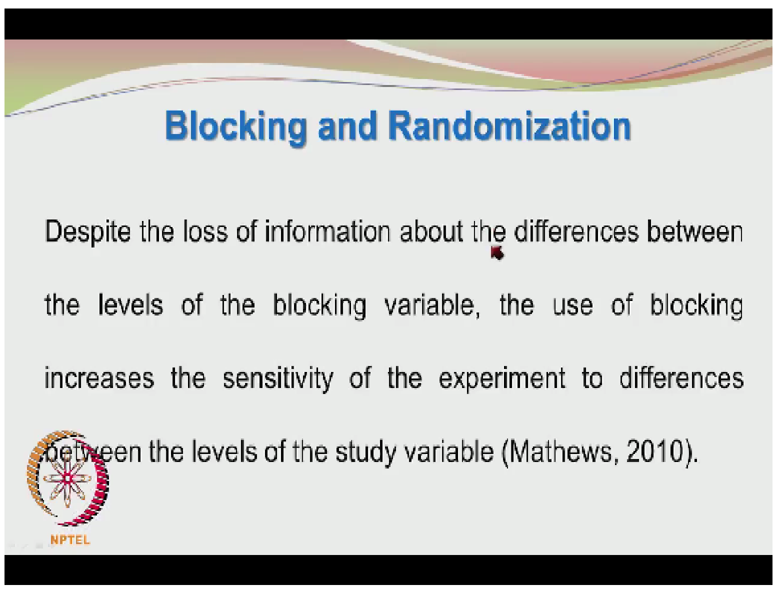
Statistics for Experimentalists

Prof. Kannan. A
Department of Chemical Engineering
Indian Institute of Technology - Madras

Lecture – 27
Blocking and Randomization

Welcome back. So we were discussing about blocking, there is no need to randomize among the blocks because you are randomising within each block and block is something which you do not have control of. It can be different plots of land; it may be different machines. So there is no point in randomising among the blocks unless they seem to have some systematic trend and you essentially have to randomise within the block. I cannot think of any situation where you would have to randomise between the blocks, alright.

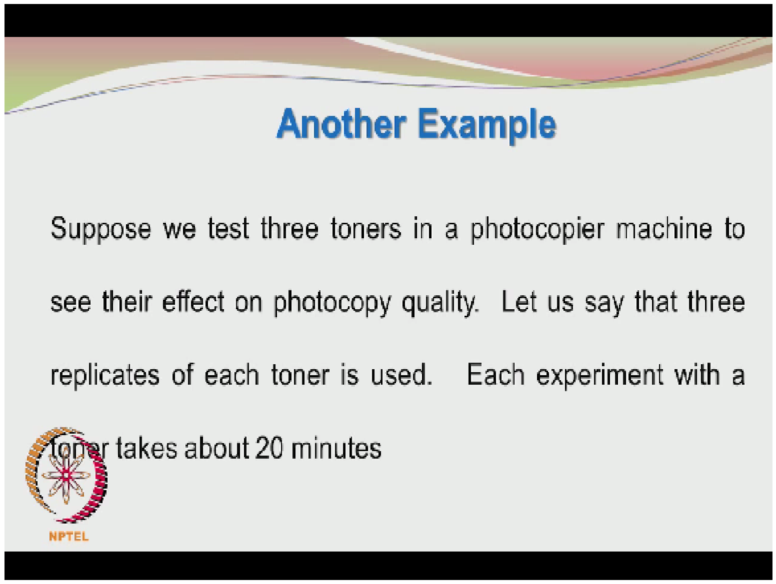
(Refer Slide Time: 01:17)



So blocking, well blocking seems to add additional complication to the analysis. On the other hand, it also increases the sensitivity of the tests, okay. So if you had not blocked, then there is danger of making the statement that no treatments are effective, that is really unpopular conclusion among decision-makers.

So probably the decision-makers want some meaningful tangible conclusion out of the experimentation where a lot of money, manpower, time has been invested and after that if the conclusion is there is no effect between the different treatments, then it is going to fall flat. So the blocking helps us to increase the sensitivity of our experimentation. Let us see how?

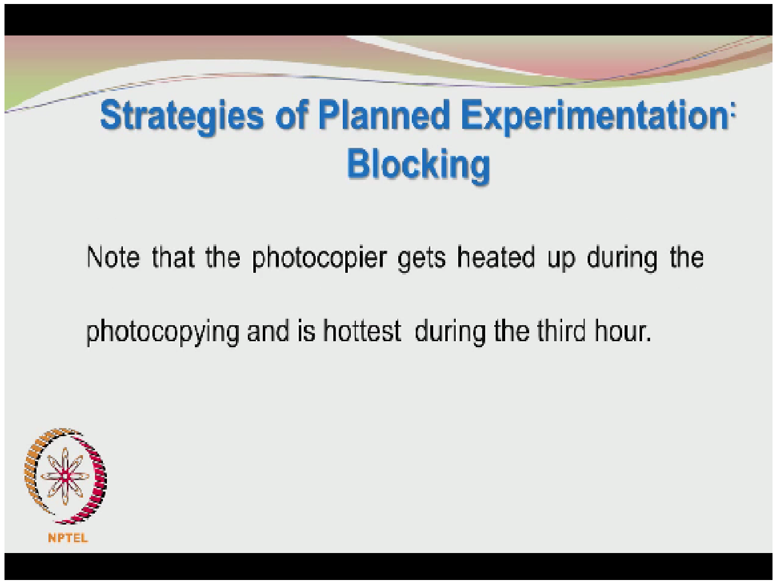
(Refer Slide Time: 02:22)



Another interesting example which I created. Suppose we tests 3 toners in a photocopying machine. We all have photocopiers in our offices and many times the toner goes out and the quality becomes very bad whether be it a photocopier or a printer. The toner is a crucial component and so let us say that there are 3 different brands of toners and we want to try copying the photocopying machine to see the effect of the toners on the copy quality.

So we will assume that there are 3 replicates of each toner and each experiment with the toner takes about 20 minutes. So we how a photocopier. You are going to try out 3 different toners and each experiment with the toner takes about 20 minutes and we want to say that 3 replicates of each toner is being used.

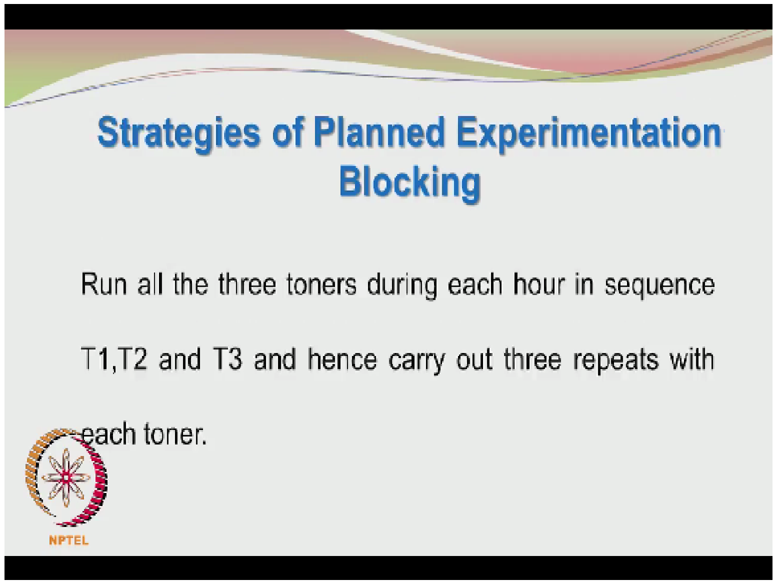
(Refer Slide Time: 03:43)



So the photocopier may get heated up during the photocopying and it is hottest during the third hour. Well there is a slightly confusing statement I made in the previous slide. So let us go back to it right now rather than discuss about it later. What is this 3 replicates of each toner that is used, okay. So what we are essentially saying now is about repetition. We have not come to blocking yet, okay.

We want to drive home the concept of blocking. So we want to repeat with each toner, okay and have an idea about the copy quality, okay. So we are talking about repetition now. The important thing is the photocopier gets heated up during the photocopying and is hottest during the third hour. This also we have encountered. That is why many of these photocopier machines are kept in air-conditioned rooms, okay, right.

