Rheology of Complex Materials
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Lecture – 38
Review of material functions 2

We are reviewing the material functions for rotational rheometry and in the previous segment of the course, we looked at some overall methods in a rotational rheometries and what are important in terms of types of flow and geometries? And the conditions which are used in terms of either constant stress or constant strain or oscillatory strain and so on.

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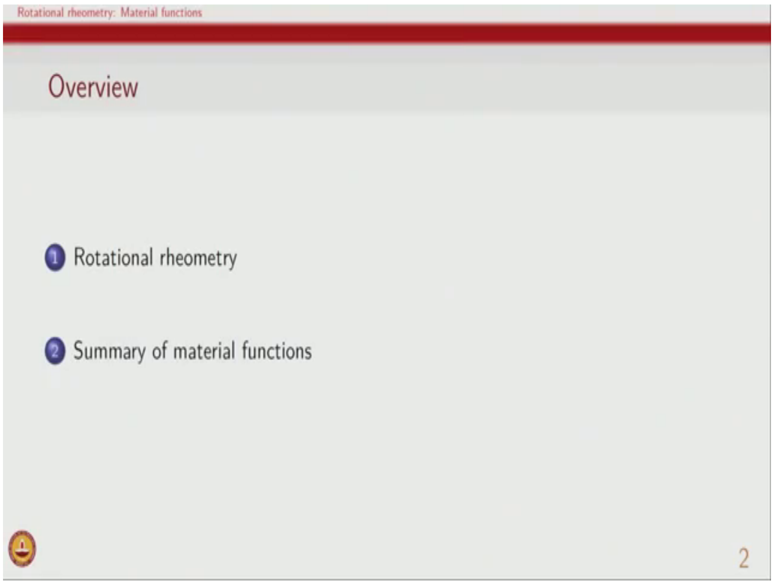

Rotational rheometry: Material functions

Overview

1. Rotational rheometry
2. Summary of material functions

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And so, we were reviewing the summary of all the material functions that we have discussed in class so far and so,.

We looked at stress relaxation and now in this segment we will look at the remaining material functions.

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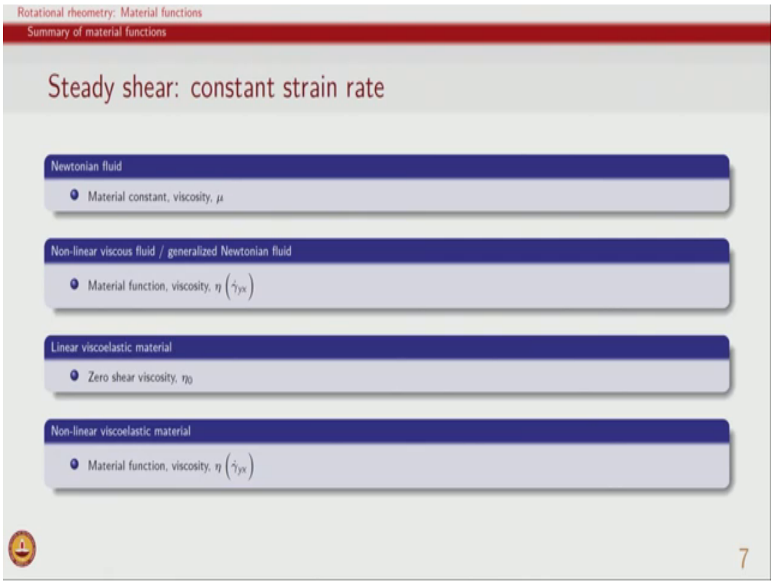

Rotational rheometry: Material functions

Summary of material functions

Steady shear: constant strain rate

Newtonian fluid

- Material constant, viscosity, μ

Non-linear viscous fluid / generalized Newtonian fluid

- Material function, viscosity, $\eta\left(\dot{\gamma}_{yx}\right)$

Linear viscoelastic material

- Zero shear viscosity, η_0

Non-linear viscoelastic material

- Material function, viscosity, $\eta\left(\dot{\gamma}_{yx}\right)$

And so, as we know the material functions are very important for quantitative response.

