Rheology of Complex Materials
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Lecture - 14
Rheometric flows

So, in the last lecture we looked at the qualitative definition of strain and we also discussed the ideas related to shear flows and extensional flows. So, today's class we will look at the overall description of the flow in terms of position of the material particle and then we will use this position of material particle to actually in detail describe the simple shear flow and extensional flow.

So, just to complete what we had done last time and also discuss it in a little more detail let us first look at the qualitative picture involved. As we will see we will call these flows rheometric flows. So, rheological properties are being measured and so there is measurement. So, therefore, rheometric and the fundamental idea behind all of rheometric flows is that it should be a control flow. We have seen two predominant mode of controlling is to have it a shear flow or shear free flow or we will also see that the further control will be in terms of something will be constant, either stress will be constant or strain will be constant or strain rate will be constant and so on.

So, there are different ways of achieving rheometric flows. Why do we do rheometric flows? Because then what happens is we can relate since there are minimal quantities involved. So, for example, in shear flows only one component of the tensor is nonzero and hopefully that component then can be related to something measurable, because in the end if we have to do rheometry which means we have to measure the rheological properties we ought to be able to measure something. So, we will see that in rotational rheometers which are the most common rheometers usually torque will be measured because the motor will be used to rotate and you can measure the torque. So, can torque be related to stress.

Similarly, the rotation rate can be measured or the amount of rotation can be measured using some let us say optical probe. So, then can that be related to strain or strain rate. So, that is why it is easier, our job will be easier if there are only very well defined flow and quantities which are well defined and only few quantities are involved. Then we can relate the theory of rheology in terms of the description, in terms of stress, strain and strain rate to the measured quantities such as torque and the angle rotated and so on.

So, that is why let us look at rheometric flows, and this we have discussed quite a few times.

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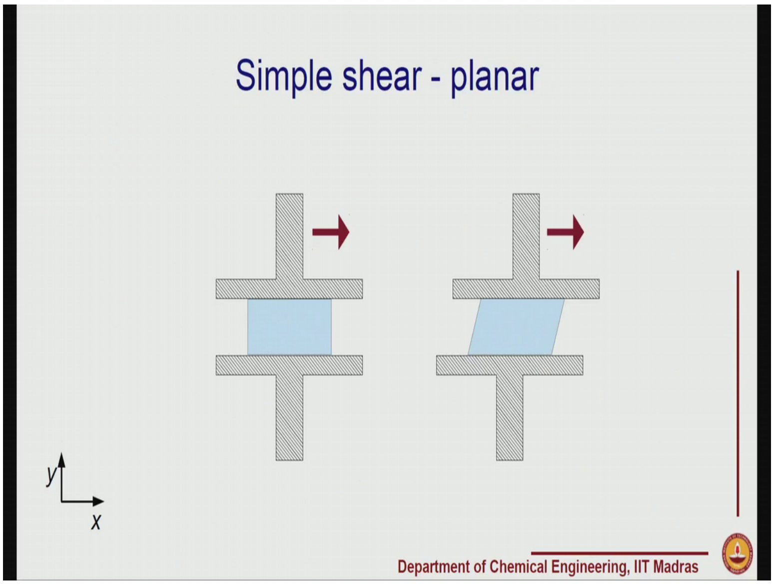

So, this is a simple shear flow right. So, what we have on the left is basically a two platens let us say and we use this xy coordinate system it is a planar shear flow which means this is a plate basically which is rectangular and in the z direction itself it is also going. Then we push the top plate towards the right and what happens is we have the material being sheared and of course, all of us are familiar with this that the layers of fluid is moving in horizontal direction in this case which happens to be x direction. So, this is an example of simple shear planar flow.

But to achieve this will be more tricky in lab conditions because if we want this flow to continue for long time then what will happen is fluid eventually will start flowing out of this here. But we would like to do this fluid flow for a significant amount of time to make sure that steady state is reached and also that we get enough data point so that we can average and get a good value and so on. So, in general to achieve simple shear in planar flow is more difficult. Any idea how could this be done? When I have drawn it using two different platens and take the platens and sort of shear one of them, but can you think of a possibility where you can actually achieve this simple shear flow which is planar between two plates. But at the same time this limitation of this plate running out is not encountered because sooner or later this plate will sort of go and then fluid will have no where right then it is not a planar flow anymore right.

If you go for torsional flow and. In fact, that is the flow which is used more often because it is easier. In fact, I was trying to motivate why torsional flow is easier because fluid actually does not really move from one point to the other, but if at all you were to do this planar flow it is possible for example, to do something like this you could for example, put roller, set of rollers and let us say have a belt which is sort of these rollers are rotating and then there is a continuous reservoir here of fluid, so you have fluid here and you have fluid here right.