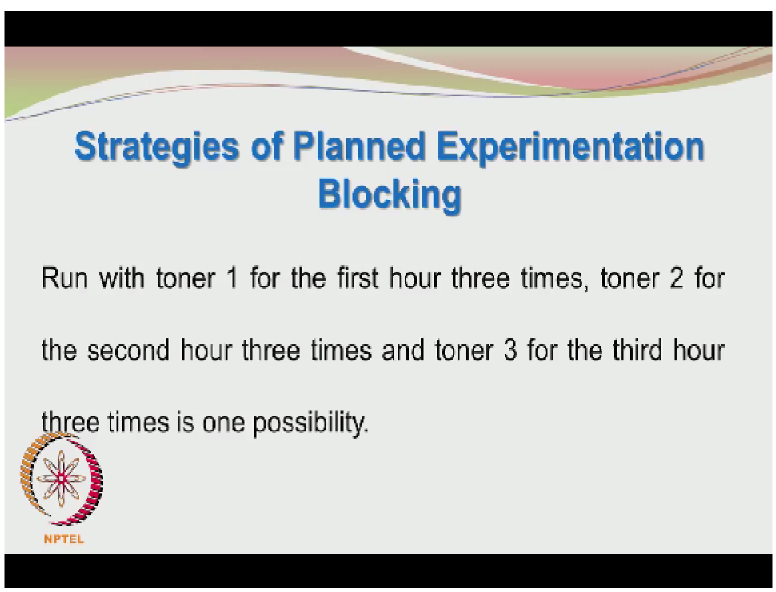
(Refer Slide Time: 05:01)



So there are different ways of doing this experiment, okay and we will do in one particular way first. What we will do is first hour, we will keep toner 1; second hour, we keep toner 2; third hour, we keep toner 3, because each toner takes about 20 minutes and we are going to have 3 repeats. So 20*3 is 60 minutes. So first hour is kept reserved for toner 1 and 3 times the toner is being tested by repetition. Second hour, we use toner 2 and we do it 3 times for repetition.

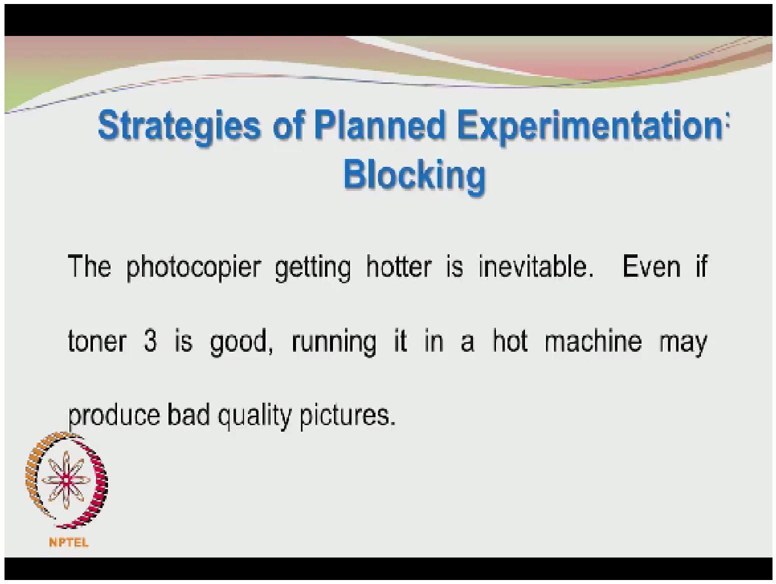
Third hour, we do toner 3. Now you can imagine what may happen? What may be an uncontrollable factor?

(Refer Slide Time: 05:59)



So this is one possibility, run with toner 1 for the first hour 3 times; toner 2 for the second hour 3 times and toner 3 for the third hour 3 times.

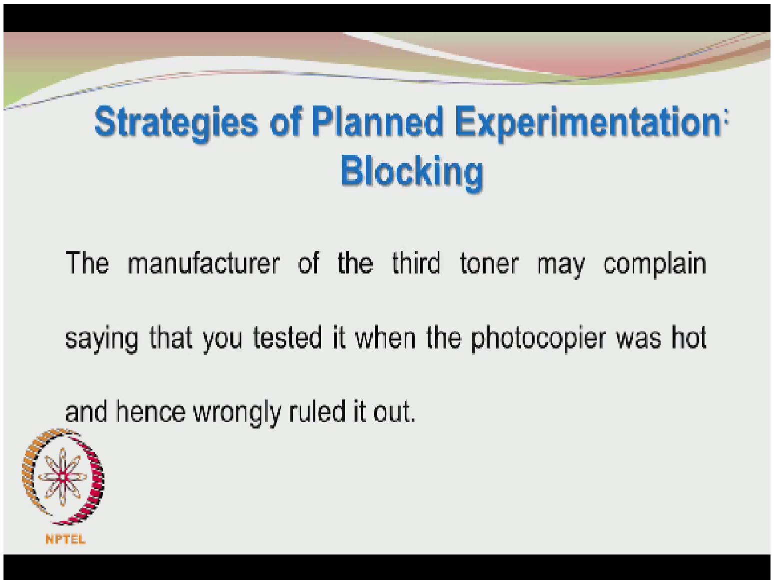
(Refer Slide Time: 06:12)



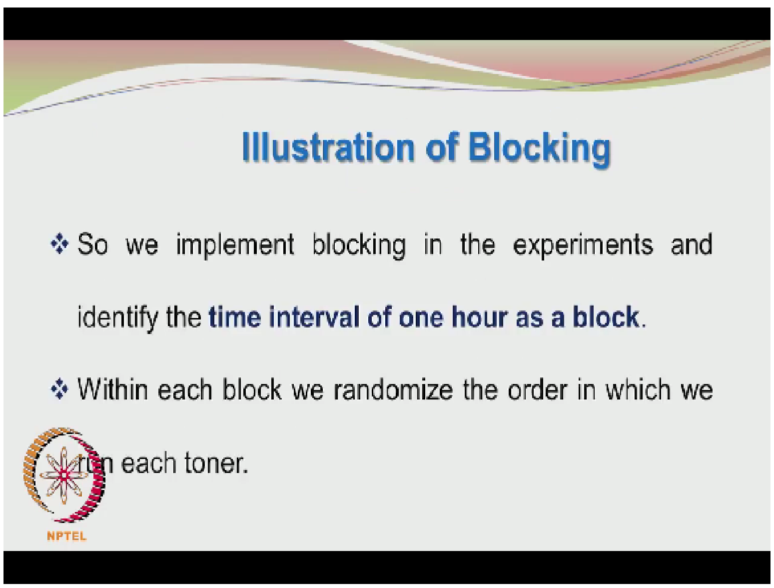
So a photocopier is going to get hot and once it gets hot, the photocopy quality may deteriorate or suffer. So even if the toner 3 is a good one, running it in a hot machine during the third hour may produce bad quality pictures, okay. Then you may conclude that toner 3 is not good. Well that is a wrong conclusion. The manufacturer of that particular brand of toner is going to see red and make a complaint saying that look my toner is as good if not better than the other toners and how can you say that it is bad.

So that is a problem by doing the experiments in the routine fashion. So he can justifiably claim that you tested his toner when the photocopier was hot. In other words, you used the toner in unfavourable conditions and then you had a conclusion based on that and you wrongly ruled out that toner.

(Refer Slide Time: 07:21)



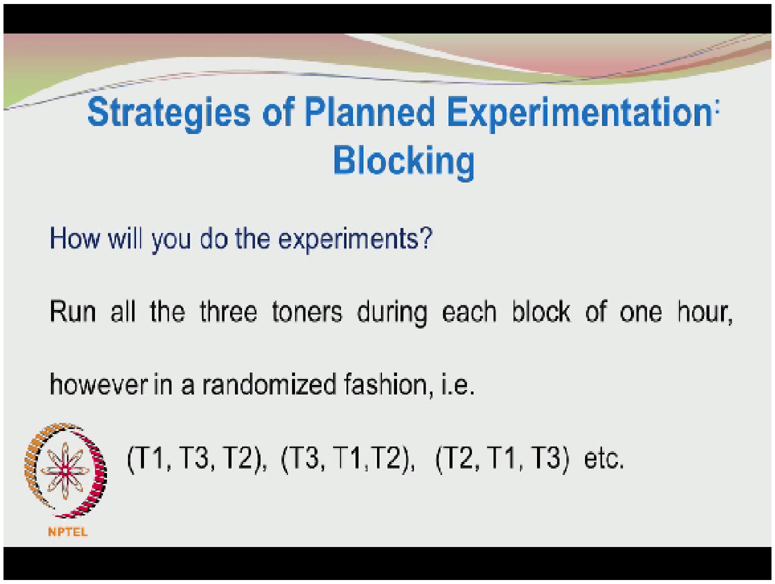
(Refer Slide Time: 07:25)



So what we do it we have to implement blocking in the experiments and identify the time interval of 1 hour as a block. So we implement blocking in the experiments and identify the time interval of 1 hour as a block. Within each block, we randomise the order in which we run each toner. So within one hour, we run the toners in randomised fashion. So the time interval is blocked out.

