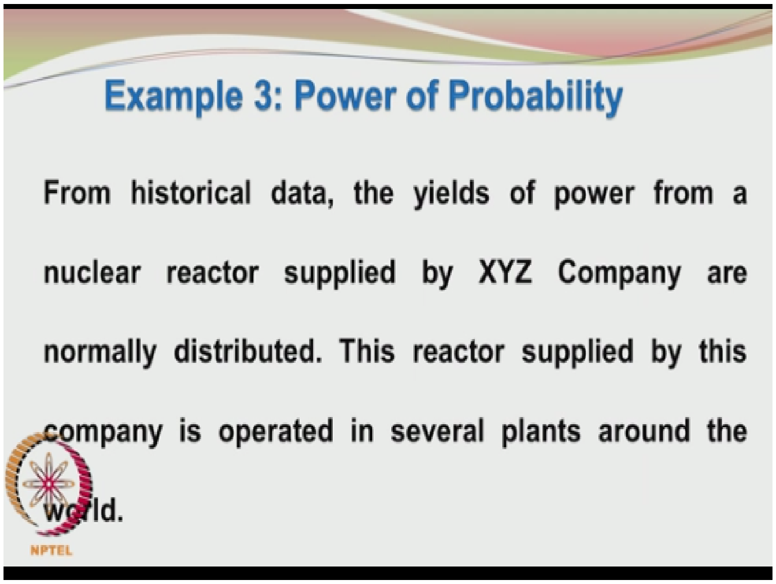
So, when you expressed them in a standard normal form, it becomes quite straight forward, X1 bar may be in turn normalized by subtracting mu1, X1 bar – mu1 divided by sigma1/root n. Let me just correct that typo and so we have Z1 = X1 bar – mu1 divided by sigma1/root n1. Z2 is X2 bar – mu2 divided by sigma2/root n2 and if you look at the X1 bar – X2 bar, you can create it as another random variable with mean mu1 – mu2 and standard deviations square root of sigma1 square/n1 + sigma2 square/n2.

(Refer Slide Time: 16:17)



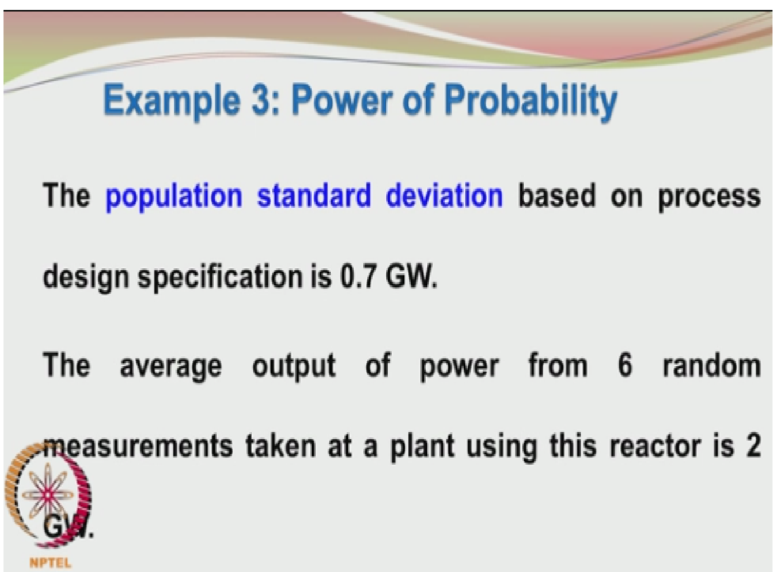
So, the random variable combination X1 bar + X2 bar may be expressed as shown here. So, this is a very nice way of putting it in a compact form and then we may use the standard normal probability tables to do the necessary calculations, right. Now let us look at example 3, the problem statement goes on like this. From historical data, the yields of power from a nuclear reactor supplied by XYZ Company are normally distributed.

(Refer Slide Time: 17:28)



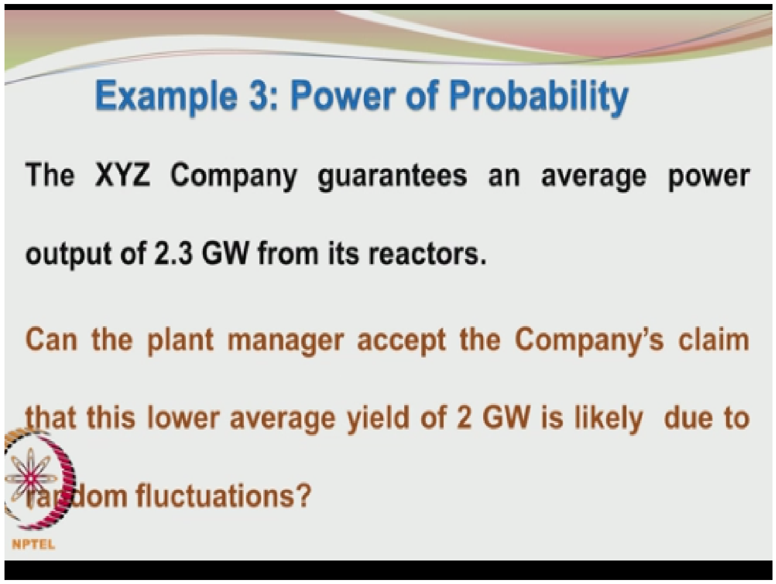
This reactor supplied by this company is operated in several plants around the world. The population standard deviation based on process design specification is 0.7 gigawatt. The average power output of power from 6 random measurements taken at a plant using this reactor is 2 gigawatt. However, the XYZ Company had guaranteed an average power output of 2.3 gigawatt from its reactors.

(Refer Slide Time: 17:49)



Obviously, the client organization using this reactor is getting an average power output of 2 gigawatt and it is concerned because it is supposed to produce 2.3 gigawatt, but it is producing only 2 gigawatt and that may lead to loss, okay and then the company is contacted, the company says do not worry the thing is normal, it is only a random fluctuation or a random variation even if you are taken the means, the differences because of random fluctuation.

(Refer Slide Time: 18:07)

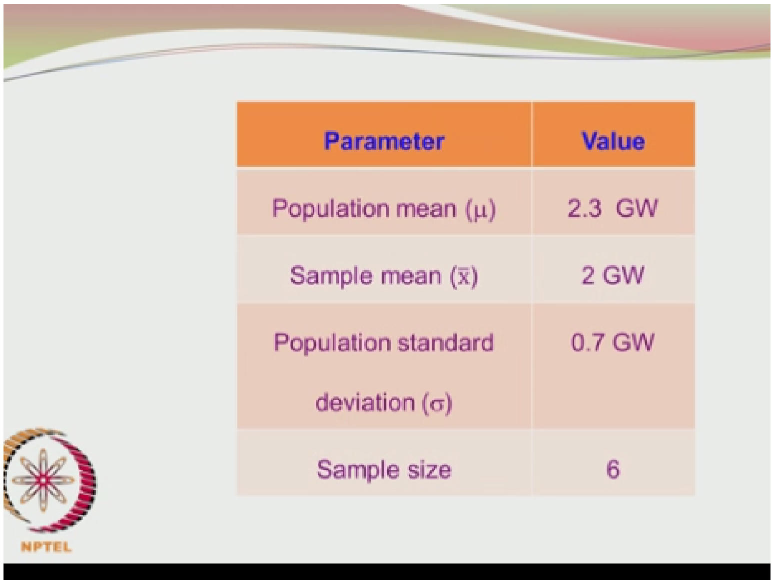


But the company said if it is random fluctuation on the positive side, if we had got 2.6 gigawatt that would have been nice, but we are getting only 2 gigawatt whereas you are promising 2.3 gigawatt. So there is an issue here and we have to see what is the probability of the average power output from the plant being 2 gigawatt even though the actual mean value is 2.3 gigawatt. Coming again, what we have to do is there is a distribution of the sample means and the mean value is 2.3 gigawatt.

So from this sampling distribution of the means probable distribution, what is the probability of picking up a sample with a mean power output of 2 gigawatt? If the probability is quite high, then the probability of occurrence of such kind of event is quite high. So, we can only attributed to random effects. We cannot say anything more.

However, if the probability of picking up a sample of mean power output of 2 gigawatt is pretty low from sampling distribution of the mean of 2.3 gigawatt, then we have to question the supplier. So, we have to look at the sampling distribution of the mean. Since we are talking about the mean power output we are referring to the sampling distributions of the means and we also have a probability distribution.

