Multiphase Flows: Analytical Solutions and Stability Analysis
Prof. S. Pushpavanam
Department of Chemical Engineering
Indian Institute of Technology – Madras

Lecture - 31
Tutorial Session: Solution of Assignment 4 on linear stability

So, good morning, in this class, what we will do is to look at this computational assignment. It is actually problem set 3 but I have call it problem set 4 online and what I will do is I try to work through a couple of the problems. And as I go long feel free to ask me any doubts that you have about the assignment and then later on about the course until this point of time we can discuss some of those things.

Then I will talk about the new assignment uploaded online yesterday, the computational one and explain how you can write a code to simulate Swift–Hohenberg equation, so that we can test some of the things that we have done in the theory. So, this is only work if you ask me your doubts. So feel free to ask and do not have to worry about the video going on, that we can edit anyways.

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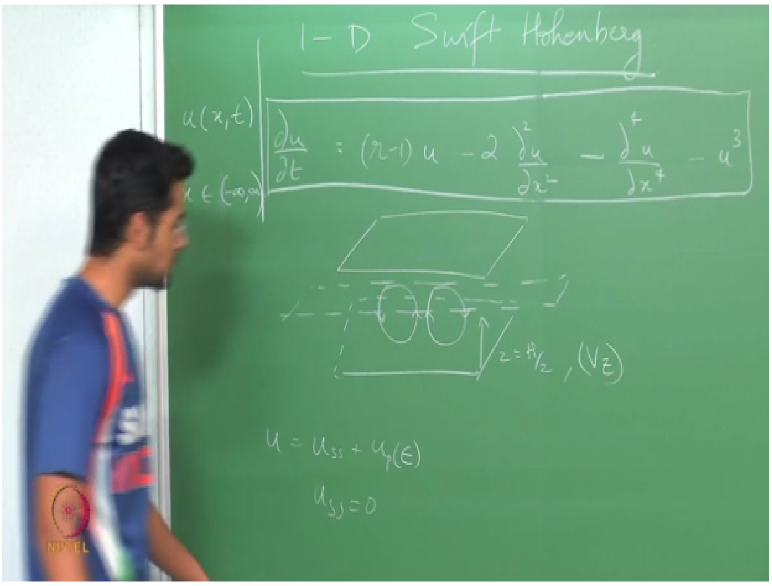

Alright, the first question was the 1 – D Swift–Hohenberg problem. Right, so u is the function of x and t and x extends in this problem from – infinity to+infinity. And this equation is simplified you could see averaged version of the Rayleigh–Bénard equations. So, using the method of multiple scales what has been done here is essentially they have looked at the set of Rayleigh–Bénard and using a suitable averaging contents everything into this simple equation.

So, quantitatively this will not, cannot be expected to give you the same results. But what has been done is all the physics of the Rayleigh–Bénard problem have been captured with the simple equation. The idea being that by studying this equation we can understand a lot of basic features about pattern-forming systems.

And this has been derived in a book by Cross and Greenside, which is the book that I have used to put together this assignment and the next one as well and that is a really good book for the course. So, I will give the reference a bit later. So, they derive it from heuristic perspective and then from a rigorous mathematical point of view also. So, keeping that in mind this parameter r is similar to the Rayleigh parameter.

So, we want to find the value of r for which the system is unstable and there is a fourth order term which will come from the coupling of I think temperature and velocity and then you have a second order term. So, everything fixed together similar to the Rayleigh–Bénard, if you just look at it from that angle. Okay, so without, so you can be interpreted as the vertical component of velocity.

So for example, if you say I mean say you want to compare this with some experimental data. Then if I have my Rayleigh–Bénard cell I need it to infinity in both directions. Then as you already know at some point I will have maybe some kind of roll something will be happen. And if it is stable there will be no convectional at all. So, u is almost the cut out along the z direction as H/2 and it is the Vz.

The z component of velocity that I have cut off through the center of that. So, now it is only a and I have considered to be only a function of x. So, I have ignored the y direction variation say that cell is quite narrow, so that we have looking at the x direction. So with this interpretation we will move on to the analysis which is actually quite simple no because it is just one equation.

So, the first step is of course to linearize the problem. So by that we would write U as U steady state+U let us call it perturb, right. And U steady state is basically 0 here, it is a homogeneous state and that makes sense with the fact that there is no flow in that. So U steady state is 0 and if you were to substitute here and retain terms I will call, multiply this by epsilon for magnitude.

