Rheology of Complex Materials
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Strain measures and convected rates: conceptual framework
Lecture - 47
Strain and convected rate 4

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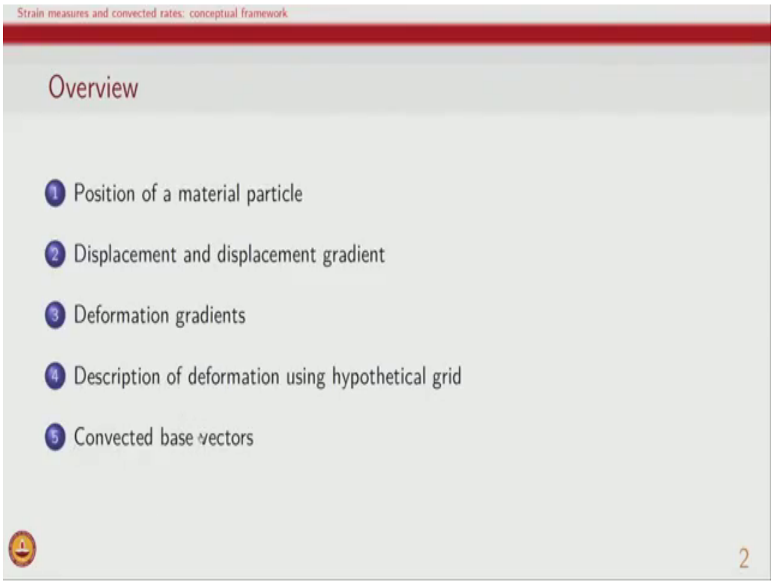

So, in the previous lecture, we had looked at the base vectors and how these base vectors were familiar to us in rectangular or a cylindrical coordinate system. In this part of the lecture, we will look at the base vectors and also we will look at how do they contain information about deformation, and also what can we learn from looking at rate of change of these base vectors.

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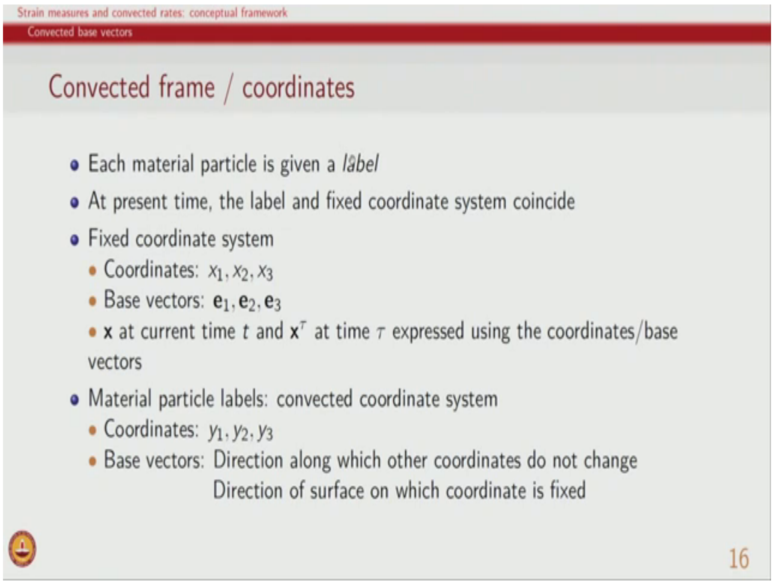

So, just to reiterate what we discussed in the last lecture was the fact that we define convected frames in terms of label of each and every material particle and these labels are fixed to the material particles. So, when material particles moves the label or the coordinate system also moves. And then we could define two alternate set of base vectors, one which are dependent on changing in the direction in which coordinate changes, the other one in which the surface constant surfaces which direction they are oriented. And these are called the tangent vectors and the reciprocal vectors.

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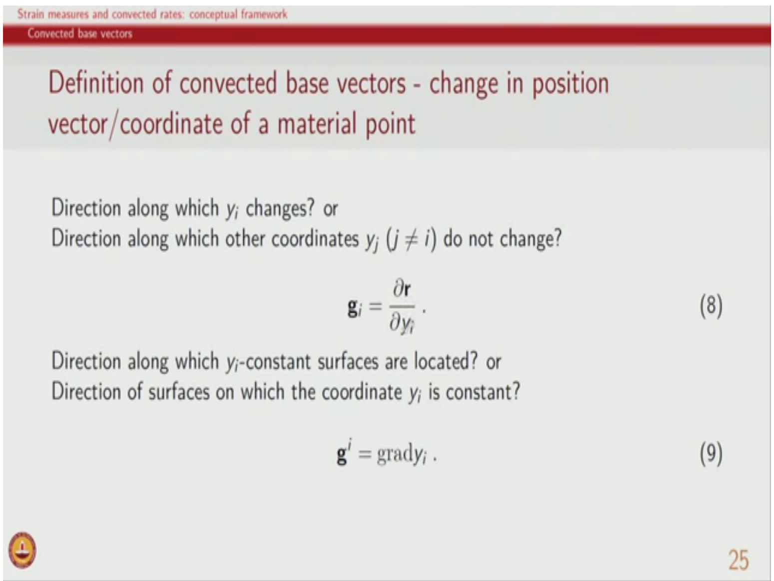

And so we have defined the tangent vectors as change in position with respect to the coordinate, and the reciprocal vectors as the gradient in the coordinate.

