Rheology of Complex Materials
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Lecture – 58
Strain, convected rates and non-linear models

In the course on rheology we have seen several aspects of rheological measurements and the analysis of rheological response using material functions and in the process we also looked at several models and from time to time we have reminded ourselves that the two most predominant modes of rheological observations are related to steady state characterization which includes steady shear or steady extension.

And the other most common one is related to oscillatory shear stress relaxation creep, but all of them at small deformations or in the linear viscoelastic range and we know that for engineering applications deformations can be arbitrarily large and therefore, the how does the material microstructure respond to large deformations is something that has to be understood for us to say that we understand the rheological response of these materials.

So, therefore in these segments of lectures we will review and try to understand how do we discuss the non linear response non linear rheological response of materials and what are the tools that are needed for us to describe the non linear response as has been discussed several times before. So, what we will do is start with definitions of strain we have seen earlier that we define strain through infinitesimal strain tensor which is only valid for small deformations.

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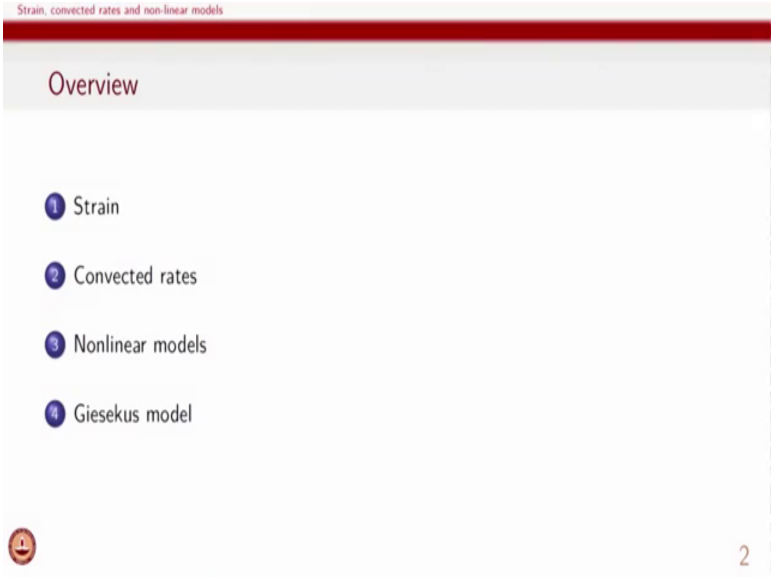

But for large deformations we had gone through and looked at qualitatively what strain tensor should be for large deformations we will define it in this segment of lecture and then we will also look at the convected rates we have emphasized this time in again that for the rates to be frame invariant and for rates to be proper instead of using partial or substantial derivatives we need frame invariant rates and especially for quantities such as stress and strain these frame invariant rates are very useful in determining physically meaningful rates.

So, as an example of that we will look at the upper convected and lower convected derivatives which are quite commonly used in describing the non linear response of materials. Then we will quickly review the non linear models which are commonly used. This discussion will be preliminary and for advanced learners of course more discussion has to be followed in terms of many of these non linear models their origin and how and under what situations are, they very useful. And finally as an example of a non-linear model we will review the governing equations as well as response of the Giesekus model. So, now going on we had defined strain earlier qualitatively.

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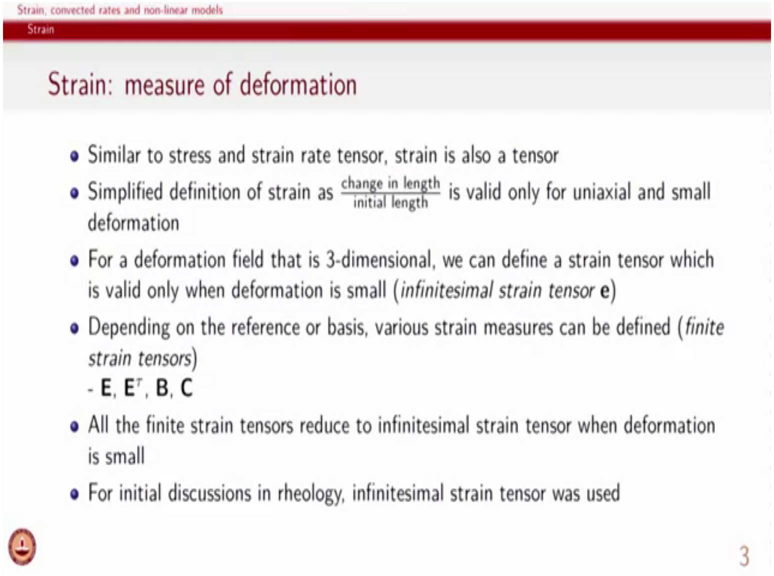