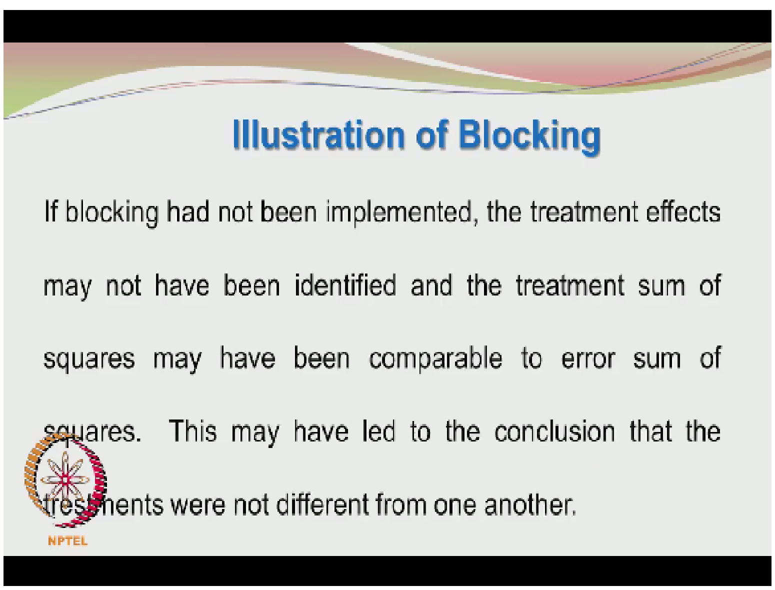
The time interval effect is blocked out and each timeslot of 1 hour we run each of the 3 different toners. So we are essentially having the repeats but we are now using the concept of blocks. So we are having 3 toners. We are having 3 blocks. What are the blocks? The time interval of 1 hour is a block. From let us say 8 to 9 is 1 block, 9 to 10 is another block, 10 to 11 is another block and within each block, the toners are not run as T1 T2 T3, T1 T2 T3, T1 T2 T3. We are doing in a randomised fashion. T1 T2 T3, T3 T1 T2, T2 T3 T1 and so on. So we are randomising within each block.

(Refer Slide Time: 08:59)



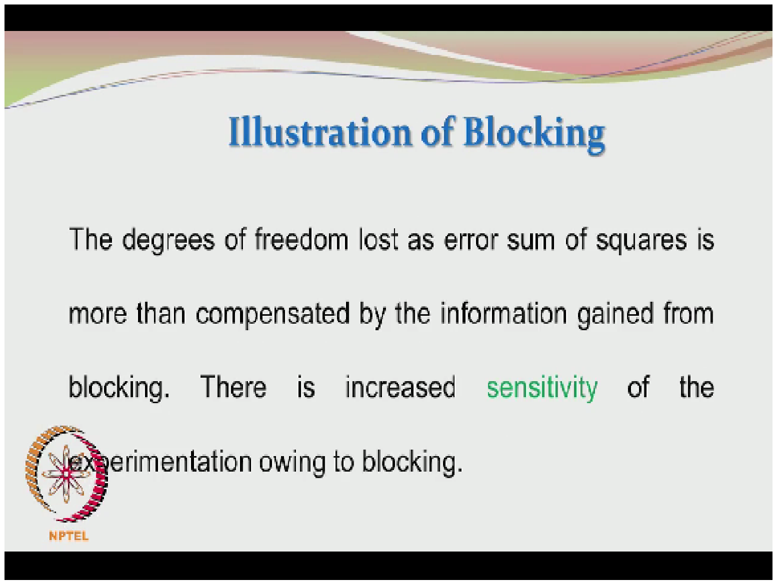
So that is what the next slide says. Run all the 3 toners during each block of 1 hour, however, in a randomised fashion T1 T3 T2, T3 T1 T2, T2 T1 T3, etc., okay.

(Refer Slide Time: 09:21)



So if the blocking had not been implemented, the treatment affects may not have been identified and the treatment sum of squares may have been comparable to error sum of squares. Now we are getting into the analysis of variance mode. So you can realise that we are always comparing the treatment mean square with the error mean square, mean square treatment/mean square error. If the 2 are compatible, then the treatment affects are comparable to noise or random effects and so there is no effect of the treatment, okay. This is what we do in the analysis of variance exercise.

(Refer Slide Time: 10:11)

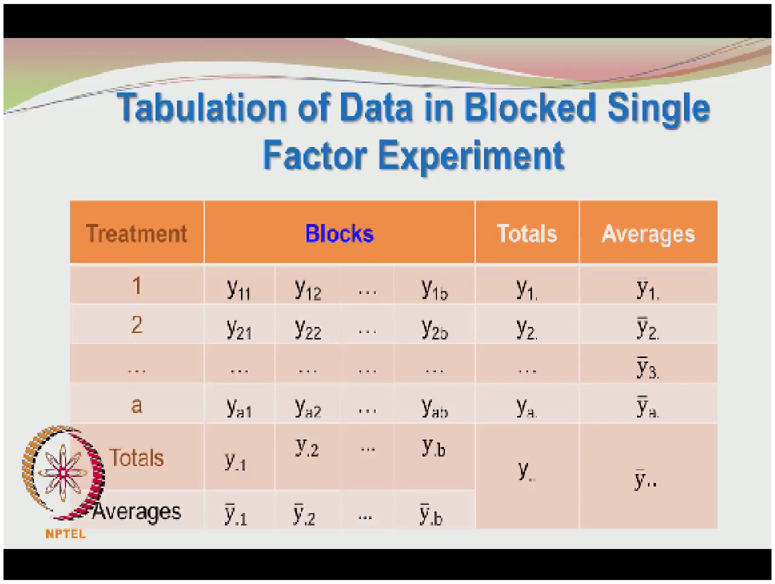


When you do blocking, okay, you lose some of the degrees of freedom in the error component. The treatment degrees of freedom are not altered. Only the error degrees of freedom get reduced as a result of blocking. See the blocking is an additional entity coming into the picture and the total number of degrees of freedom is the same. So that is conserved, the total number of degrees of freedom is conserved and the treatment degrees of freedom is also not going to be changed.

Then you are going to have the remaining degrees of freedom apportioned or shared between the blocking and the error component. If there were no blocking component, the errors degrees of freedom would have been quite high but because of blocking, the blocking will consume certain degrees of freedom and the errors will be left with lesser number of degrees of freedom.

So in this way losing degrees of freedom for the error is not a very good thing but on the other hand by introducing blocking, you are increasing the sensitivity of the tests which is a very good thing. So between loss of degrees of freedom from the error and the gain in the sensitivity of the test because of blocking, it is better to go for blocking.

(Refer Slide Time: 11:48)



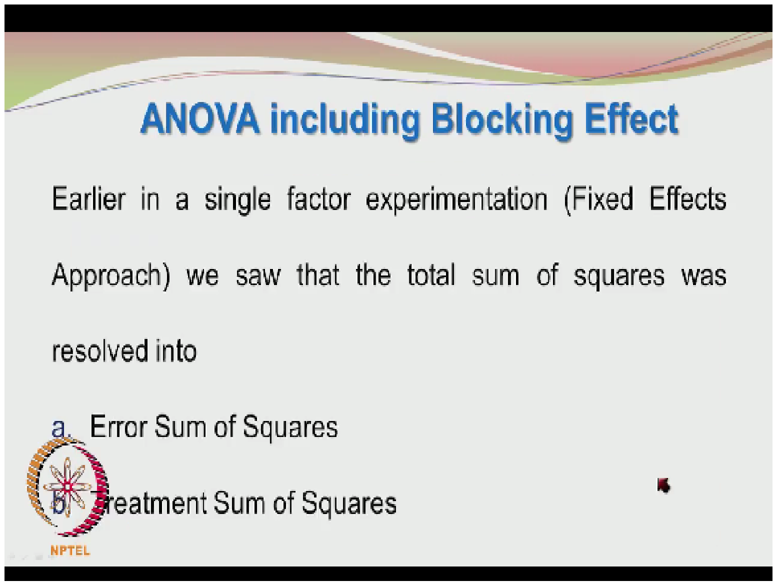
So let us have the usual table or tabulation of experimental data. So you have the treatments in the first column and then you have these observations. These are not repeats, they are experimental observations obtained from different blocks. y11 is the observation from the first treatment from the first block. This is block 1, block 2 and this is block j if you want and this is the beat block, okay.

So you are having a total of b blocks as I said earlier. You are having a treatments. So y21 for example refers to the second treatment observation from the first block and yab would represent the experimental observation of the 8th treatment in the bth block. So i stands for treatment and j stands for block in the treatment observation yij. You should also know that y1. Means, I am adding up all the experimental observations across the b blocks for the first treatment.

So I am summing up over all the blocks for the first treatment, summing up over all the blocks for the second treatment y2.. If I look at it here y.1, I am summing across all the treatments for the first block. I am summing across all the treatments in the second block. I am summing across all the treatments in the bth block. So I have y.b and then I can take the respective averages y bar .1, y bar .2, y bar .b.

These represent the average of the values for a particular block after taking all the treatments into consideration. For example, in this case, I am taking the average across all the observations horizontally, y bar 2. for example, represents the average of all the observations taken across all the blocks for the second treatment. Then you have the grand sum y.. and then you have the global average y bar .., right.

