Statistics for Experimentalists
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Lecture - 33
Fractional Factorial Design-Part B

So, it was said that main factor A was aliased with BCD.

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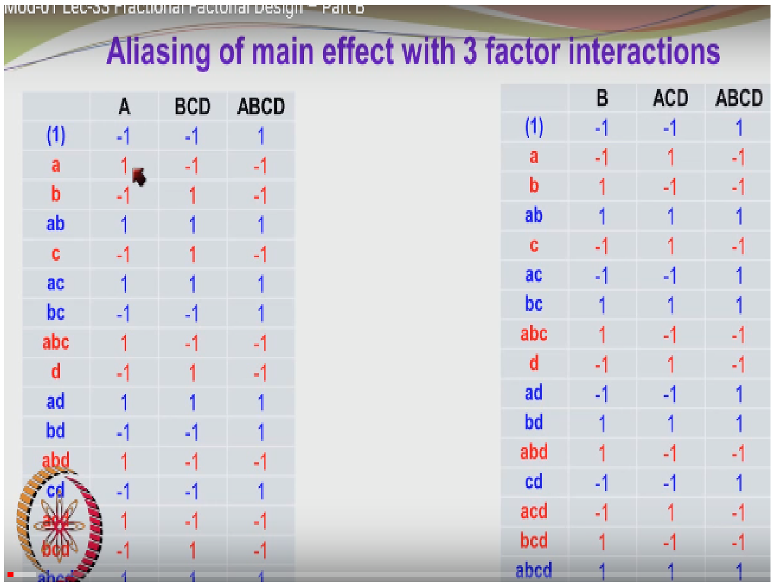

If we look at this particular table, we can see that the blue entries for A and BCD or the entries in blue for A and BCD are exactly identical-1 here-1 here 1 here 1 here 1 1-1-1 1 1-1-1-1-1 1 1. So, that is matching perfectly and if you also want to look at the red okay before we go to the red entries let us look at the blue entries and all these blue entries are corresponding to the blue entries of the design generator ABCD.

Now looking at the red entries red entries correspond to the negative 1 or-1 entries in the design generator ABCD. So, when you look here you can see the red entries are related in such a way that is A=-BCD. So, with that relation 1 for A and-1 for BCD and from the second setting b if a is at a lower level-1 BCD is at +1. So, that corresponds to the second fraction in the second fraction we are all taking the negative values for ABCD.

The second fraction corresponds to the negative entries in the ABCD column and that is the reason why you have the relation A=-BCD for the second fraction the first fraction is A=+BCD in the second fraction A=-BCD. We also know that B was interacting or aliased rather with ACD interaction looking at the blue entries we can see that all the blue entries are matching whereas the red entries are related by B=-ACD-1+1 1-1-1 +1.

So, the red entries correspond to the second fraction the aliased effects are related by a negative value in the principle fraction we are having the main effect aliased with the 3 factor interactions directly in the second fraction the main effects are aliased with the 3 factor interactions with a negative relationship. Similarly, for other 2 factor interactions 2 factor interactions are also aliased with each other that table I do not have.

But you can easily show that for example ab column would be exactly matching the cd column for the principal fraction whereas ab column would be related by related to-cd in the second fraction I leave these things as self exercises.

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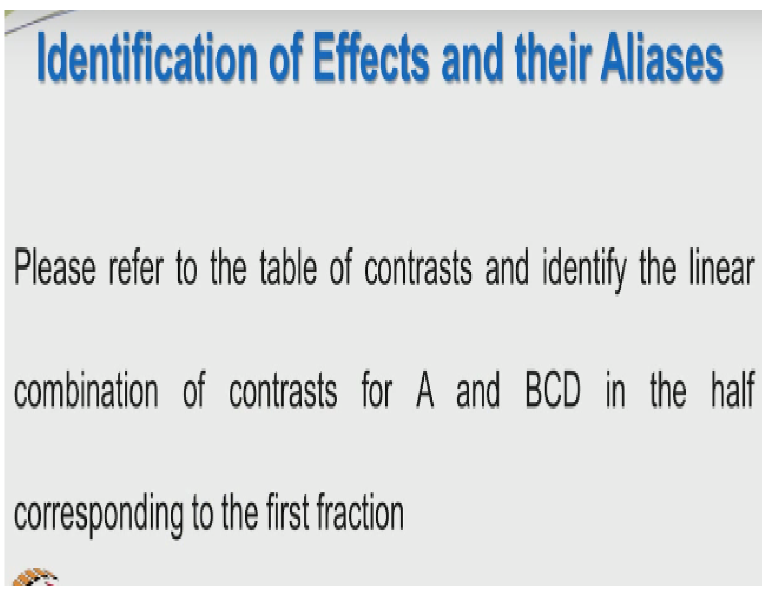

So, let us again refer to the table of contrasts and identify the linear combination of contrast for A and BCD in the half corresponding to the first fraction. So, please look at the table linear combination of contrasts for A and BCD we just now saw that A and BCD are identical, so you cannot separate the 2 contributions and you have to live with both of them. So, if you look at the contrast for either A or BCD.

The effect obtained from the contrast is going to represent both A and BCD. So, you will have-1-1 +ab+ac-bc +ad-bd-cd and abc +abcd I request you to write it down and then compare it with what I am going to show in the next slide.

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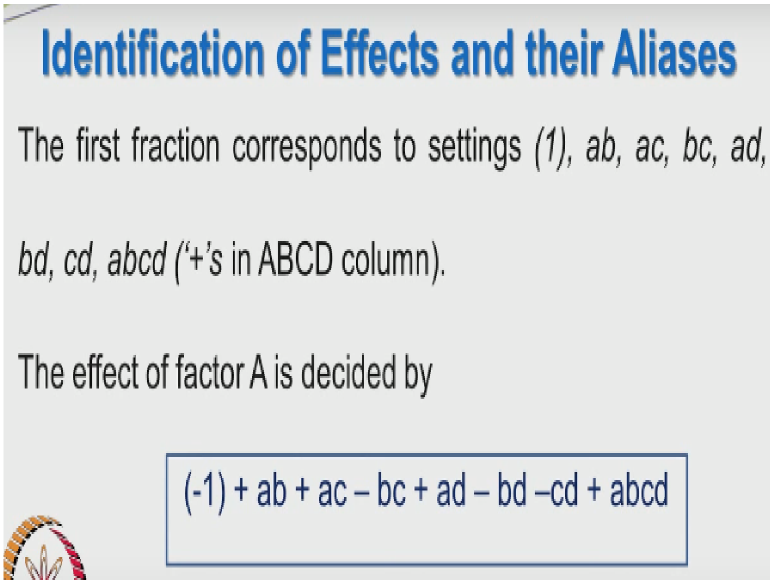

So, its effect of factor a is decided by-1+ab+ac-bc+ad-bd-cd+abcd please note that there are only 8 entries here 3 5 and 3 so that makes it 8.

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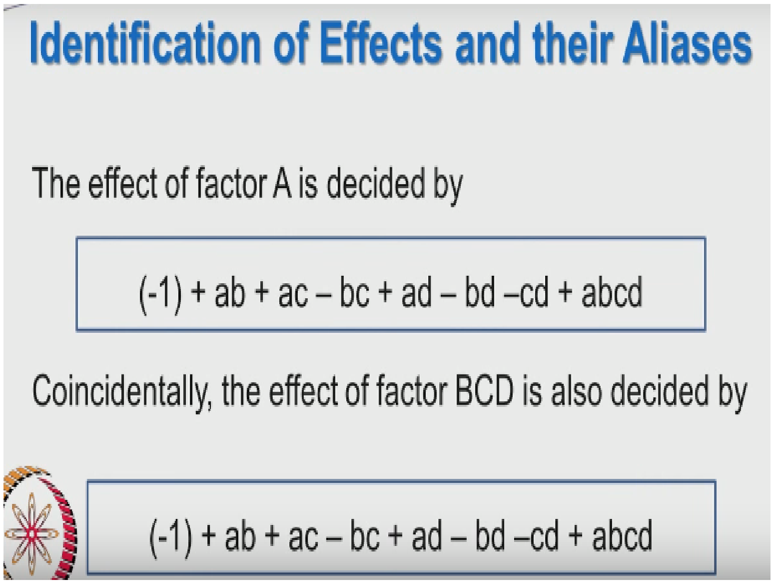

Not only affect our factor A is decided by the contrast as shown here the same contrasts defines the effect of factor BCD also. So, these 8 entries are representing the combined effects of A and

BCD.

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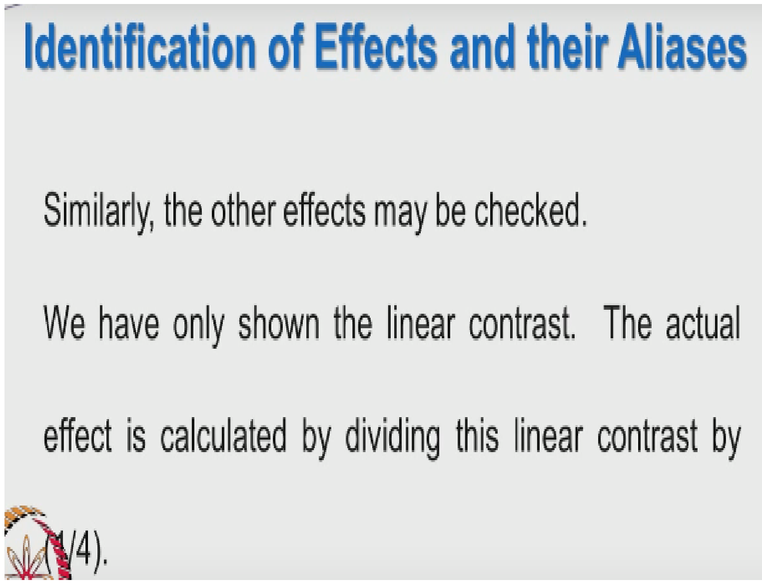

Similarly, we can check for the other effects. And the linear contrast was only least shown in order to calculate the effect you can you have 4 positive entries and then 4 the negative entries you are finding the difference between the 4 positive entries and the 4 negative entries and when you average hence you have to divide by 4.

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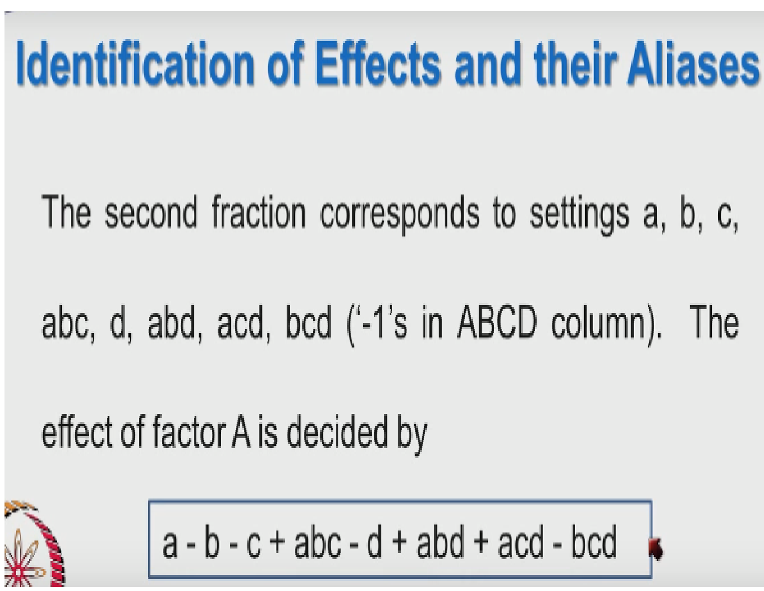

And if you look at the second a fraction please look at the red entries 1-1-1 so the a b c abc d and so on. Please write it again on the paper the calculation for the contrast of a corresponding to the second a fraction so you will get a-b-c +abc-d+abd+acd-bcd you may verify this and this is A is not only uniquely determined but A will also be matching with-BCD so the effect of factor- BCD is also decided by the same contrast.