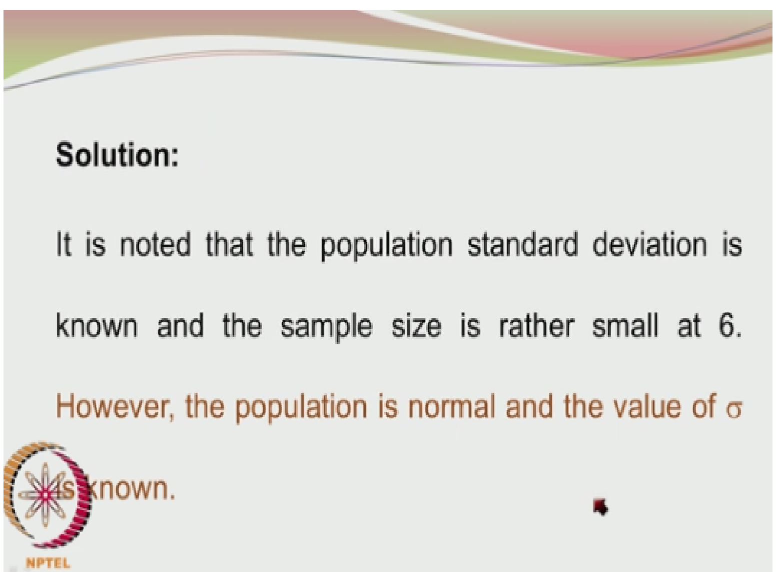
(Refer Slide Time: 21:00)



Parameter	Value
Population mean (μ)	2.3 GW
Sample mean ($\bar{x}$)	2 GW
Population standard deviation (σ)	0.7 GW
Sample size	6

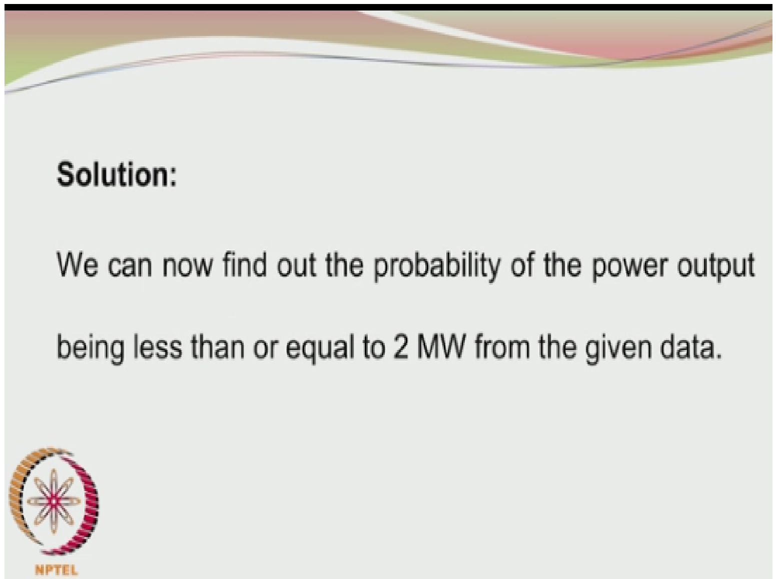
So, the population mean has given as 2.3 gigawatt, sample mean x bar, I am using small x bar because sample has been taken and it is value known that that is 2 gigawatt only. Population standard deviation based on design specification is 0.7 gigawatt, having the same units as the mean power output and sample size is only 6. So, it is given that the population is normal and the value of sigma is also known which makes life easier for us.

(Refer Slide Time: 21:23)



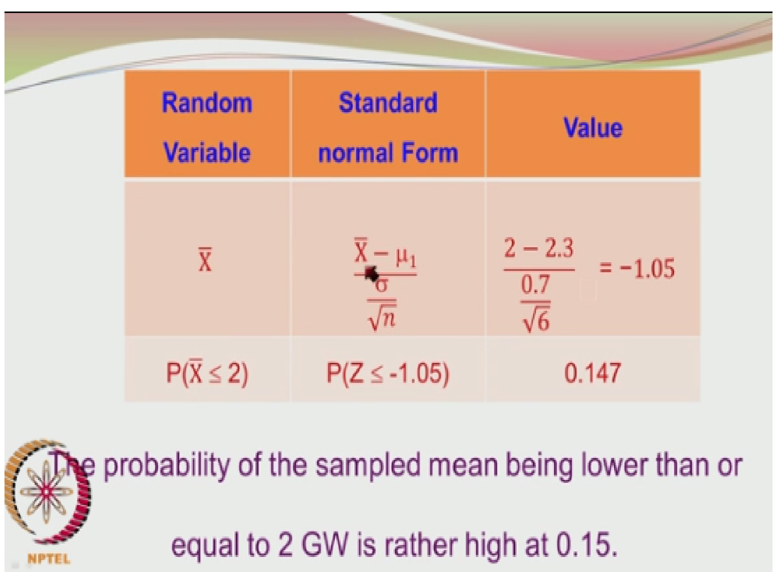
So, we have to find out the probability of the power output being < or = 2 megawatts from the given data and X bar, I am normalizing it again X bar – mu1/sigma/root n, 2-2.3 divided by 0.7/root 6, let me sort of check it out. So, I should be doing -0.3 * root 6 divided by, so I am getting -1.04978, -1.05 is okay and so, what is the probability that X bar would be < or = 2, which is equivalent to asking what is the probability of the standard normal variable Z < = -1.05.

(Refer Slide Time: 21:34)



And the probability is 0.147, so the probability of the sampled mean being lower than or = 2 gigawatt is rather high at 0.15, okay. So, the company is saying the mean power output is 2.3 gigawatt. It is not stopping there; it is also saying that the standard deviation of the normal distribution is 0.7 gigawatt. Now, we are taking about the sampling distribution of the means, the probability distribution of the sample means.

(Refer Slide Time: 21:43)

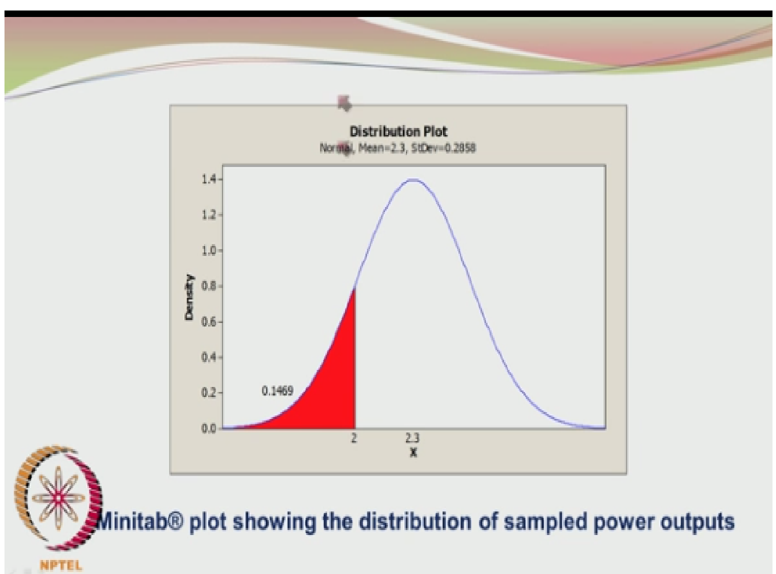


And the probability distribution of the sample means is centered again at 2.3 gigawatt and having the spread given by 0.7/root 6. So, what is 0.7/root 6? 0.286. So, there is a spread of 0.286 gigawatt around this particular sampling distribution of the mean. So, the standard deviation is 0.286 gigawatt.

The company is getting only 2 gigawatt, so when we do the calculations for the probability of this occurrence, namely the occurrence of 2 gigawatt or lower when the sample is taken from a sampling distribution of the means centered at 2.3 gigawatt and standard deviation of 0.286 gigawatt. The probability comes to 0.147 which is rather high. So, you really cannot question the supplier because 0.15 is a good reasonable chance of occurrence of this kind of event.

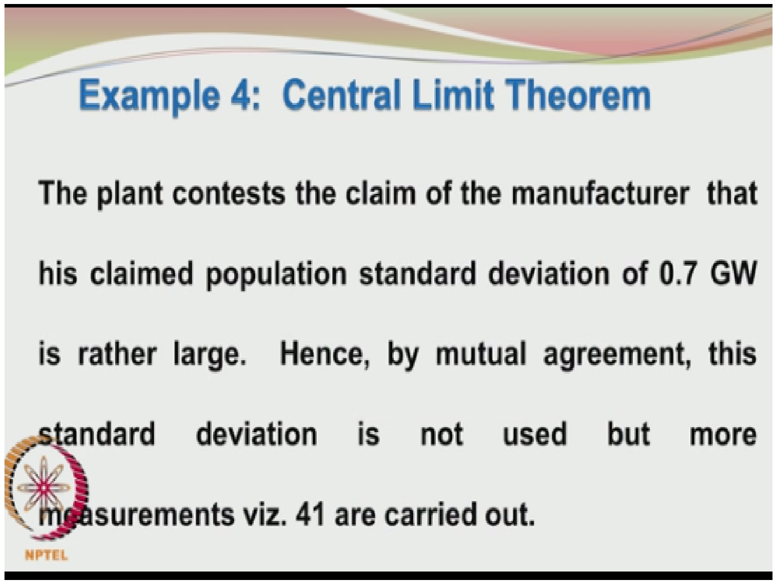
So, if you do the plotting with the Minitab, this is the normal distribution centered at 2.3 gigawatt and having the standard deviation of 0.7/root 6 which is 0.286 gigawatt. So, this is the spread and I am looking at the probability of occurrence of 2 gigawatt or lower from this probability distribution and I am finding the probability, the area under the curve in the shaded region which comes to 0.147.