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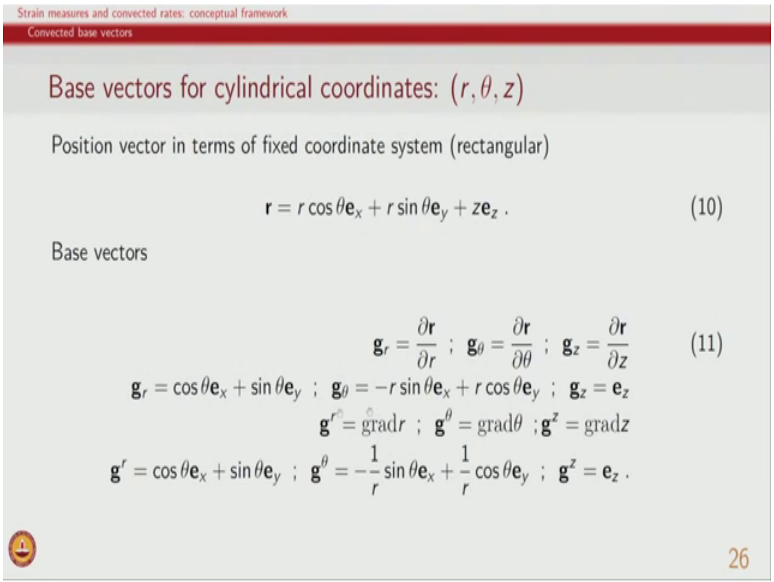

And then we had seen these quantities for system like cylindrical system and the important observation was that the base vectors are not necessarily unit less. And of course, in this case the base vectors also were orthonormal to each other, so g r, g theta and g z, and of course, g r was also normal to g theta and g z. What we will see in what we have also already defined in the last lecture is that in general the g i will only be perpendicular to g 2 and g 3 or g j where j is not equal to i; and these three will not be orthogonal to each other necessarily.

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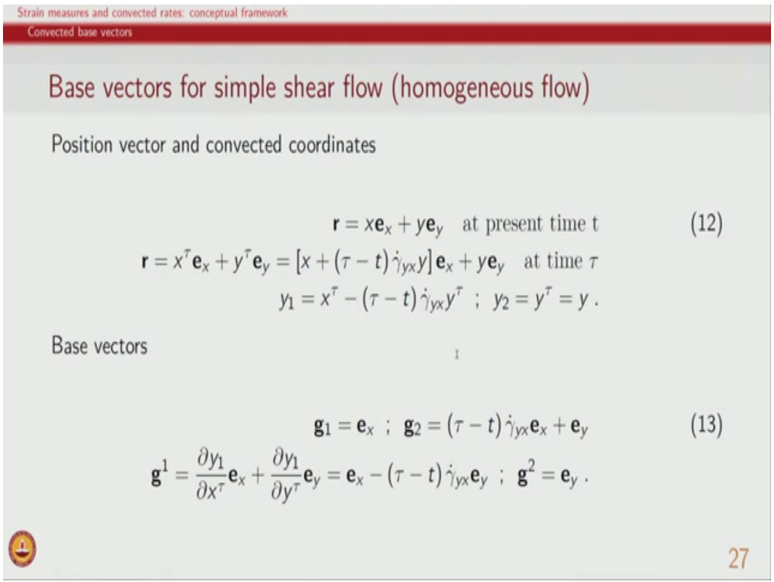

So, now let us look at the same idea of base vectors for a case of simple shear flow. ah