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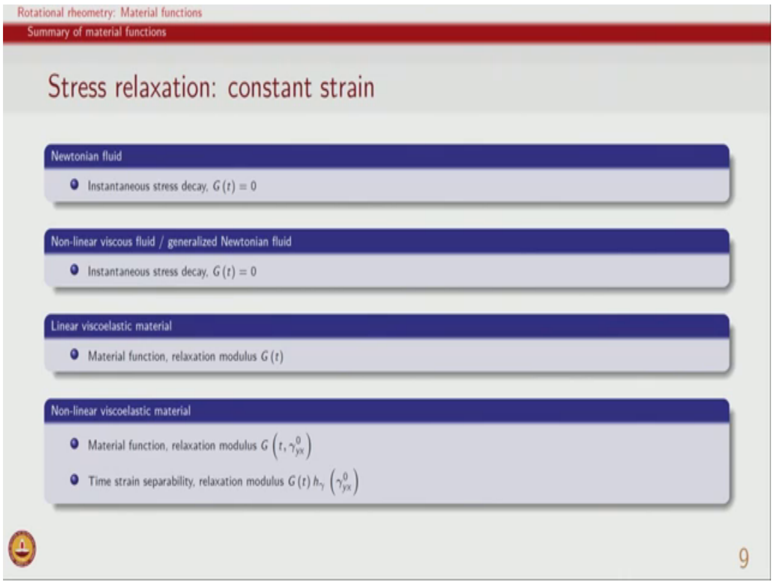

Rotational rheometry: Material functions

Summary of material functions

Stress relaxation: constant strain

Newtonian fluid

- Instantaneous stress decay, $G(t) = 0$

Non-linear viscous fluid / generalized Newtonian fluid

- Instantaneous stress decay, $G(t) = 0$

Linear viscoelastic material

- Material function, relaxation modulus $G(t)$

Non-linear viscoelastic material

- Material function, relaxation modulus $G\left(t, \gamma_{yx}^0\right)$
- Time strain separability, relaxation modulus $G(t)\, h_\gamma\left(\gamma_{yx}^0\right)$

And so, the stress relaxation modulus in both linear and non-linear regime was discussed in the previous segment.

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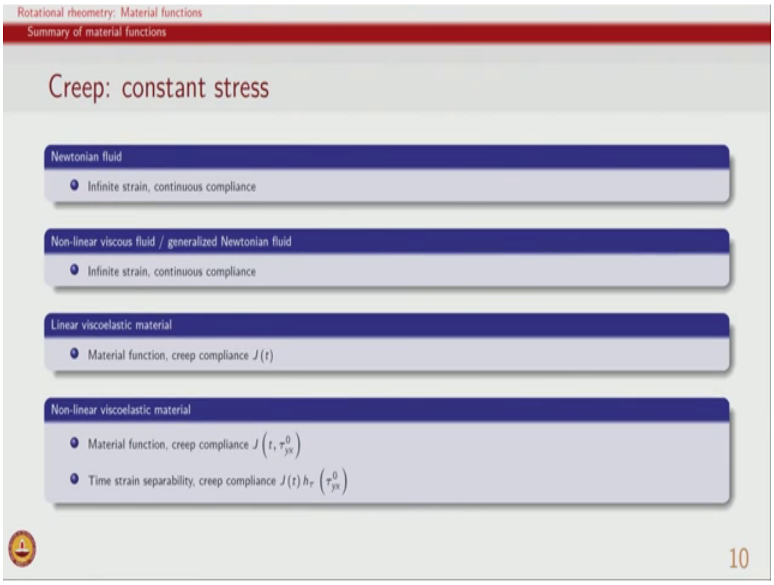

Creep: constant stress

Newtonian fluid

- Infinite strain, continuous compliance

Non-linear viscous fluid / generalized Newtonian fluid

- Infinite strain, continuous compliance

Linear viscoelastic material

- Material function, creep compliance $J(t)$

Non-linear viscoelastic material

- Material function, creep compliance $J\left(t, \tau_{yx}^{0}\right)$
- Time strain separability, creep compliance $J(t)\, h_{\tau}\left(\tau_{yx}^{0}\right)$

So, now we can summarise other material functions and for example, we have already defined creep, which is a constant stress measurement. So, in case of a Newtonian fluid we saw that the material has basically continues to creep. So, therefore, there is a continuous compliance or infinite compliance; the material keeps on deforming when a constant stress is applied.