(Refer Slide Time: 24:20)



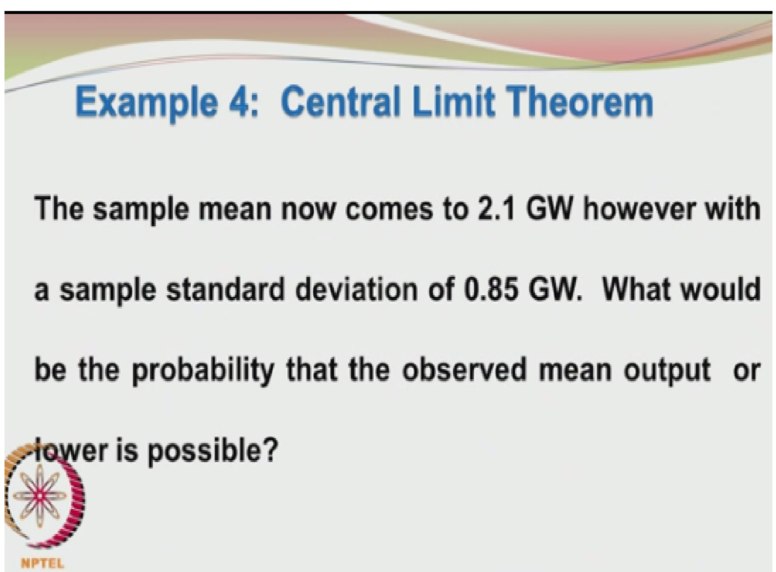
So, moving onto the next example, the plant contests the claim of the manufacturer that his claimed population standard deviation of 0.7 gigawatt is rather large. Hence, by mutual agreement, this standard deviation is not used but more measurements names 41 are carried out, okay. So that 0.7 gigawatt is thrown out of the window and you are no longer even thinking of the population being normally distributed that is not mentioned in the problem statement.

(Refer Slide Time: 24:51)



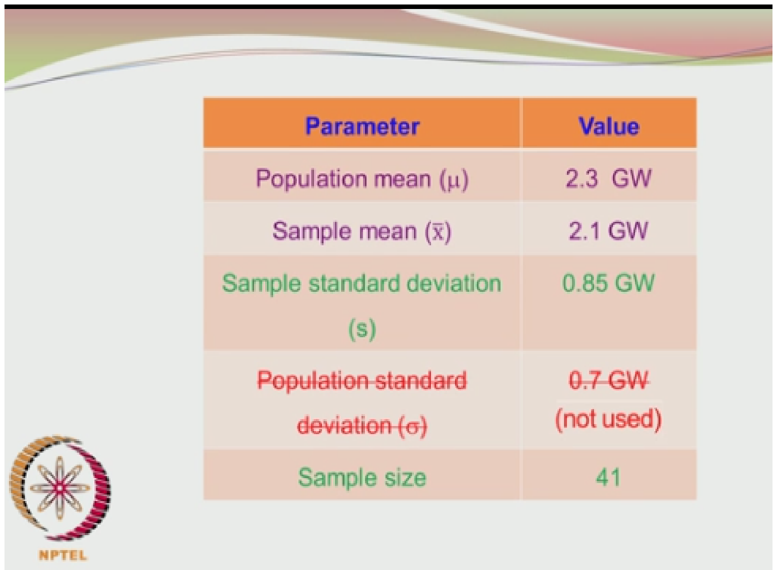
Whereas in the previous problem statement, it was given that the population was normally distributed. But you are also taking a large sample size of 41. The sample mean now comes to a slightly higher 2.1 gigawatt, but the sample standard deviation is 0.85 gigawatt. The sample standard deviation is even higher than the design specification value of 0.7 gigawatt. What would be the probability that the observed mean output or lower is possible?

(Refer Slide Time: 25:38)



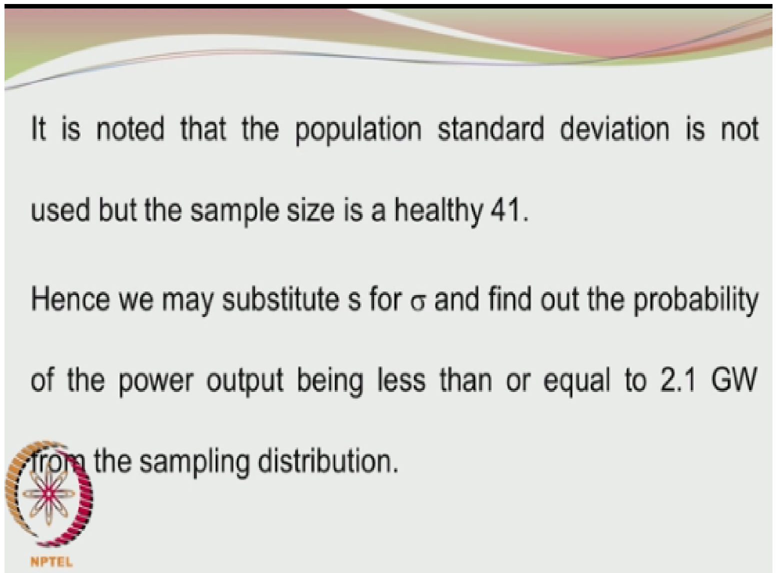
What is the probability of this occurrence? That you can get a sample mean of 2.1 gigawatt or lower that is what we have to find now. So, conditions are slightly changed. Population mean value mu is 2.3 gigawatt, sample mean X bar is 2.1 gigawatt, sample standard deviation s is 0.85 gigawatt, sample size is 41. We are no longer using the population standard deviation of 0.7 gigawatt.

(Refer Slide Time: 26:18)



So we are not supposed to use sigma, but we can use s, the sample standard deviation. When s is used that is permitted because the sample size is quite large, we can even continue with the normal distribution according to the central limit theorem. The central limit theorem says that irrespective of the population probability distribution characteristics. If a large sample is taken typically > 30, then the resulting sampling distribution of the means is also normal.

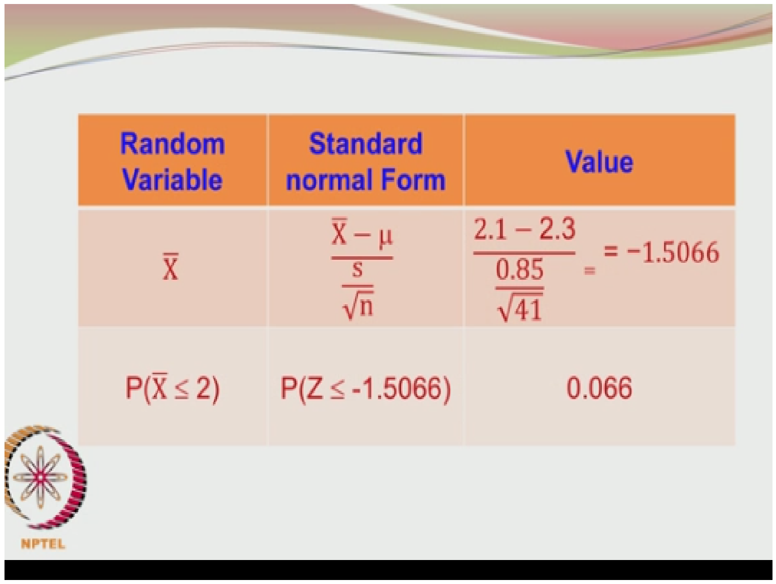
(Refer Slide Time: 26:39)



In the present case, the parent population we do not have to worry about because the sample size is quite large and so the central limit theorem will apply and so the sampling distribution of the means is going to be normal and since we are going to use s, because sigma is not available for use. The s value may be substituted for sigma in the calculations and we are also having a large sample size of 41 to account for it.

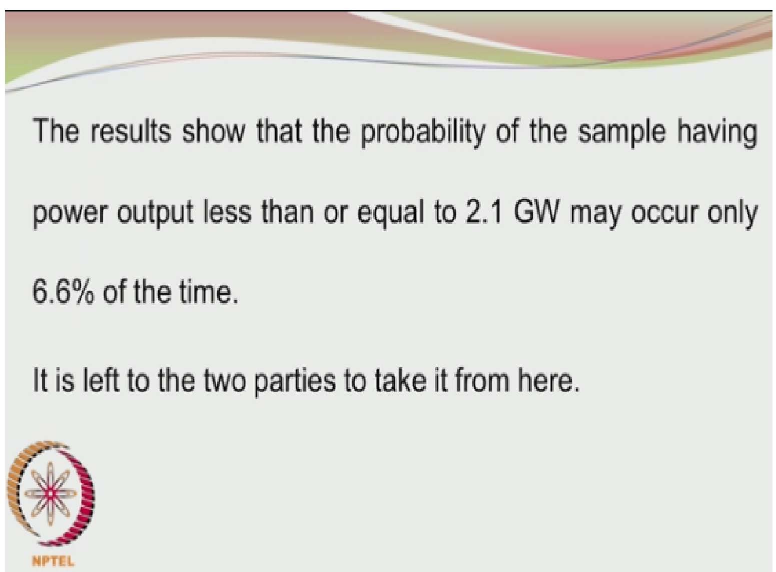
So, the problem calculations are quite straight forward instead of using sigma, here we use s, we have X bar – mu/s/root n and that is 2.1 – 2.3 that is -0.2 * root 41 divided by 0.85 that comes to -1.5066, -1.51. So, the probability of X bar < 2 is equivalent to probability of Z < -1.51 and the probability has now considerably reduced to 0.066. So the results show that the probability of the sample having power output < or = 2.1 gigawatt may occur only 6.6% of the time or the probability value is 0.066,

(Refer Slide Time: 28:02)



So, we stop here and left to the 2 parties to take it from here, okay. So showing this on the normal probability distribution, here we have a standard deviation of 0.13275, how did that come about that was s used is 0.85 divided by root 41. So, 0.85/root 41 is 0.13275. The mean value hypothesized or taken as 2.3 gigawatt, so that is what we have here. So, the probability of occurrence of 2.1 gigawatt are lower is given by the area of the shaded portion and that is 0.066.