So, we will quickly summarize saying that similar to stress and strain rate tensor strain is also a tensor and of course, we are familiar from school times that change in length versus initial length is what strain is and so from a point of view of our course the kind of things that we have to remember is we have been saying that at time t is equal to tau and at time t there are material different configurations and at this time we had indicated position of a material particle using x tau while at present time here using x. And what we are interested in knowing if I take any two material points and sort of a material fiber which joins them what we are interested in knowing is what happens to this material fiber after sometime.

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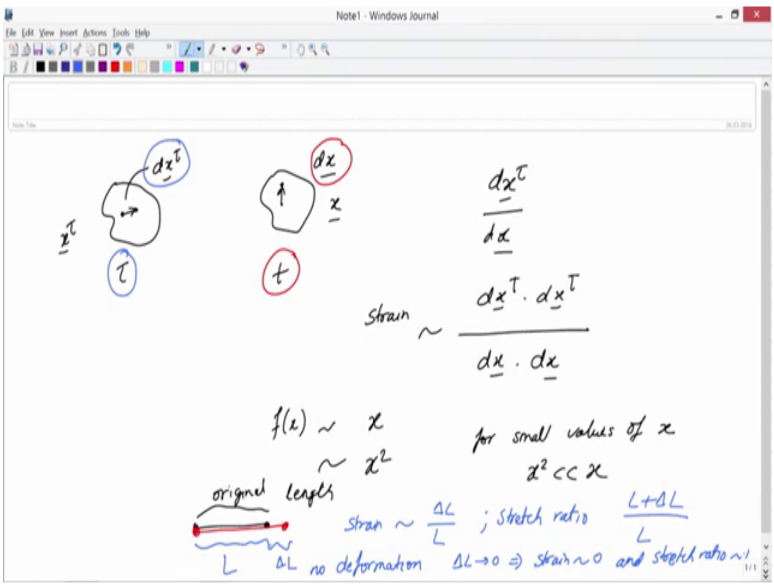

And so therefore what we are interested in knowing is what happens to d x tau and d x and importantly what happens to its length. So, because to again conform to what we intuitively say is strain material and of course given that the deformation field itself is 3-dimensional we have basically a strain tensor which has 9 components and of course, we work with infinitesimal strain tensor in discussion of linear visco elasticity.

And so depending on the reference or basis in this segment we will define these other strain tensors and these strain tensors will have the quality that they will reduce to infinitesimal strain tensor whenever deformation is small and which is again an expected thing, that these strain measures are valid for arbitrarily large deformations. But whenever deformation is small or for linear viscoelastic response they will reduce to the infinitesimal strain tensor.

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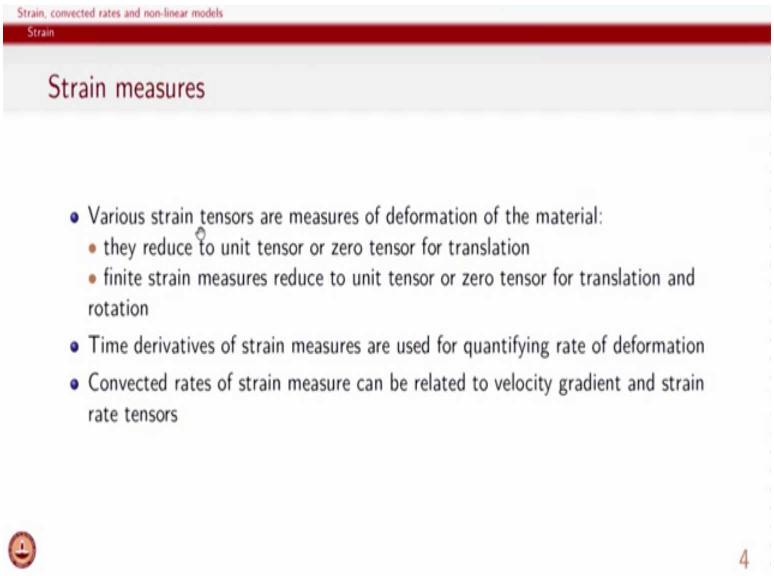