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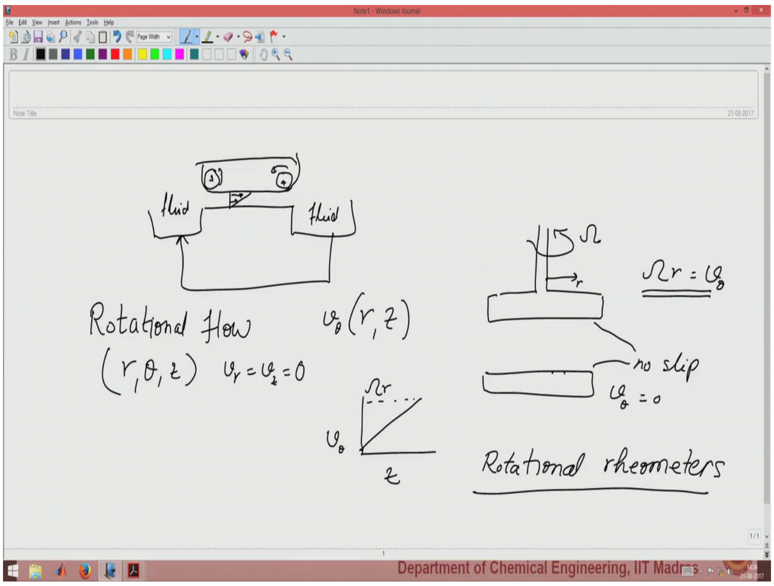

And if this belt keeps on rotating then basically we will achieve a planar shear flow because here because this top belt is moving you it will move the bottom plate is stationary fluid will go here and then again it will have to be pumped back here. So, you can see it is a very elaborate arrangement to just achieves planar shear flow in a lab condition because the fluid has to go somewhere it has to be replenished it has to be brought back.

So, this kind of a setup is far more complicated, but in terms of arguments in the class or to learn concepts this is the most simplest one because to describe this 2D is very easy. So, that is why in class we will consistently use planar shear flow, but in labs its most often we will never do planar shear flow, but like he mentioned earlier we could do rotational or what is torsional flow. So, in this case we take fluid in between the platens and now this is circular cross section. So, therefore, cylindrical coordinate r and z is used to describe this geometry and one of the platens we can rotate.

So, now fluid does not have to really go anywhere, but it is being sheared. Now, the bottom most layer will remain stationary because the bottom plate is fixed. The topmost plate will have velocity which is r omega right depending on the distance from the origin as well as whatever is the omega which is the rotation rate. So, therefore, in this case what we will have is so rotational flow, so rotational flow I will just we will have basically v theta and this will be a function of what quantities given that we have it rotating it will be a function of r. Omega is anyway a parameter because that is the rotation rate, but in terms of coordinates because this is a cylindrical coordinate system we will use. So, therefore, we will describe this using r theta z which is a cylindrical coordinate system and we will say that v r and v z are 0 right because we are only twisting the top plate and therefore, it is only rotating.

So, in that case what else is this. So, v theta is the only nonzero component and what is it a function of it, it will vary not only with r it will vary with z also because as this top plate is rotating. Let us say this is rotating with some rate omega then if r is the distance then omega r is the velocity right this is the velocity at any location, but then bottom plate velocity is 0 because of no slip condition. So, we will always use this no slip. In fact, here as well as here right.

So, the top plate because its rotating their velocity everywhere will be omega r well bottom plate velocity will be 0. So, clearly when you go from z to bottom plate to top plate velocity will increase. And again it will be a similar velocity profile the way we had drawn earlier v theta as a function of z if you plot it will increase right, it is a linear velocity profile the topmost value this will be omega r bottom plate value will be 0. So, therefore, this is also an example of shear flow and this is what we is used more often in laboratory conditions. So, these are all called rotational rheometers.

So, in one of next class we will see what are all different possibilities in terms of rheometers, but rotational rheometers are workhorse as far as rheology is concerned. Pretty much 90 95 percent of measurements are done using rotational rheometers. More often than not when somebody says they have done rheology it is quite likely that they have done rotational rheology. But we will see that there are limitations to what you can do in a rotational rheometer. So, therefore, there are other rheometer also. So, in one of the future classes we will learn about different types of rheometry.

So, now let us go on and look at the next possible in terms of a simple shear in class we will only look at the planar shear flow.

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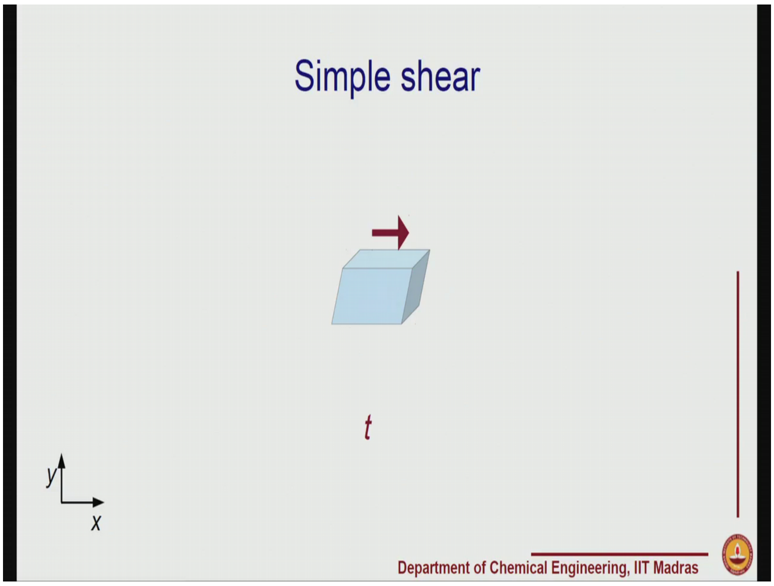