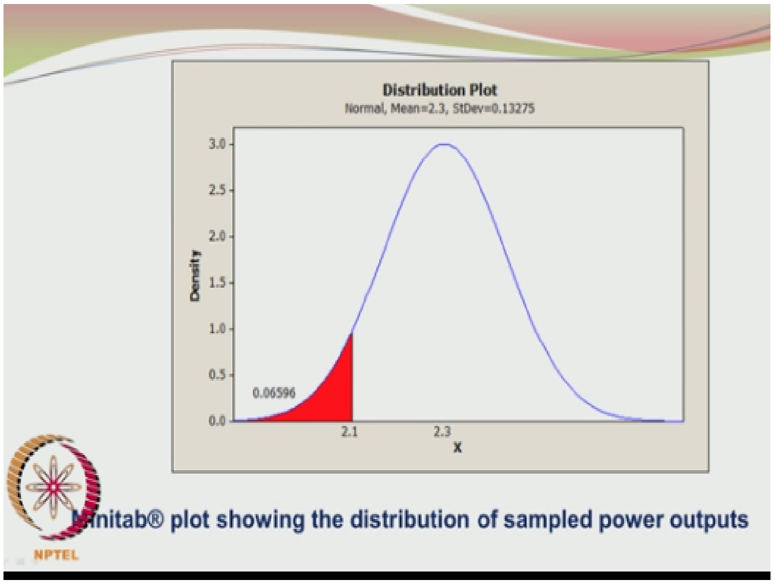
(Refer Slide Time: 28:45)



So, the probability is 0.066. So, the 2 probability distributions are plotted as shown in this figure generated from Minitab, so you are having 2 probability distributions. The first one is centered at 2.3 gigawatt and has a standard deviation of 0.2858. How did this 0.2858 come about, it was 0.7

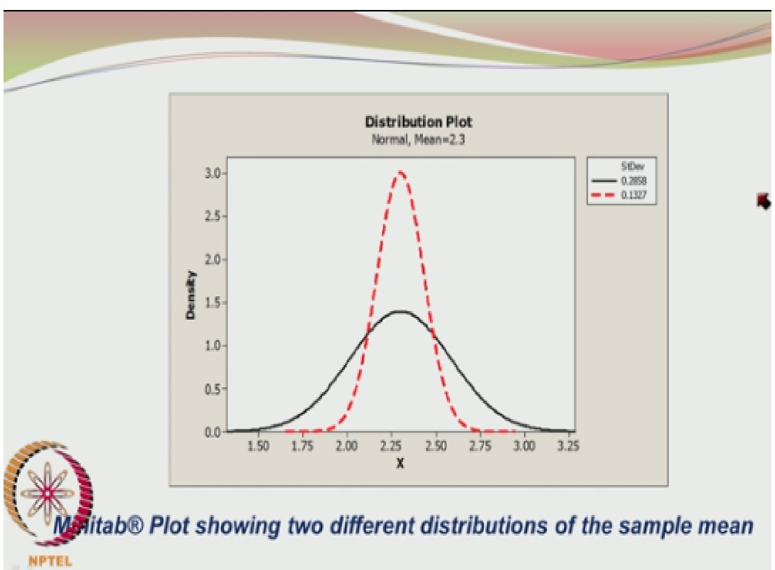
gigawatt divided by root 6. The design specification of sigma was 0.7 gigawatt and the sample size was 6 in the first case.

(Refer Slide Time: 29:03)



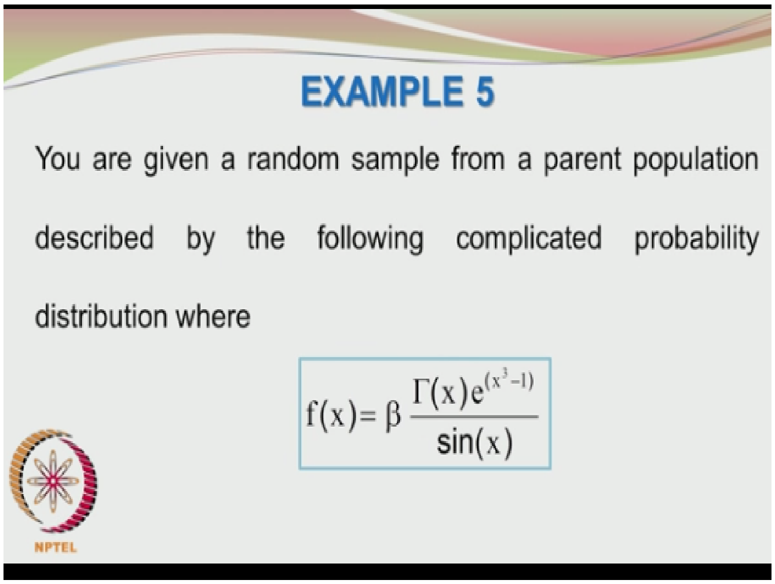
So, we are having 0.7 divided by root 6 which is 0.2858. The second distribution shown is having a lesser spread and it is also centered at 2.3 gigawatt. It is based on a sample standard deviation of 0.85 gigawatt which is higher than 0.7 gigawatt and still the spread this smaller because of the larger sample size. So, instead of using sigma/root n, we are using s/root n2, where s is 0.85 gigawatt and n2 is 41.

(Refer Slide Time: 29:50)



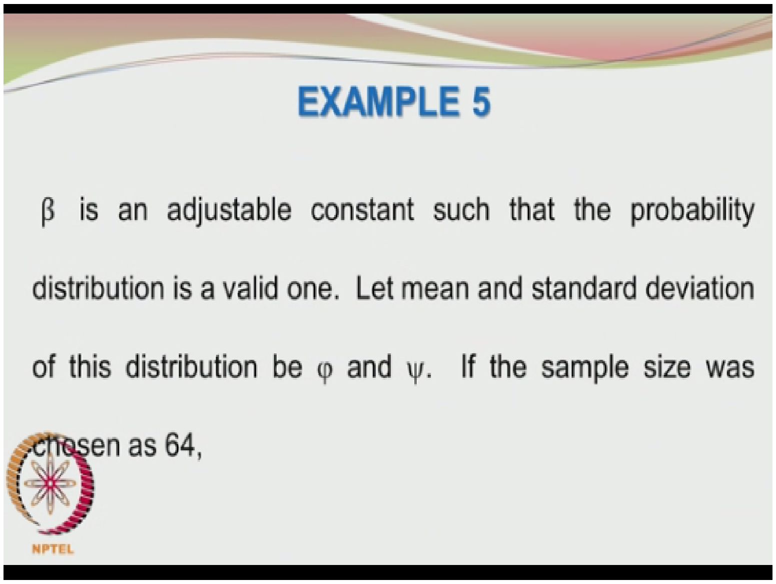
So, 0.85 divided by root 41 is 0.1327 which is more than half of the earlier spread value of 0.2858. So you can see as lesser spread here and also the probability value declined. The next problem is you are given a random sample from a parent population described by the complicated probability distribution function where f of x is beta * gamma x e power x cube – 1/sin x.

(Refer Slide Time: 31:36)



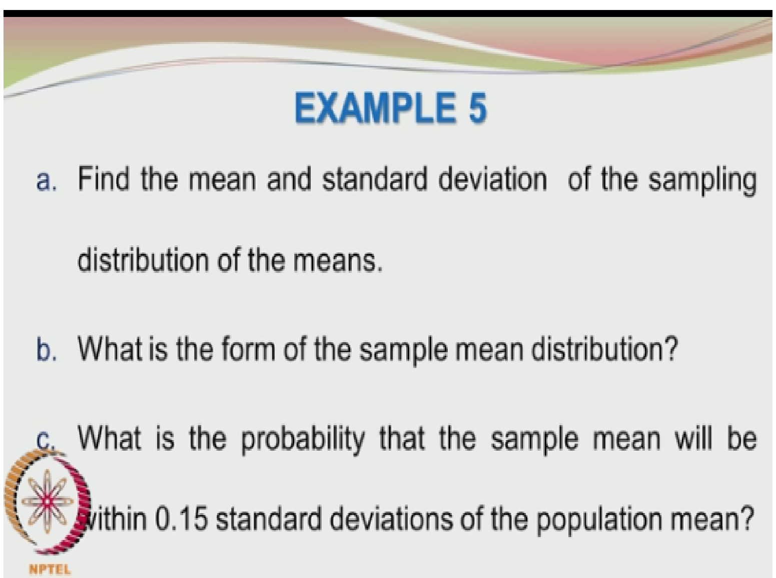
So, beta is an adjustable constant such that the probability distribution is a valid one, you know what it means. The area under the curve for any probability distribution function should be one continuous probability density functions described by a smooth curve and the area under such curves should be = 1. So, we adjust the parameter beta such that this is a valid probability distribution function.