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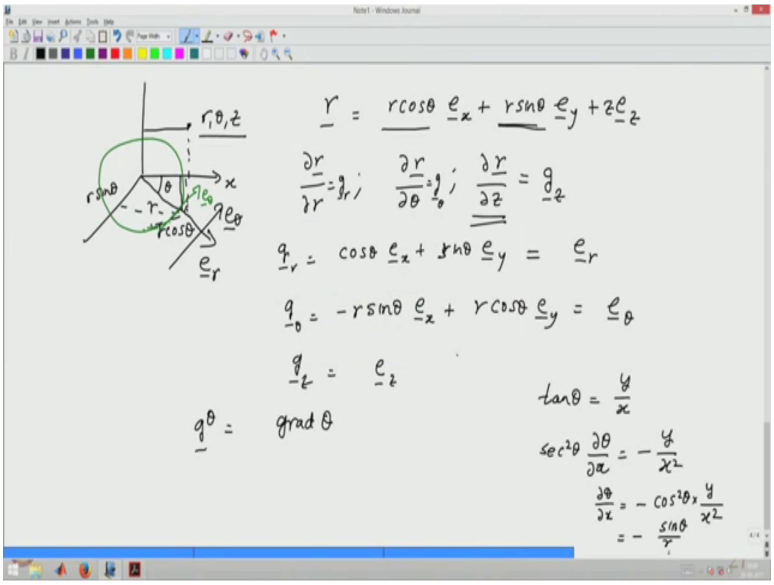

Since, we have already defined and looked at what happens to the grade in case of simple shear flow, it would be easy for us to just keep that picture in mind.

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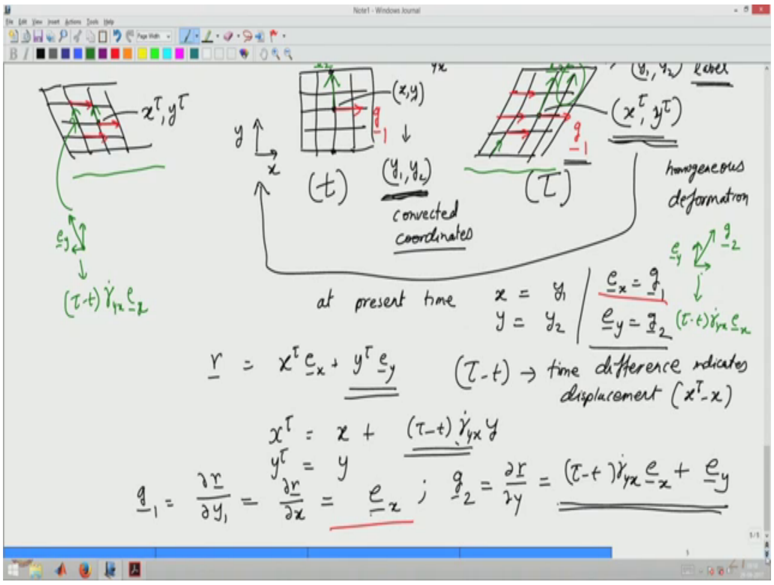

So, what we had said was grid if there is a parallel plate and top plate is moving, and at the present time if we embed a grid like this in the material system, at any time in the future the grid would have deformed. Because in general the top part of the fluid is moving to the right, and therefore, the grid would deform like this. And so what we are interested again now in defining is what are the base vectors and the reciprocal base vectors.

So, again just to reiterate the definitions of base vector one of them was the tangent vectors, where which is the direction along which y 1 and y 2 changes. So, in this case, let us just define this as y and x. So, this is our fixed coordinate system. And in this fixed coordinate system of course, the any point is denoted as x and y. Of course, we will also denote the same point as y 1 and y 2 which are the convected coordinates. So, these are the convected coordinates.

And of course, the same point would have moved to another location because of the flow which is being imposed. And we of course, have denoted this new point as x tau y tau and of course, this description is in the same coordinate that we have described earlier, but the same point of course, remains y 1 and y 2 because this is the label of the material particle. So, no matter what happens to the material particles it is remain, its label or its convected coordinate remain fixed.

And, so, now, the question is which is the direction along which y 1 is changing and so clearly the direction along which y 1 is changing is this; and similarly that direction along which y 2 is changing is this. So, in the at the present time, the direction along which y 2 changes is this; and the direction along which y 1 changes is this. So, therefore what we have drawn are g 1 and g 2, and we can evaluate these also. Similarly, where what we what we have drawn is g 1 in both cases, and the other vectors that we have drawn are g 2. So, we can see clearly that at present time, at present time, x is equal to y 1 and y is equal to y 2; and also e x is equal to g 1, and e y is equal to g 2. But at any other time g 2 and g 1 will keep on varying because the material is undergoing deformation.

So, it is our interest to now find out expressions for g 1 and g 2 based on the definition. And since these each and every material particle is moving and the overall shear deformation can be written in terms of gamma dot y x as we had written earlier, so we can define the overall position of a material particle at any time instant tau as x tau and y tau. So, therefore, at any arbitrary time tau, the position vector is nothing but x tau e x plus y tau e y. And of course, we know how to express this x tau and y tau in terms of x and y and the time interval tau minus t given that this is at time tau and this is at time t. The tau minus t the time difference tells us to time difference is an indicator of how much is the displacement, displacement for each and every material particle. And so this we have already done that x tau therefore, is going to be x plus tau minus t gamma dot y x into y.

