Chemical Reaction Engineering 2 (Heterogeneous Reactors)
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Lecture 28
Inter and Intraphase effectiveness factor continued

In the last class last 10 to 15 minutes we have derived one equation for interphase, intraphase together, right and in the graph I have drawn I have mentioned that is Eta Phi square that must be Eta 0 Phi square in the graph x axis Swami did not draw the graph at all, did you draw the graph x axis, x axis is Eta Phi square I have written there that must be Eta 0 Phi square, right.

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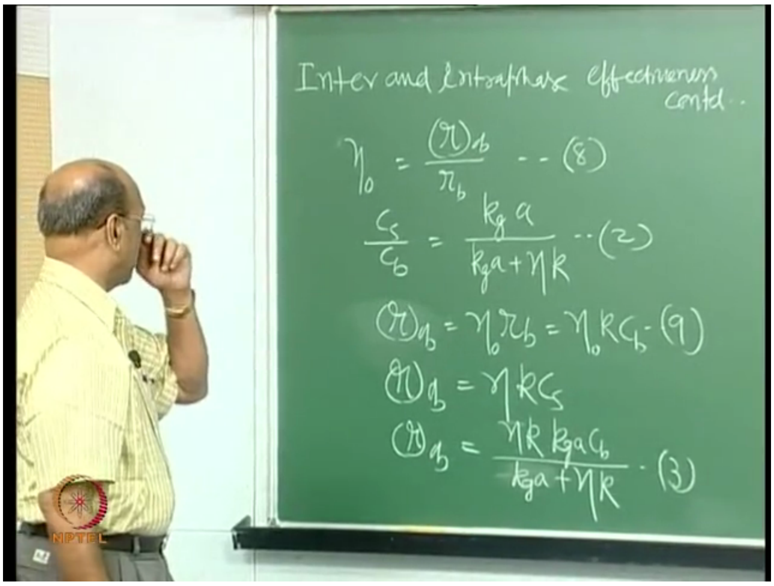

Inter and Intraphase effectiveness contd..

$$\eta_0 = \frac{(r)_{ob}}{r_b} \cdots (8)$$

$$\frac{C_s}{C_b} = \frac{k_g a}{k_g a + \eta k} \cdots (2)$$

$$(r)_{ob} = \eta_0 r_b = \eta_0 k C_b \cdot (9)$$

$$(r)_{ob} = \eta k C_s$$

$$(r)_{ob} = \frac{\eta k\, k_g a C_b}{k_g a + \eta k} \cdot (3)$$

So I think again I will quickly derive this also can be derived in another way in terms of Eta 0 starting starting with Eta 0 this is again inter and intraphase effectiveness continued, okay what we have done in the last class last minute last 10, 15 minutes we have done that so in hurry we have done but I think you know all the equations are right but only thing one step I have to proof that why that r ob L square by De C b equal to Eta 0 Phi square, where Eta 0 is overall effectiveness factor, okay we have used Eta bar for external, Eta for internal intraphase and overall is Eta 0, okay so that is what to just.

The first definition we had Eta 0 equal to r ob by r b this equation did I write first time no not okay ya good. So this is the one which missing, so Eta 0 is defined in normal fashion that but this is overall from film to centre of the particle so the entire dimension is taken for defining this. So then you have the equation already in terms of C s by C b I do know C s by C b you have an equation already please tell me that k g a divided by k g a plus Eta k, okay.

Now substitute this equation what was this number, this one is 2 and this one what was the last number we just continue this we will put as 8, okay good. So from this from 8 we can write r ob equal to Eta 0 r b which is if you take for example first order reaction and that reaction also it will be same no problem Eta 0 k C b for first order reaction, correct no this is r b r b is rate based on but isothermal only we are talking please remember that, okay this overall intraphase that is interphase plus intraphase together we have been talking only overall I mean only isothermal condition so that is why so this is no temperature there it is only k into C b.