(Refer Slide Time: 32:14)



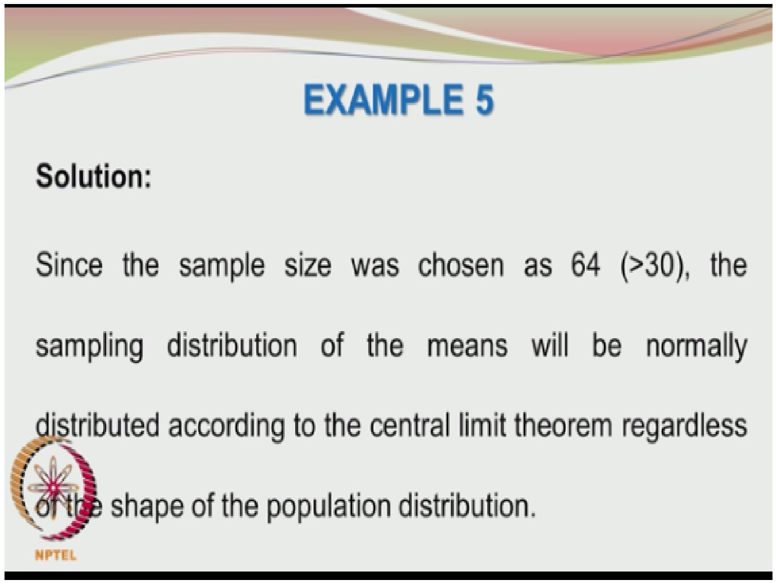
Let the mean and standard deviation of this distribution be phi and psi that means the variance of this distribution psi squared. If the sample size was chosen as 64, find the mean and standard deviation of the sampling distribution of the means. What is the form of the sample mean distribution? And what is the probability that the sample mean will be within 0.15 standard deviations of the population mean?

(Refer Slide Time: 32:30)



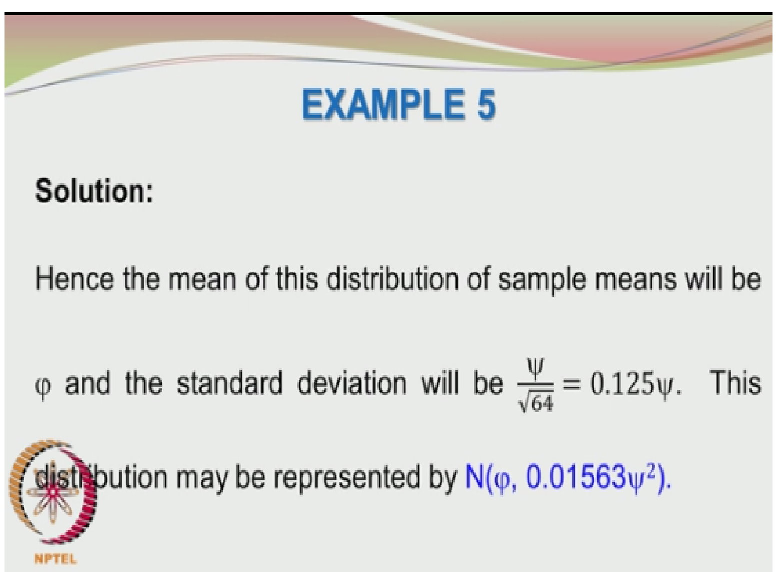
So, since the sample size is quite large at 64 which is > 30, the sampling distribution of the means will be normally distributed according to the central limit theorem regardless of the shape of the parent population distribution. So, now the problem is quite straight forward. The mean of this distribution of sample means will be phi and the standard deviation will be psi/root 64 which is 0.125 psi.

(Refer Slide Time: 32:46)



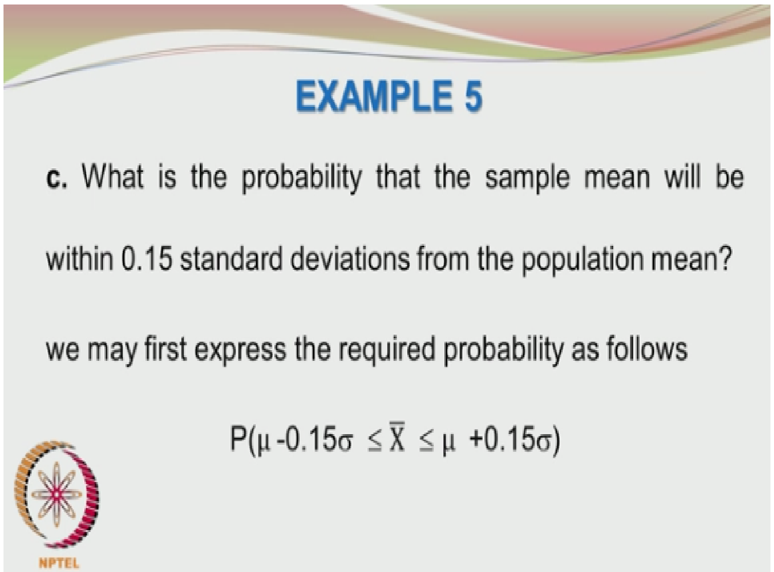
So, 1/root 64 is 1/8 which is 0.125. So the standard deviation of the sampling distribution of the means would be 0.125 psi. This distribution may be represented by a normal distribution of mean phi and variance which will be square of this 0.01562 psi square, okay. 0.125 square, let us confirm 0.125 square is 0.015625, so that is fine. What is the probability that the sample mean will be within 0.15 standard deviations from the population mean.

(Refer Slide Time: 33:03)



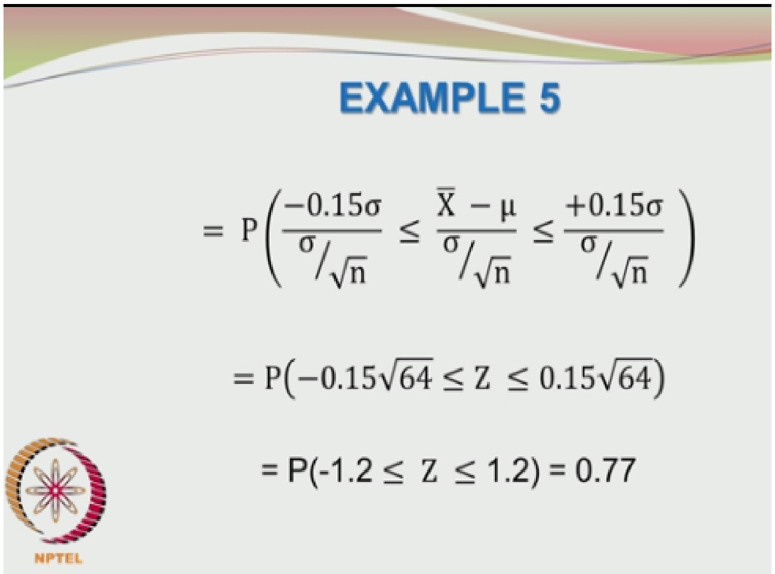
So, the problem can be expressed in the following way. Probability of the value of the random sample being 0.15 sigma distant from the population mean. So, probability of mu which is the population mean and also the random sample probability distribution mean mu –0.125 sigma < or = X bar < or = mu +0.15 sigma. So, the random sample which we take may have a value either lower than mu or higher than mu.

(Refer Slide Time: 34:01)



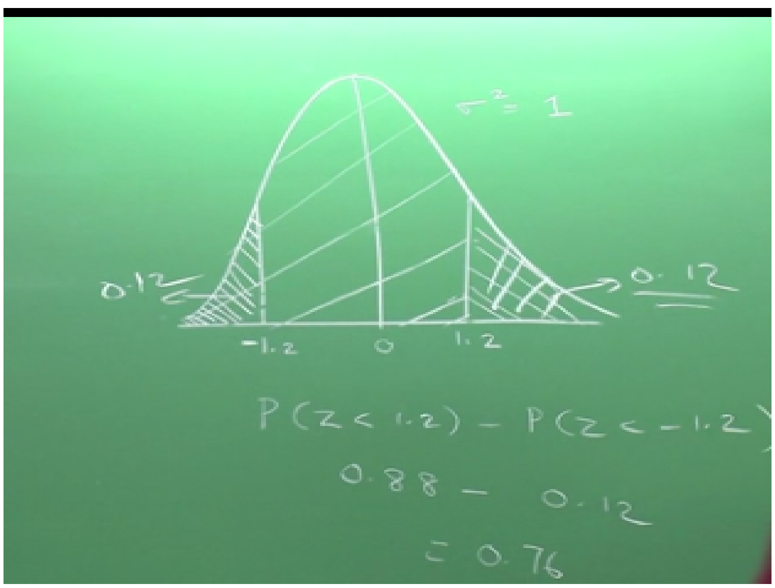
And it may lie either on the right hand side of mu or on the left hand side of mu. So, now it is easy to normalize and how do we normalize, we just subtract mu from x bar and divided by sigma/root n, we do it in all the other 2 sides of the inequality and then we get -0.15 sigma/sigma/root n, +0.15 sigma/sigma/root n and this works out probability of -1.2 < or = Z which is the standard normal random variable < or = 1.2 and this comes to 0.77, okay that can be read of from the standard normal probability charts.