So, a point which is located at y is equal to 0 basically here is not going to move at all and so therefore, for it x tau will remain x. A point which is much further away in y will move more and more and similarly if we change the velocity of the top plate; that means, we are changing the strain rate and therefore, the x tau will be much further away from x. So, x tau minus x is the displacement. And, so, similarly in this case because it is a flow which is simple shear y tau remains y. And the other thing you can notice is this is an example of homogenous slope because g 1 and g 2 remain the same at all different points. So, even though there are different material points in the system g 1 and g 2 remains same.

And therefore, since g 1 and g 2 as we will see we will contain information about deformation, this is an example of a homogenous deformation. So, based on these definition, now we can calculate g 1, and so g 1 is nothing but del r by del y 1 which implies that it is del r by del x. And, so, therefore, we can calculate it to be e x, and so therefore, it is not surprising that we have found g 1 to be in the direction of e x. Similarly, g 2 can also be found, and g 2 will is again del r by del y and therefore, we can see that the g 2 that dependence is little more complicated based on the deformation which is taking place in the material.

And so the g 2 has dependence on x itself. So, we will get this term tau minus t gamma dot y x e x and then given that y itself is also dependent we will get e y. So, therefore, g 2 is a combination of e x and e y at different instance of time. At present when tau minus t is 0, we see that g 2 is equal to e y and that is what we have already written that for present time g 2 is equal to e y.

Now, we could repeat this exercise for sometime in the past also. So, if we have some time in the past, then the same grid would have would look like this. And again the same point that we were considering which is x y point would be at some other x tau y tau. And again looking at the directions g 1 will remain again all the same, so all of these points g 1 is same as e x which is what we have found here. And of course, g y g g 2 will depend on the time tau, and in this case because tau minus t is going to be negative we are going to have we can resolve these two.

For example, at any point here we can say that this is the e y, and then this vector is tau minus t gamma dot y x e x. And, so therefore, the resultant vector is what is drawn here. We can do the same exercise in case of any time in the future also. This quantity is again e y. And some quantity here this quantity is tau minus t gamma dot y x e x, and therefore, the resultant vector is g 2 which is what we have drawn here.

So, therefore, based on the fact that it is a homogenous flow we find that g 1 and g 2 are not dependent on position itself. So, all throughout the material g 1 and g 2 are identical. And in one of the cases the tangent vector is coincides with the fixed base vector while the reciprocal set actually coincides with the y vector, but g 1 and g 2 are of course, containing information about the deformation. So, if gamma dot y x is zero which means there is no deformation on the material then g 1 will be e x, g 2 will be e y; or g 1 is e x and g 2 will be e y.

So, clearly as was indicated to us when we looked at shapes of the grid points we saw that the grid itself keeps deforming when the material is deforming. Now, we have seen that even if we calculate the base vectors according to their definitions the base vector keep on changing based on the deformation. And depending on the time evolved or depending on the rate of deformation the g 1, g 2, and these base vectors will keep on deforming. So, therefore, they contain information about the deformation itself. Now, let us just go and look at the.

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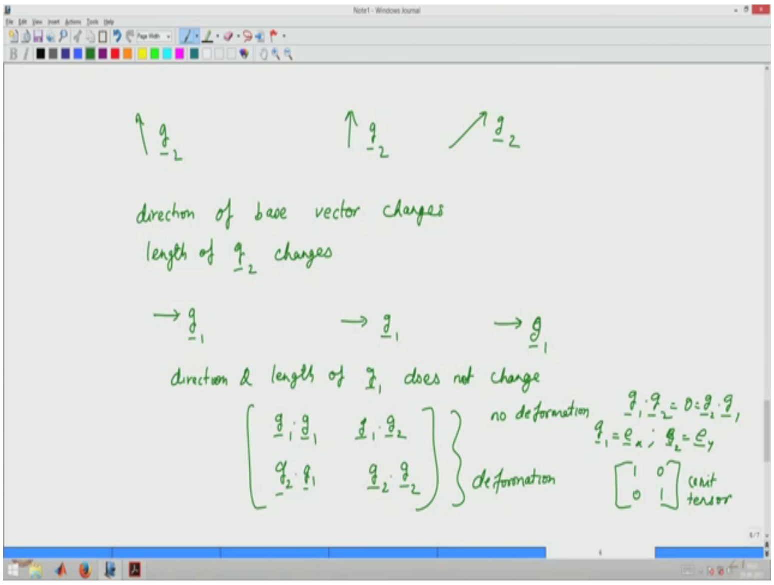

So, two things we are saying for example, the direction of g 2 itself that we had drawn at different instance of time, so the direction of base vector in keeps on changing. The other thing to notice is also the length of the base vectors keeps on changing. Because, in this case, the grid points are located on the same, and therefore, this distance is going to be very different, so this distance or this distance is going to be different compared to the distance between this point and this point.