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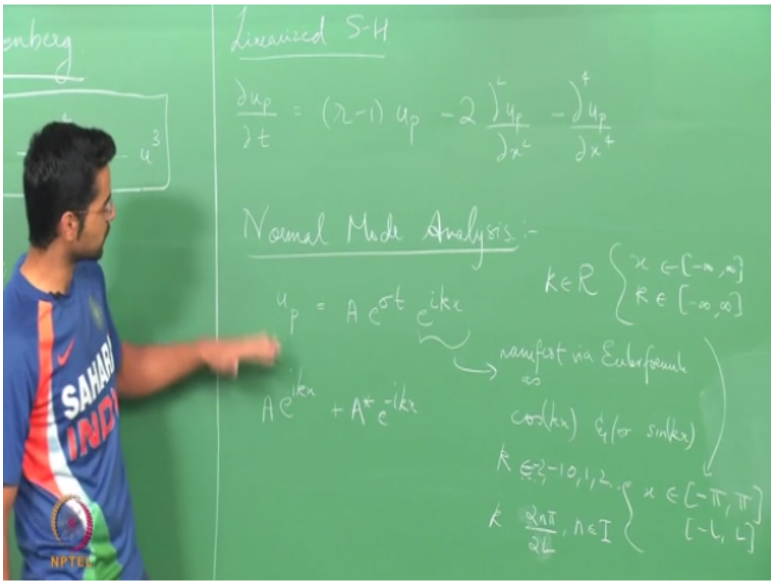

Retain terms of epsilon then we would get the U cube which is the non-linearity does not survive in the linearization. Now the way this equation has translated the entire nonlinear term has simply be knocked off. But this will not always happen such the kind of thing happens only if your base state is identically 0. If I had a non 0 Uss then I would have a term like Uss squared times U perturb and other terms as well.

So, you need to be careful when you are linearizing. Only if the base state is 0 you can write it down by inspection but on other cases it is better to substitute always and make sure. So, this is the perturbed equation. Now the next step is to take a form for the perturbation or the normal mode analysis. So I will say few things about that.

So, typically what we have been doing throughout the courses at this stage writing Up as some constant A which is the initial condition of the perturbation. We take an exponential form and time because it is a first order system and then this is the most important part where we write down something like e to the pi ikx and this means you are considering the Fourier amounts.

Okay, so first of all I just go through it because some people have asked me this question. So, the first thing is that this is a complex factor but then Up has to be real. So, how does it resolve? Whether it resolves with the Euler identity that relates complex exponential terms to cos h and sin h.

So, when we look at the Fourier series in this manner we actually need to consider k's from okay because we are looking at we are not looking at a system where k would be discrete right now. If we have the entire effects belongs to the real number line, so if x is a continuous variable from – infinity to+infinity, k should take all values from – infinity to+infinity. So, we are looking at both k's negative k and positive k.

Even though the dispersion curve you always plot positively it is understood that we are also considering the negative k. Now what happens is the positive k term combines with the negative k term to give you a real function and the real function will have the form of cos and sin. So, eventually this complex guy will manifest itself via the Euler formula as something like this would be the dependence finally.

You will get cos kx and or depending on the initial conditions sin kx. So, if you want to go plot you have variation you cannot plot this, you can plot cos kx and sin kx. So, this coefficient A is also complex that has to be kept in mind. So, this A is not a real thing it is complex because this is also complex. So, when this A with the positive k combines with the A with the negative k gives you these real parts.

And you will find that the A corresponding to the negative k is actually A star. It will be the complex consecutive of A. So, in other words I have a mode this will be 1 mode, this mode will have to be added with A star, where A star is the complex consecutive of A and that will be obvious when you look at the equation because this when you write instead of k if you take minus k it is basically looking at this same equation and replacing i/– i.

Which means that any unknowns will just be the consecutives of what we get in, any unknowns here will be the consecutives of what we get here. That is clear? So, this is how even though we looking at the complex modes ultimately we are going to manifest in a real fashion. And this will always happen in all these problems. So there is something to do with the physics and all these just the way the Fourier amounts work.

“Professor-student conversation starts” A is 10:20 combination of both the imaginary part from A+bi and that one e power ikx can be written as cos kx+I sin kx. So there will be some when you multiply that. No see, this is a complex term, this is its complex conjugate, right. If I added I want to get a real. No, this is some complex number I can write it as A+ib. This is the same complex number conjugated I write it as A – ib.

Then I will added then I will get 2a and the sin part of it, I mean if you what you are saying you will add the cos parts the sin parts will also get added and ultimately give you real thing. If you still have a doubt just go back to some of your old books on Euler identity and how you solve the equation. I mean, this all of this business comes in this equation. So, you see how the solution here is done or you can directly write sin x, cos x, sin y, cos y solutions or you can proceed with complex exponentials and then see how the 2 are resolve, so that should clear any doubts here. **“Professor-student conversation ends”**

So, ultimately when you write this it means you are looking at both cos and sin and that which of them come depend on initial condition boundary condition. That is one important point. The other important point is that here k is real when this is the case. So, if x goes from –infinity to +infinity k takes all the values on the real number line from –infinity to +infinity.

But if x belongs to some interval like say –pi to +pi with periodic boundary conditions. So, when would this come up suppose I am doing a numerical simulation, so if I want to simulate this numerically which is what we are going to do in the next problem. I cannot very well have an infinite long domain I will be waiting for ever the problem to get solved. So, one way to do it is to take a very large domain.

So, maybe – 100 to+100 where the characteristics wave length is of order 1 then that might work and other way to look at it is I will take a small domain like –pi to +pi and impose periodic boundary conditions to simulate what would happen in the infinite case. Because when it is infinite our argument is it has to be periodic. If it is not periodic then it has to blow off to infinity at one of the extents.