We will also see that if there is no deformation for example, if there is only rigid body translation or there is rigid body rotation then these strain measures will also reduce to expected quantities. For example we will see that E and E tau will reduce to 0, while finger strain tensor such as B will reduce to unity. And so now, these unit tensor or zero tensor they will reduce to and the time derivatives of these strain measures are what are useful in terms of applications in non-linear description of rheological response and for these time derivatives we will use convected rates. And again for advanced learners we can show that how the convected rates of strain are related to in fact, strain rate tensor and velocity gradient tensor.

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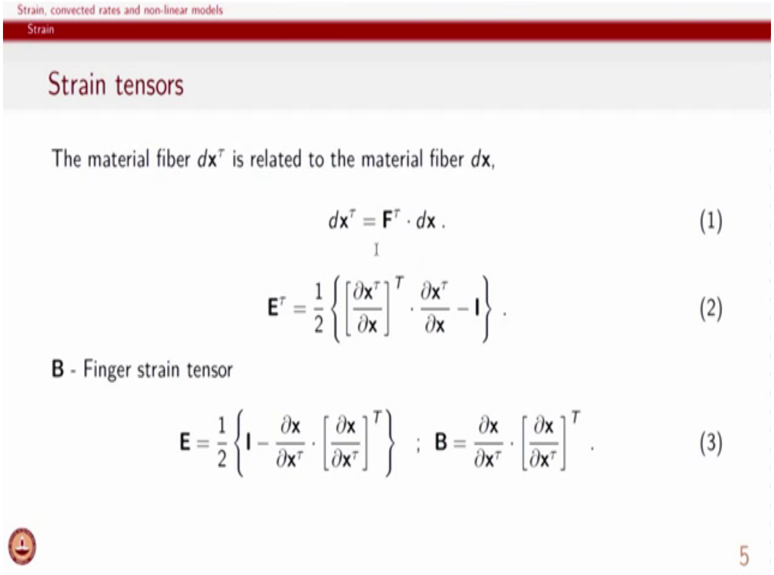

So, let us now define the strain tensor itself. So, as we have said there is material fiber at time tau and this material fiber has to be related. So, at time tau we have a material fiber which is at time tau and this material fiber has to be related to at present time which is t. And in fact, it is the relationship between the two which defines and this is something called deformation gradient which we have defined earlier and remember that we have also talked about that if you want to measure strain it is not only important to look at quantity like this, which is just saying that how is this material fiber changing with respect to the present material fiber. We actually need to know a quantity which is of this kind.

So, where the length of the material fiber is important and so these are the type of quantities which are involved in defining strain. So, therefore the strain tensor is defined as the from the deformation gradient as F T transpose dot F T. So, from deformation gradient we can define the strain tensor and we subtract the unit tensor so that this strain tensor reduces to 0 whenever we have rigid body translation and rigid body rotation.

So, for rigid body translations and rotations this tensor will reduce to unity so that the overall E tau will reduce to 0. Similarly we commonly define finger strain tensor which is given as this. In this case the basis is used as any time tau and the present time with respect to any time tau is what is used for defining the deformation gradient and again this strain tensor E will reduce to 0 whenever we have rigid body translation and rotation. And of course, B will reduce to unity whenever we have rigid body translation and rotation.

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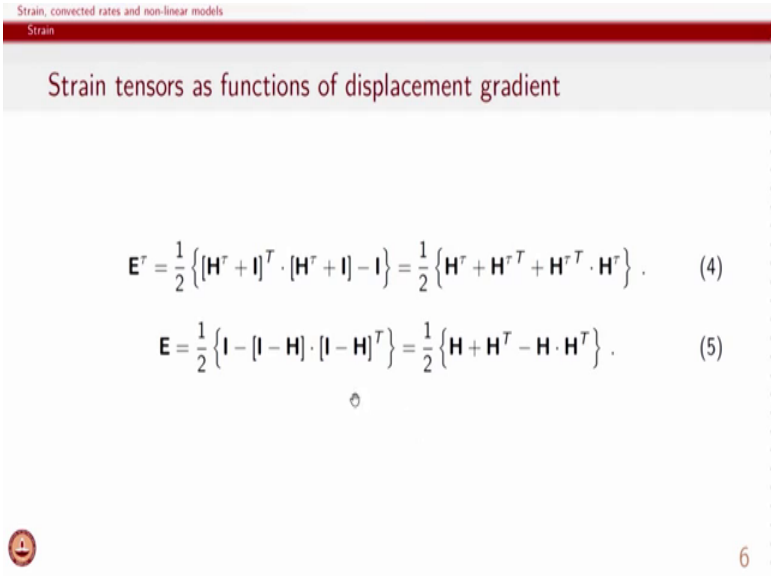