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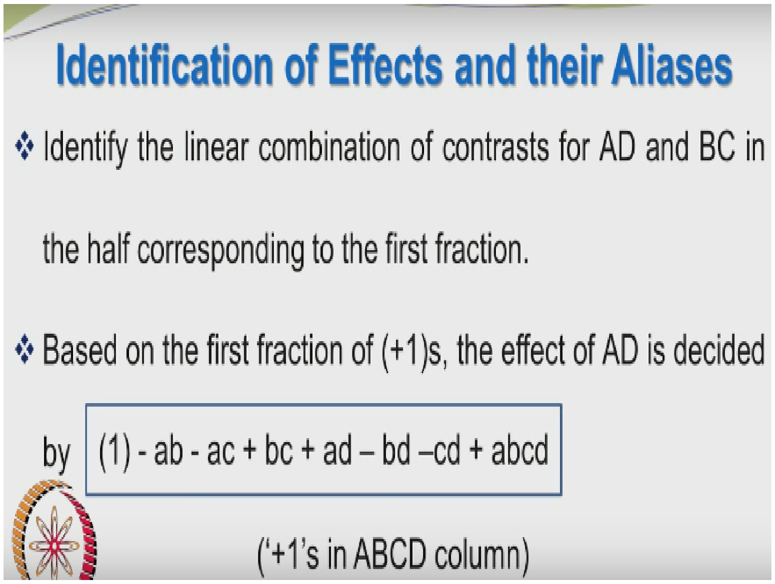

So, for AD the 2 factor interaction you go back to the principal fraction given in blue color and AD you have to go all the way back to the original table so it would be 1-ab-ac+bc+ad-bd-cd+abcd and so that is the effect of AD but that is not only the contrast unique contrast for AD the same contrast applies for BC as well.

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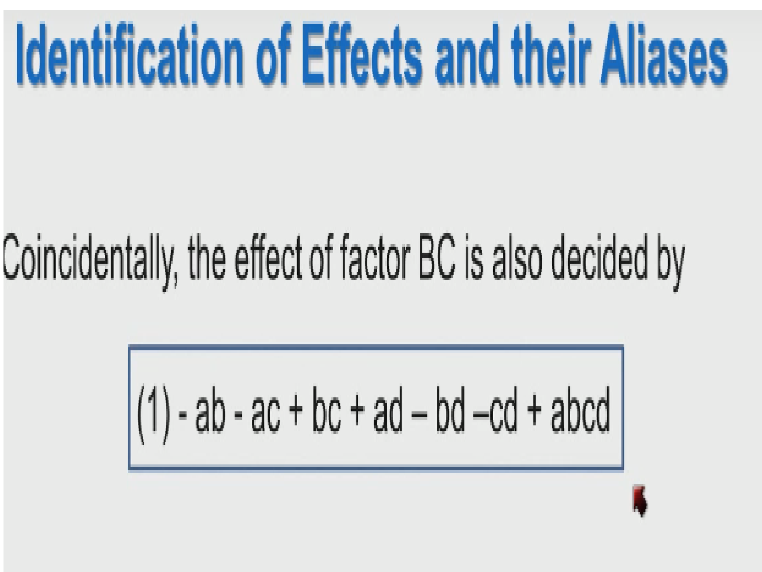

You can go back and check it out that the contrast in the first fraction are the same for both BC and AD. So, similarly you can find the aliases for other 2 factor interactions. So, now the important question is how do you find the effect of A and the effect of BCD or how do you resolve the combined effects of A and BCD or the separate the BC with AD. So, what I am trying to say here is we know that the main factors are aliased with 3 factor interactions.

And 2 factor interactions are aliased with other 2 factor interactions we want to separate out this combined effect so that we uniquely get the effects of A B C and D and also AB BC CD and so on. So, with this in mind obviously the resolution is to do the first fraction and analyze it and then do the next fraction analyze it and in order to segregate the different effects you combine the 2 fractions.

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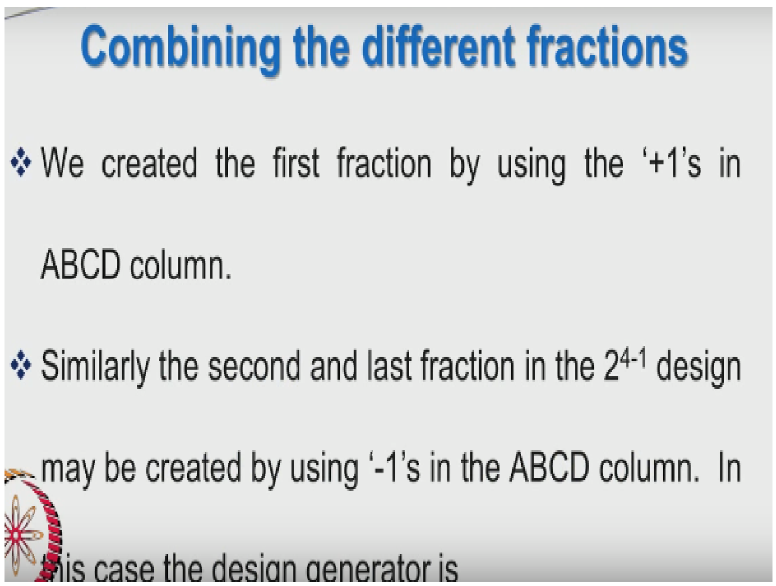

Now sort of summarizing we created the first fraction by using the +1 in the ABCD column. Similarly, the second and last fraction here in the 2 power 4-1 design was created by using-1 in the ABCD column in this case the design generators I=-ABCD the second fraction the design generator was I=-ABCD in the first fraction the design generator was represented by I=+ABCD I only represents the matrix of pluses.

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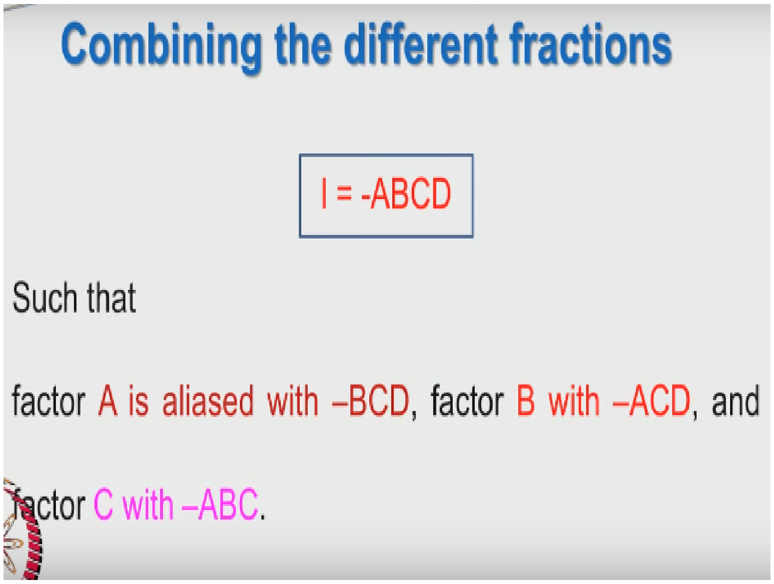

So, you have factor A is aliased to-BCD factor B is aliased with-ACD and factor C with-ABC how is it possible we just put the A here-A Squared A squared will be all +1 so we can take it out *any numbers that number. So, we can just take out a square and we have A=-BCD similarly B=-ACD and C will be=-ABD. So, that is a typo I will just correct it immediately right, so C is with=-ABD.

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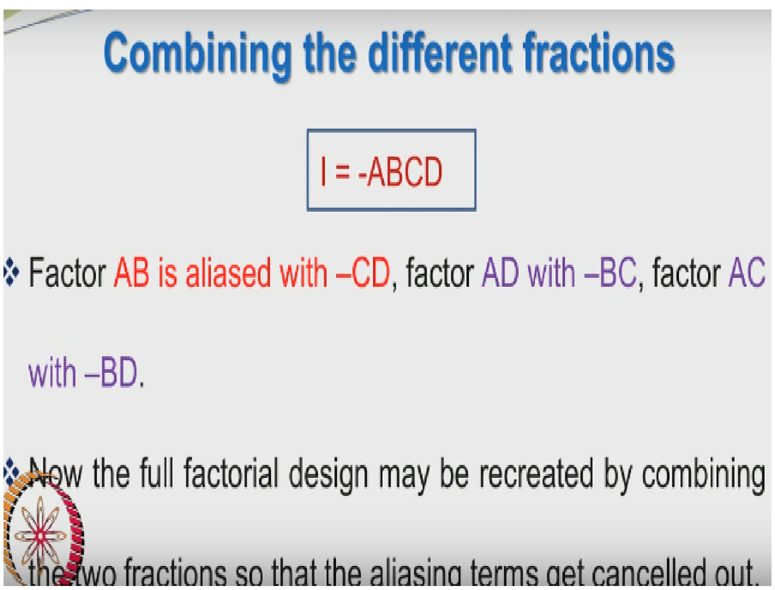

So, now the full factorial design maybe recreated by combining the 2 fractions so that the aliasing terms get cancelled out.

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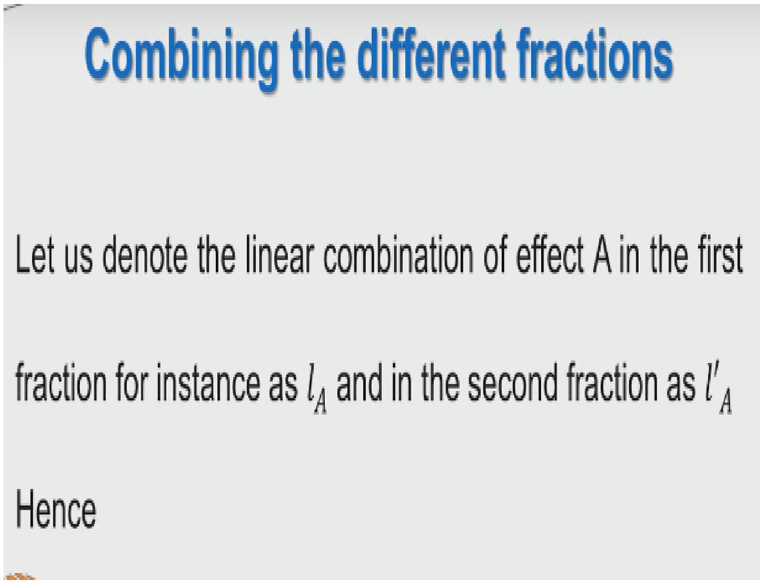

And let us denote the linear combination of effect A in the first fraction as LA and in the second fraction as L prime A. This is the linear combination we know that in the first case A was aliased with BCD and in the second fraction A was aliased with-BCD and when you combine the 2 fractions the BCD in the first fraction will cancel out with the-BCD in the second a fraction. So, you will be able to recover A alone.