So, realizing that it must be periodic we could take a periodic domain and of course taking a periodic domain restricts the modes that we would get. So, that is my point here. If we take a domain like this –pi to +pi then k will long to a discrete set and so on. So, you have the integral values. So, because –pi to +pi, so it will be sin x, sin 2x, sin 3x. So, as the wave has to fit properly in the periodic domain it cannot fit half of it.

If you had –L to +L then k will belong to 2n pi/L, actually sorry I think this have, you will have – n pi/L, yes it is 2n pi/L. 2n pi/2L I guess, because the 2L is the length of the domain. So, again the n takes values and will belong to the integers. So, the idea is that if the domain is infinite the k will vary over all the real numbers, if the domain is finite and periodic and then k takes discrete special values.

So, this is the same thing that happens in Fourier series and Fourier integral. So the Fourier series is how you stood up a function on a finite domain. So, when you write the Fourier series you sum over the series and the series sums over integer values of n that is what happening here. When you do Fourier integral you do the Fourier integral for the function on an infinite domain.

Then you have to integrate across all the k values because k is not discrete any more. It is the whole real number. So, with this understanding we are writing ikx. So, if x is infinite k takes all real numbers and your dispersion curve is continuous. If x is finite in a finite periodic domain k takes only discrete values along that dispersion curve. So, what it means is if I take a domain – pi to + pi my k can only be 1 and then 2.

k cannot be 1.5 because 1.5 that mode will not fit in that – pi to + pi. So, if I controlling the size of the domain you can exclude modes or include modes in the periodic case. If you take the infinite case all the modes will come. So, that is something that you should remember and thoroughly understand before you try to simulate the problem.

Because when you take a periodic domain you will actually be restricting the modes that are entering the problem and then that should that can lead to different results unless you are aware. Okay, that was one point, the other point is with someone ask me is now when we do this, okay let us carry on and I will come back to it. So, we have got this form now and so will go and back and substitute it here and get the equation for the dispersion curve.

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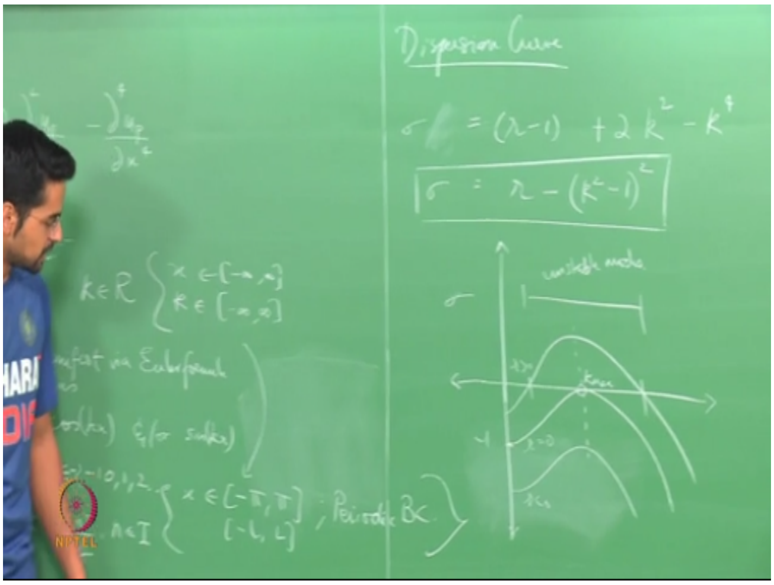

You all agree? Those of you all have done it? So, this problem is very simple because you start off with that but with only one special direction because you are knocking off time and space we will just get the algebraic equation which is the dispersion curve. It gives us a dispersion curve immediately.

Now at this point some of you all ask me the question as what happened to the boundary conditions because we had a it and then we certainly I mean, reach the solution and we did not seem to have said anything about the boundaries. But the fact is that once again when you writing it to the power ikx it implies certain boundary conditions. So, what it implies is either that you have –infinitive to +infinite and the domain is basically going to be periodic.

If your thing was a finite domain then taking it to the power ikx again forces it to be periodic boundary conditions where this –L to +L will control which modes comes. So, the length 2L of the domain controls which of the k modes are allowed and what are those periodic boundary conditions? Let me just, I will just write this, so periodic boundary conditions as simply u at u and all its derivatives.

So, n is 0 means just the values are equal. When n is 1 it means the first derivative, so it is 0, 1, 2, 3. So, you equate the values and all the high derivatives until you get 4 boundary conditions because I need 4 boundary conditions in the x direction. That clears? It is with respect to these boundary conditions, periodic boundary conditions that we are using the form it to the power ikx when x is on a finite domain –L to +L.

And if it is on infinite domain then the idea is like splitting it up into the Fourier modes. So that is fine. Now we can, we will have the dispersion curve, okay. So, in case 0 what will happen? I get r – 1, so let us look at the case of r = 0 because we already know that is going to be the critical thing. So, when r is 0 basically k will be, when k is 0 and r is 0 sigma will be =-1 right? – 1, so it will be somewhere here and then it does something like that.