So, just to rewrite the strain tensor we could write them in terms of displacement gradient which we have defined earlier. So, this is a matter of algebra to try to write these quantities which is the deformation gradient in terms of displacement gradient and then we can show that these strain tensors are related to deformation displacement gradients. And we can see here that there are terms here which are of the order of displacement gradient itself and then there are terms which are of the order displacement gradient squared.

So, this is very similar to a function where if f of x is there, then what we have is some terms are of the order x and some other terms are of the order x squared. And if x is very small then we can ignore the x squared term. So, for small values of x x squared will be much less than x and therefore can be ignored.

So, we can see here that if displacement gradient is very small, in other words if deformation itself is very small then the strain tensors will reduce to this and it should not be a surprise to us that in fact, half times H plus H transpose is nothing but the infinitesimal strain tensor. So in fact, we can work with some of these details and then just get familiar with what happens to these strain tensors for couple of examples and to take the most common example that we have used in this course which is Simple shear flow where there is one dimensional flow with shear in y direction so that we have only one component of velocity gradient nonzero and of course, the overall strain is basically an integral from any time to present time gamma dot y x d t to prime. And therefore, we can write the anytime configuration of a material point in terms of configuration at the present time.

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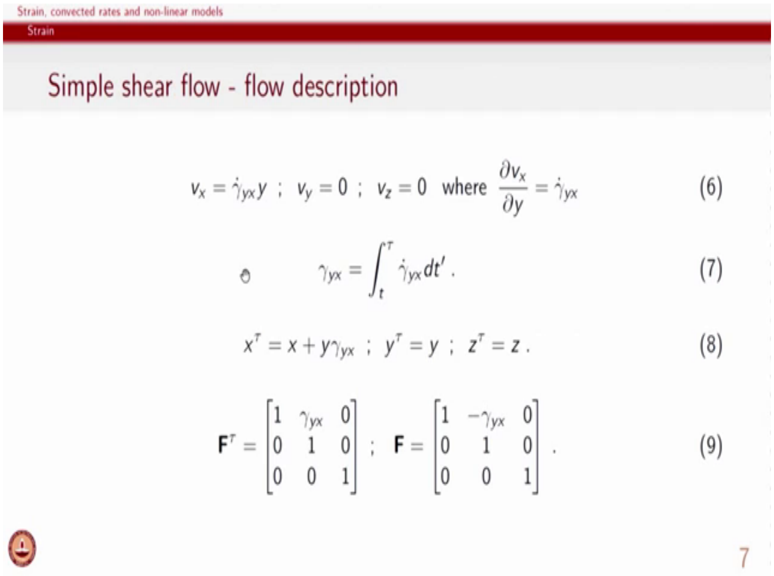

So, based on these and definitions of the deformation gradients we can see that the deformation gradient is for sym for simple shear has only 1 non-diagonal nonzero element while all the diagonal elements are 1. If the strain is 0 for example, is if v x itself is 0 then, del v x by del y will also be 0 and gamma dot y x will be 0. In that case both F tau and F will reduce to unity. So, for rigid body motion in which case gamma dot y x is 0 we also have gamma y x 0 and in that case F tau and F will be unity. Otherwise of course, gamma y x and minus gamma y x inform us as to how much is the amount of shear that is being imposed on the material.