Now substitute ya substitute now for this C s okay sorry this is Eta 0 k C b okay one more equation I think what you have there so that is where I have been searching that ya r ob equal to r ob equal to what is that you have given written there not that the beginning r ob equal to k g a C b is also equal to Eta k C s Eta k C s, so r ob is also Eta k C s, good substitute now what is this equation number 1 no first one okay you do not have to write that equation number substitute equation 2 here and tell me what is Eta what is r ob Savita not able to follow still this equation you have, this equation you have, what is that you have here C s, right like Savita many people also may not understand this C s C s substitute here and then what you get for r ob ya r ob equal to this whole thing right Eta k Eta k here is this ya for C s I have to write C b k g a C b divided by this one k g a plus Eta k so this is the one now.

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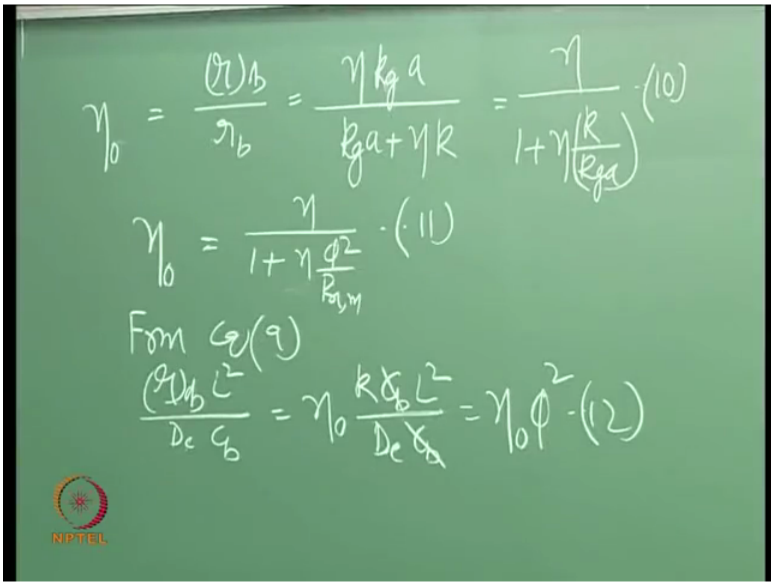

$$\eta_0 = \frac{(r_o)_b}{r_b} = \frac{\eta k_g a}{k_g a + \eta k} = \frac{\eta}{1 + \eta\left(\frac{k}{k_g a}\right)} \cdot (10)$$

$$\eta_0 = \frac{\eta}{1 + \eta \frac{\phi^2}{B_{a,m}}} \cdot (11)$$

From eq (9)

$$\frac{(r_o)_b L^2}{D_e C_b} = \eta_0 \frac{k C_b L^2}{D_e \cancel{C_b}} = \eta_0 \phi^2 \cdot (12)$$

So substitute this in Eta 0 ya you do that later but I think substitute what is Eta 0? Eta 0 equal to r ob by r b or in this equation also this is equation 3 no not 3, 9. Karthik what was this equation number? This is already there that is what I am 3 ya this is 3 we are mixing that and this now. So now substitute for Eta 0 and then give me the equation what do you get? That is equation 8, substitute equation 3 in 8 with r b also r b also you know r b is k C b substitute for r b and r ob what do we get? 1 by Eta plus plus 1 you will get ya everyone is giving his own so I will give my own.

So this is Eta k g a k g a plus Eta k excellent that is one thing now divide this is also not 1 by just to make some 1 plus Eta k by k g a there must be some meaning no when writing that, what is k by k g a? Yesterday we have done something else, it will become Phi square yes that becomes Phi square by Biot number. So Eta 0 equal to Eta by 1 plus Eta Phi square by Biot number mass, okay good.

So now here you can substitute for Tan h Tan h Phi by Phi for slab or otherwise for other one spherical particle you have 1 by 3 by Phi into whole that that you have to remember examination is coming in the next week, okay. So that also you can substitute for any shape know you can substitute because we have not taken any shape when we are developing this equation.