As we discussed earlier at any given time t where if we take a snapshot basically the top surface is moving and the bottom surface is stationary. So, we might visualize the fluid to be of this and if we look at take a snapshot in the future and we will continuously use this tau as a symbol which we have used in the last class also to indicate any arbitrary time, when tau is equal to t its present tau greater than t implies its future. So, therefore, sometime in the future of course, we would expect that this to be moved even further and therefore, this to be deformed more similarly if we go for some time in the back past, then this block of fluid because it is undergoing simple shear flow would have deformed little less. So, it is important for us to visualize the deformation this way.

In the end actually what we will have to do is we will have to see a macromolecular solution or a colloidal system or whatever is a material of interest undergoing this kind of flow. So, whichever rheometric flow we talked about its important for us to visualize what is happening to the fluid, what is happening to the sheets of fluid, what is happening to an individual fluid element and so on.

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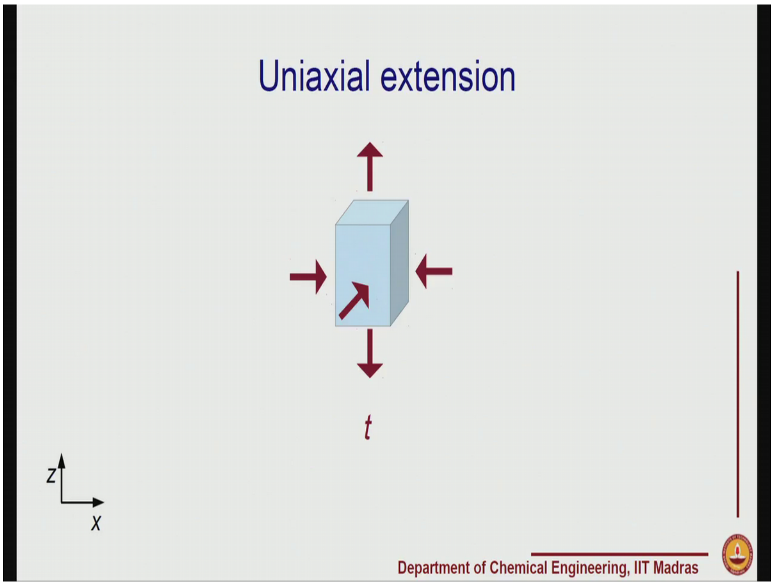

So, now let us do the same thing for uniaxial extension which we discuss briefly in the last class right. So, uniaxial extension we will use rectangular coordinate system. Again this is something which is more difficult to achieve in a lab setting. A rectangular block of liquid is difficult to achieve it is not possible I mean in case of solid we can of course, cast it into a solid bar of square cross section and then we can do the testing. In fact, most of the dog bone testing the cross sectional area is a rectangular cross sectional area. So, in case of solid this is quite usual.

But just for understanding in the classroom setting we might use this kind of a setting where we are talking about cuboidal block of fluid and when its subjected to uniaxial tension we are basically pulling it along z direction and then since we are pulling it along z direction and its incompressible fluid it will flow in x and y direction. So, therefore, x direction as well as y direction it will flow in.

And now if you take a snapshot of this fluid in some time in the future it would have extended in the z direction right. So, it would have become much elongated in the z direction while its dimensions in both x and y would have reduced considerably and so this tau greater than t implies it is sometime in the future.

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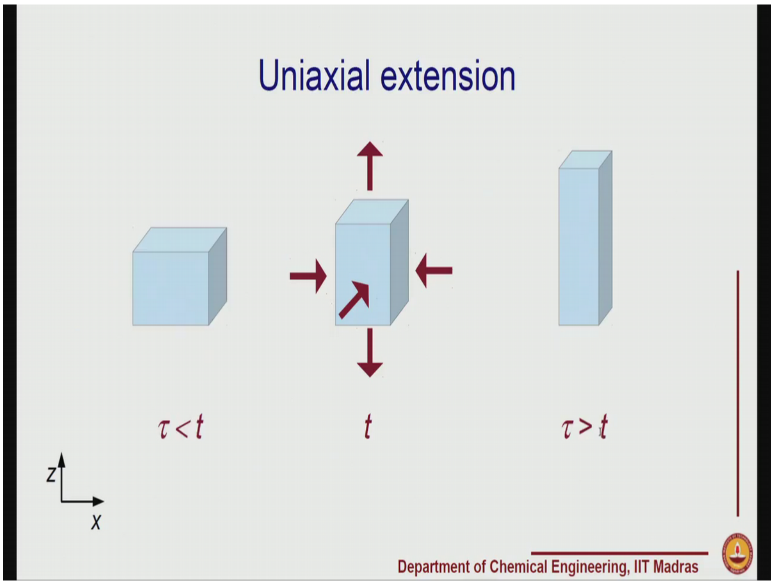

Similarly sometime in the past would imply that the dimensions in the z direction would be smaller and correspondingly the dimensions in the x and y will be larger.