So, this is the distance that we had talked about earlier and now there is a distance change. So, therefore, length of this vector g 2 is also changing. At of course, the present time, the length is unity and that is why we have g 2 is equal to e y at the present time, but at any arbitrary time length of g 2 changes. Of course we can see the g 1 itself does not change. So, g 1 remains in this direction g 1 remains in this direction. And, so clearly direction and length of g 1 does not change.

So, what we can see here is how we can keep track of deformation by keeping track of length of this in different base vectors. Now, we have two base vectors and they keep on changing. So, we have actually four possibilities of how their lengths can be captured. So, we could look at the length of the vector themselves which is what we have talked about. And then we can also look at the length of their contraction. So, therefore, these four numbers actually indicate what is the deformation that is happening in the material. In case we have gamma dot to be 0, if gamma dot is 0, then there is no deformation in the material g 1, g 2 both do not change.

And therefore, what we will get is for no deformation, g 1 will be perpendicular to g 2, and therefore this will be 0 and similarly g 2 dot g 1. But of course, g 1 will be equal to e x and g 2 will be equal to e y, and therefore this set of numbers that we had talked about will reduce to 1 0 0 1. So, clearly what I have shown here is matrix which contains information about deformation. If there is only there is no deformation, then it reduces to unit tensor. So, therefore, by keeping track of these quantities which will define as g i js.

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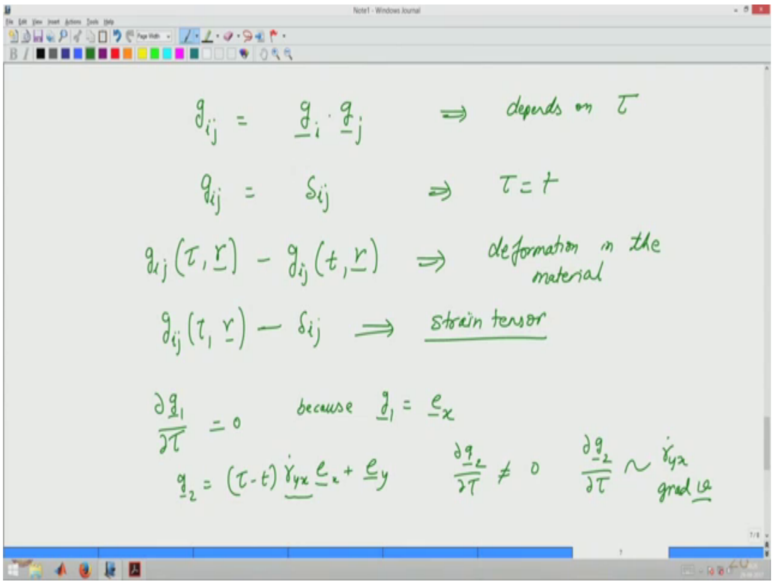

So, we will define gi j as g i dot g j. And of course, this depends on time. At present time, g ij is equal to delta ij, if tau is equal to t. So, what we could do is we can define g ij which is at any particular time tau, and of course, it also depends on what is a position minus g ij which is at present time t for the same position, this gives us an idea about deformation in the material. This of course, is nothing but delta ij, therefore we can define a strain tensor. So, this can be a definition of strain tensor. So, this we will define formally later on, but now its good idea for us to get comfortable with the fact that when deformation is there, the length as well as direction of these base vectors changes, and therefore by keeping the matric g i dot g j, we can keep track of the deformation.

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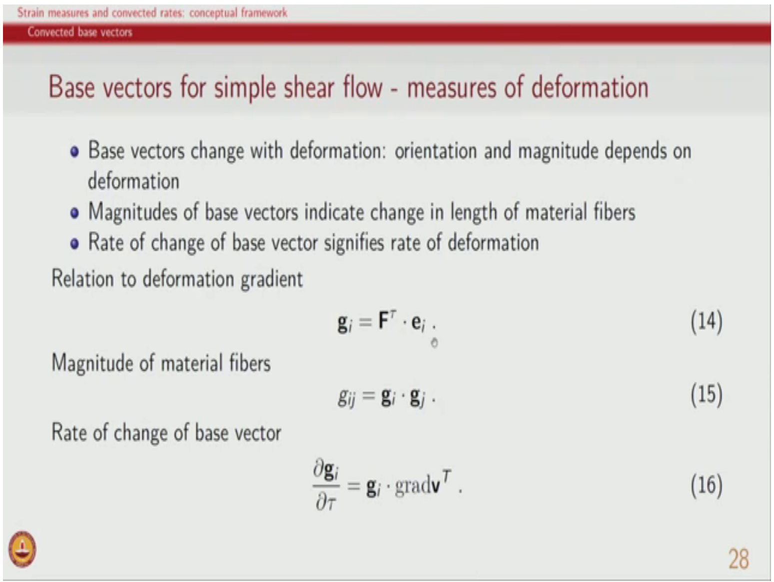

Now, the other thing which we can summarize is base vectors are already changing with deformation, and therefore their orientation as well as magnitude depends on deformation. And the magnitude of base vectors indicate change in length of the material fibers. The last question we can look at is what happens to the rate of change of a base vector.