So, it becomes critical here at any higher value it will cross. Right, so basically you have a region of unstable modes, right. This is for r = 0 this is for r > 0 and r < 0 it would have all been negative. And this peak remains the same throughout which you can find out what it is and I will call it, I mean this portion is called kmax. So, the kmax gives me the maximum value of sigma for all values of r which means that the peak of the curve does not change with r.

That is not always the case as you would know when you do the assignment and this problem it is true. So, what it tells me is that this mode corresponding to kmax is the fastest growing mode. But there is also a range of modes along with it all of which are unstable. And the number of modes which are unstable keeps increasing as my super criticality keeps increasing as r becomes larger and larger than the critical value 0.

So, that is as far as the first few points of the assignment were. Any doubts about this?

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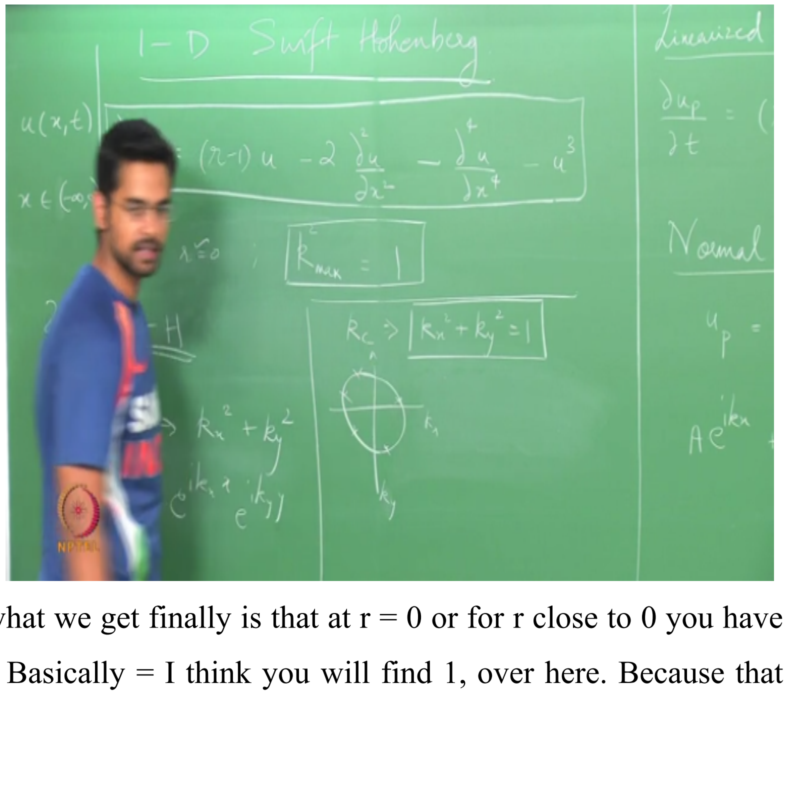

So, essentially what we get finally is that at r = 0 or for r close to 0 you have the critical k or the kmax, right. Basically = I think you will find 1, over here. Because that is where sigma becomes 0 when r is 0. So kmax is, this is basically 1. So, the idea now is if you try to interpret pattern formation what will happen is if you have r just above 0 may be 0.1 or 0.5 also then you will see a mode manifesting which will be the mode k = 1.

If you do the non-linear simulation of the equations, you will find that from the base state 0 you will get a new steady state which has the same wave length as this k = 1. Which means the solution will look very similar to sin kx, with some magnitude A like A sin kx something like that the new non-linear state will close to r = 0. For r much > 0 higher non-linearity's come in and then the steady state can change.

But close to this onset the kind of wavelength you will see in the pattern and this case it just 1 to mention will be a wave length of k = 1 means lambda will be 2 pi. So, if my domain is – pi to + pi for example if I look at the thing in – pi to + pi whether it extends to infinitive or not if I just look at the thing – pi to+pi I will just get 1 sin wave and this is basically sin, I mean this could this is sin x I could also get cos x because that is also equally valid.

So, then I could have got, I am not very good at drawing this but basically got the idea. This sin is 0 at both ends you would get cos whether the derivatives are 0 at both ends, it is just the 90 degree phase shift I think I have got it right, whatever but you will get cos x, so that is fine. Now the second part of the question was what happens when you got to the 2 dimensional Swift-Hohenberg.

So, in the 2-D Swift-Hohenberg basically you will have these partial derivatives but now extended in x and y. So, if you go through the analysis what you will find ultimately is that you will get the same story except instead of k square I will have to replace it by kx square+ky square and if you not be a surprise because we have done this and seen this in many problems now in the course.

So, this kx, I mean this comes because I take e to the power ikx x e to the power iky y. So, that is the kx that is the ky. So, the interesting thing in now is that in the 2-D version of the problem my critical kc or whatever is actually given by kx square+ky square = 1. And that is very different from what I had before that just k = 1 in the 1 – D case. Now, I have kx square+ky square = 1.