Now, remember when we talked about strain measure we said that we will use present time as the basis. So, therefore, you can see how it is helpful to keep this in mind because the present is what will determine, what kind of deformation this material is undergoing. We compare present with past, we compare present with future and then see whether the material is undergoing any deformation, because in case of fluid like materials the basis will always be the present state because there is no such in thing called stress free state which is uniquely determined. So, therefore, there are multiple options. So, we might as well use the present state which is well defined state.

So, this is as far as again something which cannot be achieved. So, easily, but again using a rotational rheometer. We can achieve another type of extensional flow by axial extension. When we saw this example it is getting pulled in one direction and two directions it is contracting. Now, you could do biaxial extension in this in which case let us say you pull in x and y and z it shrinks, exactly opposite of this. So, it is like in x direction you pull, y direction you pull and z direction it will shrink.

It is important to say that just because this shape is there does not imply that this is how the fluid was to begin with maybe earlier there was some extension which was going on and therefore, fluid was like this in the past. So, somewhere in the past when tau is less than t will be time 0. So, tau is equal to 0 is the start of the experiment. So, tau should be used to denote the time usually. So, we should not use this t to denote running time that is why we are using tau as a symbol, whenever we say t it means present time and again just to emphasize present time we say because in case of fluids we cannot really define what is which tau should I take as basis. So, we take present.

So, therefore, we can use the rotational rheometer the way we talked about and we can in fact, achieve an extensional which is called biaxial extension.

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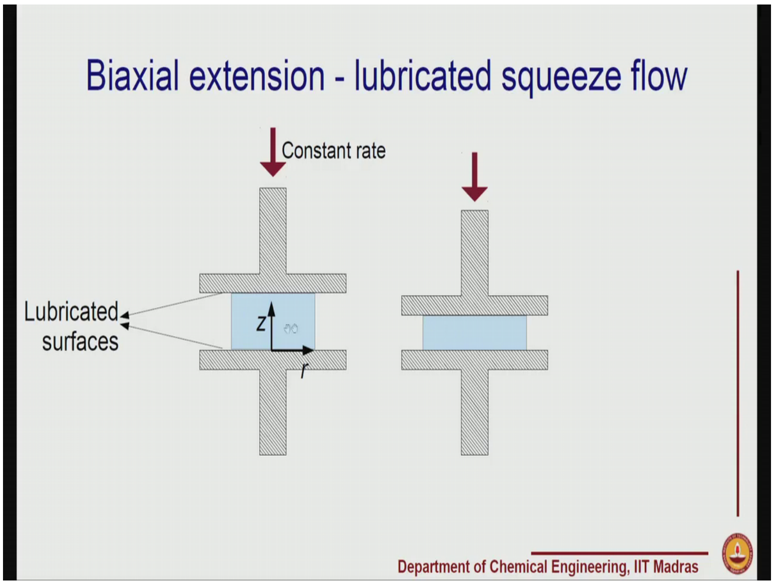

So, in this case we take a fluid between the two plates which is very similar to what we saw earlier and in this case we actually push the one of the plates and we have actually these surfaces lubricated which means fluid will slip. There is no contacting between, so that this velocity is coming down. So, fluid will also come down, but it will slip. So, that is why you can see in the next some instant of time the fluid actually the separation between the plate has reduced, but the diameter of the fluid has increased. But more importantly you can see that all the fluid the z direction there is no difference.

If you remember earlier when we had plotted v theta we had said that there will be a velocity gradient right. We had said that the top plate will move a higher velocity and this is an example of shear flow, but now we do not really have any distinction everywhere the fluid has radial velocity, but that radial velocity is not a function of z because there is lubricated surface. So, that is why this is called lubricated squeeze flow. We are squeezing the material between two plates, but since the plates are lubricated there is no slip is completely violated there is perfect slip at the top and the bottom surface and therefore, fluid actually gets extended.