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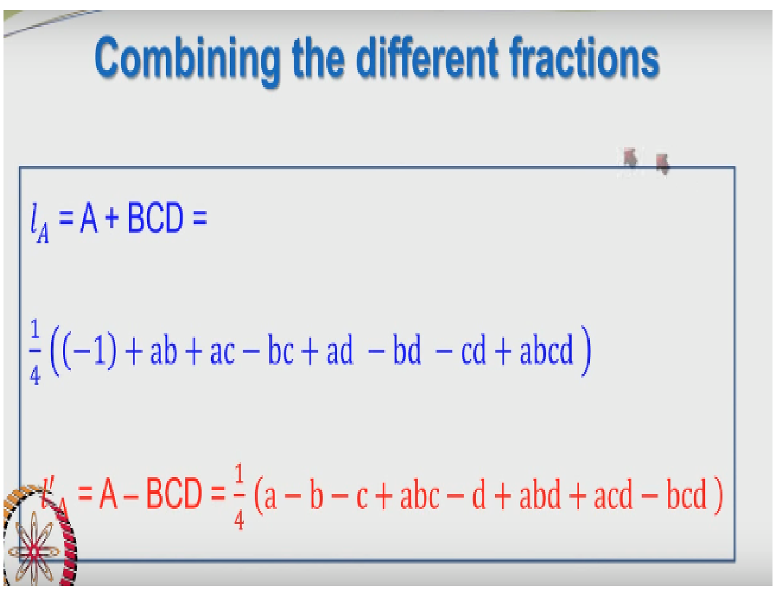

So, you have lA=A+BCD and that is given by this entire contrast divided by 4 l prime A is given by A-BCD and that is the contrast corresponding to that. And when we combine lA with l prime the you will get 2A+BCD-BCD +BCD and-BCD will cancel and you will be having to 2A. So, 2A is represented by 1/4 of these sum of these 2 entities. So, A will be 1/8 of a linear combination of both of these.

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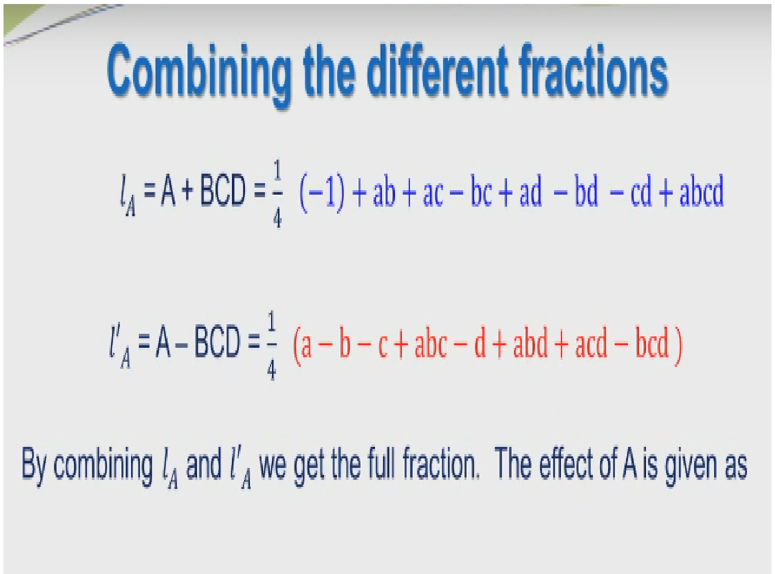

So, La is given by this and lA prime is given by this we add the 2 so we get 2A we get 2A and that was=1/4 of this total sum bracket is missing. Let me just put the brackets here so that there is no problem here to go so we have these entities and the effect of A is given by this.

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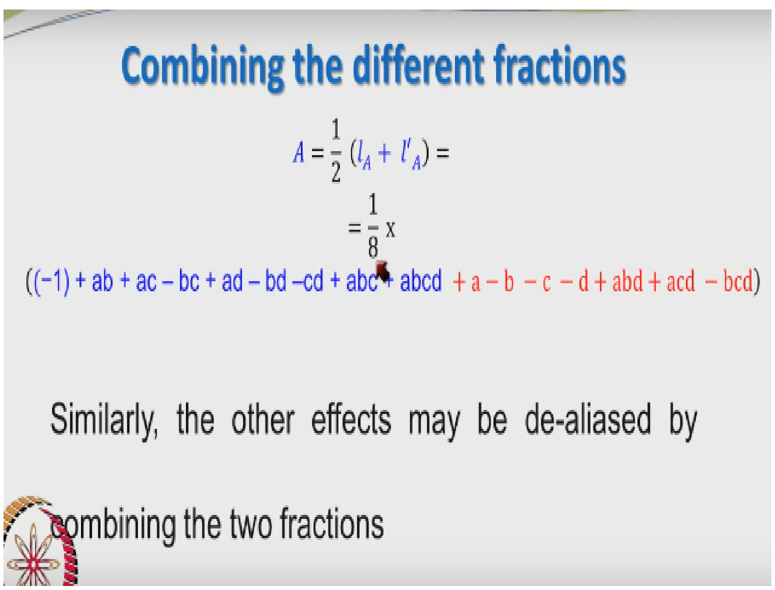

And that would be 1/8* this entire thing if we had done the 2 power 4 designed out this fractional approach this is the contrast we would have had and since there are 8 pluses and 8 minuses we are taking the average and then for that we are dividing it by 8. Similarly, the other effects may be de-aliased by combining the 2 fractions.

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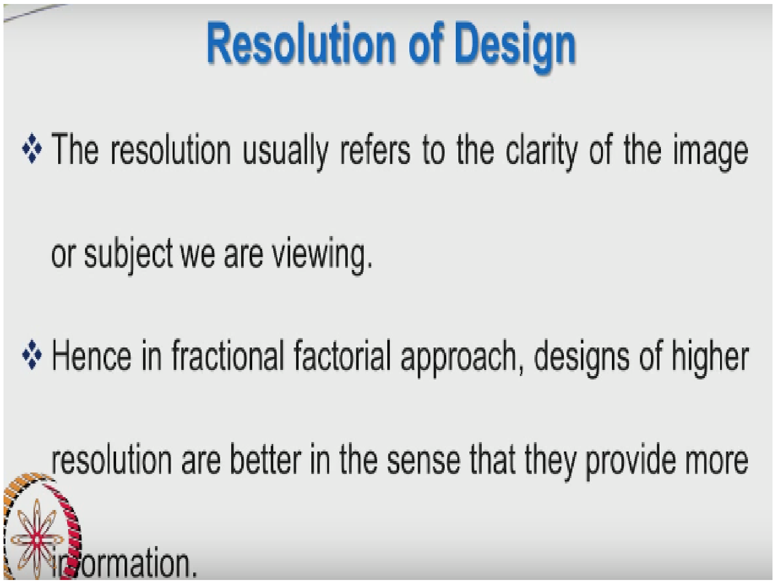

Now we come to an important concept called as resolution and the resolution of the fractional factorial design is important when we talk of fractional factorial designs of a higher resolution they refer to clearer designs or those which hide less and give more information. So, what is the meaning of resolution.

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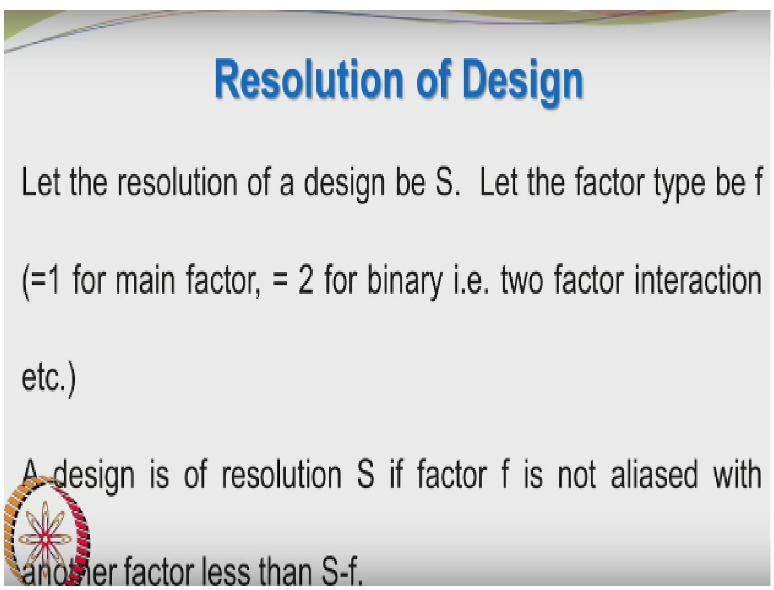

So, what is the meaning of resolution let us denote the resolution of a design by S by the symbol S. Let the factor type or order of the factor be F=1 for a main factor or 2 for a binary factor and a design of resolution S if factor f is not aliased with another factor<S-f it is quite simple. Resolution is denoted as S and the order of the factor is denoted by f we calculate S-f and say

that a design is of resolution S if factor f is not aliased with another factor< S-f.

So, we will plug in some values for S and f and you will soon see what this means.

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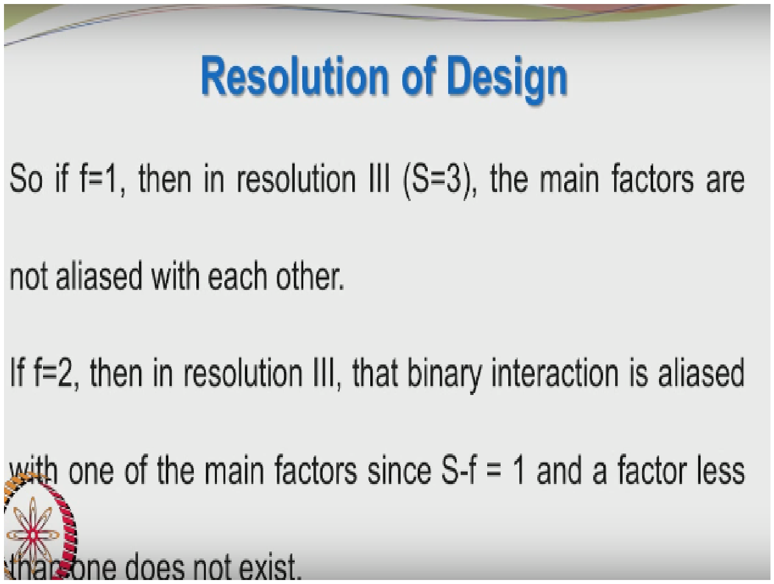

If the order of the factor is 1 that means it is the main factor and the resolution used is 3 we use roman numerals to represent the resolution S=3 then 3-1=2 it also means that the main factors are not aliased with each other. So, if f=1 in resolution 3 we have S=3 the resolution 3 is represented by roman numerals so 3-1=2 and that means if you have 3-1=2<2will be 1. So, the main factor is not aliased with another main factor.

Okay so be a bit careful about this we are only saying that the main factor cannot be aliased with the factor of order<<S-f which means that the main factor is not aliased with another main factor. This also means that the main factor is aliased with a factor of order 2. So, when you have a design generated I=ABC factor A will be aliased with BC factored B will be aliased with AC and factor C will be aliased with AB. This can be easily obtained from the design generator.

So, you can see that the main factors are not aliased with each other, but the main factor 1 main factor is aliased with another factor of order 2 okay. So, this is for resolution we can obviously have resolutions of higher orders like 4 and 5. If you look at f=2 then in the resolution 3 S-f=1 and so binary interaction is not aliased with the factor<1. So, the binary interaction is not aliased with the factor<1 means it is aliased with the factor 1.