So, for example, if we go back and look at our example, we had seen that g 1 is given by e x itself; and clearly with time that is not changing. So, therefore, in the simple shear example the rate of change of del g 1 by del tau is 0, because g 1 was actually e x all the time; however, if we look at the value of g 2 that keeps on changing with time. So, since g 2 actually is dependent on time, we can now look at what happens to g 2 as a function of time.

So, let me just write this again of course, this indicates that the material particles are getting displaced in the x-direction, and the overall deformation therefore is simple shear deformation. And now this g 2 keeps on changing as a function of time. So, therefore, this del g 2 by del tau is not 0, and you can see here that if we take a derivative of del 2 what we have is basically this gamma dot y x. So, we will see that del g 2 by del tau is related to gamma dot y x which is nothing but related to gradient of velocity. So, in terms of summary of these base vectors and how they are measures of deformation, and rates of deformation the relation to deformation gradient is just based on the finding out component based on the definition.

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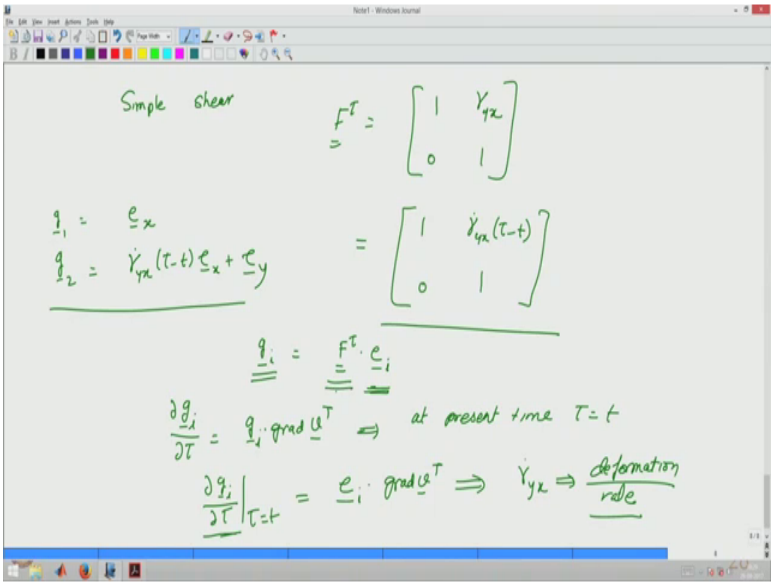

And we had defined this for a simple shear case. The f deformation gradient was defined as 1, gamma yx, 0, 1, where this gamma yx is nothing but gamma dot yx into tau minus t. So, let me just right now for our benefit g 1 and g 2, so g 1 is nothing but e x, and g 2 is nothing but gamma dot yx tau minus t e x plus e y. So, we can see how these quantities g 1 and g 2 and the deformation gradient F contain the same information and in fact we can in tensor notation, we can write this as the base vector is related to the deformation gradient. And we can find the appropriate base vector by looking at whichever fixed coordinate unit vector.

And we also saw that the magnitude of the material fiber keeps on changing as the function of time. And in general for different types of deformation, g ij will also be a function of position. Since, we looked at simple shear, which is an example of homogenous deformation, we saw that g ij was also independent of position. And then we also looked at the rate of change of the base vector; and rate of change of base vector is basically leading to is related to gradient of velocity. We can also see that at the present time since the rate of change of is related to the at present time basically tau is equal to t. So, therefore, we can evaluate this del gi by del tau at tau is equal to t; and we know that at tau is equal to t, this is e i and that is gradient of velocity. So, therefore, gamma dot yx will be the indication of deformation rate. And so the derivative of base vectors gives us deformation rate, and the base vector itself gives us deformation.