So, which are the velocities in this case? v r is there and then v z is also there right. So, v r is there and v z is there and of course, v theta is 0 right and since we are saying constant rate here what can we say about we z, will it be same at all points because the bottom plate is not moving right. So, the fluid which is next to the bottom plate is not in fact moving in z direction only the top plate is coming down. So, will v z at the bottom point here will v z be there, v z will be 0 right because the bottom plate is not moving at all only the top plate is coming down. In fact, v z is a function of z, it is not a function of r right because whether its point here or here all of them are moving same way. In fact, what we will find is v z is a function of z and what about v r, of course, fluid is moving in this direction right its moving radially, its function of r only there it is not a function of z because whether it is in the centre or whether it is close to the plate it does not matter. So, v r is not a function of z and in fact so this is a function of r.

So, by just this description can you make out whether this is shear flow or extensional flow. Why would you say its combination or why would you say its extension. Take a look at the, we have already written the description right all of us agree that v r will be a function of r only and v z will be a function of z only. So, is this an example of extensional flow or is this a shear flow.

So, we can do it both ways we can look at the expressions and try to decide or we can go back and look at the graphical picture and then intuitively try to decide. So, we can use both these to try to answer this question.

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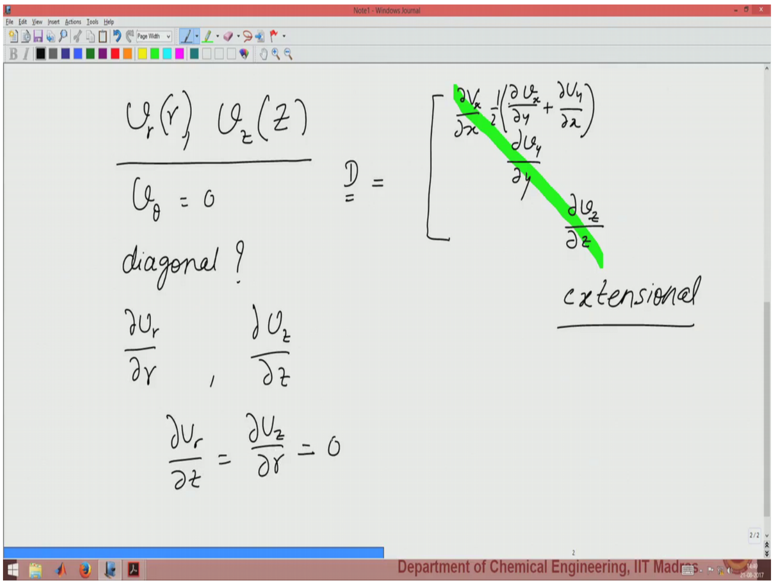

So, if you look at this there is a velocity gradient because I see that is where if you remember maybe what I will do is I will first write this in rectangular coordinate system you try to think of same thing in, so we had terms like this right these are the diagonal components. And what are the off diagonal components v x, those are what signifies shear right. So, now, you can try to justify both ways. Of course, if we want to write for D right then this is half plus del v y by del x right and so on.

So, you can see that the diagonal components. What are these? Extensional right, in this case will we have diagonal components, that is the question we have to try to answer right. So, in this case which will be the diagonal types of terms? Del v r by del r and del v z by del z right. So, clearly in terms of velocity gradient description also we will see that this will be a shear free flow because terms like del v r by del z or del v z by del r will be 0. So, therefore, this is also an example of a shear free flow. If you look at a disk of the material what is happening to the disc is the following.

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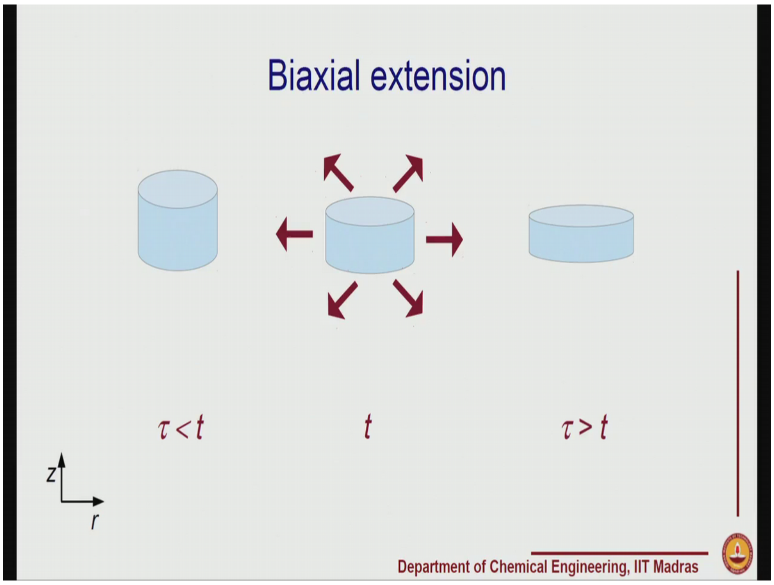

If you take a disc of the material it is sort of flowing in r direction and it is also getting squished in z direction and at any future time it is basically getting extended in r direction. So, that is why it is a planar extension by axial extension. And how would be the flow in terms of at some time in the past? The same blob of fluid would basically have more height in z and little less in radius.