You cannot have a factor< 1 okay the rule says that for resolution 3 the binary interaction cannot be aliased with the factor<1. So, it cannot be aliased with the factors 0 it does not exist, so the binary interaction aliased with 1 of the main factors. I think this rule is a bit difficult to remember on the long run, but it is quite useful and anyway you do not need to remember this rule.

Because you can always look at the design generator and see how the main effects are aliased and the binary interaction effects are aliased. So, that is not really required all you need to have is the design generator the proper appropriate designed generator for the resolution and these are available in standard textbooks. So, you do not have to even remember all of them.

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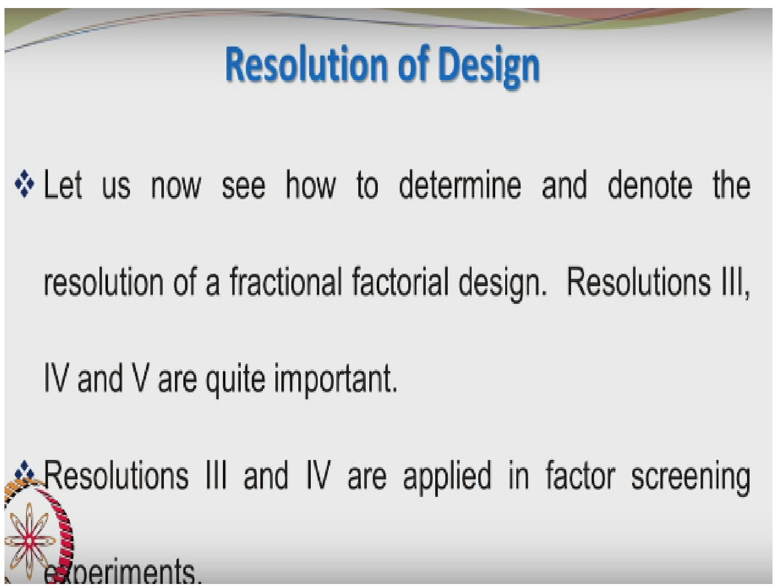

So, there can be resolutions of 3 4 and 5 and resolutions 3 and 4 are applied in factor screening experiments factor screening means you are considering a large number of factors and by doing fractional factory design you are screening out some factors or eliminating some factors.

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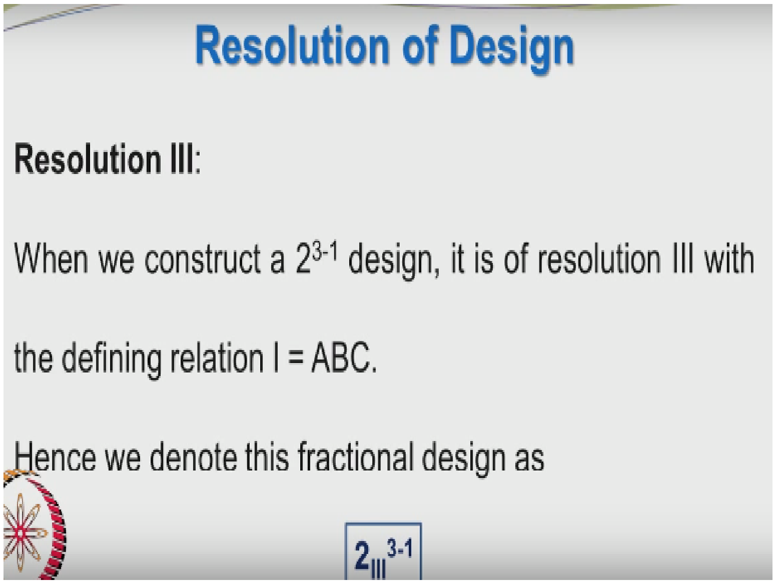

So, when you have a resolution 3 and you are constructing a 1/2 that means its represented by 2 power 3-1 design and the complete notation for 1/2 fraction of a 2 power 3 design of resolution 3 is given by 2 3 roman numerals a subscript and 3-1 as superscript. This tell us we are conducting a 1/2 fraction of a 2-level factorial design of resolution 3 and the number of factors in our consideration is 3.

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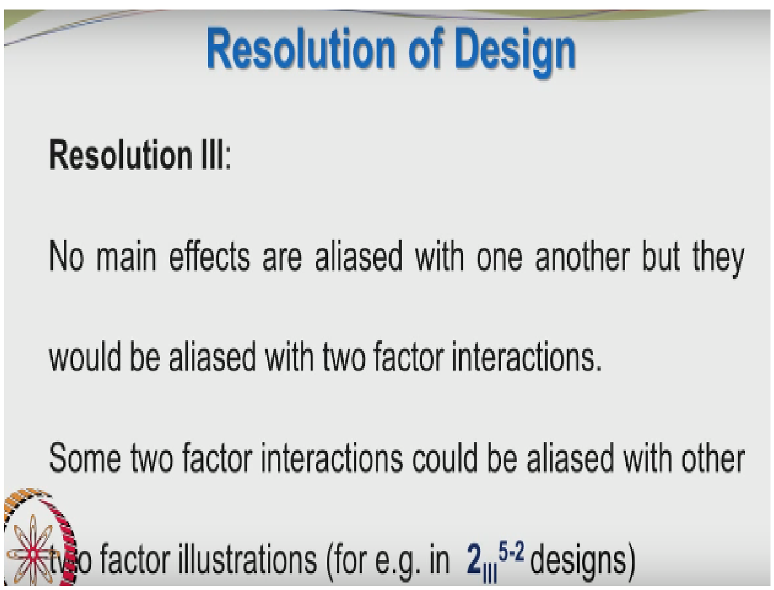

Resolution 3 the summary is no main effects are aliased with one another but they would be aliased with 2 factor interactions that is what the rule also said S=3 and f=1 and so it cannot be aliased to 3-a factor< 3-1. So, main effect cannot be aliased with another main effect but they would be each main effect would be aliased with another 2 factor interaction. So, quite clear some 2 factor interactions could be aliased with other 2 factor illustrations.

Sorry let me just correct the typo here it should be 2 factor interactions some 2 factor interactions could be aliased with each other could be aliased with other 2 factor interactions and this happens in a 2 level factorial design of resolution 3 involving quarter fraction of full 2 power 5 design or let me put it again we are looking at a quarter fraction of 2 level factorial design of resolution 3 involving 5 factors.

That means essentially, we are doing only 8 experiments in such cases some 2 factor interactions could be aliased with other 2 factor interactions.

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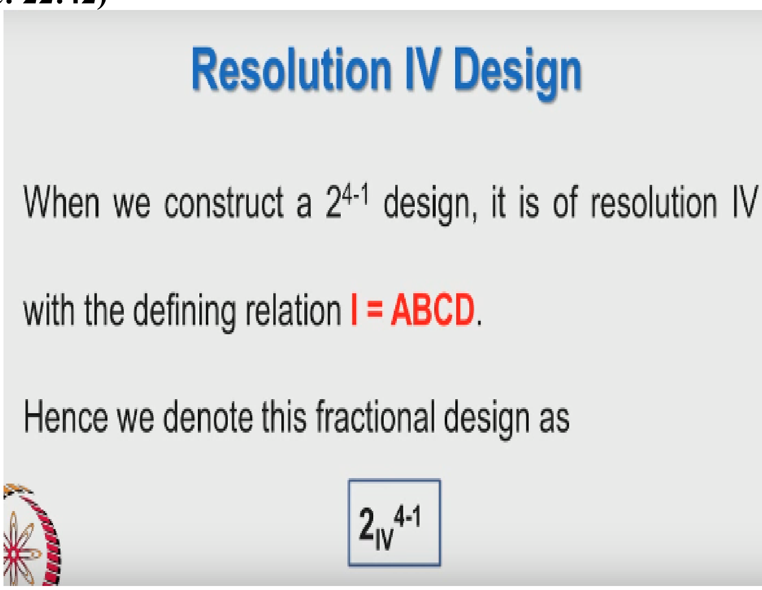

When we construct 1/2 of a 2 power 4 design it is often resolution 4 with defining relation I=ABCD this is what we saw in the beginning of the demonstration of the fractional factorial design. So, the design generator I=ABCD and that is defined as 1 1/2 fraction of resolution 4 for a 2-level factorial design involving 4 factors.

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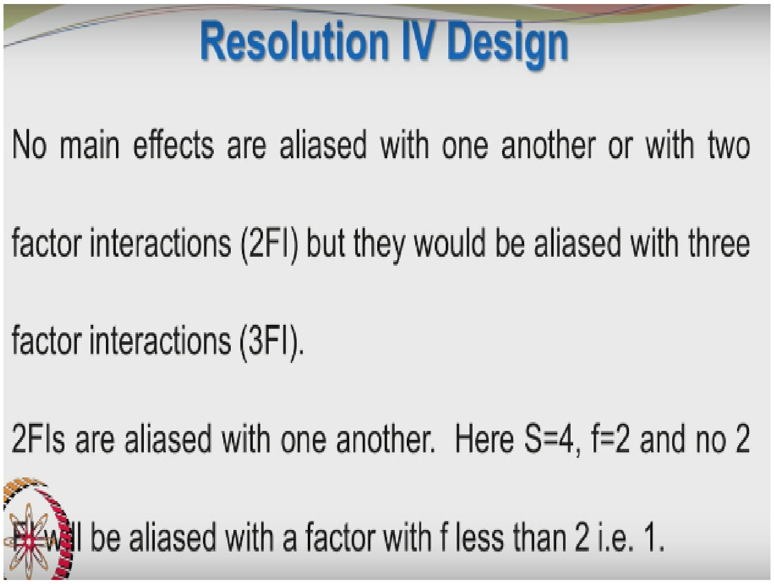

In this case we know by now no main effects are aliased with each other but are even with 2 factor interactions. This is very good now the resolution has increased so the main effects are not aliased with the other main effects or main factors and they are also not aliased with any other 2 factor interactions, but they would be aliased with 3 factor interactions on the other hand 2 factor interactions are aliased with other 2 factor interactions.

Okay this again we saw here S=4 the resolution is 4 and the number of f=2. For example, the binary interaction and so S-f would be 4-2 which is 2. So, no 2-factor interaction could have aliased with a single factor so hence a 2 factor hence a 2-factor interaction would be aliased with other 2 factor interaction.

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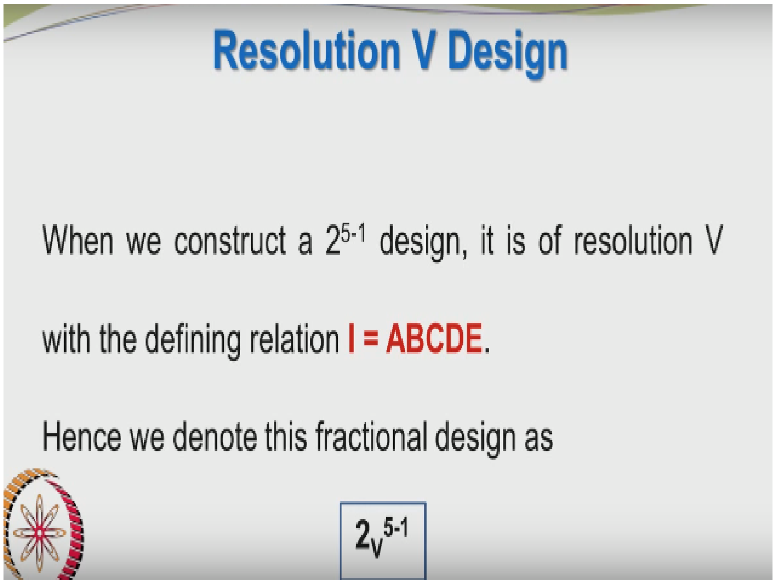

When we have a resolution 5 design we construct 2 power 5-1 it is of resolution 5 with this defining relation I=ABCDE remember there are 5 factors starting from A up to E. Here we denote this fractional design 2 of resolution 5 2 level factorial design of resolution 5 involving 5 factors and the fraction is a 1/2 fraction.