Similarly, a non-linear viscous fluid also or generalize Newtonian fluid, thus an infinite strain and continuous compliance of the material. So, both Newtonian and non-linear viscous fluids have similar responses and in in the sense that they would offer no elastic contributions in case of creep and therefore, infinite compliance.

Linear viscoelastic material on the other hand, we saw through an example of standard linear solid model. That there is an exponential increase in strain or the creep compliance was increased with time and became constant.

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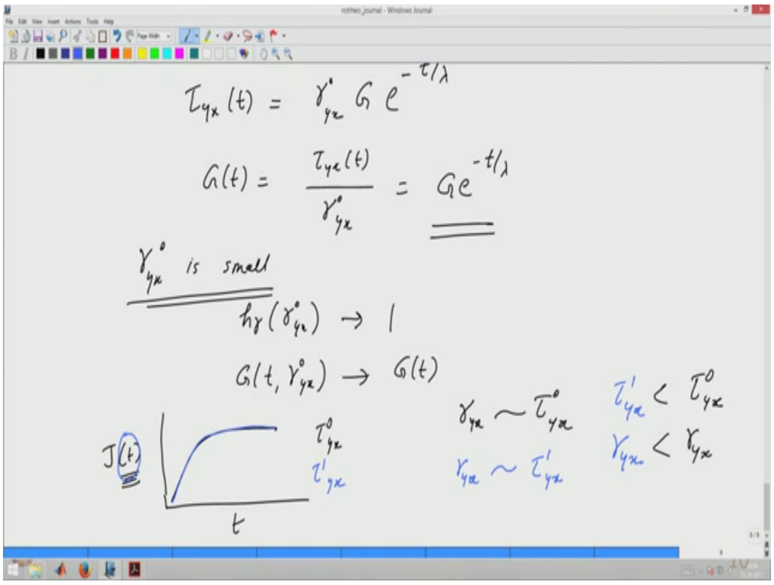

$$\tau_{yx}(t) = \gamma^0_{yx}\, G\, e^{-t/\lambda}$$

$$G(t) = \frac{\tau_{yx}(t)}{\gamma^0_{yx}} = \underline{\underline{G e^{-t/\lambda}}}$$

$$\underline{\underline{\gamma^0_{yx} \text{ is small}}}$$

$$h_\gamma(\gamma^0_{yx}) \to 1$$

$$G(t, \gamma^0_{yx}) \to G(t)$$

$$\gamma_{yx} \sim \tau^0_{yx} \qquad \tau'_{yx} < \tau^0_{yx}$$

$$\gamma_{yx} \sim \tau'_{yx} \qquad \gamma_{yx} < \gamma_{yx}$$

So, for linear viscoelastic material it is quite often possible to observe an increase and then reaching constant value. Of course, the compliance is only a function of time and so, if this experiment was done at one particular stress. And we repeat the same experiment at another stress and measure again the compliance. What we would see is it pretty much follows the same curve because the overall dependence is on time only? This is because the gamma is different in the 2 case..

So, in 1 case, the gamma y x will depend on whatever was the stress which is applied; in the other case, let us call this tau 1. So, the gamma the stress strain which is observed will be based on the whatever is the constant value. If tau 1 y x then tau not y x; then, clearly what we have is less strain is being applied and therefore, we will also have less strain. And so, we will what we will observe is gamma y x will be more then the gamma y x which is measured in case of lower stress.

And so, in case of linear viscoelastic materials, the creep compliance is only a function of time. And in the non-linear regime, when the stress which is being applied is it is higher value then the creep compliance, but also become a function of stress itself.