So, therefore, it will have a little more height and less radius. So, this is an example of biaxial extension. So, here what we can do is extensional flow can be achieved using a rotational rheometer which is as I said quite common, otherwise to do extension in general is a far more difficult exercise. So, to achieve extensional flows is more difficult.

And one aspect of that we will see is to achieve a constant rate of extension here, in fact we will see that the position of a material point will have to change exponentially. In this case when we achieve a constant rate of shear the plate has to move at a constant velocity that is easier to manage. In this case to have a constant rate of extension the plate will have to move exponentially you see in one of the things that we would try to do for example, is do an experiment at a constant shear rate or a constant extension rate right. If you remember your solid mechanics always the tests are done at some constant strain rate it will say 1 mm per minute or some such rates are there and that rate is used to tell basically at what rate to apply the loads right.

So, let us say we are interested in doing constant strain rate experiments. So, which means we will have to either do a constant shear rate and constant shear rate will be automatically just if specified if top velocity is fixed. And similarly here constant rate will be specified if the plate moves the position of the plate changes exponentially in time. The rate of the plate motion is constant. So, therefore, when in terms of position control it is far more difficult.

When plate moves with a constant velocity position changes linearly with time, actually how we will achieve this is take the fluid between two plates and then separate like this. So, therefore, a position of the plate is also changing. So, to get an understanding of this little more clearly we need to go back and actually look at the derivations of how to describe the simple shear flow more precisely because so far we have just been saying in terms of conceptually.

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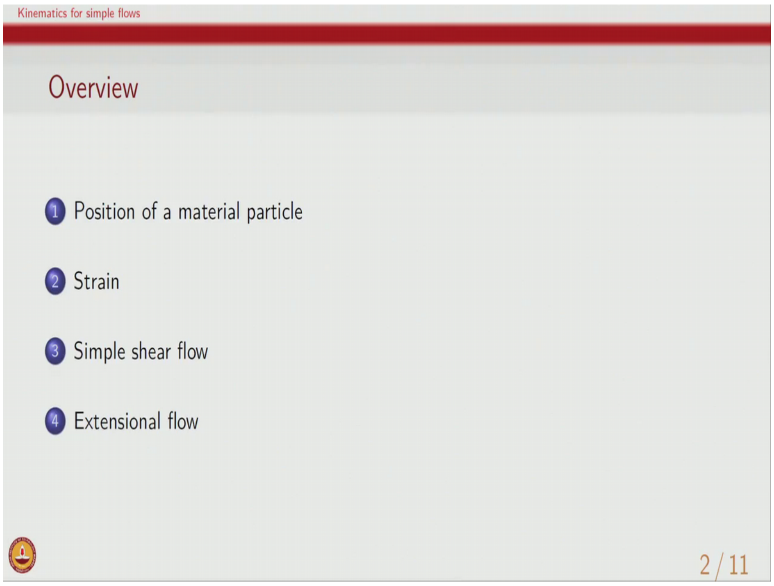

So, now let us look at description of simple shear flow.

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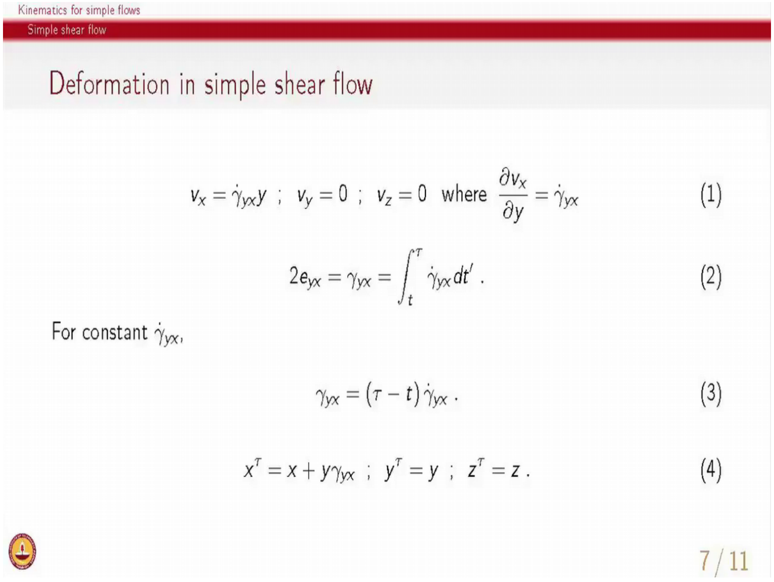

What we have is this description. Again we will in class we will only look at planar shear flow which means we have rectangular coordinate system and velocity is linear with respect to y. So, again let me just draw this.