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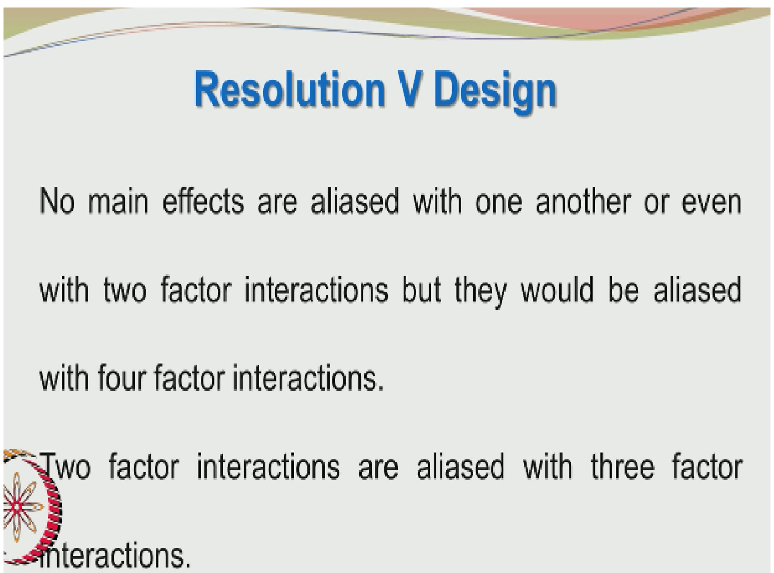

In the resolution 5 design you can easily understand that the no main effects are aliased with 1 another are even with 2 factor interactions, but they would be aliased with 4 factors interactions that is a very big benefit 2 factor interactions are aliased with 3 factor interactions.

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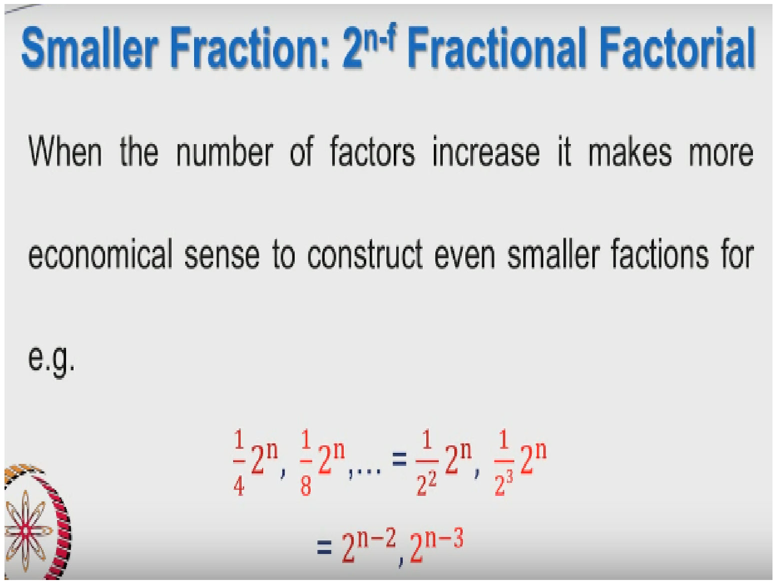

We can even construct a smaller fraction with the number of factors increase it is tempting and makes more economical sense not to just consider a 1/2 fraction but even consider a 1/4 fraction and if the number of factors are really large a 1/8 fraction may also be suitable. And that we may represent it as 1/4 2 power n 1/8 2 power n, 4 can be written as 2 power 2, 8 can be written as 2 power 3 and so this will become 2 power n-2 this will become 2 power n-3 as shown here.

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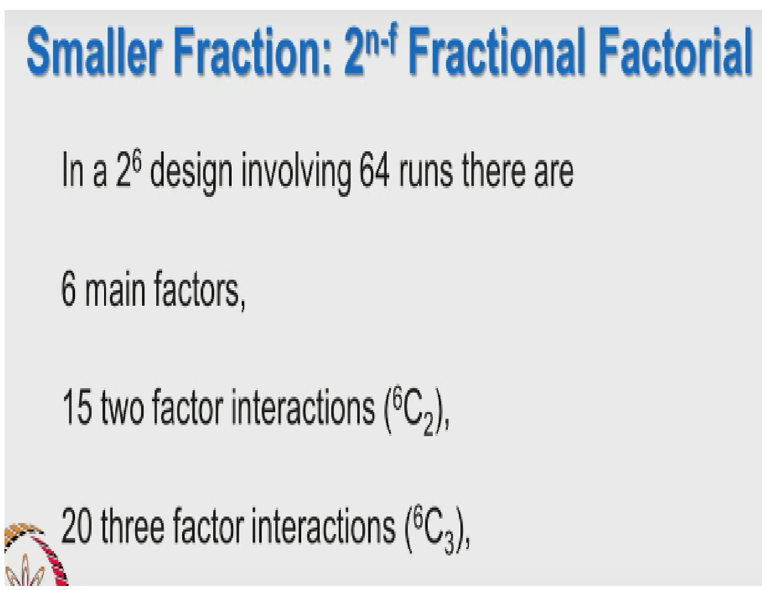

A 2 power 6 design involving 64 runs there are a 6 main factors 6C2 2 factor interactions 6C2 would be 6*5 divided by 2 which is 30/2 which is 15 and then you also have 6C3 3 factor interactions 6*5 30 30*4 120 120/6 is 20 23 factor interactions.

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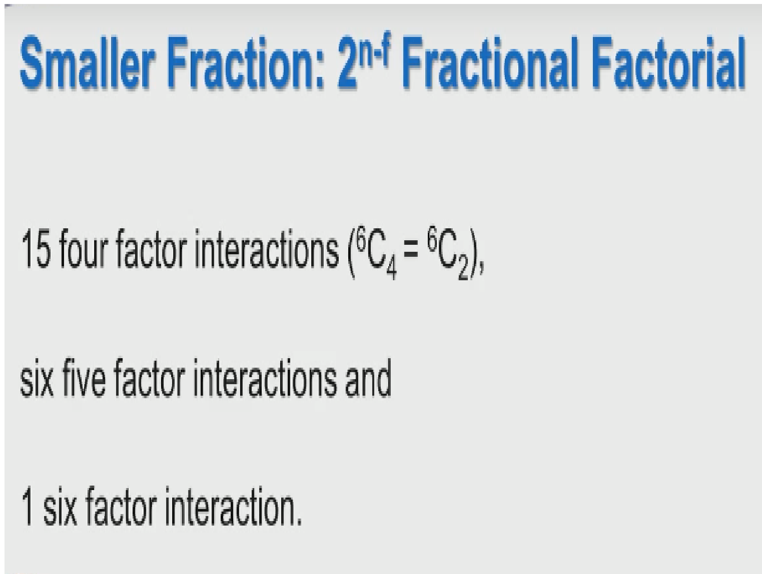

15 4 factor interaction 64=62 6 5 factor interactions and 1 6 factor interaction. Let us see whether the total adds up to 63 6 +15 21+20 41 41+15 56+6 32 62 rather+1 63 remaining 1 corresponds to beta 0 in the model.

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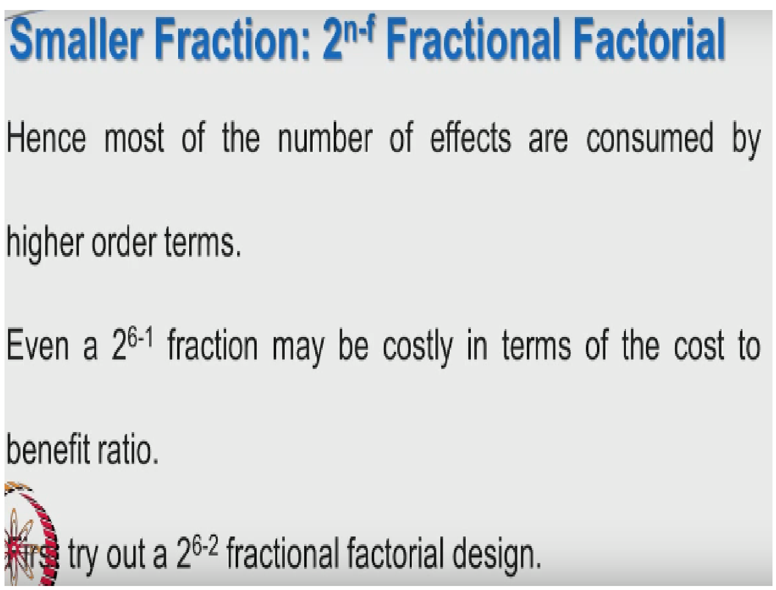

So. we can see that most of the number of effects are consumed by higher order terms even a 2 power 6-1 fraction may be costly in terms of the cost to benefit ratio. So, we may even consider a 2 power of 6-2 fractional factory design that means a quarter fraction of a 2 power 6 fractional factorial design.

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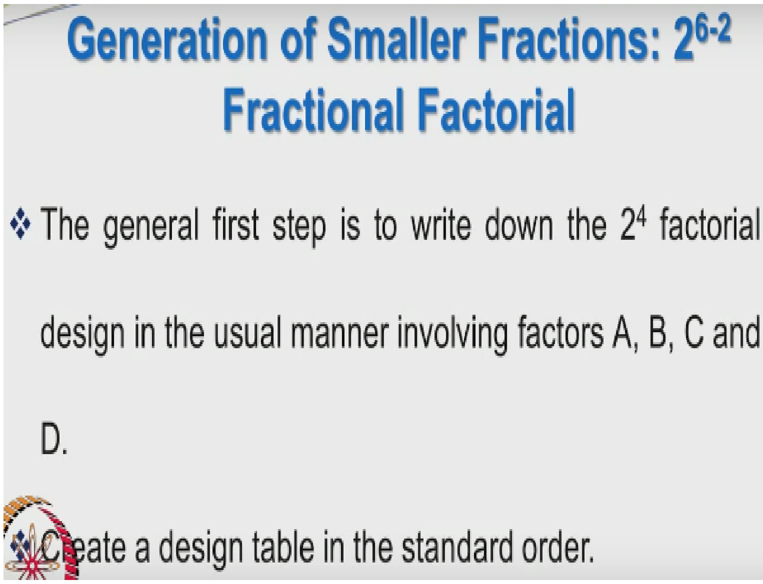

So, how do we do that the general first step is to write down the 2 power 4 design in the usual manner involving factors A B C and D create a design table in the standard order.

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A	B	C	D	E = ABC	F = BCD
-1	-1	-1	-1	-1	-1
1	-1	-1	-1	1	-1
-1	1	-1	-1	1	1
1	1	-1	-1	-1	1
-1	-1	1	-1	1	1
1	-1	1	-1	-1	1
-1	1	1	-1	-1	-1
1	1	1	-1	1	-1
-1	-1	-1	1	-1	1
1	-1	-1	1	1	1
-1	1	-1	1	1	-1
1	1	-1	1	-1	-1
-1	-1	1	1	1	-1
1	-1	1	1	-1	-1
-1	1	1	1	-1	1
1	1	1	1	1	1

And so, this is the standard order and we are imagining as if that there are only 4 we are having a 2 power 6 case which is involving 64 experiments we are looking at a quarter fraction. So, since we are considering a quarter fraction that means that is 1/4. So, consider the first 4 factors for convenience A B C and D write down the design matrix and the standard order and the standard or this is the standard order.