This is actually k square = 1 and then I took the square root. So, the point now is that the first question that I ask is the mode therefore that will manifest is that mode unique is the wavelength that you will see unique is the kind of pattern that you are going to see unique in this case it was, I mean, okay there was it could be sin x it would be cos x but in an infinite domain or a periodic domain those things do not matter.

It is basically the same physical pattern you have seen. It is just phase shifted. But here I can actually get an infinity of possible modes which look differently and which look different in space mathematically they have a different representation in kx and ky apart from the phase shift. So, what is that mean? If you understand why that is, so if I, can be a value of kx say 0.5 I can always find the value of ky to go along with that.

So, in that sense I can get what actually happens is in the plane of kx ky, I have an entire contour where it is maximum. So, all these points in the plane of kx ky are critical.

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So, physically what does it mean? If I have a part of the domain just a piece of the infinite plane and I look at some manifestation of the modes. So, I do the non-linear solution give it to the perturbation close to the onset. Suppose I take kx = 0 and ky = 1 that is one possibility or let me do kx = 1 because that is this basically falls back to the 1 – D case.

It is almost like you have the 1 – D problem and I added a y direction and said nothing will change in y that is this case and that is also equally unstable. So, what we will get here is that there is a periodic variation in the x direction of wavelength 2 pi and in the y direction nothing is changing. So, you will just get stripes, right where this thing is means that > 0 and this means it is < 0.

So, if you take any cross section you will just get the sin x or the cos x and you will get this kind of stripe pattern. So, that is clear how that can. Now if I looked at the opposite case where kx is 0 and ky is 0 then it just a rotation of this pattern. You will just have like horizontal stripes, sorry ky stripes, you have horizontal stripes. This is also equally valid as that.

If I took the intimidate case like this is also equally valid I would get and the perpendicular vector will be oriental along the 45 that is how. So, now what you see is that and I can go on with this which I want thankfully. So, what you will see is that you have a whole range of patterns all are which are equally possible in terms of the stripes. The basic thing is the stripes.

Now this is where the symmetry breaking which is more often discussed by physicists but which is relevant to all of us is particularly important. So, the reason why you have all these different possibilities is actually very simple and obvious once you think about. And that is that suppose I were to say that I am only going to consider the 1 dimensional problem.

So, I only have kx = 1 and ky = 0 means I am only going to get this pattern, okay, so fine. But now the question is if I look at the physical system what is that to tell me what the x direction is. So, if you look at the stable and that is my experimental system which is the x direction. So, you do not know that you can choose the x direction as you please because the system is isotopic.

And we were discussing a system like that in infinite fluid so nothing is particularly special about the fluid or about the boundary conditions or about the governing equations in any particular direction x and y. So, that was totally, I mean it was just our choice of the axis and coordinate system to allows us to proceed mathematically. But when we come back to look at the physical system there is no x direction.

So, because of that you would immediately release that it has to be that all orientations of this should be equally unstable because I could choose any direction as the x direction. So, any rotation of this pattern is equally valid because I could have taken that is the x direction. And this is simply saying the same thing in a mathematically way.

So, all of these patterns are just rotations of this pattern and the fact that all the rotations are equally unstable tells you that the original problem had rotational symmetry. Which means that I could if the xy axis was there and I rotated it in any way does not change the problem or if I took the x y axis I rotate till the coordinate system by theta and I made that substitution.

So, I replaced x/x * y/y *, the rotated axis I would get exactly the same equations. The problem would not change at all. So, that in variance to rotational transformation is what we mean by rotational symmetry of the, not of the pattern, rotational symmetry of the base equations and the base state.

So, because of that what happens is that if I have a pattern like this I can confidently expect that all rotations of this pattern will be equally unstable or equally possible in my new steady state. So, that is an important thing. However, you must realize that this pattern is not rotationally symmetric because this is the pattern that has a clearly well-defined orientation in the y direction and the x direction.

If I rotate it and make it this these 2 patterns are not identical. So, there is the rotational symmetry is being broken when I went from the base state to the new state. So, the base state was flat homogeneous that is rotationally symmetric. It is just, everything is just 0. But when it came to a new state, the new state broke that symmetry. So, because this is no longer invariant to rotation, alright.

But because there is no specific directions in my problem because the original equations and base state was rotationally symmetric, the new state all its rotation should be equally valid that is the idea. None of those states are rotationally symmetric but any rotations the whole family of them should be equally unstable. So, that is the broken symmetry which I have written down in the solution and given some references, you can read more about it.

The same idea comes when we look at the phase shifts that is also a manifestation actually of broken symmetry. Because let me look at the 1 – D case, I remember I said if you look at – pi to+pi in the infinite problem I could get a sin wave. But in truth because the problem is infinite I could get any phase shift of that sin wave. So, basically if sin x is a possible pattern sin x+any phi will be possible where phi is a phase shift.

Or basically the pattern of sin x translated by any amount is in equally valid pattern to emerge in the new steady state. Why? Because again if you go back to the base problem there is no reason to, there is no place which I can fix as the origin. I can take anything as the origin. So, if I shift my origin along the plane basically that is translating the sin x curve. So, in the 1- D problem I did not have this question of rotation what I had was translation symmetry.