(Refer Slide Time: 35:04)



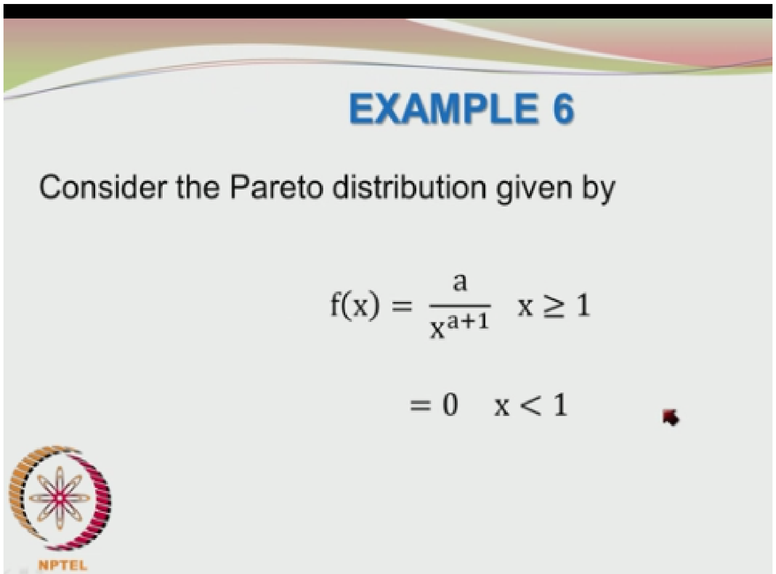
I hope now you are comfortable using these charts and you should be able to figure out how we get this 0.77. I just illustrate this on the board, so you have the standard normal distribution which is having a mean value of 0 and variance sigma square = 1 and we have to find the area under the curve 1.2, -1.2. So, what we can do is probability of Z < 1.2 – probability of Z < -1.2.

(Refer Slide Time: 36:06)



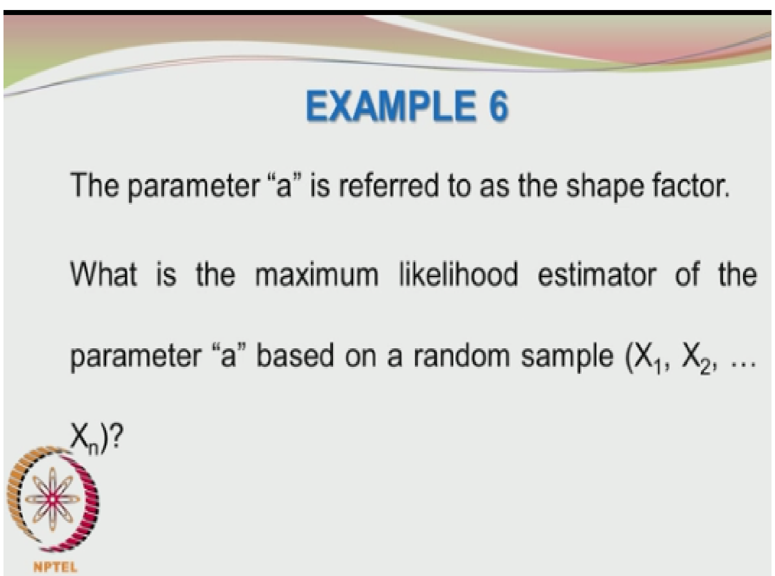
So, first what we do is we find the area under the entire curve and then, from this total area, we subtract out this area and we get the required probability. If I remember right this comes to around 0.88 and then this would be 0.12. If the entire area is around 0.88, then this area would be 0.12 and by symmetry, this area would also be = this area would be 0.12. So, 0.88-0.12 is 0.76, I am just doing it from memory and you can also see the answer is to 0.77.

(Refer Slide Time: 37:50)



Let us move onto the next problem. Here, we have the Pareto distribution, quite an interesting function. This was the problem I had taken from the Ramachandran and Tsokos book and f of x = a/x power a+1, x > or = 1 =0 for x < 1. So, the parameter "a" is referred to as the shape factor. What is the maximum likelihood estimator of the parameter "a" based on the random sample X1, X2, so onto Xn.

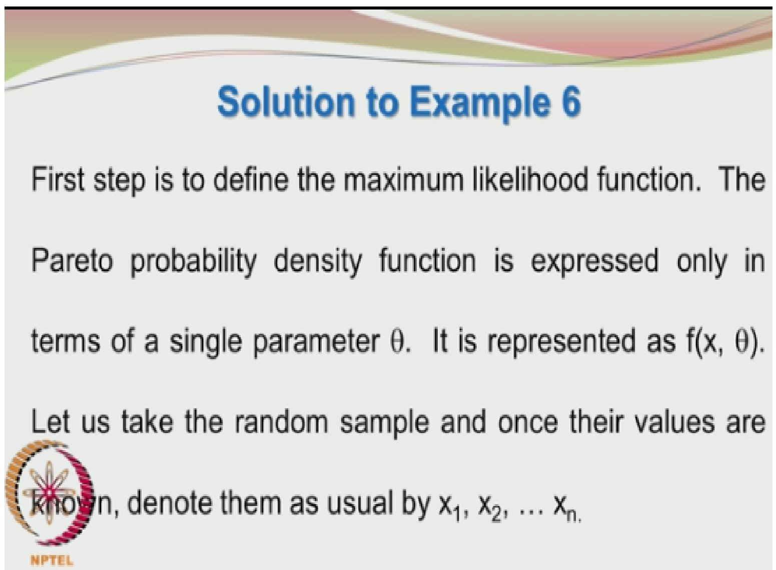
(Refer Slide Time: 38:10)



Some of you may ask we do not know the value of a and we do not know whether this is a valid probability density function, so finding the area under the curve from 1 to infinity a/x power a+1 dx should tell us the value of a. So, what is the additional need for finding the value of a. I leave it to you, okay. The hint is you cannot find out a using this method for the simple reason that no matter what value of a, you plug in there, the integral 1 to infinity will be = 1.

I mean do the integration, you can find out this will be X power -a-1, so it will be -1/x power a and a would cancel out and so, when you go from 1 to infinity, it would be 1-0, 1 power a is always going to be 1. I requested to do the integrations yourself and confirm that no matter what the value of a is, the a will cancel out and so, this area under the curve will always be = 1. So, let us move onto the actual problem.

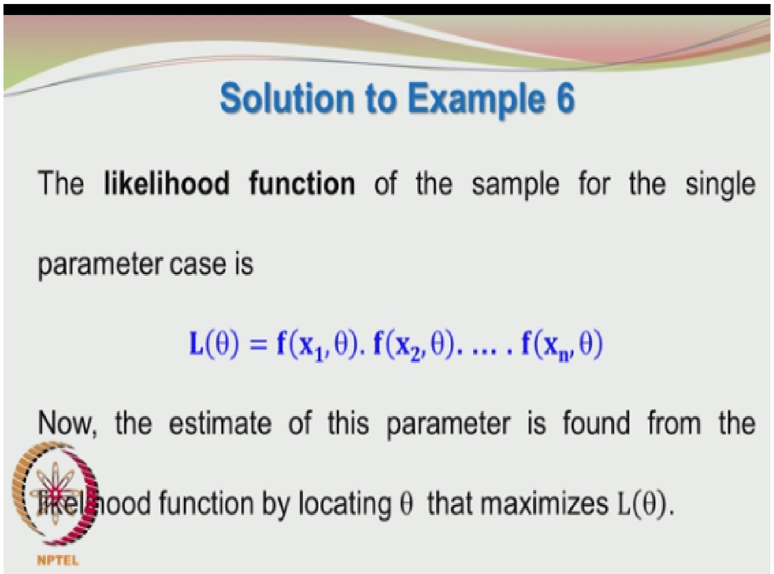
(Refer Slide Time: 39:49)



We have to define the maximum likelihood function. We are using the method of maximum likelihood parameter estimation method to find out what a is? The Pareto probability density function is expressed only in terms of a single parameter theta. It is represented as f of x, theta. Let us take a random sample and once their values are known, will denote them by X1, X2, so onto Xn.