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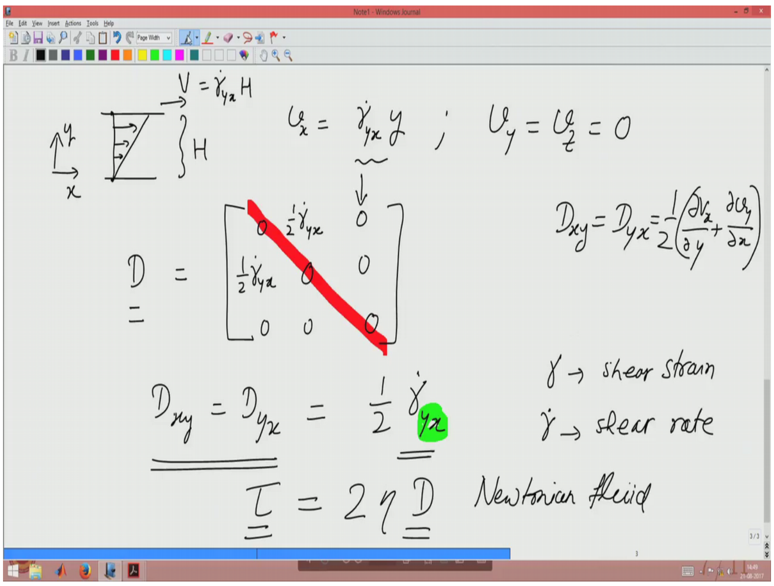

We have the top and the bottom plate and the top plate is moving with certain velocity so that we get the linear velocity profile which is a simple shear flow and we write v x as gamma dot yx times y. So, at y is equal to 0 the velocity is 0 and at y is equal to h let us say we will call this separation h, so at y is equal to h velocity is the velocity of the plate.

So, velocity of the plate is nothing but gamma dot yx into h and of course, we also have v y is equal to v z is equal to 0 right both the velocities are 0.

A note on this usage in general we are only describing a one dimensional situation right now, because this flow is one dimensional, but we know that there are 9 components of strain rate tensor and 9 components of stress. So, to remind ourselves that we will always use this gamma dot yx. So, right now gamma dot yx is the component. If you have to think in terms of overall strain rate then del v x by del x is 0 or nonzero, 0 right; because del v x an is not v x is not a function of x. Del v y by del y is 0, right because v y itself is 0. Del v z by del z, 0. Del v x by, so this is remember D xy equal to D yx is half del v x by del y plus del v y by del x right. So, what is D x y? Half of gamma dot yx. What about these components. They will all be 0.

So, the answer to your question is D xy is equal to D yx because the tensor is D and sometimes to make it easier people use this gamma dot as a term right because gamma is used for shear strain commonly used symbol and therefore, gamma dot is used for shear rate whenever we do one dimensional discussion. So, to just to be consistent with that we are using the symbol gamma dot, but really the overall tensor that we are interested in is actually the strain rate tensor.

So, D xy and D yx are equal to each other and both of them are half times this gamma dot symbol. Just to remind ourselves that it is not a one dimensional its part of the overall 3 dimensional description we use this basically this symbol yx. Just to remind us that it is a simple shear flow, but in 3 dimensional description is necessary. Especially if you recall we had talked about normal stress differences earlier and in shear flow what you would expect for a Newtonian fluid is normal stresses to be 0 because Newtonian fluid the stress is proportional to D.

In fact, the expression for this is the expression for Newtonian fluid. So, if some component of D is 0 automatically that component of stress is 0. So, since normal stresses are all 0 normal strain rates are all 0, normal components of tau will also be 0. So, therefore, in case of rheology even if we are doing one dimensional flow we always should keep in mind the 3 dimensional picture. So, that is why with that reminder only we are using this gamma dot y x.

Now, what we will do next is in the next lecture we will look at this description in little more detail in terms of what is meant by strain rate, what is meant by strain and more importantly how do I describe position of a arbitrary material particle. So, if you are able to describe all these things then we would have described the simple shear flow in complete details.

Similar to simple shear we will also see deformations for extensional flows and in each case of course, we have the intuitive deformation that we have already studied earlier and so in this case that deformation happens to be shear strain gamma yx. This shear strain can and its derivative can be expressed in several different ways and so it is a derivative of e yx where e is the infinitesimal strain tensor since it is a shear strain it is referred to as gamma yx and of course, the derivatives of e yx and gamma yx are referred to as either e dot yx or gamma dot yx dot implying the rate of change with time and we know that this is same as the velocity gradient del v x by del y and we could also express this in terms of the strain rate tensor which is D yx.