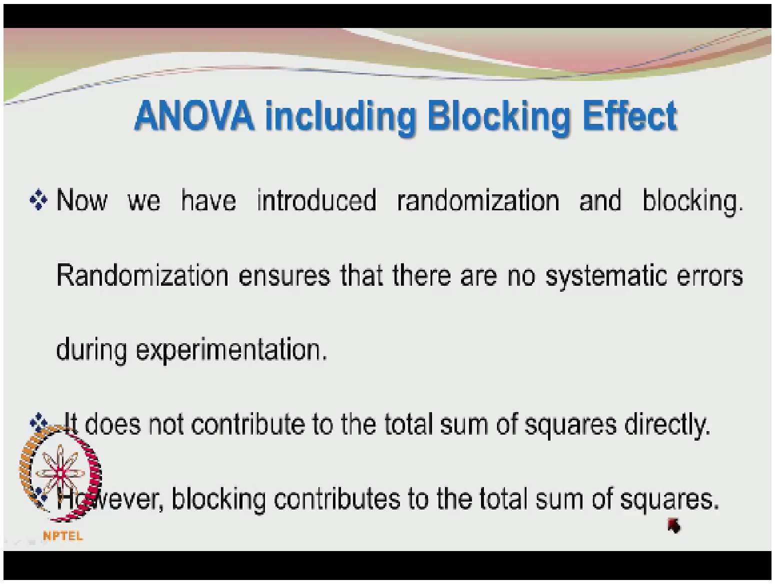
(Refer Slide Time: 14:51)



So earlier when we were not considering the blocking. In a single factor experimentation, we saw that the total sum of squares was resolved into error sum of squares and the treatment sum of squares. Now we have introduced randomisation and blocking. Randomisation ensures that there are no systematic errors during experimentation. In fact, we are buttressing or supporting the concept of random effects, okay.

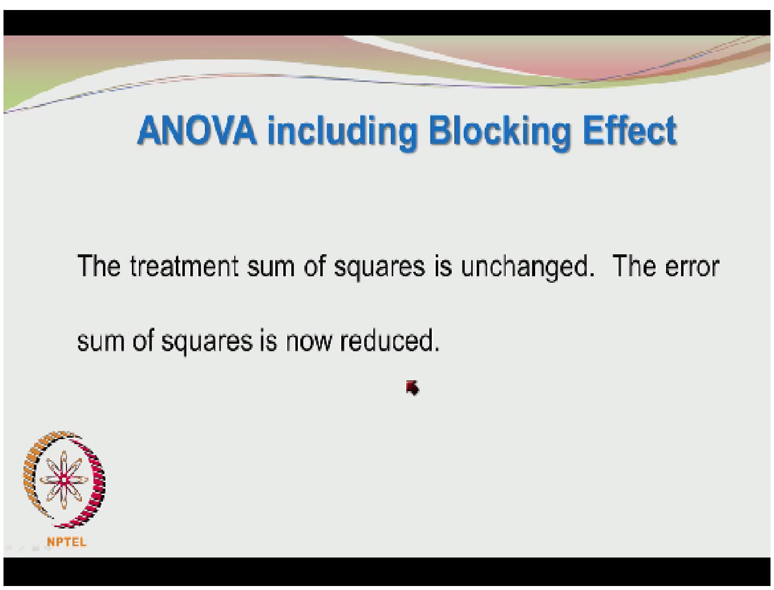
So it is making sure that the error sum of squares is mainly because of random effects and not any systematic effects. So it does not directly contribute to the total sum of squares, okay.

(Refer Slide Time: 15:39)



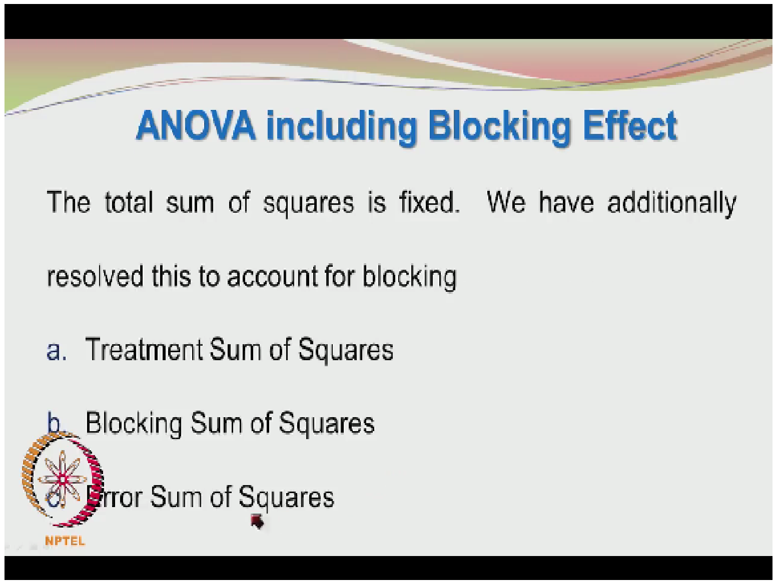
Blocking however contributes to the total sum of squares directly. Let us see how? The total sum of squares is fixed, that is not going to change. The total sum of squares is only based upon the individual observation value minus the global average value and this deviation yij-y.. bar is squared to and then added up to give the total sum of squares. This is fixed. This is resolved into treatment sum of squares, blocking sum of squares and error sum of squares. So let us see how the splitting is done.

(Refer Slide Time: 16:28)



The treatment sum of squares is unchanged. The error sum of squares is now reduced because earlier the error sum of squares was alone present but now once you have the blocking sum of squares coming into the picture, sum of squares earlier taken up by the error, will be now taken up by the blocks.

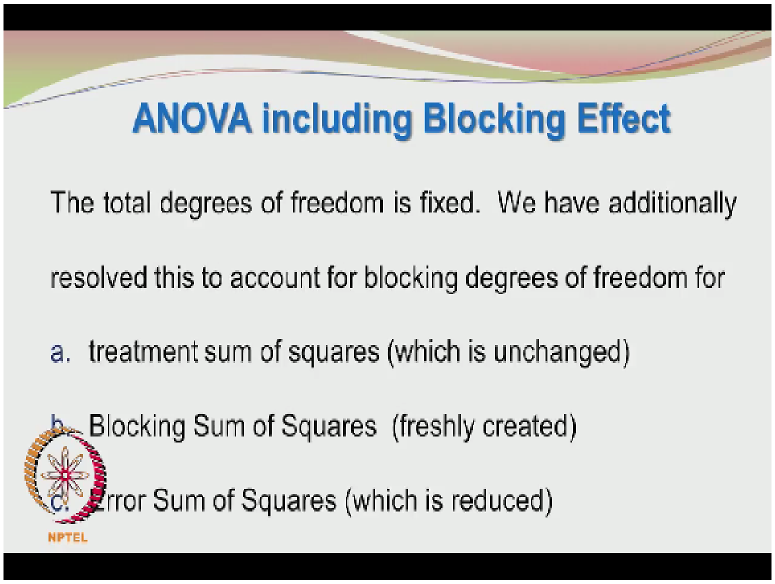
(Refer Slide Time: 16:33)



In simple terms, blocking reduces the sum of squares of the error. Since the sum of squares of the error is reduced and in the F test, the error sum of squares or the error mean squares is coming in the denominator, since the mean square error is reduced, the F value will increase. So there is a greater chance of you rejecting the null hypothesis because the F value has increased and the F statistic is more likely to lie in the rejection region.

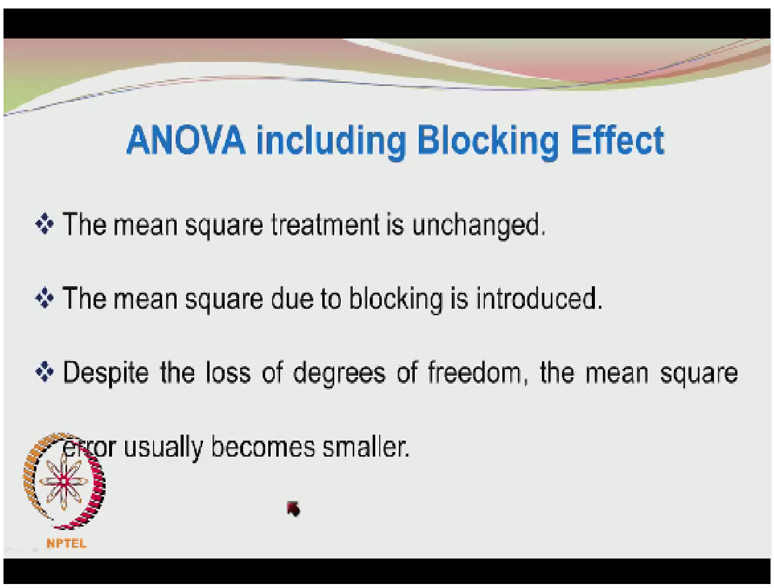
You can see the logic, the mean square error has reduced, the mean square error is coming in the denominator and since it is coming in the denominator, the F value which is based on mean square treatment/mean square error will increase. If the F value increases, there is a stronger chance of you to reject the null hypothesis and make a statement saying that the treatment is indeed having an effect, okay. So the test sensitivity has increased.