So only thing that is left you know what should be proved was that r ob C b De by by De r ob L square, right r ob L square by ya that is equal to Eta 0 Phi square I said, can you proof that Eta 0 Phi square, what do you start with this is the equation to start with ya in equation 9 you can substitute and then get equation 9 you have r ob multiplied by L square divided by karthik searching? So what do you get, all that is over mathematically coming there ya so that is one way and here also you can use equation ya this equation (divide by L square) sorry multiply by L square divide by De and what is else the other one? C b both sides anyway C b also we get cancelled you will also get Eta 0 Phi square, okay.

So anyway let me do that so from equation 9 we have r ob L square De C b equal to this if I take Eta 0 k C b L square same thing De C b, C b C b will get cancel, this k L square De is nothing but Phi square so that will be Eta 0 Phi square ya because there are mistakes even in Carberry sometimes he writes this one as Eta, sometimes he writes mistakes as Eta 0, Eta 0 is the correct one for overall.

So in that graph you should have Eta not, okay Eta not and please think carefully about that graph that is graph is looking good but I think it is not that easy to generate I may ask you how do you generate that graph you know you have Eta 0 versus the plot was Eta 0 versus Eta 0 Phi square for various Biot numbers for various Biot numbers, how do you really generate that graph? Make a question mark there you know how do you generate that graph? I want you to think, good ya I think somewhere numbers have gone 9, so this was not there right it was not there yesterday so this is 10, this is 11, this is 12, okay because I realized it that you know you also should have Eta 0 and Eta related that is the reason why I wanted to do that otherwise yesterday it was not obvious, Eta 0 I have not mentioned so that is the reason why I thought again I will do it, okay.

So now next one what is left was is taking all the equations what you have done till now that means interphase effectiveness factor, the temperature profiles, concentration profiles you know the particularly temperature difference concentration difference and also intraphase temperature difference, concentration difference you have the relations, right can you get some more information from this?

Like for example I told you thumb rule is for many of the processes in catalytic reactions the controlling factor is mass transfer through the pores and heat transfer through the film, okay these are the thumb rules but how do you know that what I am saying is right? You simply believe me so you say yes you have to believe me because you think that you know if you do not believe if you write wrong things in examination then I will not give marks, I think atleast for that reason you have to believe but we have to proof that, you can also proof that without trying to use any new equation all the equations is already with you till now in your notes whatever equations what you have.

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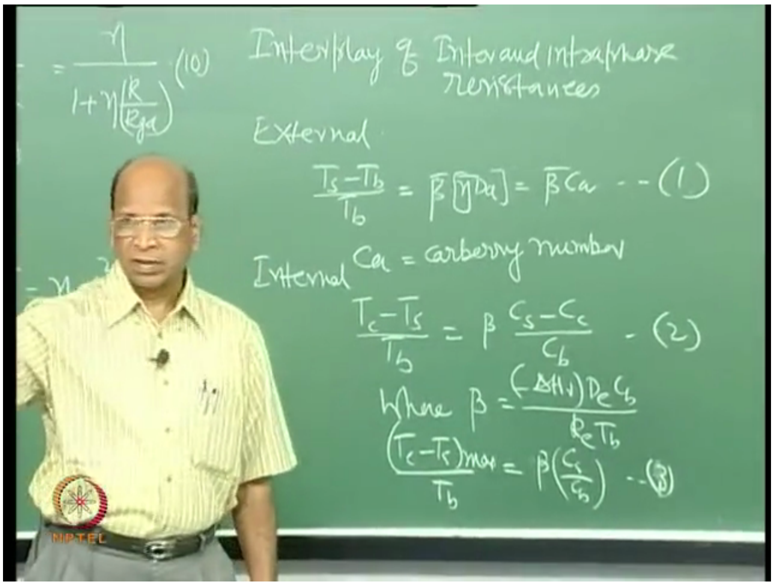

$$= \frac{\eta}{1+\eta\left(\frac{R}{R_{gA}}\right)} \quad (10)$$

Interplay of Inter and intraphase resistances

External

$$\frac{T_s - T_b}{T_b} = \beta[\eta Da] = \beta Ca \quad --(1)$$

Ca = carberry number

Internal

$$\frac{T_c - T_s}{T_b} = \beta\,\frac{C_s - C_c}{C_b} \quad -(2)$$

$$\text{Where } \beta = \frac{(-\Delta H_r) D_e C_b}{k_e T_b}$$

$$\frac{(T_c - T_s)_{max}}{T_b} = \beta\left(\frac{C_s}{C_b}\right) \quad --(3)$$