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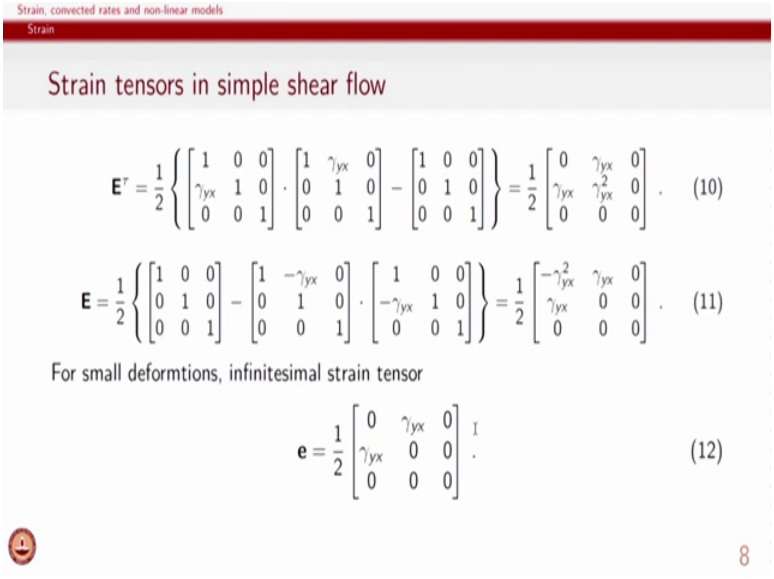

So, when we look at the strain tensor in simple shear flow again by working with algebra based on the definitions we can see that it is basically multiplication of the transpose of the deformation gradient along with itself and then subtraction of the unit tensor and we get for a simple shear flow the overall strain tensor is given by half gamma y x gamma y x squared. So, this is the square term which is the unusual term compared to whatever we have been used to. In class from earlier times including our initial classes and strength of materials given that we are looking at small deformations the strain infinitesimal strain tensor is what we are familiar with. So, we can see that if gamma y x is very small then in that case the second order term can be neglected and we get the infinitesimal strain tensor.

So, whenever gamma y x is very small the square term can be very can be neglected with respect to the gamma y x and therefore it reduces to infinitesimal strain tensor. And again this is intuitively what we are more comfortable with that given the only non diagonal term one non diagonal term is nonzero. So, therefore this is an example of simple shear deformation.

But when simple shear deformation happens over arbitrarily large quantity then we have this gamma y x squared also and this term also contribute to the deformation. You can see also that even though the material has been subjected to the shear deformation in the diagonal terms there is a representation of a or there is an existence of gamma y x. So due to the simple shear being imposed on the extensional element or the diagonal element there is a presence. And so, it therefore is intuitive

now that whenever we have large deformations and non-linear response, non-linear rheological response of materials even though we are imposing only simple shear flow the material can respond and give us non the normal stress differences. So, even though only shear stress is being imposed on the material we also will have normal stresses generated in the material.

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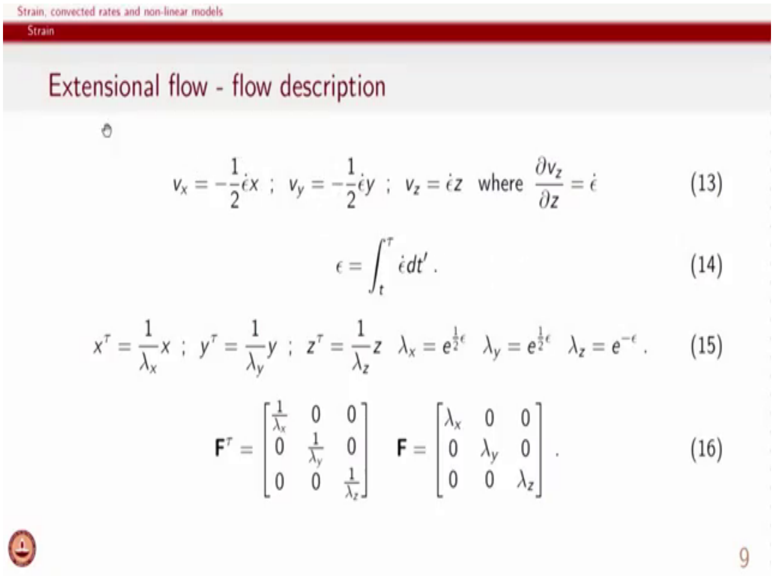

And So, the similar continuing on we have now the extensional flow flow description where we saw that the overall velocity is only in the z direction. So, this is like where we are pulling the material in the z direction and it contracts in x and y direction. And we have seen this information earlier in the course and so where we can define the configuration at any time tau in terms of the present configuration and generally for these extensional flows we define the two ratios, ratios of these two configuration as lambdas so called stretch ratios.