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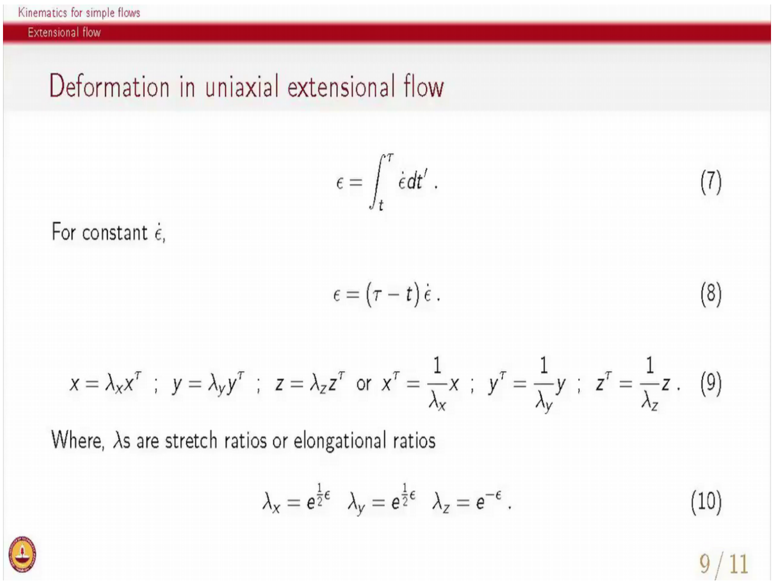

Continuing on we can see the deformation and its description in uniaxial extensional flow. As we have seen before the motion involves exponential in the sense that position changes exponentially where epsilon is the strain which is being imposed on the material based on a constant strain rate epsilon dot. The lambdas here are called the stretch ratios and so the position of material particle at time tau is expressed in terms of position at time t in terms of these lambdas.

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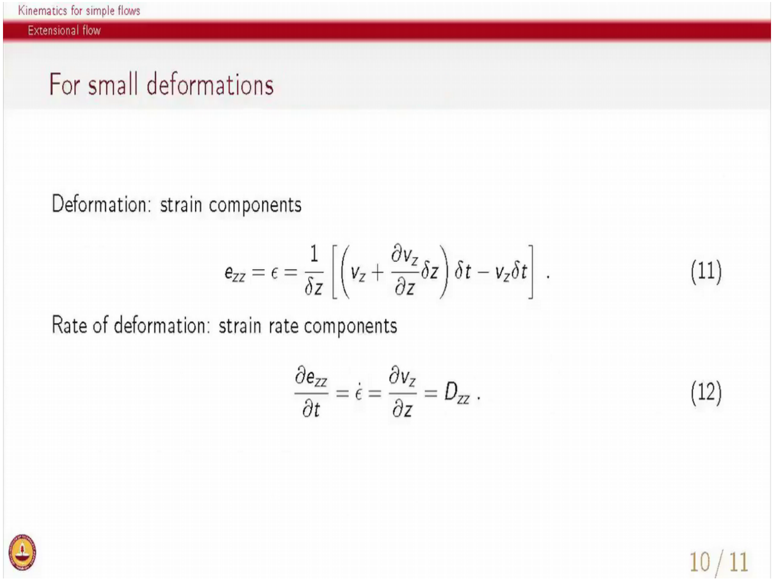

Similar to the shear case the strain in uniaxial extension also can be reduced to what we know intuitively as the small strain where it is basically the deformation which is the length at time t plus delta t minus length at time delta at time t. So, therefore, we could define usually this find this as epsilon. And again we can write this in terms of different components or derivative of e zz with time which is epsilon dot and we know that this is the velocity gradient del v z by del z and can be expressed also as D zz.

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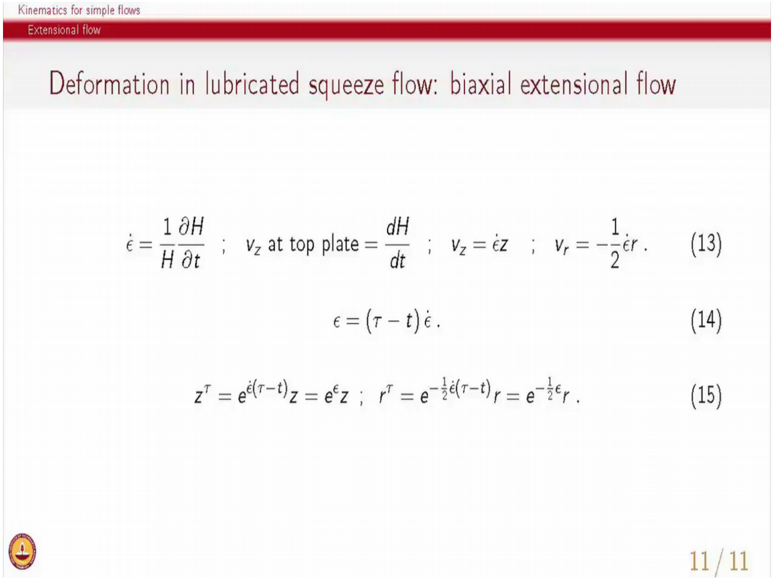

Lubricated squeezed flow which is an example of biaxial extensional flow where between two parallel plates separation is H and this is changed by squeezing the two plates together and so therefore, in this case the strain is given by 1 over H and the derivative rate of change of H. So, in this case also we can describe the overall velocity flow field where z and r direction components are there and again we can describe the position of particle at time tau in terms of position and of material particle at time t. So, this can be, we will do this as part of the exercise where we will try to solve problems related to how this description can be understood.