So that is why what we do now is this is interplay of intraphase okay inter and intraphase resistances ya see now resistances we are talking either mass transfer resistance when do you get or heat transfer resistance when do you get so that is the kind of information. So please take this as an interaction we have seen till now that we have seen till now that the general problem of diffusion reaction is quite involved so that means you know too much information you have generated it is quite involved in the derivations and all that.

Fortunately however the physical and chemical processes at work under realistic conditions favour isothermal pellets and negligible external mass transfer resistances. Let us now prove the same, Rinita what did you understand from this para what did you understand I think I gave already the answer very nice answer, read the second sentence what you have written ya why should we assume we have isothermal particle, thumb rules are there nicely see I mean here safely after doing all this if you are interested in reaction engineering and if you are interested in catalytic reactions in future life unfortunately if you get a job somewhere in chemical industry then you can happily tell this thumb rule you know when you have a catalyst particle you can always assume that the particle is isothermal because heat transfer is not controlling or the other way to tell is mass transfer control is more important in the particle pores.

Whereas if you take the external film it is the heat transfer which is important but mass transfer is not a problem, you will have sufficient amount of mass that is going through the film, okay through the film to reach the surface then afterwards the diffusion, okay.

Mathematically also we can prove that with the equations what you have already used, one equation what you have already used is this that is T s for external that means external film what you have the equation is T s minus T b by T b if you have notes you can check equal to beta bar beta bar Eta bar Da which we call now as beta bar C a where C a is Carberry number some people use this that is the spelling of Carberry car plus berry, okay so this is one equation what you have.

Similarly you have for external in fact Eta bar Da is nothing but again Eta bar Da in terms of concentrations 1 minus C s by C b, okay that is also there, good. So then the next one is we have internal that means intraphase you have T c minus T s, T c is the centre equal to this I think I have to just write (maybe I think I can also write this okay) T b equal to this is beta C s minus C c by C b ya C b where beta where beta equal to you have the equation minus delta H minus delta H r De C b by ya k e T b, similarly beta bar also you have, okay.

So now this is another equation beta that is equation number is 2 ya so we have also seen that so we have also seen that this we will get T c minus T s max by T b equal to Eta where is Eta ha beta into ya beta into C s by C b, okay so this is equation number 4 not 4, 3 equation number 3. So then, why C s minus C c because now earlier you assumed C s equal to C b but now it is not, Kavya we are now breaking the problem till T c I mean T b T s and T s to T c.

Similarly concentration is C b, C s, C c and C c is 0 for the maximum temperature difference, okay so that is why this C s will be there this C s by C b is also nothing but 1 minus Eta bar Da that also you have this C s.