Because I can translate the origin on the x axis and it is not going to change the problem. So, because the base state and the base equations were translationary symmetric when I got the new pattern all the translations of the new pattern were equally valid as a new pattern as the unstable mode. Again once again you will see that this sin mode which is the unstable mode is not translationary symmetric.

Translations will give you a different pattern but all those different patterns are equally possible because the base state was translationary invariant because the origin can be placed anywhere on the x axis. So, there is a very good book a popular account of these questions in physics, chemistry, biology, Crystallography and so on it is called Fearful Symmetry by Golubitsky and Stewart.

Stewart is a very good writer and Golubitsky has done a lot of work in this area for many years and that it is a popular science book. So, you can just pick it up and read it and it tells you a lot of different patterns in nature how symmetry place a role and you will be able to understand those things very easily now that we have done some of the math in the course.

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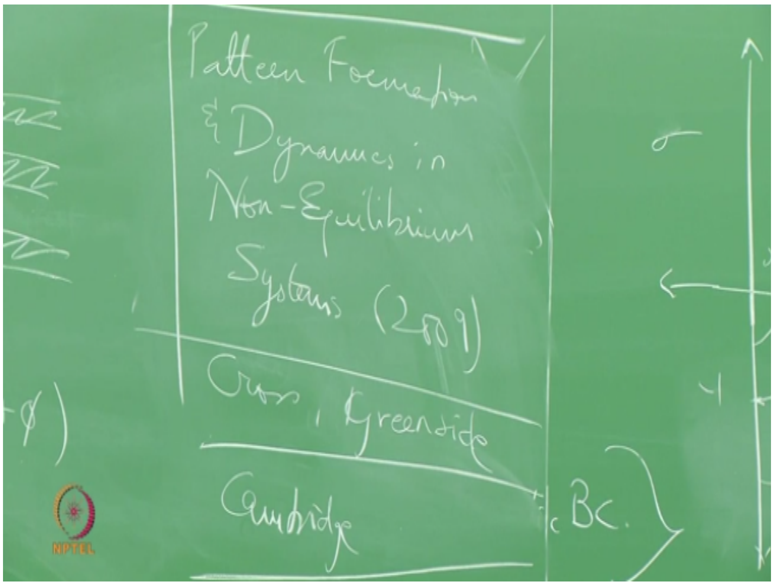

Along with this I would recommend for not only this problem but in fact the entire assignment is based on that book and the next assignment also and it explains many of the ideas and Maths really well in that book. I have given it in the solutions, so you can look at the full thing. It is a pattern formation and dynamics in Non-Equilibrium Systems. It is by Michael Cross and Henry Greenside, it is Cambridge.

That is a very good book for all the idea that I spoke about. So, now when you look at the non-linear simulations of this problem or an experiment the idea is you should be ready to expect all these different orientations of patterns. That brings us quickly to the final point I want to make about the patterns is that not only our all orientations of this stripes state possible but even super positions of different stripes can come up and there is nothing to stop that.

So, if I tell you that all the modes on this circle are equally possible then either anyone of them can emerge individually or maybe 3 or 4 of them can or 6 of them can super post together and lead to some pattern or they can all grow together and once they grow initially the non-linear terms will come in and modify the solution it is called non-linear saturation and give you a new steady state.

So, that new steady state could bear resemblance to any orientation of this stripes set it could also be given by some combination of those stripes. So, just as an exercise, if you look at these stripes and they are 90 degree rotation what you can do is you can go to a math lab and plot the contours of cos kx x* cos ky y Take that function plot its contours when you take kx = 1 ky = 0 you will get these stripes, black white black white.

Then you plot the other case then you plot their combination and see what you get. So and in some portions the say if I look at this and this is the and this portion everything is positive, right in this portion everything will be negative. Here things will be sort of canceled out. So, you will have almost the square the kind of pattern emerging simply out of these 2 combinations.

And those stripes of patterns are known to be seen like a new steady state has known to be seen in these kinds of non-equilibrium systems after the symmetry breaks, after the instability. A similar combination also gives rise to other like the hexagonal, so the Rayleigh–Bénard**.**

So, that is how you get so the thing to be no were in the whole problem either in the 1 – D problem of Rayleigh–Bénard nor in the 2–D problem you found that. You just had these stripes basically. But it these combinations of those stripes that lead to you to those states that you see in many non-equilibrium systems that you will get hexagons and different kinds of cells.

And the question as to which of these patterns would come how do I know the hexagons will come or how do I know that these stripes will come. That depends on very often on the boundary conditions which are not in this thing right now. It depends on the initial condition also but very strongly it also depends on the non-linear terms in the equation which we have totally neglected from the linear stability.

So, in that sense it is similar to the case where you get the eigenvalue to be 0 and then you cannot say anything about stability you need to go to the non-linear terms, so to understand how it is going to grow. So, in a similar way here all these modes have 0 growth rate, in fact 0 eigenvalue. So, now the question you ask me is which of them will come actually is asking me which of them is going to grow and dominant.