And you then define the remaining 2 unaccounted factors namely E and F E=ABC and F=BCD

E=ABC and F=BCD you can also say F=ABC and E=BCD there is no problem for a illustration I am taking E=ABC and F=BCD.

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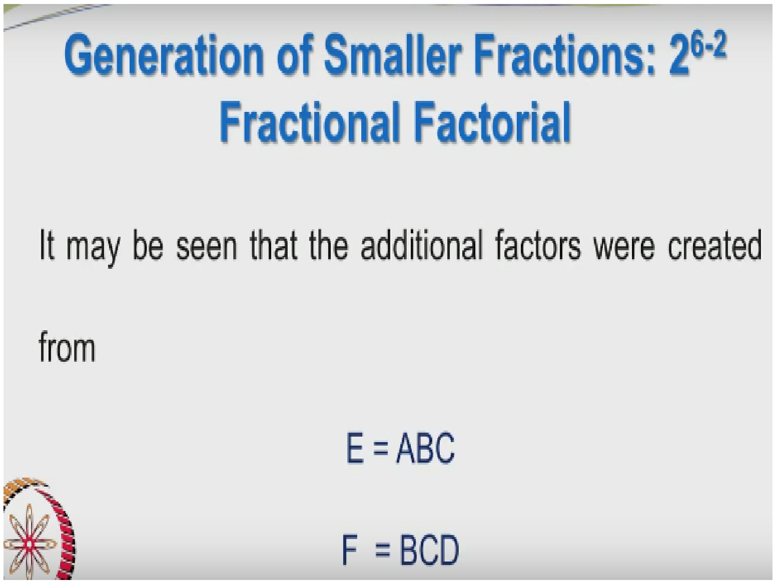

And so, you can generate smaller fractions 2 power 6-2 fractional factorial and additional factors were created from E=ABC and F=BCD so E=ABC means the setting of the design matrix for E would be-1 then I do ABC it becomes +1. So, I can fill up these 2 columns.

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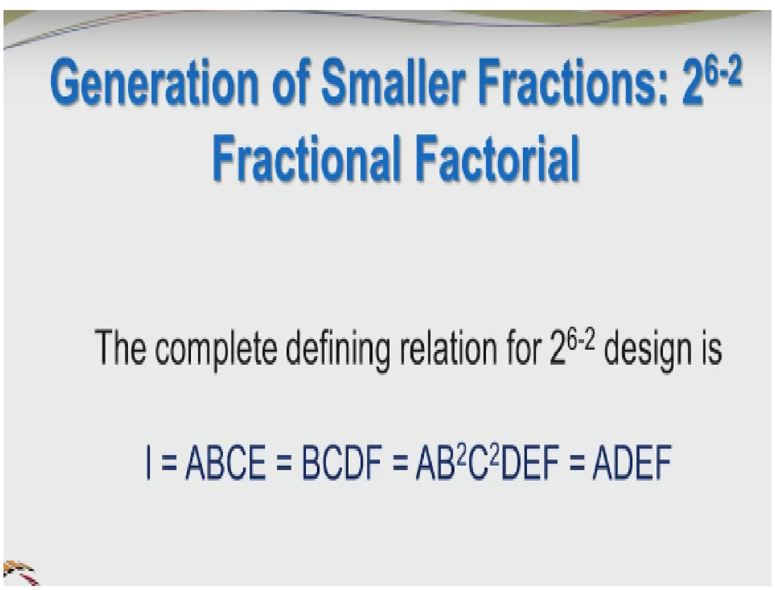

And what does that actually mean so how do I get the first fraction? how do we actually do the experiment? before we analyze. Let us see the practical issue here how do we get the first fraction there are 4 fractions here because we are considering 1/4 of a 2 power 6 design. So, you should have 4 fractions and how do you identify the 4 fractions, so you have ABC and BCD. So, you can write all those elements corresponding to +1in this column the complete set.

Okay you write the full factorial design involving 2 power 6 case and all the +1 in ABC and all the +1 in BCD will constitute the first fraction. The second maybe all the pluses in ABC all the minuses in BCD will help to contribute to the second fraction and the third fraction would be minuses in ABC and pluses in BCD that combination and the final 4th fraction would be negative in E and negative in F or negative in ABC or negative in BCD.

So, with this combination you should be able to get the or identify the 4 fraction settings. So, the complete defining relation for a 2 power 6-2 design is I=ABCE. So, If I do E squared I get I E*E would be all pluses, so I=ABCE F squared will be BCDF so I=BCDF I=ABCE. So, that is the complete defining relation for a 2 power 6-2 design I=ABCE BCDF and if I multiply the 2 I get A B Squared C squared.

Let me see what are all the repeating terms A is not repeated if I multiply these 2 so A survives B and C do not survive because B squared, and C squared will result. So, we have ADEF, so you have ADEF here.

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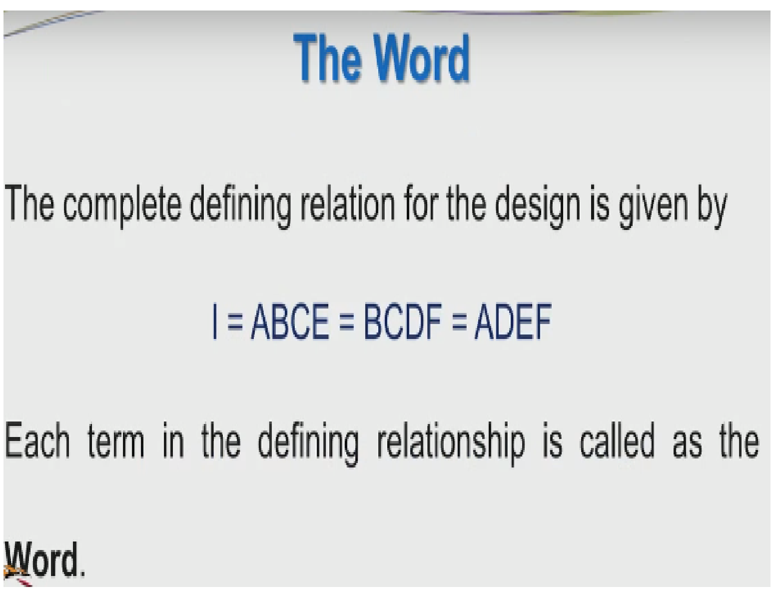

So, the complete defining relation for the design is given by I=ABCE=BCDF=ADEF each of these terms is referred to as the word.

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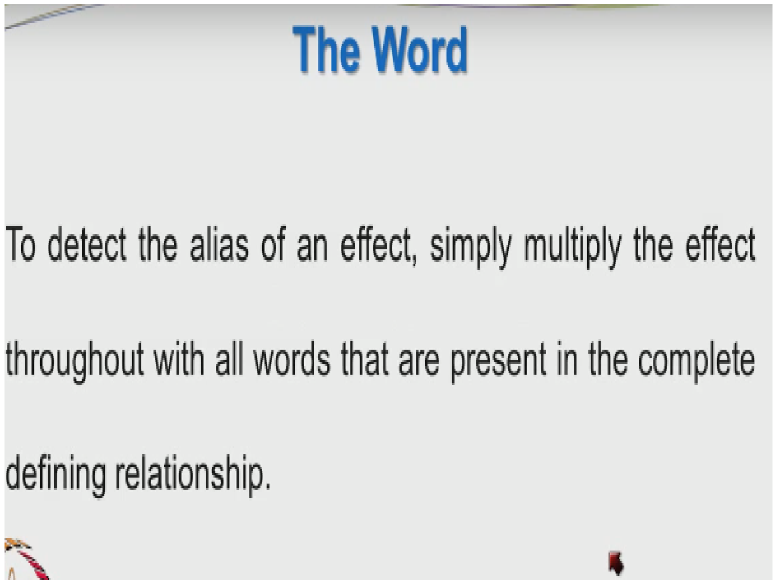

So, once you have the complete defining relation to detect the aliase of an effect simply multiply the effect throughout with all the words that are present in the complete defining relationship.

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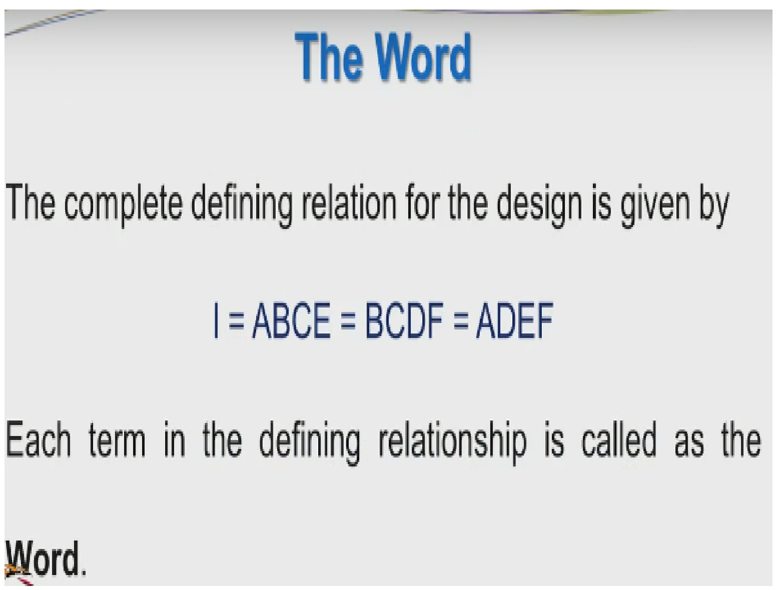

So, for this particular case I multiply A here I will get A=BCE equals ABCDF and=DEF that means A is aliased with BCE A is aliased with ABCDF and A is aliased with DEF.

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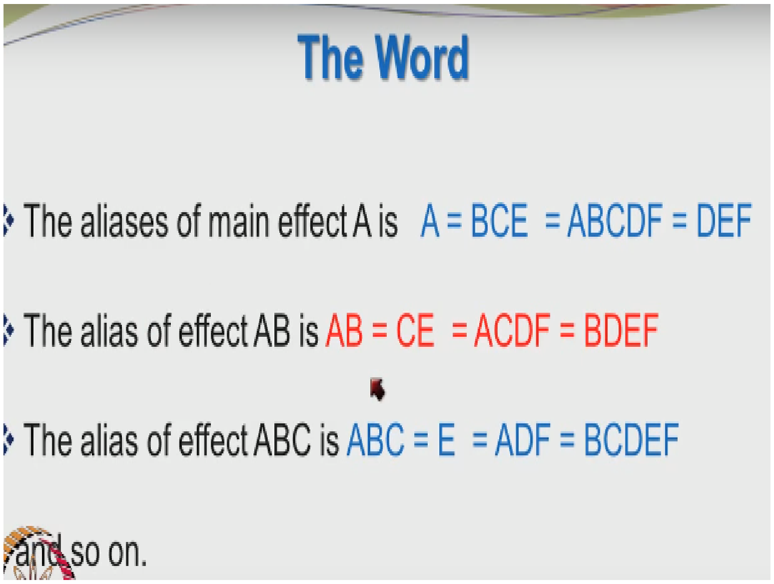

And that is what is given here. Similarly, the alias of effect AB you can find easily AB=CE=ACDF and BDEF. Similarly, for the effect ABC is aliased with ABC=E=ADF=BCDEF and so on. How did we get here you have ABC so if I put ABC here is A will cancel out and then you have BCDEF? In other words, ABC is aliased with the 5-factor interaction also.