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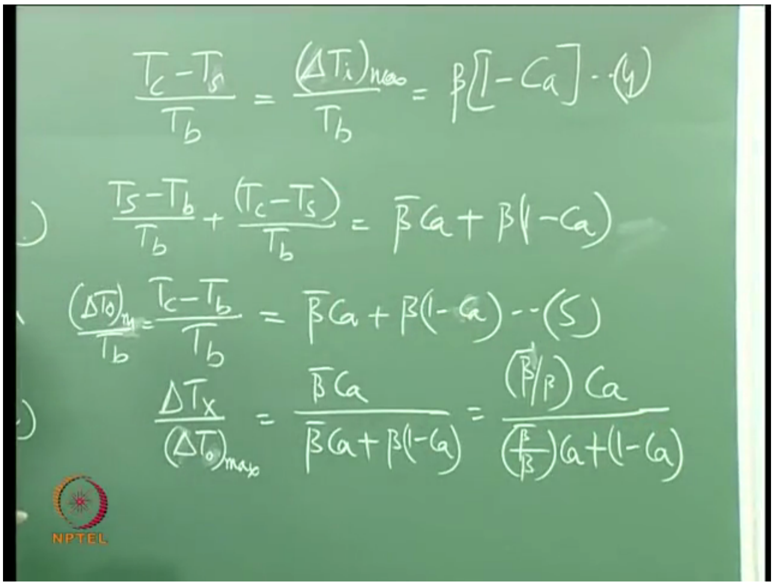
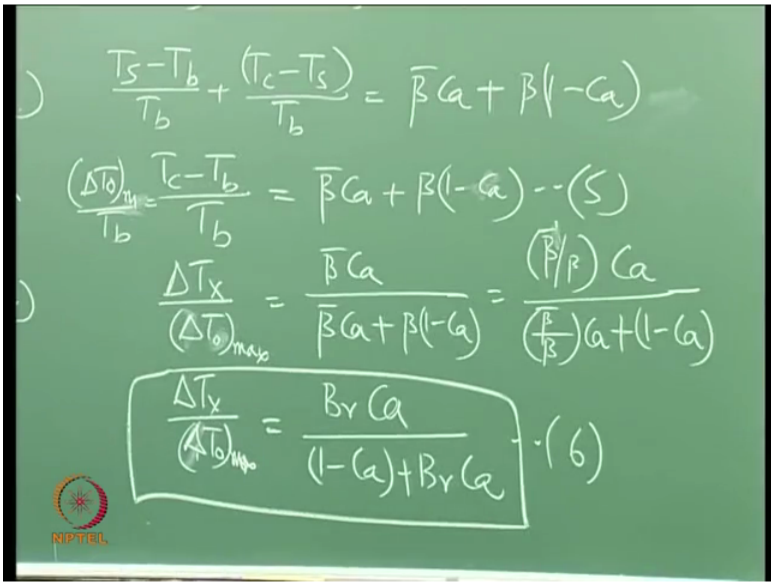


$$\frac{T_c - T_s}{T_b} = \frac{(\Delta T_i)_{max}}{T_b} = \beta[1 - Ca] \cdots (4)$$

$$\frac{T_s - T_b}{T_b} + \frac{(T_c - T_s)}{T_b} = \bar{\beta} Ca + \beta(1 - Ca)$$

$$\frac{(\Delta T_0)_m}{T_b} = \frac{T_c - T_b}{T_b} = \bar{\beta} Ca + \beta(1 - Ca) \cdots (5)$$

$$\frac{\Delta T_x}{(\Delta T_0)_{max}} = \frac{\bar{\beta} Ca}{\bar{\beta} Ca + \beta(1 - Ca)} = \frac{(\bar{\beta}/\beta) Ca}{\left(\frac{\bar{\beta}}{\beta}\right) Ca + (1 - Ca)}$$



$$\frac{T_s-T_b}{T_b}+\frac{(T_c-T_s)}{T_b}=\bar{\beta}Ca+\beta(1-Ca)$$

$$\frac{(\Delta T_0)_{max}}{T_b}=\frac{T_c-T_b}{T_b}=\bar{\beta}Ca+\beta(1-Ca)\quad\text{--(5)}$$

$$\frac{\Delta T_x}{(\Delta T_0)_{max}}=\frac{\bar{\beta}Ca}{\bar{\beta}Ca+\beta(1-Ca)}=\frac{(\bar{\beta}/\beta)Ca}{(\frac{\bar{\beta}}{\beta})Ca+(1-Ca)}$$

$$\boxed{\frac{\Delta T_x}{(\Delta T_0)_{max}}=\frac{Br\,Ca}{(1-Ca)+Br\,Ca}}\cdot(6)$$

So you have same thing I am writing here T s by T b equal to delta T ya here I have to also write ya here this one is just nothing but delta T i max I will write, okay delta T i max i that means internal internal temperature by T b so okay so this also I am writing as max by T b equal to beta 1 minus Carberry number so this is 4, okay actually that is Eta bar Da, okay so which is as ya I think here also shall I write Carberry number equal to Eta bar Da otherwise you will get confuse later what is this good.