(Refer Slide Time: 18:16)



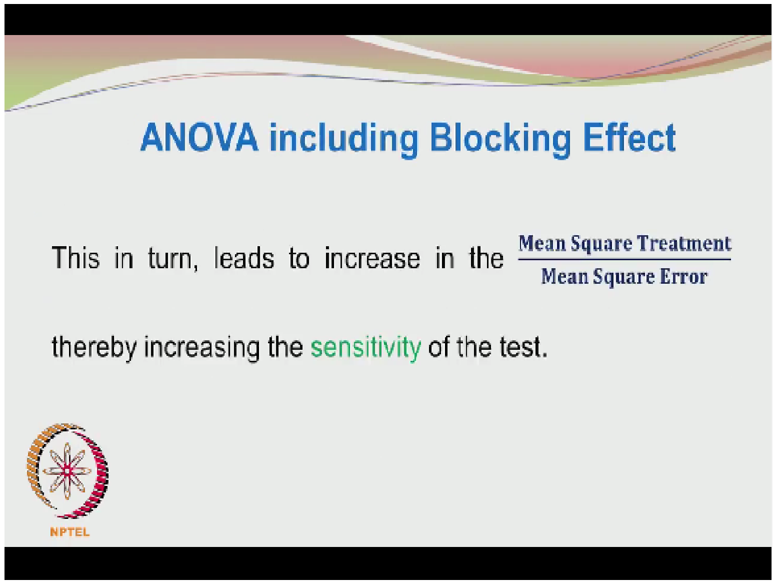
The total degrees of freedom is also fixed. So the treatment sum of squares degrees of freedom is unchanged. The blocking sum of squares will consume some degrees of freedom and the error sum of squares degrees of freedom will be reduced, okay.

(Refer Slide Time: 18:33)



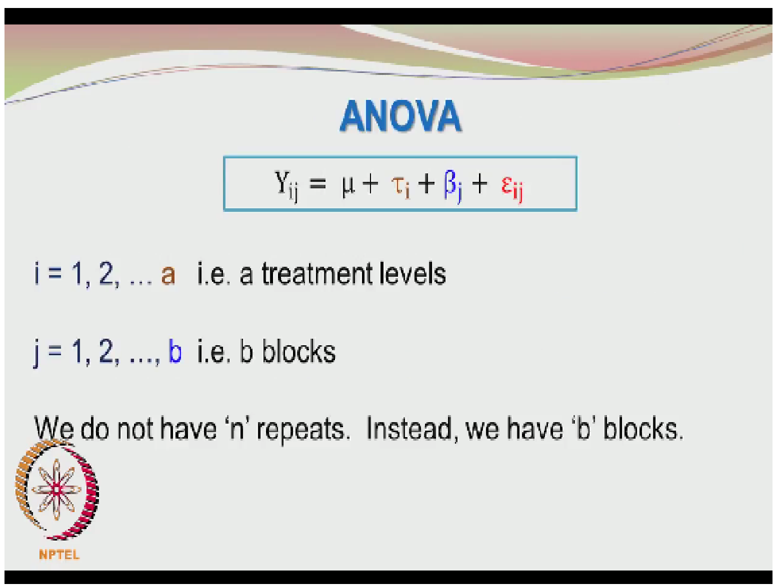
So despite the loss of degrees in freedom, the mean square error usually becomes smaller as a result of blocking.

(Refer Slide Time: 18:46)



So as I said earlier the mean square error reduces as a result of blocking and so this ratio increases and there is a greater chance or likelihood of the statistic lying in the rejection region.

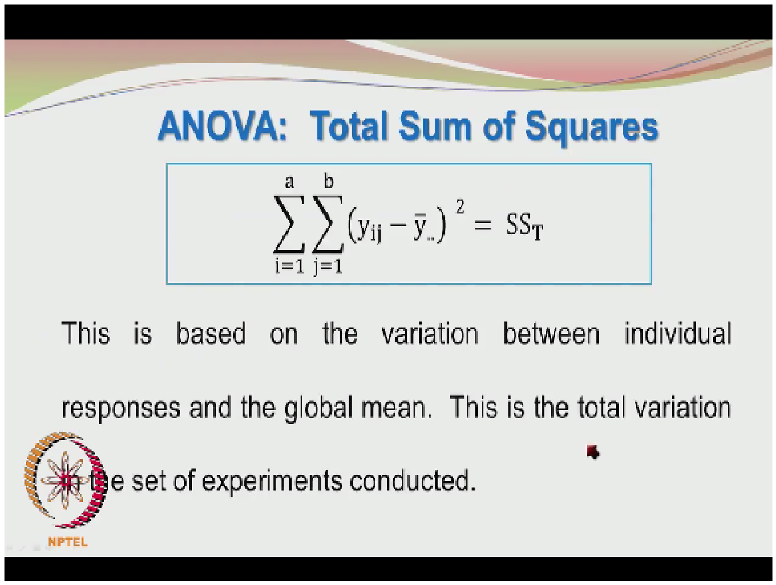
(Refer Slide Time: 19:08)



Let us go back to the linear model, the simple one. The response yij is a sum of the global mean. The treatment mean the blocking contribution and the random component. Tau i is the additional value given by the i treatment to the global mean value, okay. What is the value addition? The I treatment is giving over the normal average value. Beta j is the blocking contribution and epsilon ij is the error contribution.

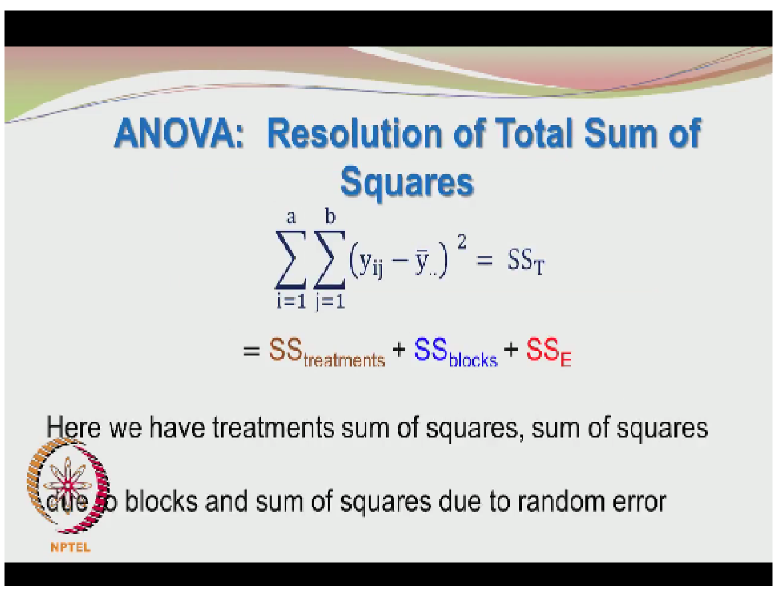
Earlier if the beta j was not present, then this was somehow merged with the error component and so the error component looked larger, okay but now we have reduced the error contribution by bringing in the blocking contribution. Well so j=1, 2 so on to b blocks. i=1, 2 so on to a treatment levels. We do not have n repeats. Instead we have b blocks.

(Refer Slide Time: 20:25)



The total sum of squares is given by the summation over i=1 to 1 j=1 to b yij which is the individual observation-y bar .. which is the global average and this is squared to give the total sum of squares. So this is the total variation in the set of experiments conducted.

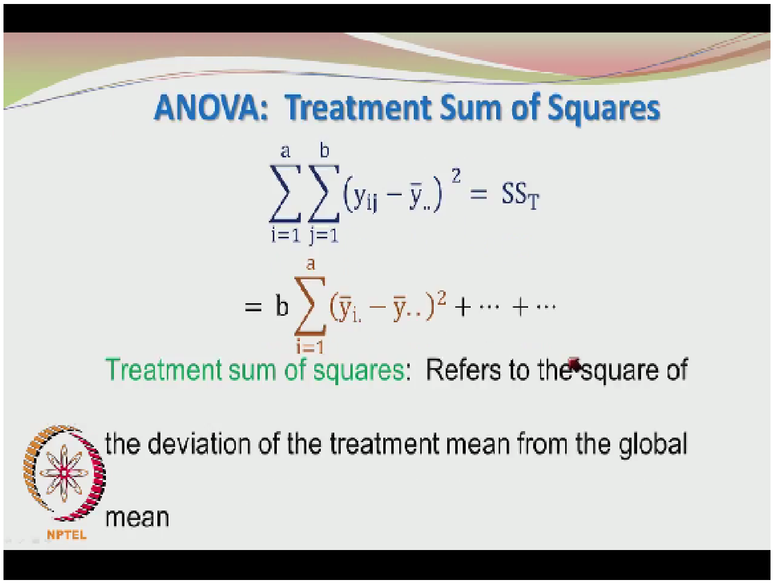
(Refer Slide Time: 20:55)



Then this is resolved into sum of squares due to treatments, sum of squares due to blocks and sum of squares due to error. So we need to see how this particular square term is getting split into

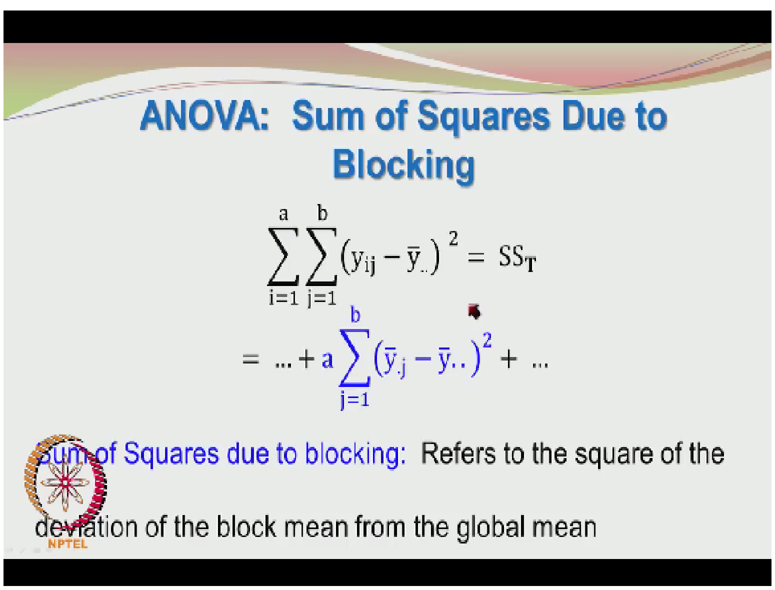
3 different sum of squares.