So, for to understand that we need to look at these different modes the different stripes and look at the effect of non-linear terms on their growth. So, that is called weakly nonlinear theory and that will tell you which of these patterns will come. So, those of who you are all interested in that can again look at this book and he has a discussion about those things. And those are the more advance topics in the area of non-equilibrium dynamics pattern formation.

So, now you can go back to your other problems that you have done. The Rayleigh–Taylor problem, the Rayleigh jet, Rayleigh-Plateau jet instability capillary instability and look at the same thing over there. Look at the base state identify its symmetries and then see that in fact your new steady state or the modes that you get should all obey this idea of broken symmetry.

That all they are the whole family of translations rotations and so on should be equally unstable and then it will not be surprising to you anymore. You have any other doubts in the assignment I can address that now and then I want to talk a bit about the computation assignment I have put up. So, if you all have any, if you do not have any doubts on assignment I will directly do that otherwise you will have a doubt I will continue with the assignment.

It is oscillating. Why mean that is the system is like that the dynamics are oscillatory, so it is a 2 – D system. So, it can oscillate and in the sense that there is a base steady state you solve the non-linear equations. So, now the base steady state was actually unstable possibly at that case the eigenvalues were had a complex part. So, it was complex positive and then you had an oscillatory kind of behavior.

If it is stable it will oscillate and die out to 0 if it is unstable it will oscillate and reach a new steady dynamic maybe we are limit cycle maybe it will be an oscillation, I mean it will be a steady state. The main idea is that it all of the things should have died out the steady state which we were looking at should have been stable under all conditions because the eigenvalues of – epsilon and – 2 epsilon.

But that does not happen because of non-normal growth. And I have given in the solution I worked out a bit and given some references where you can read more about it. Any problems with that reaction diffusion problem, so that is clear in that problem you need to look at the rate. I have not specified any form you should continue with the unspecified form, so that means where I have written, right.

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So, when you linearize it about Uss you do not know what of r of Uss+epsilon u perturb. I mean you do not know it in the algebraic expression form. But that is the whole idea, anyway doing a Taylor series, so if you want to approximate the value of this function about Uss epsilon. This will just be r of Uss+r prime of Uss into epsilon Up. So, this is a constant depending on the Uss.

So, you do not know what uss, it does not matter. So, this is some constant number you can replace it by a if you want and then the rest is just epsilon Up, so you will get aup basically from this term into rss. Basically aup you will get from this term in the linear equation and then you will find that if r prime of Uss is positive the system will be unstable and if r prime go negative of the system would be stable.

So, diffusion also if you look at it diffusion would only be stabilizing. So, this value of sigma will be given by simply r prime of Uss itself and the diffusion just stabilizes it. Because sigma will be r prime of Uss – k square or something like that. So, in this problem diffusion is stabilizing. We might look later at a situation where diffusion is actually destabilizing and that happens when you have 2 chemical species and gives rise to what it called Turing patterns.

But I want to talk about them now because I think we will do it in the course. If you have any other doubts I have written the solutions said down, so you can go through that and then we can discuss them later especially about the classification of the different states you can just read that part, alright.

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So, now the next thing that we need to do on our, this will be on a computers. So, again we are going to look at the 1 – D Swift to Hohenberg and to now simulate the non-linear equation, alright and then see whether in fact if I perturb the system for r > 0 whether that I will get a new steady state, will the new steady states pattern be similar to sin x or cos x then how the effect of boundary conditions will change the instability and so on.

So, all these questions that you are trying to answer analytically in this 1 – D problem it is quite simply to go and see for yourself numerically. So, it is like a numerical experiment. We can see whether the theory matches the computation. So, the equation is this similar thing. I have put up the assignment online with the list of questions and exercises and there I have given a short description of how you can write a numerical code to simulate it.

So, I will just discuss some of those things here so that might make it easier, alright. So, that is the governing non-linear equation. Now there are the next now here the important question comes of boundary conditions because I cannot simulate and infinite domain. So, what I recommend is you go from – L to+L and then you apply periodic boundary conditions which are those conditions they are just Un of – L or+L, right.

So, U the values of U and all the derivatives will be equal at both ends of the domain, okay. Now this is where the important thing about that mathematics business comes in which I was talking to you about.