So now we will add these two equations that is 4 and then 1 ya 4 and 1. So when you add that you will have T s minus T b by T b plus T c minus T s by T b equal to beta bar C a plus beta 1 minus Ca this is equation okay let me simplify that some more. So this becomes T c minus T b by T b equal to beta bar Ca plus beta 1 minus Ca so ya this is equation number 5, okay. Now there is another thing here ya this one also please add here this one as delta T x by T b I do not want write another equation there itself in the first place itself you can write x means external good and this one this one can be written as this is delta T not max T c minus T b by T b that is overall no T c to T b that is why T 0 is overall, m refers max, okay.

So now if I take the ratio of 5 and 1, 1 as delta T x by delta T not max equal to beta bar Ca divided by beta bar Ca plus beta 1 minus Ca which also can be written as beta bar by beta into Ca by beta bar by beta Ca plus 1 minus Ca ya so one more step here that is delta T x by delta T max equal to we will write another number Br into Carberry number by 1 minus Ca Carberry number plus Br Ca so this is equation number 6, where Br is anyone heard Br is what number Narsimha Reddy? Brinkman number, Brink Brinkman number, how do you know all these numbers? This is what is the problem most of the time how do you know means we do not know how do we know, transport phenomena good okay good.

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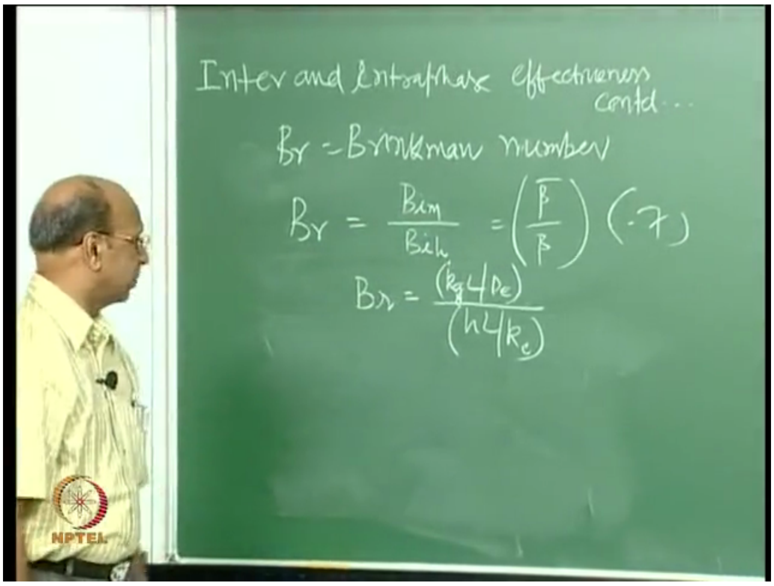