(Refer Slide Time: 21:18)



The treatment sum of squares is the same as we encountered in the previous single variable experimentation without blocking. So we have b, number of blocks, i=1 to a, treatment mean-the global mean, how different is the treatment mean from the global average that is squared. Then we have the additional contributions from the blocking and from the random error. Let us see the contribution from blocking next.

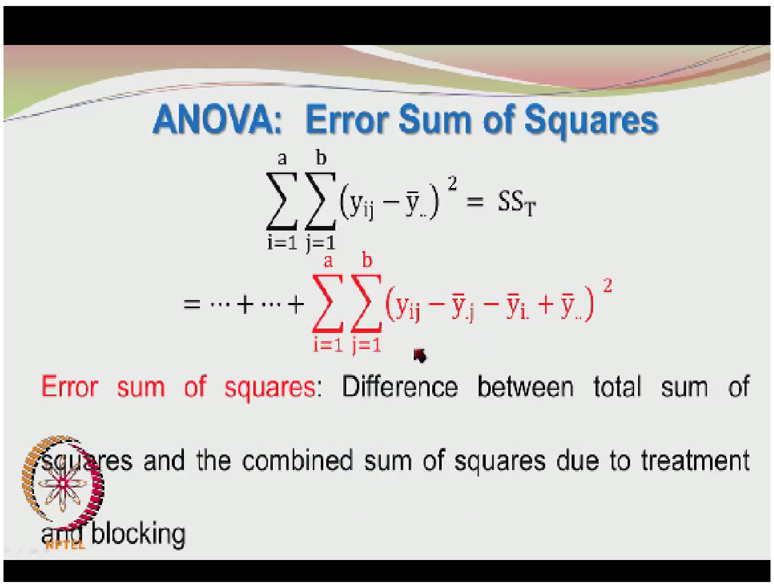
(Refer Slide Time: 21:57)



The blocking is given in terms of a j=1 to b y bar .j-y bar.. squared. So for each block, you are summing across all the treatments, that is why you have y. here. If you go back to the table of observations, you will see what I mean. We are now looking at y.j, so what is that? y.j means we are summing across the index i. So you are having y. a given value of j, it can be 1, it can be 2 or it can be up to b.

So we are summing across all the treatments for a given block, for a given jth block, we are summing across all the i treatments. We are summing across all the a treatments, i running from 1 to a. So this is also very clear. Where was I. So it would be summing across all the treatments for a given block that how is it different from the global average y bar… So this difference we square. So again this is quite easy to understand.

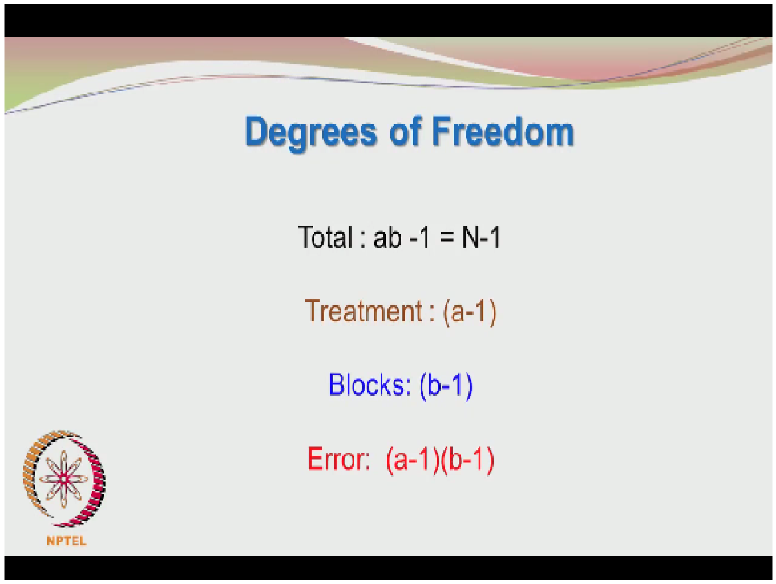
(Refer Slide Time: 23:48)



Then we have the error sum of squares which is given by yij-y bar .j-y bar i.+y bar .. whole squared. So it is the difference between the total sum of squares and the combined sum of squares due to treatment and blocking. So we are having the total sum of squares and from that if you take out the contribution from treatments and the contribution from blocks, whatever sum of squares is left, is the error sum of squares. It is easy for us to remember it this way.

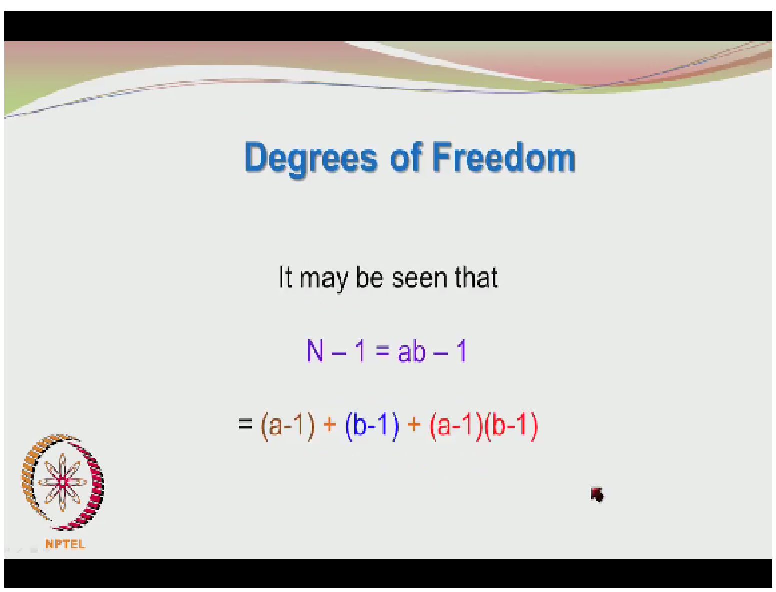
We have removed effect of treatments, we have removed the effect of blocking. So there is some more variability and that is only because of random effects.

(Refer Slide Time: 24:46)



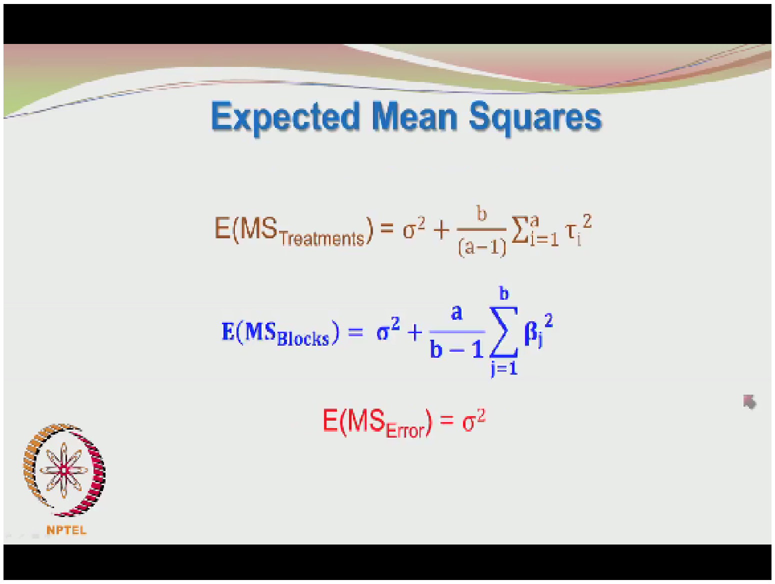
So if you look at the degrees of freedom, the total degrees of freedom would be total number of experiments-1. You subtract -1 because you are calculating the global mean from the available ab data points. So you have ab-1 which is n-1. You have the treatment degrees of freedom which is a-1. The blocks degrees of freedom which is b-1 and if I subtract from the total degrees of freedom the treatment degrees of freedom and the blocks degrees of freedom, I get error degrees of freedom as a-1*b-1.