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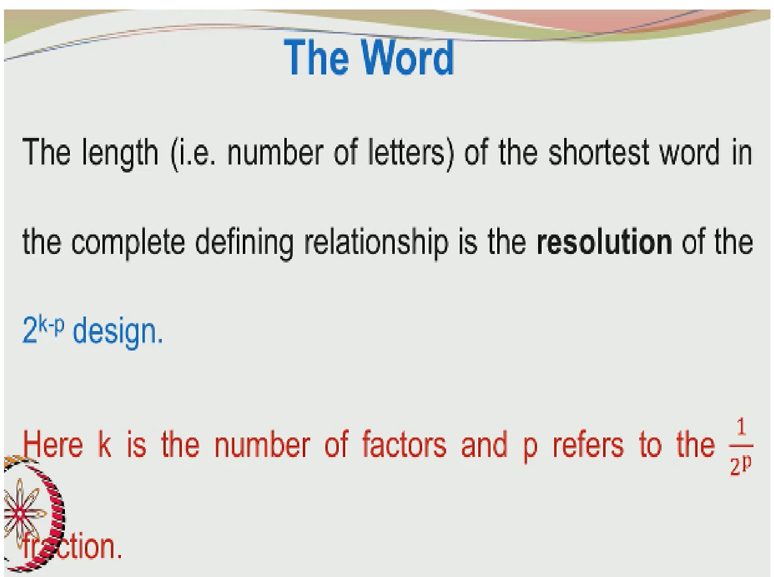

The length the number of letters of the shortest word in the complete defining relationship is the resolution of the 2-power k-p design k is the number of factors and p is the order of the fraction and so the shortest word in the defining relationship gives you the resolution. So, this is the shortest word that would be 4.

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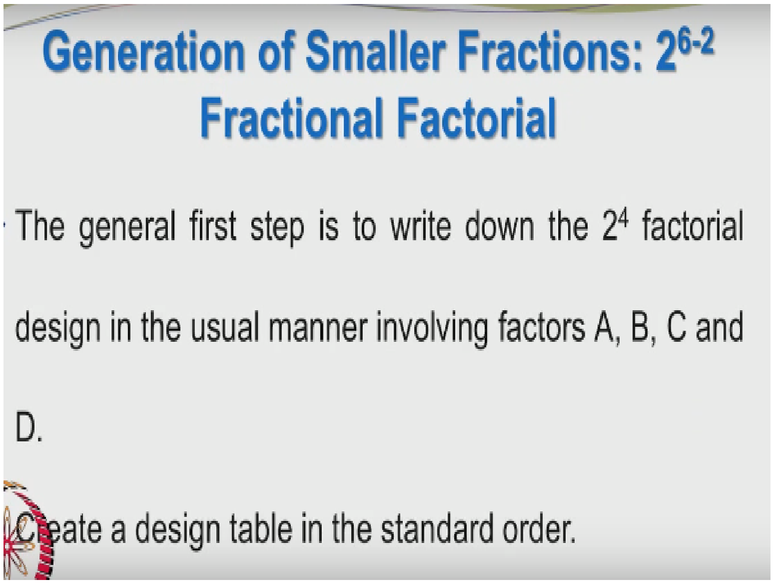

So, we were constructing a resolution of 4 in this particular case please do not say the resolution is 3 the resolution is 4 because you have to have the generator I and then the different generators So. you have I=ABCE=BCDF=ADEF and the shortest word here is of length 4.

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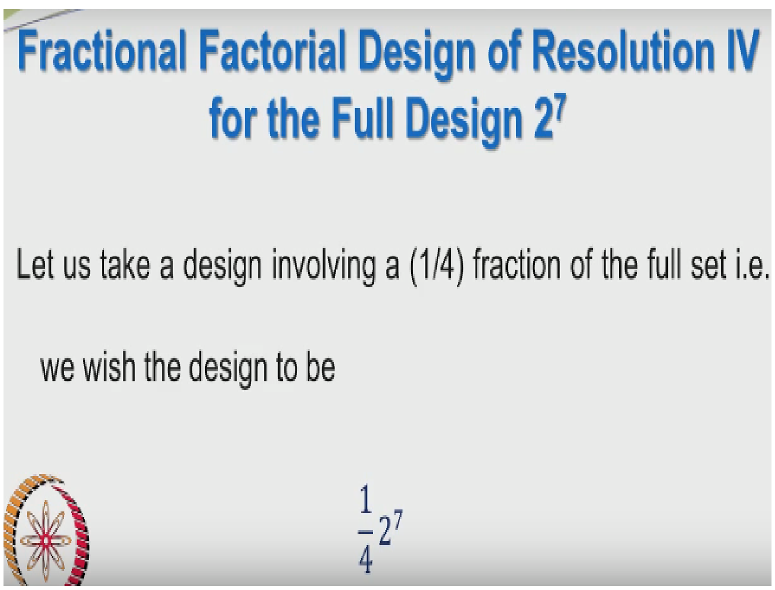

So, now let us take 1/4 fraction of 2 power 7 2 level factorial design 2 power 7 would be 128 experiments and you are going to have a quarter fraction that means each fraction would have 32 experiments.

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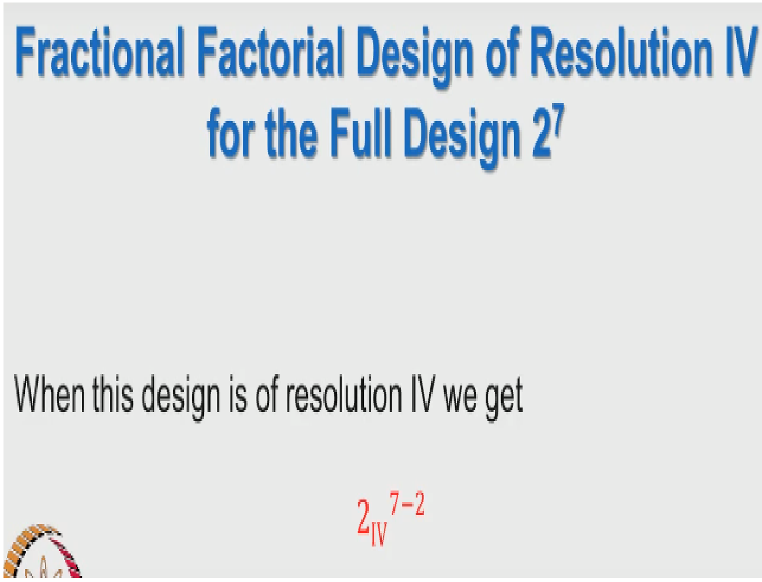

So, then this design is of a resolution 4 we represent it as 2 level factorial design of resolution 4 involving 7 factors and performed through 1/4 fractions 4 fractions are involved.

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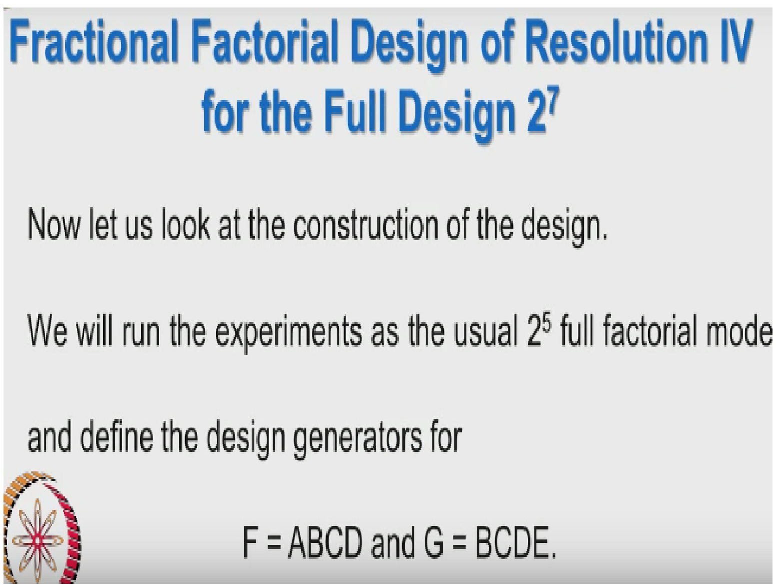

So, to look at the construction of the design we will have 32 runs. So, we will first run it as a usual 2 power 5 design how many factors we have 7 what are those factors ABCDE that would be 5 F and G corresponding to a 6th and 7th factor for convenience let us start with ABCDE and run it as a proper 2 power 5 design and what are the design generators. We define the generators as F=ABCD and G=BCDE 2 remaining factors are F and G.

So, F we alias with ABCD and G we alias with BCDE.

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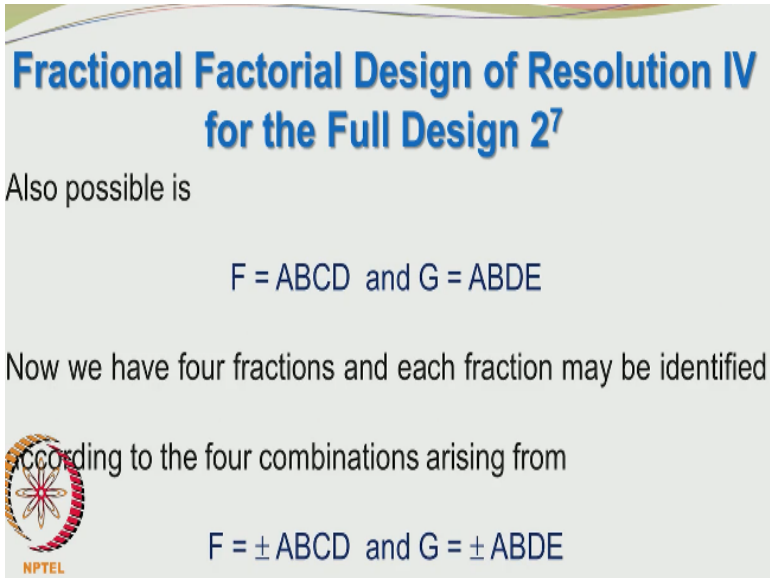

We can have yeah ABCD and ABDE we can have F as ABCD and G we can have it as ABDE. So, we cannot really have constrained to specific cases so other possibilities also sometimes there. Now we have 4 fractions and each fraction may be identified according to the 4-combination arising from + or - ABCD and + or – ABDE this is what we saw earlier in the earlier example.

So, the first fraction would be the entries corresponding to +ABCD and +ABDE. And then it will be +ABCD entries and +sorry-ABDE entries then you can have-ABCD and +ABDE-ABCD and-ABDE will complete the last fraction.