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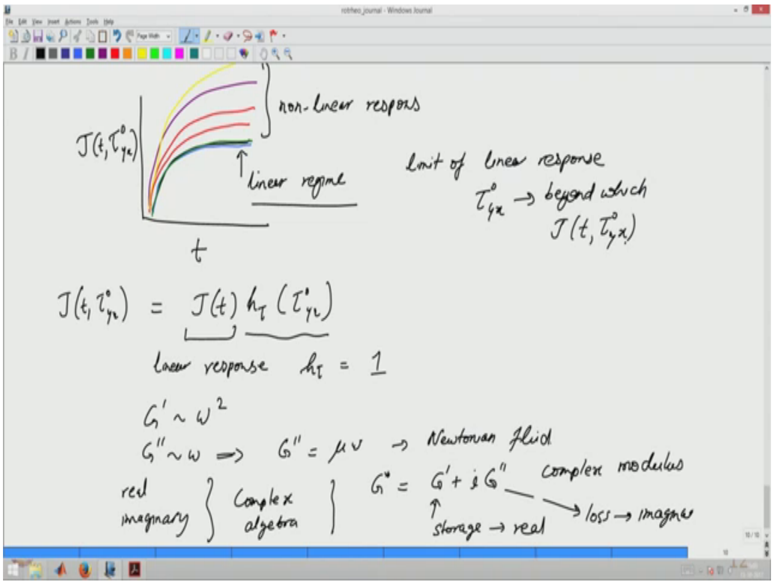

limit of linear response

$\tau^0_{yx} \rightarrow$ beyond which $J(t, \tau^0_{yx})$

$$J(t, \tau^0_{yx}) = \underbrace{J(t)}\,\underbrace{h_\tau(\tau^0_{yx})}$$

linear response $h_\tau = 1$

$G' \sim \omega^2$

$G'' \sim \omega \Rightarrow G'' = \mu \nu \rightarrow$ Newtonian fluid

real
imaginary } Complex algebra }

$G^* = G' + i G''$ complex modulus

G' ↑ storage → real

G'' → loss → imaginar

So, therefore, at different stresses what we would see is the fact that the overall compliance would be a function of the value which is of the stress which is being applied.

So, at low trains we might see that at low stress we might see that multiple measurements are basically small similar values of J f t, but when stress is further and further increased then we may see some response like this. And therefore, the modulus the compliance will keep on changing and so, this is the non-linear regime. In which case the stress dependency is also there while these set of curves are in the linear regime. And of course, this would be expected. So, there is a limit of linear response, in terms of the value of tau not y x, beyond which the compliance is a function of time as well as the stress itself.

So, therefore, it is quite usual to measure even if one is interested in a linear viscoelastic response of the material. Generally, different values of stress would be applied and the creep compliance would be measured and it will be ensured that the creep, creep compliance is not a function of stress. And then that response can be characterized as the linear viscoelastic response. And as we saw in case of the times strain separability for relaxation modulus. In case of creep compliance also, the overall creep compliance can be written as 2 functions which one of them is a function of time alone. While the other one is a function of the stress.

And so, this is the overall non-linear compliance and we know that in the linear regime if a linear response is observed then the damping function corresponding what is called the damping function in case of relaxation modulus, this will become one. And so, then we only have the creep compliance which is a function of time.

So, this is as far as the material functions for creep in the both linear as well as non-linear viscoelastic materials. So similarly, we have the response described as proportional to time for Newtonian and generalized Newtonian fluid, while it is a function of time and function of time and stress for non-linear viscoelastic material.