(Refer Slide Time: 25:22)



So N-1 total degrees of freedom which is equal to ab-1 and if I takeout from ab-1, a-1 and b-1, I will have leftover degrees of freedom as a-1*b-1 which is the errors degrees of freedom.

(Refer Slide Time: 25:47)



Now we have to look at expected values of the mean squares. Why do we need to do that? Again it is very very interesting because we can attach a lot of physical significance to the mathematical results, okay. Another good thing about statistics is whatever mathematical calculations you perform, you are going to get some results and these are not abstract results but they carry a lot of physical meaning as well.

So I think mathematics applied to real life may be termed as statistics. Again this is my own invention. I do not know whether it can be fully justified. Anyway let us get on. Expected mean square treatments=sigma squared+b/a-1 sigma i=1 to a tau I square. What this means is the expected value of the mean squared treatments is sigma squared. If you remove this or ignore the second component, we are in deep trouble because sigma squared is the error variance.

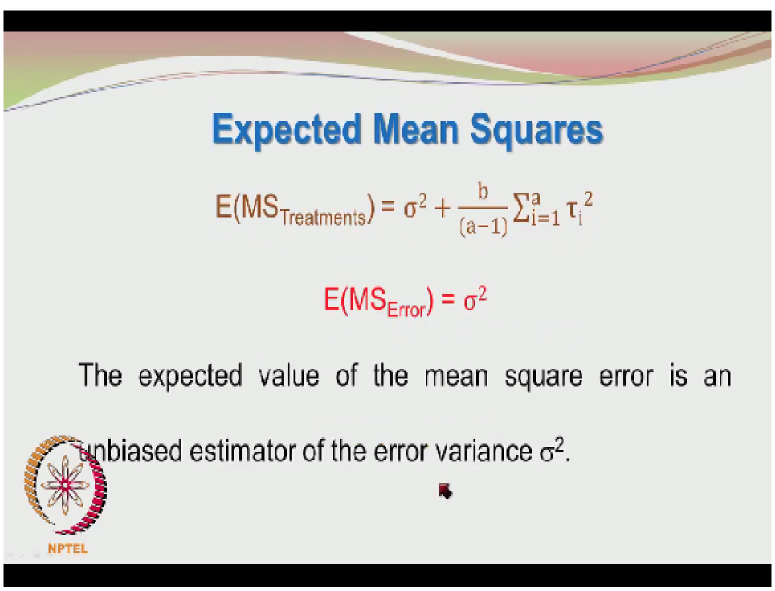
So if the variability because of treatments leads to the error variance, then the treatments are not effective at all. They are only adding to the random noise, okay. That is disturbing. So fortunately we have this additional term b/a-1 sigma i=1 to a tau i squared. You may recollect that tau I is the contribution or value addition given by the ith treatment. So if you look at the original linear model, you have yij=mu+tau i+beta j+epsilon ij.

So tau i is the ith treatment value addition and fortunately that is significant in many cases. So the expected value of the mean squared treatments is a variability which is an addition over the random variability and that is given by this term. So this is expected and this is good. We want this variability to be present so that we can claim that the treatments are effective, at least some of them are effective.

Then you have the expected value of the mean square blocks which is given by sigma squared+a/b-1 sigma=1 to a beta j squared and if the block effects were not there, which is very good because the blocks are not really different. They are all identical. Then you are actually performing experiments because the blocks are identical, then you are doing identical blocks experimentation and that would represent genuine repeats.

So the expected value of the mean square blocks is sigma squared+the additional contribution due to blocks. Anyway this may also be there and now very nicely, the expected value of the mean square error=sigma squared which is the random noise effect. Earlier the error contribution was confused with the blocking information also if it had existed and with this resolution, you are able to separate out the expected value of the mean square blocks from the expected value of the mean square error.

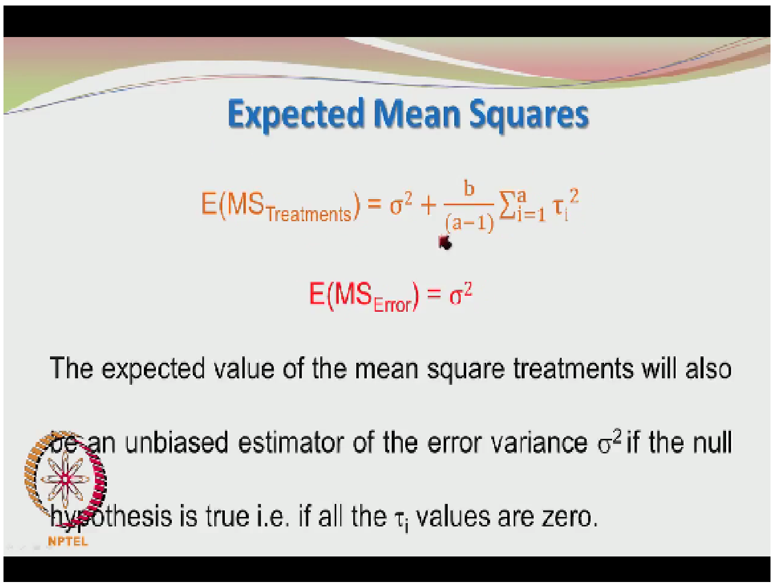
(Refer Slide Time: 29:32)



So you can see that the expected value of the mean square error is an unbiased estimator of the error variance sigma squared. When the expected value of statistic is the population parameter itself, then that statistic becomes an unbiased estimator. Again we have seen this concept a few classes back. So whatever we have learnt previously are finding applications in the design of experiments.

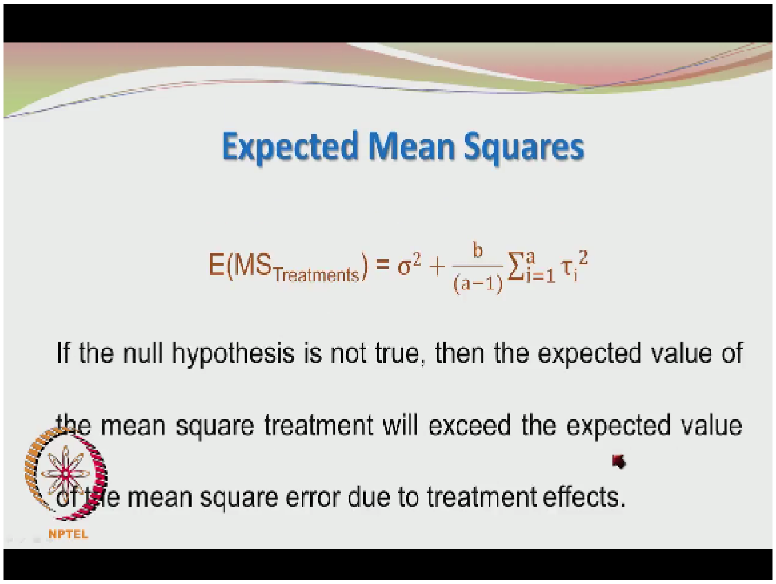
So the expected mean squared treatments also would be an unbiased estimator of the error variance sigma squared provided all the treatments were ineffective. So tau 1=tau 2 so on to tau a=0 that means there is no additional contribution from these treatments. So these things will vanish. Only then when the treatments were ineffective, the expected value of the mean squared treatments would be sigma squared.

(Refer Slide Time: 30:41)



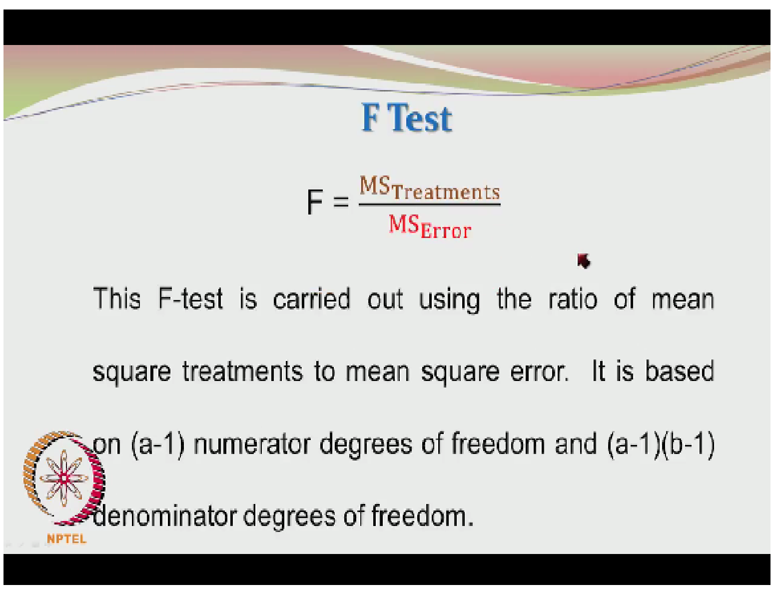
However, that is again not usually the case and so we have to work with the expected value of a mean square error which is sigma squared that is the unbiased estimate of the error variance sigma square, right.