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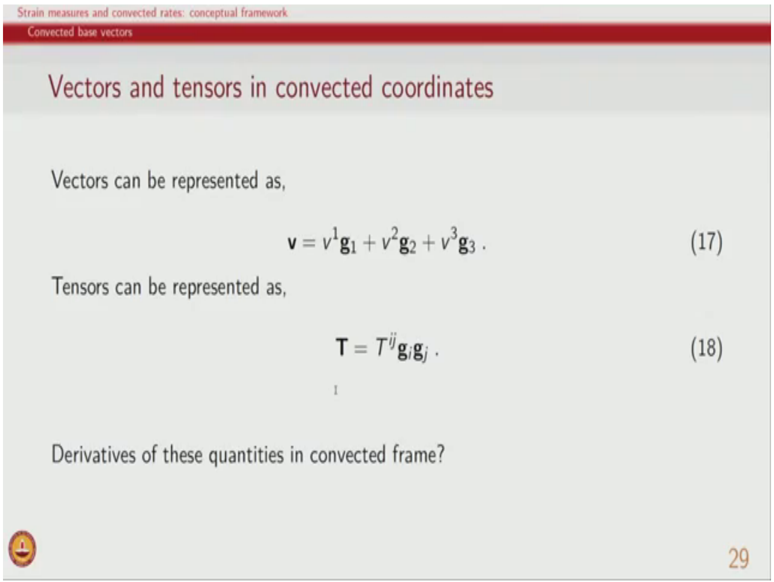

So, using these base vectors which we have defined and we also seen that how they capture the overall the deformation as well as deformation rate in the material these base vectors can be used to denote the vectorial and tensorial quantities which are involved in rheology.

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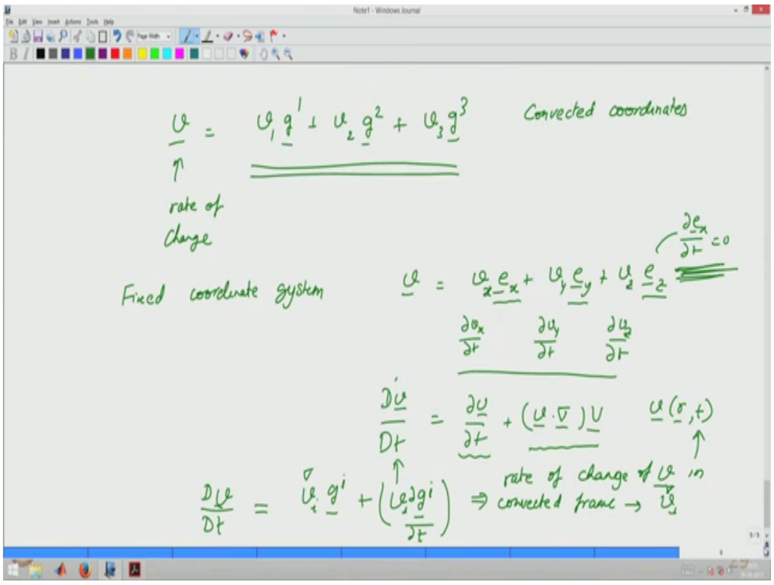

So, for example, velocity can be written in terms of reciprocal base vectors as v 1 g 1 plus v 2 g 2 plus v 3 g 3. So, what shown here is a of course with respect to the tangent base vectors and so just to keep the notation consistent, we were the components are used as superscript if the base vectors are subscript. And, so therefore, in this case since the base vectors have superscript, we use the components as the subscript. And, so this is an indication of the vector in the base convected coordinates. And the most important feature of why we are interested in all of these in rheology is we would be interested in evaluating rates of various quantities.

So, therefore, if we are interested in the rate of change of velocity, then what this is saying is that we can evaluate it in the convected coordinate system. So, if we do this in a fixed coordinate system, let us say using v x e x plus v y e y plus v z e z. And, so, if we are interested in let us say rate of change of this, then we evaluate it in terms of del v x by del t and del v y by del t and del v z by del t. And of course, these quantities are fixed, the unit vectors such as this is 0.

And, so, therefore, when we evaluate quantities in fixed frame, we may not get physically meaningful behavior that we are looking for. For example, if we are looking at acceleration, then this partial derivative of velocity does not signify the acceleration. And therefore, we use the material derivative to signify the acceleration of the material particle which is the total derivative d v by dt which we note is del v by del t plus v dot del v. And this is the consequence of the fact that velocity is a function of position as well as time.

So, when we change time, and we are interested in change in velocity with respect to time, velocity not only changes with respect to time, but since the position itself changes the velocity may change with respect to position. And, so, therefore, this is the inertial term, and the Eulerian term which together make up the material derivative of velocity. So, similarly now we have the vector expressed in terms of the base vector.

So, in this case when we let us say want to evaluate the derivative of velocity, what we will see is that this derivative of velocity will be related to the derivative of the derivative of the components themselves and this we will indicate using symbol, so that we distinguish it from ah. So, let us say we use the symbol into g i plus into derivative of this. So, given that the base vectors themselves also change the function of time which was not the case when we had the fixed coordinates system.

We will see that the overall rate of change of quantity the way we evaluated will have to be accounted for along with the rate of change of the base vectors itself. So, if we are interested in rate of change of v in convected frame which is this quantity vi with the triangle.