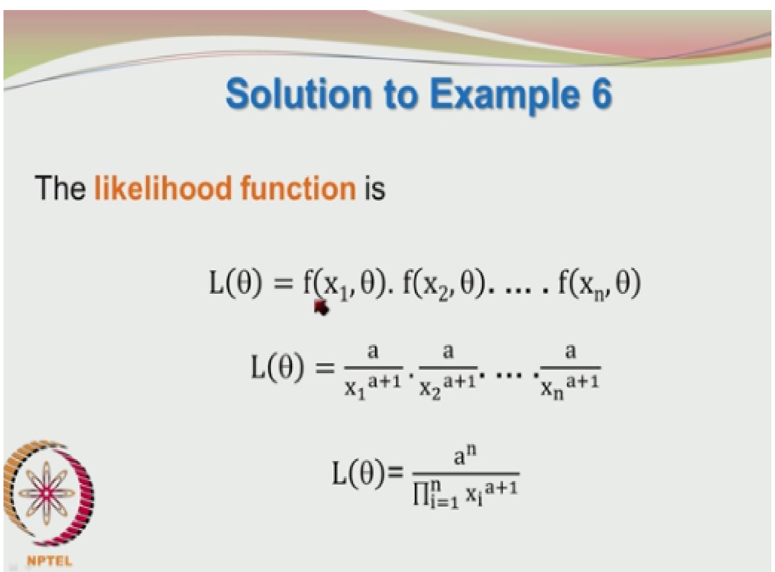
So, the likelihood function of the sample for the single parameter case is L of theta = f of X1, theta * f of X2, theta so on to f of Xn, theta. So, we have to estimate this parameter by maximizing this relationship. So, first let us get the relationship, L of theta = f of X1, theta * f of X2, theta so on to f of Xn, theta and that would be a/X1 to the power of a+1 * a/X2 to the power of a+1 so on to a/Xn to the power a+1.

(Refer Slide Time: 40:19)



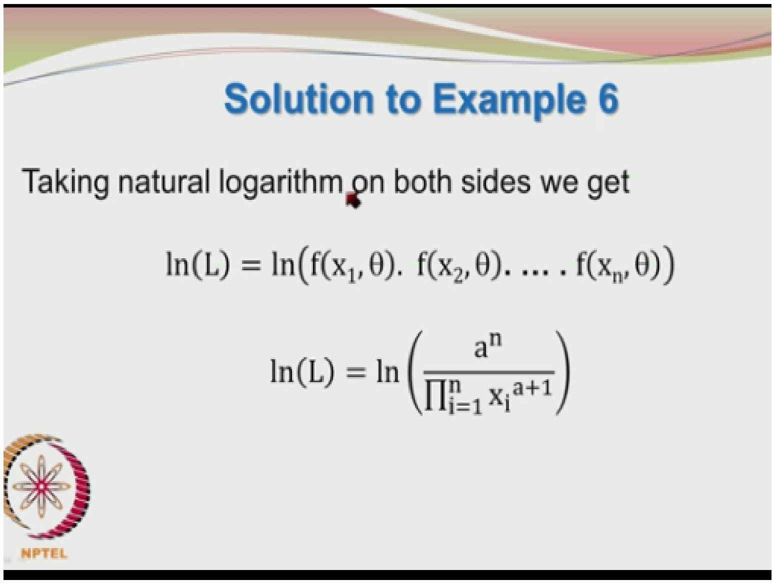
So, L of theta = a power n, because I am doing it in n times and this is the product of all the X values to the power of a+1 and when we take natural logarithm on both sides, we get ln of L = ln of f of X1, theta * f of X2, theta, so onto f of Xn, theta. So, ln L = ln of a power n/the product of the entities Xi to the power of a+1, i running from 1 to n. So we take ln L, we have this we can split into 2 parts, ln of a power n becomes n ln a.

(Refer Slide Time: 40:43)



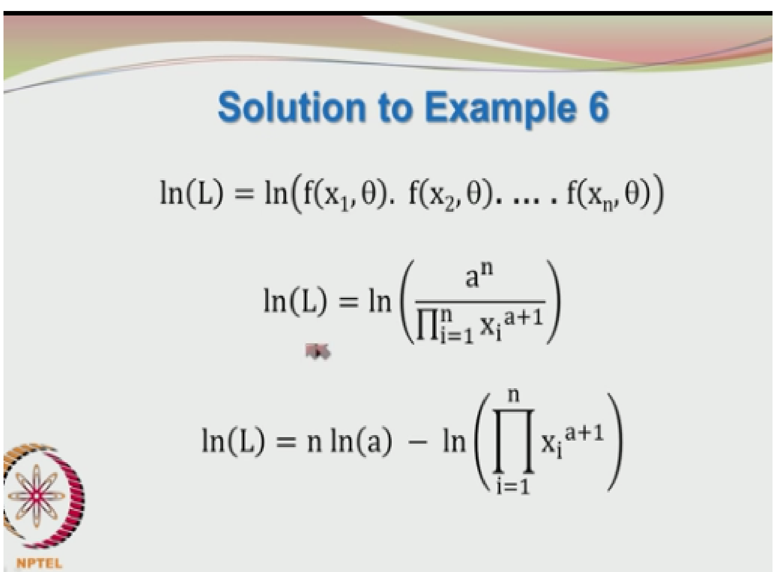
And then this becomes ln of product of Xi's to the power of a+1. So again this is quite simple, you will get ln of L = n ln a, we saw this earlier. How did this get simplified? You know that the log of product of entities, the sum of ln of those entities, so the a+1 is common here and you can put a+1 here and then you get sigma i equals 1 to n, ln of Xi. The next step is to differentiate this function with respect to a and then equate it to 0.

(Refer Slide Time: 41:14)



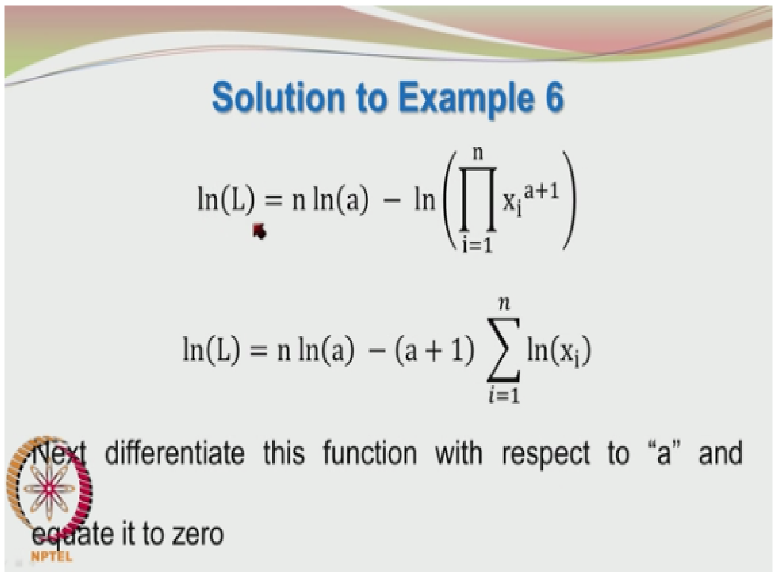
And when you differentiate with respect to a, this becomes n/a and here, we had a+1, there was no a inside, so that became quite simple, -1 * sigma ln Xi. So, the estimated parameter a is given by n divided by sigma i equals 1 to n ln of Xi, so quite simple. Let us move onto the next problem. Use the method of moments to find the parameter estimators of the following probability distribution function.

(Refer Slide Time: 41:35)



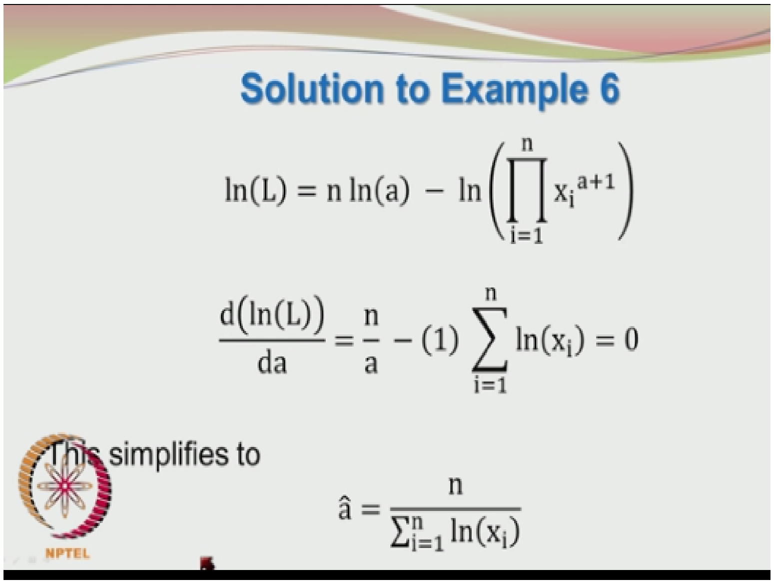
F of x = 1/B-A = 0 otherwise. So, we have to estimate both A and B. We are going to use the method of moments, so f of x = 1/B-A and the first moment E of X is obtained from the distribution in the following manner, expected value of X = A to B, x dx/B-A which is x square/2, so B square – A square/2, B+A * B-A/B-A, so B-A will cancel out. So, we have B+A/2 and expected value of X square, the second moment is given by x square dx/B-A x cube/3.