(Refer Slide Time: 30:59)



If the null hypothesis is not true, that means there is at least 1 treatment which is effective and which is different from the others, then the expected value of the mean squared treatments will exceed the expected value of the mean square error which is sigma squared by this particular contribution. So this contribution is very very important for the treatments to be effective or rather to put in another way, the treatment is effective or different from one another because this term does not go to 0.

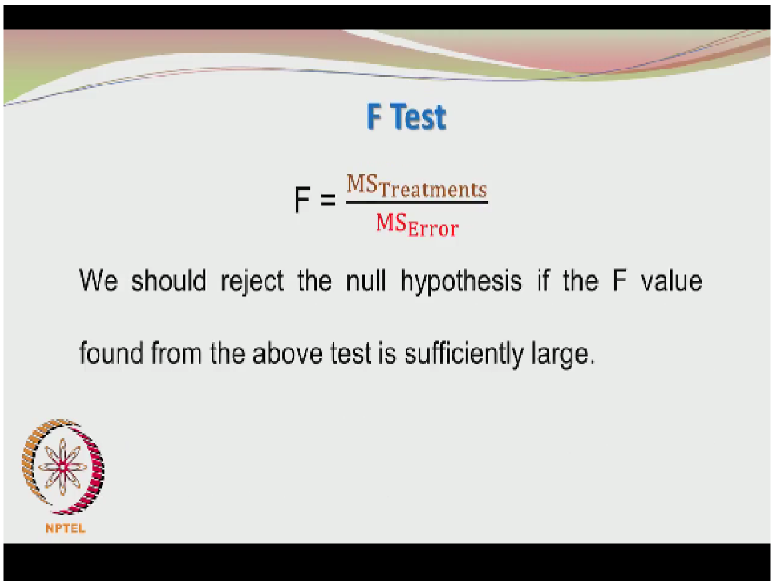
(Refer Slide Time: 31:41)



Now you can carry out the F test, a more sensitively F test based on the mean squared treatments by mean square error and now it is important to note that it is based on a-1 numerator degrees of freedom as before and a-1*b-1 degrees of freedom for the denominator, okay. So who earlier you were having 1*n-1. Now you are having a-1*b-1. If all the blocks were identical, b will be equal to n and now we are having let us say if our identical blocks a-1*n-1.

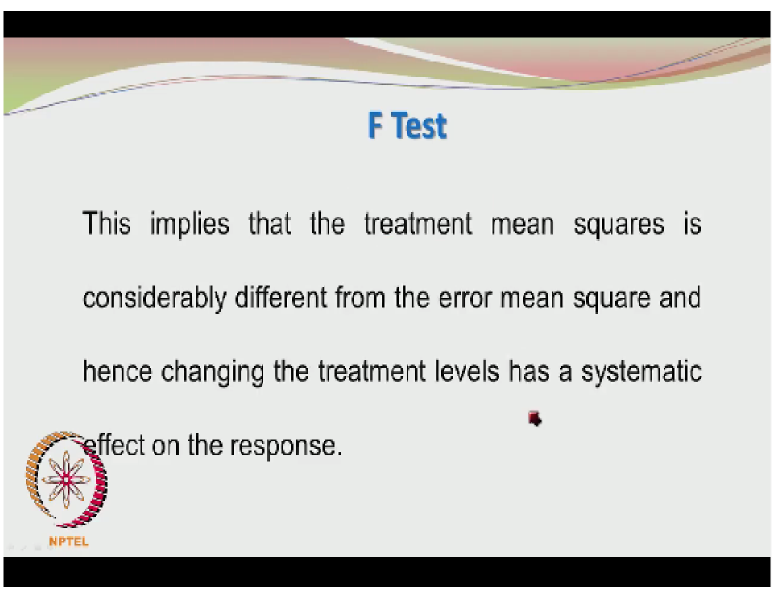
So this is smaller than 1*n-1. So we are having smaller number of degrees of freedom for the error which makes it somewhat less sensitive but on the other hand, the mean square has considerably reduced which is very good and so our test has become on the whole more sensitive.

(Refer Slide Time: 32:52)



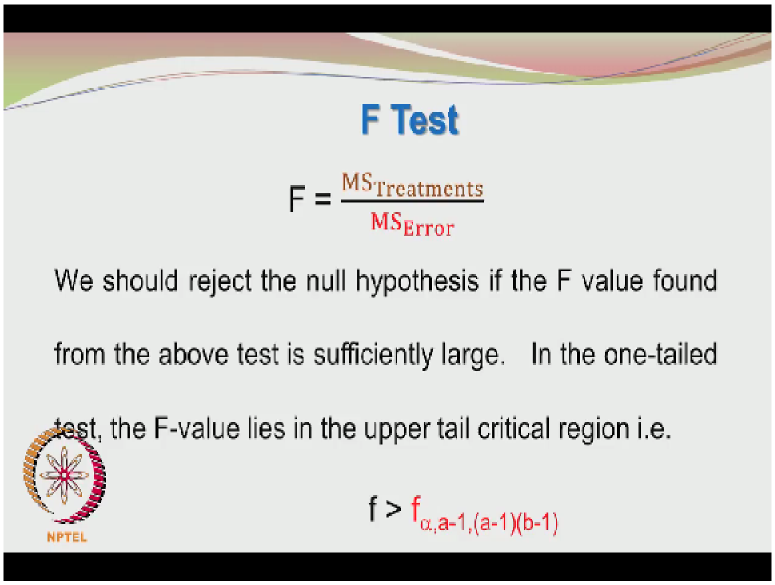
So we should reject the null hypothesis if this F value is sufficiently large.

(Refer Slide Time: 33:02)



So if the mean squared treatments is considerably different from the mean square error, it means that the error mean square is very small when compared to the treatment mean square. The treatment is having such a large variation or variability produced in the experimental observations which completely dominates over the error variability and so the treatments are effective in altering the response of the process.

(Refer Slide Time: 33:40)



So we again as before, if the F value lies in the upper tail critical region, I think let us use small f to be more precise. Let me make the change because we are (()) (33:59) value of the F statistic if that f value after the measurements have been done and the errors and the treatment sum of squares have been calculated, we will get a value which we call it as small f and that happens to be greater than F alpha a-1 a-1*b-1, then you reject the null hypothesis because the f value is lying in the upper tail critical region.

(Refer Slide Time: 34:31)

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_o
Treatments	$SS_{Treatments}$	a-1	$\frac{SS_{Treatments}}{a-1}$	$\frac{MS_{Treatments}}{MS_E}$
Blocks	SS_{Blocks}	b-1	$\frac{SS_{Blocks}}{b-1}$	
Error	SS_E	(a-1)(b-1)		
Total	SS_T	ab-1		

So now we have the modified analysis of variance or ANOVA table. We have the treatment source of variation sum of squares of treatments a-1, mean square is sum of squared treatments by a-1, nothing is different from what we did previously. When you come to blocks, you have sum of squares of blocks, you have b-1 degrees of freedom where b is the number of blocks. You have again means where the sum of square of blocks/b-1.

The error sum of squares is having sum of squares of error which is a-1*b-1 and the total sum of squares is SST which is ab-1. So the F0 statistic becomes mean square treatments/mean square error. So this completes our discussion. What remains is to illustrate these concepts through some examples. Any book on statistical design of experiments will carry lot of interesting examples.

Actually what will happen is the experimentation is one story, the calculation is another part and the calculations by hand or even with the calculator will become more cumbersome as we introduce more detail into our analysis. Even with single factor experimentation without blocking, we could just possibly get away with doing calculations with the calculator. In other words, students can answer such kind of problems without too much difficulty in about 40 minutes of a quiz.

However, the moment you start adding blocking, the calculations become more and there are more possibilities of making mistakes when doing these calculations. Of course, the teacher will not be impressed if the student goes to him and says this is a random error, so you should give me marks, that is not going to happen, okay. He will say it is a systematic error. You made a deliberate error, okay.

So let us not get into that. We have to be friendly to the student and we cannot be giving such kind of blocked designs to the students and ask him to work it out in 40 minutes. Of course, if the spreadsheet is available during the tutorial or even in the exam, I foresee exams to become open book, open resources like they can take laptops with them or other programmable calculators, online exams with the help of computers, okay.

Then of course these things can be done but that may be slightly far off from now and so it is better to expose the students to some statistical software, okay and help them to interpret the results which are shown from these software. What the students can do is do some of these problems with the help of a spreadsheet in assignments, compare their answers with answers given by statistical software and see whether they are doing it correctly.

More importantly as the problems becomes more complicated, more number of factors, more repeats, different levels, so it becomes quite cumbersome, then the importance is to understand the results given by the statistical software, the interpretation of results and the implementation in our actual experimental work. It is very important. So we have covered one important aspect of experimental design which is experimentation with the single factor.

Even that, we introduced the concepts of blocking and randomisation. Of course with more variables, you can still have randomisation and blocking but doing blocking for more number of factors is beyond the scope of this course but those people who are interested can of course look into the prescribed textbook by Montgomery to get additional details on this. So what we will do next is to solve a few problems to drive home these concepts. Thank you for your attention.