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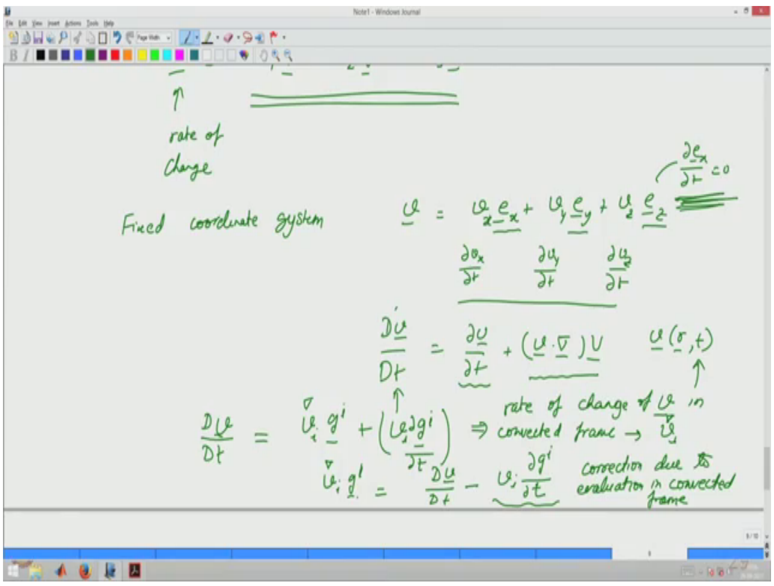

So, to evaluate this what we will have to do is to basically say that vi g i is equal to the total derivative that we calculate minus v i del g i by del t. So, therefore, this quantity is the correction because we are trying to evaluate, so this quantity is the correction because of evaluation in convected frame. So, similarly, we will also need to define tensor quantities, we have stress tensor or we have strain rate tensor or any higher order derivatives of these. And again we will define them as in the coordinate system as T i j g i g j. Snd clearly now when we differentiate this and we find the convected rate, we will have a term this similar to this velocity derivative that we did.

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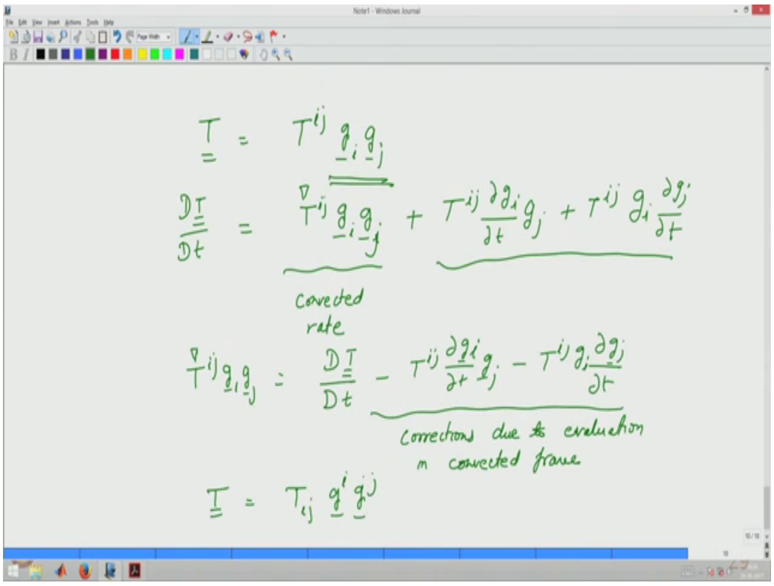

So, if we have a tensor being indicated as T ij g i g j, we will have now terms which are associated we will get three terms because this dt by dt is the total derivative that one is interested in. But we only are looking at that rate which is evaluated in the convected frame which is going to be this. So, this is the convected rate. and then we have because of the chain rule, we will have terms like. So, we have these two additional terms.

And therefore, if I want to get the convected rates again I will have to account for the fact that the total derivative which is calculated is not eh convected rate, but the additional terms which we have calculated will have to be subtracted before we can get the convected rate. So, we will therefore, when we look at convected rate, we will see this additional corrections correction terms which are due to evaluation in convected frame. And of course, we have defined two alternate set of base vectors.

And right now what I have described only is using the base vectors which are tangent based vectors. So, an alternate set of convected rate can also be defined using the base were the definitions in the reciprocal bases. So, therefore, we have convected rates defined which could be in tangent base vectors space or on reciprocal base vector space. So, so derivatives of these quantities in convected form will be very useful for rheology.

So, in the next course next section we will define these derivatives. And once we have defined them and given all the background related to convected coordinates, and how they vary and how they are embedded, and therefore, contain information about deformation, we are now ready to define the strain measures formally as well as convected rate formally.