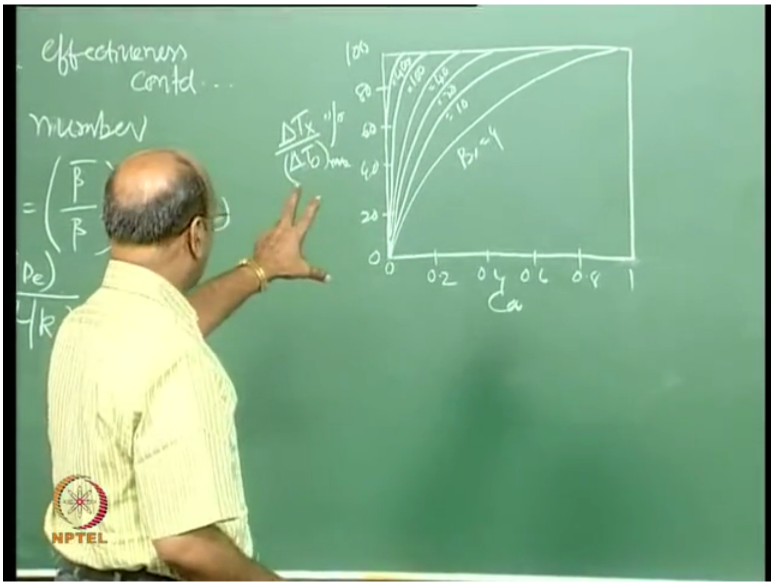

Br is Brinkman number Brinkman number this Brinkman number can be beautifully proved I will just leave it to you all the information with you all the equations are with you, okay Brinkman number using all these things you have mass Biot number by (())(28:36) which is nothing but here beta bar by ya you have all that you know what is beta bar all the things are with you, definition of beta bar you know, definition of beta you know, okay using those two quantities you try to get this ratio as Bim and Bih and you know what is Bim, okay Bim is just I am giving for easy thing this is k L by De and k g k g and the other number heat h L by k e. So you have to arrange this beta bar by beta in this format then you will get Biot number, okay good.

So now I think it is nice you proof that it is very nice very nice because I think you have to use again relationships from external that treatment for external effectiveness factor and then do it ya which ratio is Br oh sorry ya correct correct correct see Abhinav reminded you know Abhinav is today controlling mass transfer controlling, okay thank you okay good this is the equations 6 may be this is 7 this is only definition Brinkman number this is ya these two definitions so anyway.

So now what do we do after deriving the equation we have to plot that is our life all the time after plotting you have to try to find out what is happening in that plot sometime research also is boring all the time you find something plot the graph and explain the graph and publish, okay. So that is why unless you have some kicks in any problem I told you know simply increasing, simply decreasing will not give you any thrill, okay.

So now this is delta x by delta T i sorry delta T o max what is the other parameter I have to plot here? That is that equation 6, what is the other parameter Ca Ca is nothing but Eta bar Da okay Ca is nothing but Eta bar Da and Eta bar Da also you know which you can measure and all that but anyway that is not required here this is Ca and you have this side 0 to 1, Eta bar Da cannot be more than 1 I do not know whether you have checked earlier, Eta bar Da all the plots are between 0 and 1 now go back atleast you will go back next week anyway you have the exam, okay.

So this is 0 to 1 may be 0.2, 0.4, 0.6 no no 0.8 is there 2, 4, 6, I have to approximately put this 0.2, 0.4, 0.6, 0.8, okay 0.8 and this side this we can plot as percentage then we have 0 and 100 then I have here may be 20, 40, 60, 80. So if I plot this the parameter will be Brinkman number which is the ratio of mass to heat Biot numbers. So then you will have something like this I will put the parameter later, this is 4 Brinkman number 4, this is 10, this is 20, 40, okay 100, 400, okay 100, 400 good.

So what do you predict from this? Nothing some line and that is all what do you predict there what is D T x by De overall thing what does that tell you that coordinate? Ya how much is the external external heat transfer, okay that is why we plotted like that, okay. So now if the Brinkman is 4 then we have sufficient gradients inside and also outside that means in the film as well as in the particle but as you move towards Brinkman number higher numbers 100 so at this point for example you have almost 70 to 80 percent of the temperature drop within the within within the film, okay.

So that is why most of the time these things will come to very close to this so all the resistance will lie only within the film, okay so that is what is the message what we have to give and generally Brinkman number 4 is very rare, okay particularly for gases this Brinkman numbers will be 400, 500, 600, 800 like that, okay large numbers. So that is the reason why we neglect the external mass transfer, heat transfer is there definitely heat transfer is the controlling and inside we have mass transfer control, okay that also mass transfer control also I will tell you but quickly we will distribute now these things I think I have to give Rahul come okay there are two sets, one set I will give, another set he will give ya you give this, I will give this ya sent how many (())(35:57) extra Kavya give it to me you will get confused, you got this ya that graph what I have just plotted give it to one to him and one to her ya good.