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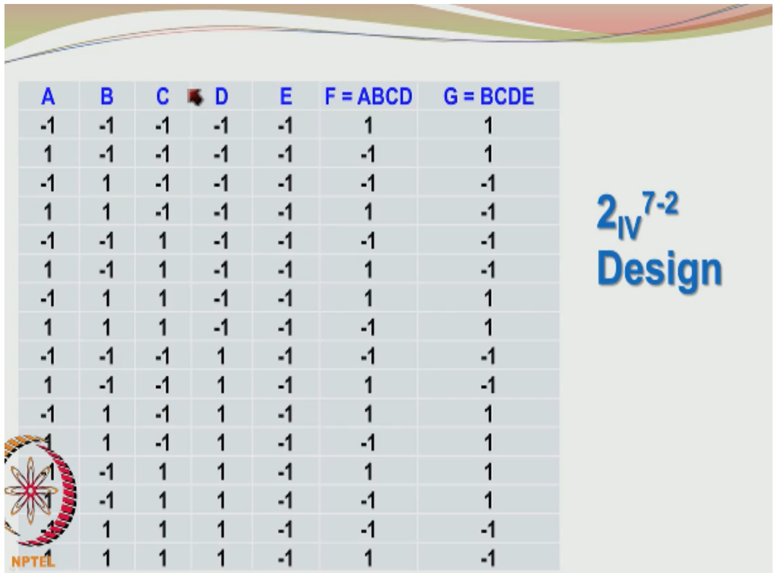

A	B	C	D	E	F = ABCD	G = BCDE
-1	-1	-1	-1	-1	1	1
1	-1	-1	-1	-1	-1	1
-1	1	-1	-1	-1	-1	-1
1	1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	-1	-1
1	-1	1	-1	-1	1	-1
-1	1	1	-1	-1	1	1
1	1	1	-1	-1	-1	1
-1	-1	-1	1	-1	-1	-1
1	-1	-1	1	-1	1	-1
-1	1	-1	1	-1	1	1
1	1	-1	1	-1	-1	1
[illegible]	-1	1	1	-1	1	1
1	-1	1	1	-1	-1	1
-1	1	1	1	-1	-1	-1
1	1	1	1	-1	1	-1

So, you write down the standard design here and then you put F=ABCD and G=BCDE.

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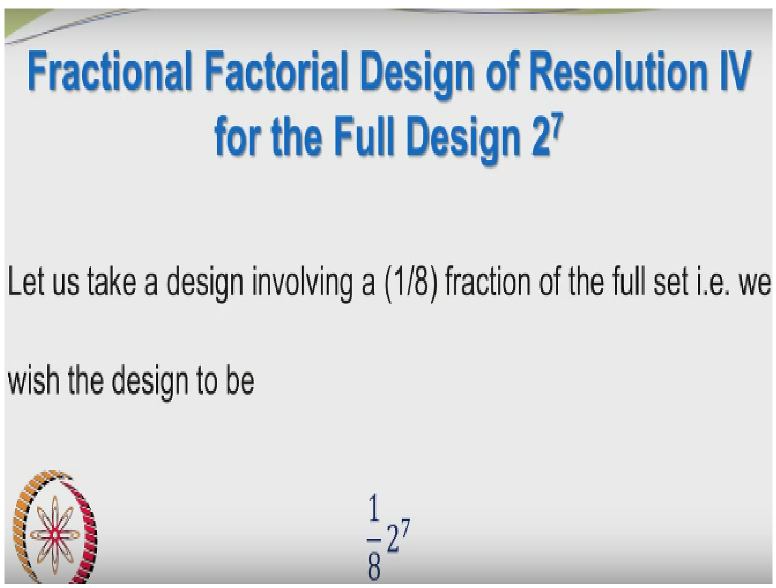

So, if you are next taking a case involving a 1/8 fraction of a full set we have 1/2 power 3*2 power 7 number of factors is 7 and were looking at 2 power 3 that means 1/8 th of a fraction.

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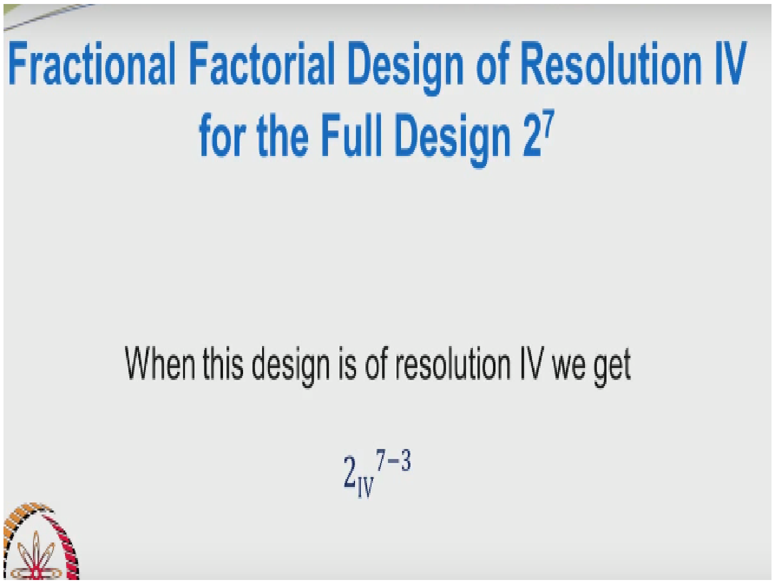

So, when you choose a resolution as 4 we represent it as 2 level factorial design of resolution 4 with 7 factors and we are considering a 1/8 fraction.

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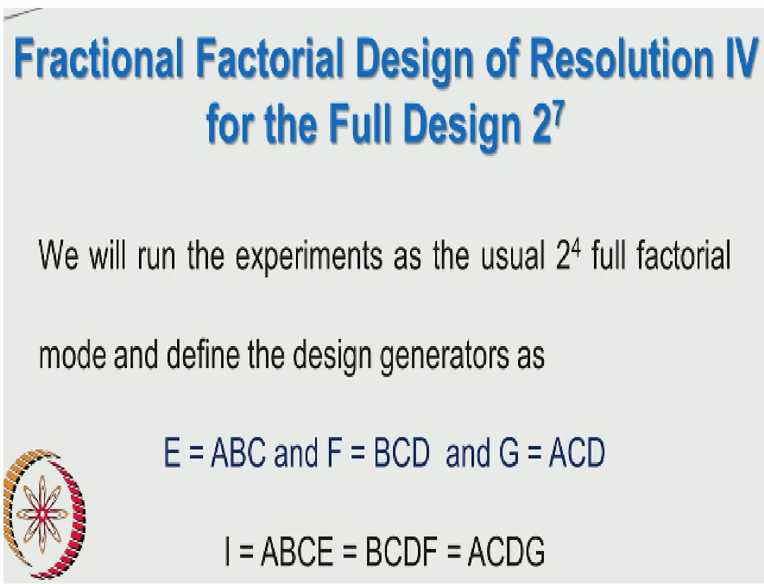

So. we will run the experiment as usual in the 2 power 4 full factorial mode and define different design generators as E=ABC F=BCD and G=ACD this is very interesting and ingenious also. So, we are first to considering only the first 4 factors ABCD and then the remaining factors are set at ABC BCD and ACD. We are not putting A=AB we are trying to get the aliasing with the highest order interaction which is possible.

So, we have E=ABC F=BCD and G=ACD. So, we have the defining that different design generators are ABCE BCDF and this is the defining the relation rather sorry. So, these are the design generators and the defining relation is given by I=ABCE=BCDF and ACDG the length of the shortest word here is 4.

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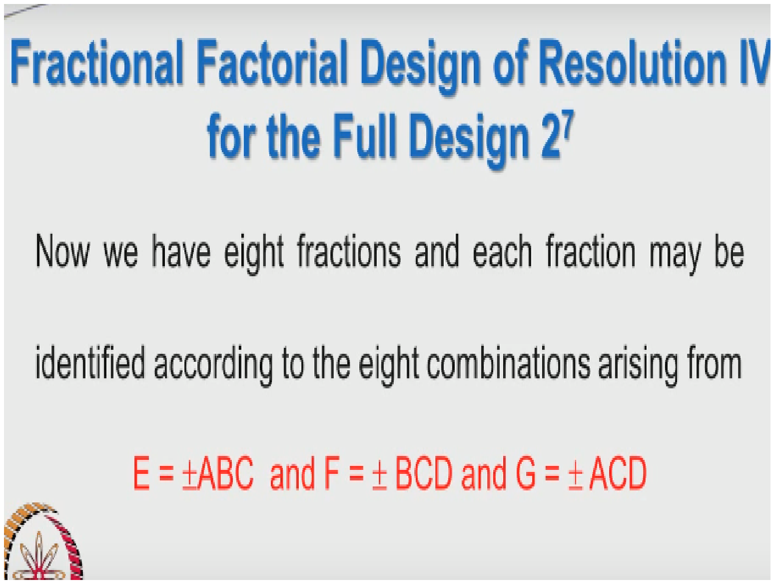

So, we have we have constructed to 2 power 4 fractional factorial design and the 8 fractions we can get by looking at these combinations is E=+ABC and F=+BCD and G=+ACD will constitute the first fraction E=+ABC F=+BCD G=-ACD will constitute the second fraction and so you can have 2*2*2 8 possibilities to complete your 8 fractions.

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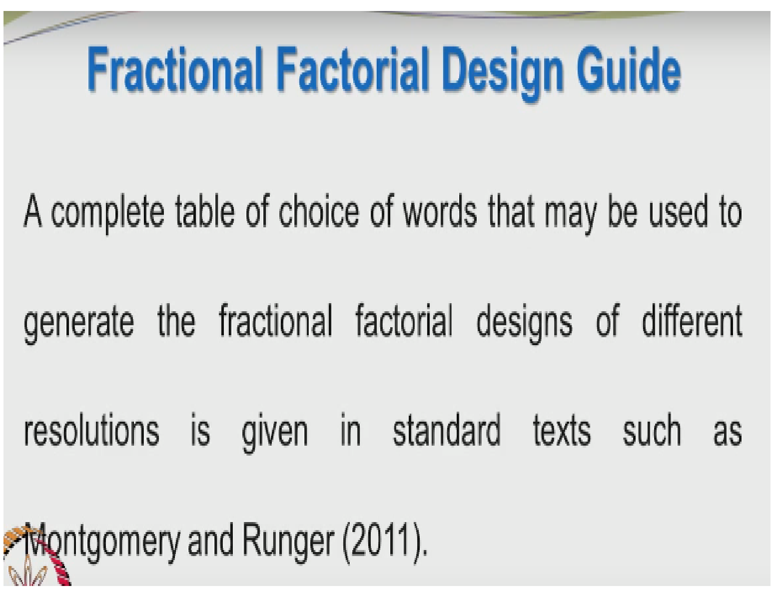

A complete table of choice of words given in standard text like Montgomery and Runger 2011 or

Montgomery 2009. So, you do not have to remember anything you have to just see the number of runs you can economically carry out in your workplace and then identify the appropriate resolution and then set up the design matrix find the contrast and calculate the effects identify which factor is aliased rather with other factors.

So, once you have done this you can sequentially conduct the different fractions and get more and more information from your set of experiments. Sometimes you may even stop after finishing the first 2 fractions saying that I have now a very good idea about the process it does not. It is law of diminishing returns so after the first 2 fractions I may not really need the third fraction.

Even if you save on 1 fraction that means you do only 3 out of 4 fractions that means you have done the experiment efficiently without overdoing them sometimes even over doing experiments is not good. The best very to understand this would be through an example and I will be covering examples for factorial designs and fractional factorial designs in the next lecture. Please go through what I have said they are pretty straightforward.

Also refer to the standard textbooks I have referred to the identify the different tables and see how you may use them the important thing is to identify the number of fractions. The resolution of the design the design generators the defining relationship and what the different fractions are sometimes when you are having 2 power 7-3 1/8 fraction you have to set up the 8 fractions correctly there are software which also help you do this one of them is Minitab.

So, what we will do is do a few problems in both factorial designs and fractional factorial designs. Thank you.