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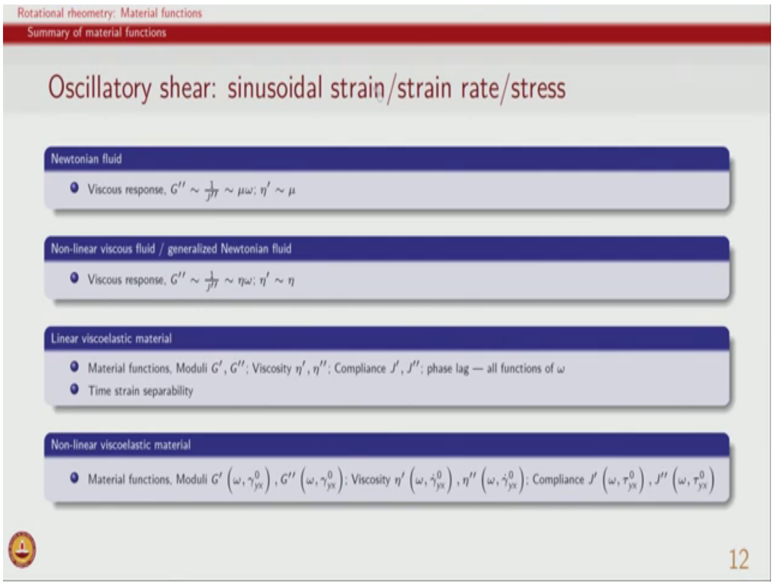

Rotational rheometry: Material functions

Summary of material functions

Oscillatory shear: sinusoidal strain/strain rate/stress

Newtonian fluid

- Viscous response, $G'' \sim \frac{1}{J''} \sim \mu\omega$; $\eta' \sim \mu$

Non-linear viscous fluid / generalized Newtonian fluid

- Viscous response, $G'' \sim \frac{1}{J''} \sim \eta\omega$; $\eta' \sim \eta$

Linear viscoelastic material

- Material functions, Moduli G', G''; Viscosity η', η''; Compliance J', J''; phase lag — all functions of ω
- Time strain separability

Non-linear viscoelastic material

- Material functions, Moduli $G'\left(\omega, \gamma^0_{yx}\right)$, $G''\left(\omega, \gamma^0_{yx}\right)$; Viscosity $\eta'\left(\omega, \dot{\gamma}^0_{yx}\right)$, $\eta''\left(\omega, \dot{\gamma}^0_{yx}\right)$; Compliance $J'\left(\omega, \tau^0_{yx}\right)$, $J''\left(\omega, \tau^0_{yx}\right)$

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In case of oscillatory here we saw that sinusoidal strain or a sinusoidal strain rate or a sinusoidal stress could be applied to the material. And depending on what is the controlled variable? And depending on what is being measured? One can define a several material functions, if a sinusoidal strain is applied and stress is being measured. Then we generally defined the material response in terms of storage and loss moduli. If strain rate is applied and stress is being measured then we generally tend to analyse the response in terms of a viscosity, the dynamic viscosity which has both in phase and out of phase components. And if the sinusoidal stress is being applied then and strain is being measured then we generally tend to look at the response in terms of a complex compliance which will have a both in phase and out of phase component.

So, the viscous response in case of a Newtonian fluid is basically dominant because, there is no elastic contribution and, in this case, we saw that the loss modulus is proportional to the frequency. We saw that storage modulus is proportional to omega square and g prime is proportional to omega for the terminal viscous response. And so, this is nothing but g double prime is equal to mu times omega for a Newtonian fluid. The proportionality constant is in fact, the viscosity.

Similarly, for a non-linear viscous fluid also the overall response is viscous only and therefore, lost modulus is proportional to frequency and the proportionality constant is viscosity. If you were to measure the properties in terms of the dynamic viscosity then the dynamic viscosity is again constant and not really a function of frequency and is directly related to the viscosities of the material.

Ah in case of a linear viscoelastic material since the deformations are very small and many of the non-linear contributions are not significant at very small deformations. What we have is the material functions are defined in terms of oscillatory shear the storage and lost moduli, viscosity which is in terms of dynamic viscous contribution and inelastic contribution. Similarly, compliance with elastic and viscous contributions and for all of these of course, we also have phase lag defined..

And all of these phase lag are basically definitions of imaginary verses real. We use this terminology of imaginary and real and the terminology which is adopted from complex algebra because we can define each of the variables as a combination. So, for example, we can define a complex modulus, in which case the storage modulus is the real part while the loss modulus is the imaginary part.