You see for example this graph this graph you take this A side A okay Anurag what is on side A? actually these are the measured profiles within the particle and also in the bulk measured, okay beautiful work by (())(37:01) ya you see A is low Brinkman number so that is why there is some temperature gradient within the film and also there is temperature gradient within the particle this radial position I say R A, R by A, 0 means centre the other side, 1 means surface if you want you can write there bulk it is written there, right.

So it is beautifully demonstrated here that low Brinkman numbers you will have the concentration gradient I mean the temperature gradients within the pellets and also within the film, okay understood appreciated, no not appreciated only understood, okay good the next side next side is Brinkman number equal to 300, now what is happening inside the particle? So that means what isothermal the particle is isothermal all the gradient is only within the film and this is so beautiful I think and they have actually measured, wonderful I say really wonderful for chemical engineers, okay so that is one information.

And the other information is this that is next graph is the graph what we have I have drawn there see anyway we have explained what is that you know most of the time you will get the Brinkman number 400, 500 like that you know ya now see the table that is very important range of parameters, what Shekhar what is that you are discussing, no tell me tell me a graph is there anything wrong ya what is the logic you tell me, film is still contributing more but inside the particle also you have, you have the temperature more gradient there.

So that is why most of the time it is only the film which is controlling for as far as temperature is concerned, okay ya Prabhu the table, what do you have in the table so what are the parameters, normally this is very good table for those people who are really want to do chemical engineering later, okay see the range of parameters are given for example diffusivities how they vary in systems like gas-gas, gas-liquid, gas-solid see now diffusivity coefficients for gas-gas it is 1 to 0.1 centimetre square per second they are large, okay and meter square also you can convert that you know 10 to the power minus 4 you will get there, okay.

So then you see liquid 10 to the power of minus 5 to 10 to the power of minus 6 but porous particle you just see 10 to the power of minus 1 to 10 to the power of minus 3 increases than liquids you know why the porosity what we are developing we making the catalyst pellet and we allow the gas molecules to enter into the pores so that is why we cannot have very very small pores where it can be more than liquid, right but liquid there are no pores it has to anyway diffuse between the molecules in the liquid so that is the reason you know these numbers and ofcourse thermal conductivity is given lambda is thermal conductivity k e what we used, okay thermal conductivity range also is given and we have the rho cp value that heat capacity.

Then we have Arrhenius numbers how normally they change epsilon, epsilon is Arrhenius number, okay and beta is internal beta what we have beta x is beta bar (())(41:01) the some other book there is a book called HH Lee heterogeneous chemical reactors so I have taken from that this graph I have collected information from various books HH Lee heterogeneous (chemical) heterogeneous reactor design, okay then Brinkman numbers also are given you see there for gas solid system how they vary 10 to 10 to the power of 4 so if you take in between value how much it be 10000 know 10 to 10000 so for many systems it may be in thousands.

So that is why that line will be almost on the y axis covering 100 percent delta T x only in the only in the film so I feel this is very beautiful information other things ofcourse dimension groups and all that is given, okay okay. So the other one I think may be tomorrow I will try to prove that you know for mass transfer always this is the internal but not external mass transfer controlling that I think I will do it, okay now run and Levenspiel gave many problems in different notations but the same thing what we have discussed in catalytic 2, 3 chapters okay 2, 3 chapters so for examination you have to really thoroughly go through that, tomorrow what I do is I just try to prove that always it is internal effeteness factor sorry internal mass transfer which is contributing internal, external mass transfer does not play a role at all because you will have easily molecules diffusing through the film so that we will proof and afterwards I will just tell you the laboratory reactors which they use for finding out observed rates, okay.

So with that catalysis will go over and then we will go to packed bed, fluidized bed, slurry reactor, (())(42:45) and you have to prepare well for the exam because exam will not be easy because you know now so much information, it is not that only module 1, module 2 like that from the beginning again till now.