(Refer Slide Time: 41:52)



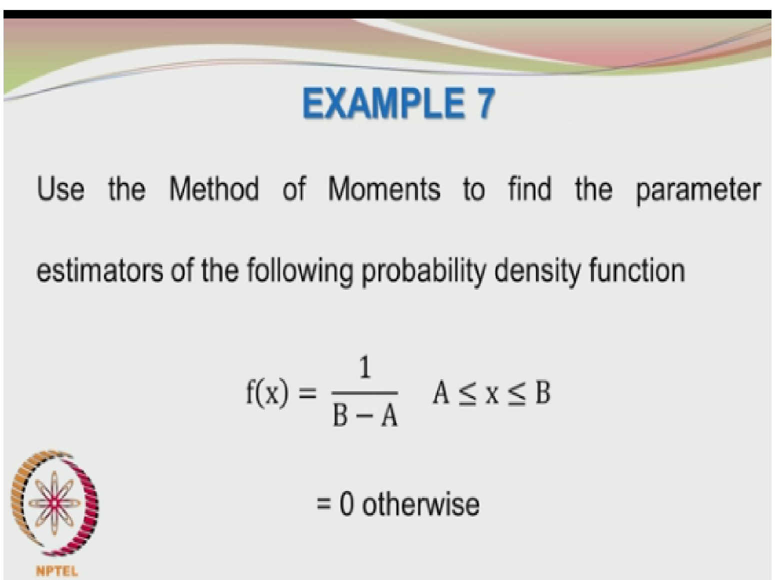
X cube/3 will become B cube – A cube and so, you are having B cube – A cube divided by B – A which is B – A/B – A * B square BA + A square and that is what we have here. So, these distribution moments may be equated with the first and second sample moments and when we do that we get m1 as 1/n sigma, i equals 1 to n, X1 + X2 + so onto Xn. We will just correct the typo. So m1 = 1/n sigma i equals 1 to n, X1 + X2 + so on to Xn = A + B/2.

(Refer Slide Time: 42:23)



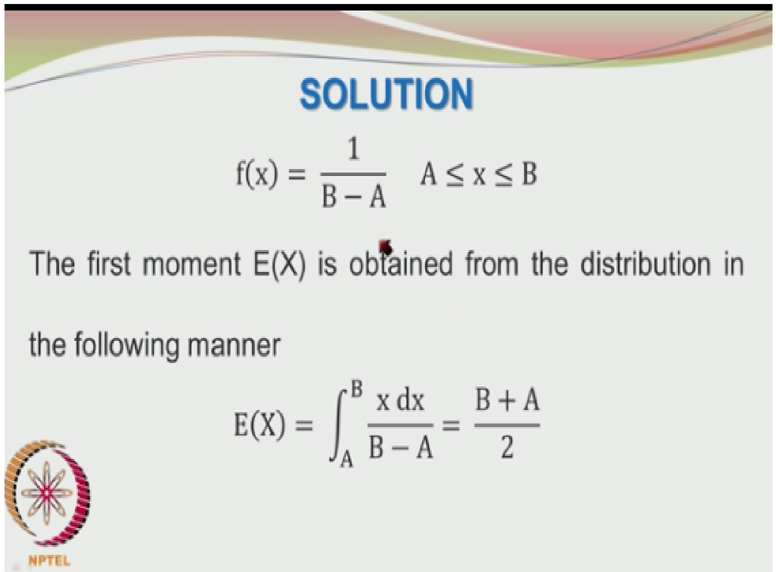
M2 = 1/n sigma i equals 1 to n, X1 square + X2 square + so on to Xn square that is A square + AB + B square/3 which is same as what we had and so we have 2 equations and 2 unknowns. The unknowns are A and B, the moments are m1 and m2, those are not unknowns, okay. So, we can write m1 = A + B/2 and m2 = A + B whole square – AB, where this can be written as A square + 2 AB + B square – AB that would be A square + AB + B square.

(Refer Slide Time: 42:48)

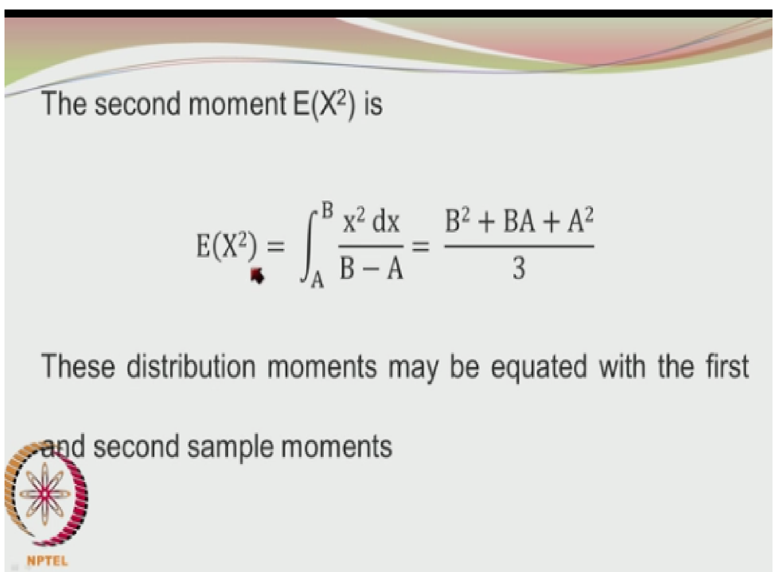


And when you have these and you solve for A and B, you get these 2 relations. I leave the quadratic equations solving to you, I hope you get the same answers as I did. So, thanks for your attention and we were doing some illustrative problems. There are lots of books on statistics and probability which have many interesting problems. I requested you to not only solve these problems independently, but also look up the problems in various books.

(Refer Slide Time: 43:10)

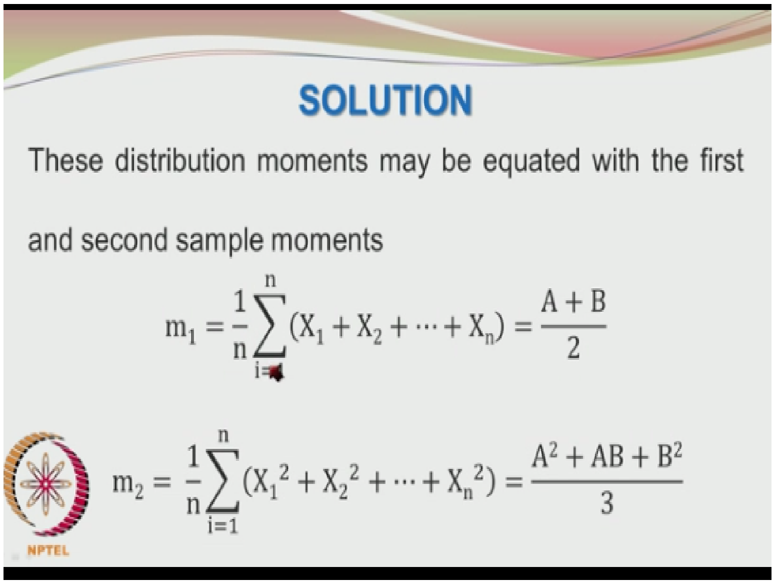


(Refer Slide Time: 43:34)

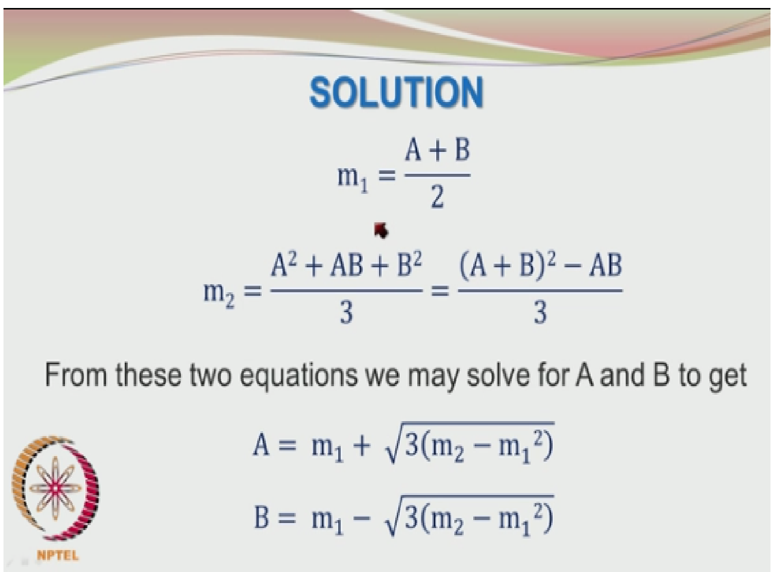


And try to solve them without any assistance either from these lectures or from the worked out examples in those book. Try to solve them on your own and if you are getting the correct answer well and good, nothing more has to be said, but if you are finding some difficulties and you are not able to get the correct answer, go through the lecture material again. See where exactly you have not understood correct your concepts.

(Refer Slide Time: 44:43)



(Refer Slide Time: 45:15)



And then hopefully you will be able to work out these kinds of problems in the correct manner. The important thing is not the actual numerical solving, but the interpretation, the assumptions made and the concepts being applied with these kinds of problems. So thanks for your attention, will see you in the next class.