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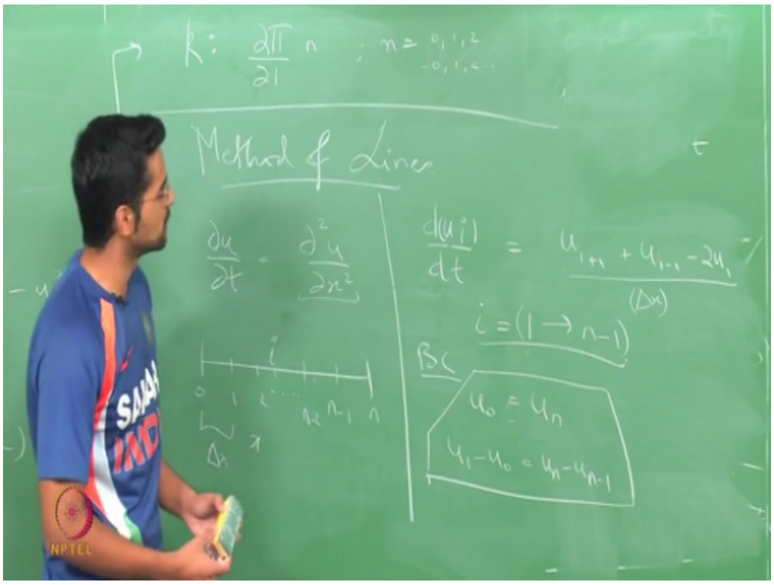

So, if your domain is this-L to+L you have a length of 2L. So, the possible case that I would get under this periodic boundary conditions it will be 2 pi/2L n, okay where n will take values of and the negative values. If I look at only sin i kx it will be+and – otherwise, you will have cos and sin with only the positive. But that the case that you will get on the dispersion curve will not be all the case.

The case will only be these discrete values, alright. So, now because we know from theory that the most unstable mode is k = 1, right. So, I will want to allow that to manifest otherwise I cannot compare the thing with theory. So, therefore, if I want to allow the k critical = 1 to manifest it should be one of these possible modes one of these discrete cell. So, therefore I should take L = pi or in general L = n pi.

So, if I go from – n pi to+n pi any integral multiple pi then I can be sure that that mode this Kc = 1 which came from the linear stability theory that mode will be allowed to come. If my L is say you can go – pi to+pi, okay. If my L is < this then that mode will not be allowed for k = 0.5 or something will come in this k = 1 will not that domain will be too small to allow sin x to come in.

Because it needs a wavelength of 2 pi if you are not going to give it a wavelength of 2 pi how can it come. So, you are not allowing it any more. So, that is an important thing you should keep in mind that the numerical simulation at some point will start failing if you do not observe that thing. This is more important when you do not have periodic boundary conditions but even in this case you can try out and see.

So, the important thing is you can keep this in mind and then keep it at domains of n pi. So, if you take n very large ultimately you will reach – infinite to+infinite then it will L will not matter anymore if you take it large enough. But then you want to avoid that much of computation. So that is the one point. Then about the numerical simulation one method of doing it is to do what is called Method of Lines.

So in Method of Lines or MoL the idea is to discretize the spatial variables, alright and leave the partial derivatives in time as total derivatives. And then what you can do is instead of a set of pds you will get a set of ods in time. And using math lab or Mathematica you can integrate those ods very easily using the inbuilt functions with robust numerical methods.

So you do not have to worry about the time stepping you just need to take care of the spatial dependence. So I will do a simple example, suppose I just had dou u/dou t = dou square u/dou x square in a diffusion equation. So that is some like this more complex version let us look at this. So I just had the simple diffusion equation.

Then what I would do is discretization space which means that if I have my x domain right I will break it up into nodes. So node 0, node 1, node 2, it is a node n, node n – 1, n – 2, right. Now the spatial finite difference discretization and what I will do is I will replace the derivative in x by its central difference finite, I mean central finite difference. So I will get dui/dt where ui is the value at the node i, okay.

So dui/dt because I have discretization space and the second derivative at i is basically Ui+1+Ui – 1 – 2 Ui upon delta x, where delta x is this difference between 0. Because this is the central finite difference of second order. So at the same way you can find central difference formulas for the fourth order derivatives also and substitute here and u cube will just be Ui cube, okay.

So now what we have is, we have a bunch of ods. So if i runs from, okay the other question okay now you have this where do I apply this equation? I apply this equation for all the interior nodes. So in this problem I would have 2 boundary conditions. One boundary condition would be U0 = Un. Let me write it here right. Boundary conditions would give me U0 = Un.

That should be the, I mean quality of the values and the derivatives would be the derivative of U = derivative of n because I have a domain like this I can use a backward difference here and a forward difference from the other side because delta x is the same I will get U1 – U0 should be = Un – Un – 1. So, actually here I you use the forward difference and there I use the backward difference. So there are other ways of doing this but this is a simple way.

You could also use the difference here and then you will have a value or what now. But if you just start, this is the simple thing you can. So, use appropriate differences, forward here and backward from the other side, so then you have got a boundary conditions and this problem you will have to repeat that for the second derivative and the third derivative using again forward difference backward difference of the appropriate formulas.

So, what you will see here is that these equations allow you to relate the values of the exterior notes U0 and Un to the interior notes. So, you can do that. So, basically, ultimately what you need to do is you need to apply this equation for i going from 1 to n – 1, okay. You apply this equation for 1 to n – 1 those are the interior notes. You do not have equations for 0 and n but that gets provided by the 2 boundary conditions.

So, you will have n – 2 ods+2 boundary conditions which are basically algebraic equations and you can use them to describe the evolution of these n notes. So, more on this is there in the assignments, so with these basics you should be able to simulate it. So, we will discuss more in later. Thanks.