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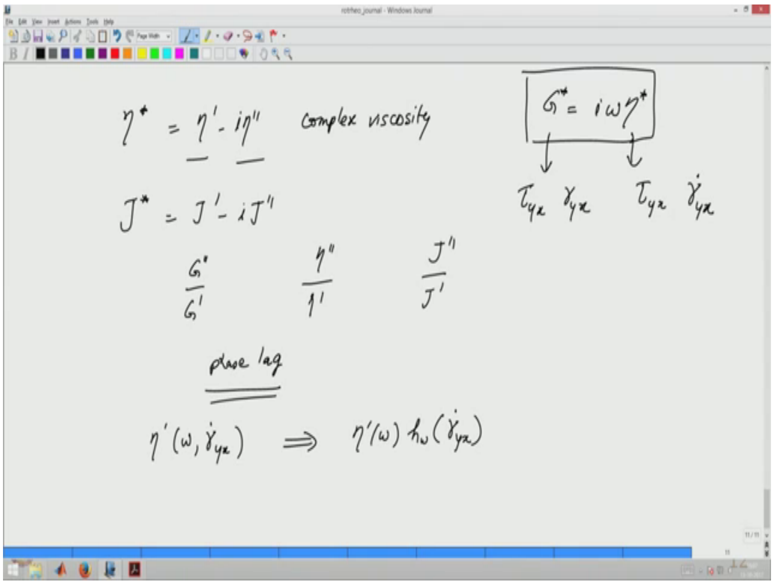

$\eta^* = \underline{\eta'} - i\underline{\eta''}$ complex viscosity

$\boxed{G^* = i\omega\eta^*}$

$\tau_{yx} \; \gamma_{yx} \qquad \tau_{yx} \; \dot{\gamma}_{yx}$

$J^* = J' - iJ''$

$\frac{G''}{G'} \qquad \frac{\eta''}{\eta'} \qquad \frac{J''}{J'}$

phase lag

$\eta'(\omega, \dot{\gamma}_{yx}) \Rightarrow \eta'(\omega)\, h_v(\dot{\gamma}_{yx})$

And so, similarly in case of viscosity also we can define complex viscosity. Again, in this case also, there is a real and imaginary part. Similarly, we define the compliance also as combination of the real and imaginary part of compliance. And so, each of these cases we can define the ratio in this case, the ratio of the loss to the storage modulus or the 2 viscosities or this and these all indicate the phase lag.

So, depending on which material function is being used to characterize, we can use the corresponding phase lag to describe the overall material response. And so, all moduli, viscosity, compliance, phase lag all of them are the functions of frequency. And in case of a time strain separability the material functions for non-linear response are basically functions of frequency and strain amplitude in case of moduli..

Viscosity is a function of frequency and strain rate amplitude and compliance is a function of frequency and stress amplitude. And for time strain separability, what we have is in each case for example, eta prime if we take which is the function of frequency and strain rate could also be written as, a function of frequency alone and another function of the strain rate.

So, therefore, there is again a separability between the time response and the deformation dependent response. And so, you can see that in case of oscillatory shear itself, there is plethora of material functions which we can define and in terms of understanding, it is important to get to know how to transform between one set of variables and another set of variables and have a feeling for how each of the material function looks like for at least simplistic materials such as Maxwell and standard linear solid model. You can use a complex algebra also to transform between these materials functions for example, G star and eta star can be related to each other through complex algebra. And remember that G star is a modulus and that multiplied with the frequency and the I we can get this relationship. And this can be obtained because G is relating the stress and strain, while eta star is relation between tau y x and gamma dot y x.

So, therefore, we can get inter relations between these material functions using complex algebra also and quite a few studies use these terminologies of real imaginary and complex algebra to describe many of these rheological functions.

And in this way, we are using some of the similar things terminologies which are also used in the electrical domain. Where, we have the sinusoidally varying current or voltages and again we can characterize the response of any complicated circuit in terms of in phase and out of phase components. In that case, we have a capacitive and resistive response capacitive indicating energy storing response and resistive indicating energy dissipative response; similarly, in rheology if; we have or in the mechanical domain, we have energy storage response which is elastic and energy dissipative response which is viscous.