So, therefore in terms of stretch ratios we can define the deformation gradients and these deformation gradients can be used to calculate the strain tensors. And therefore, the strain tensors are basically incorporate the stretch ratio square terms depending on which definition we use we can it will be 1 minus lambda x squared or 1 over lambda x squared minus 1. And the finger strain tensor of course, is only lambda x squared, lambda y squared and lambda z squared. And so, again we can try to see whether these terms what happens to them when we have deformation very small.

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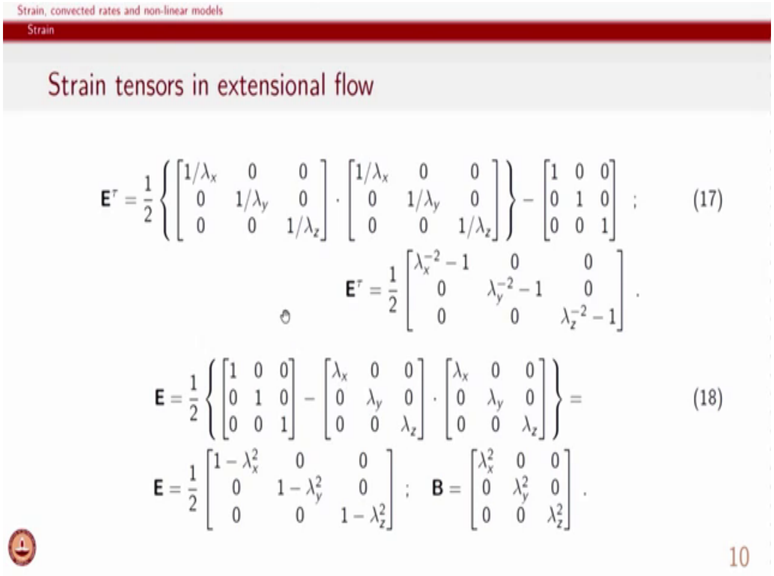

So, basically lambda x is akin to defining. So, if we have this is the original length, so if we have the original length and because if this material gets deformed and the now length changes. So, strain of course is defined in terms of the change in length versus the original length. So, strain is defined as delta l by l. While stretch ratio on the other hand is defined as l plus delta l divided by l. So, whenever deformation is 0, so if there is no deformation then delta l itself goes to 0 and therefore we have strain going to 0 and stretch ratio actually becomes 1. So, therefore, stretch ratio is also useful in terms of determining the overall behaviour of the material.

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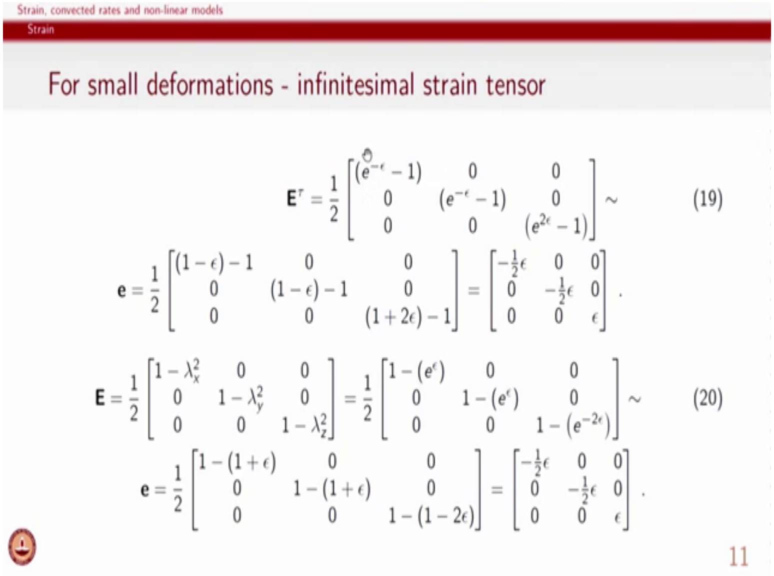

So, based on this simplification as to how strain and stretch ratio was related we can define it in terms of the strain which is epsilon and these are exponential terms and if you work with the overall algebra, if epsilon is very small then e to the power minus epsilon can be simplified as 1 minus epsilon. So, given these simplifications we can show that the overall the strain tensors both E tau and E will reduce to the strain tensor which we know as infinitesimal strain tensor.

And again this is something which we are familiar with the fact that since in the z direction there is only strain epsilon and given that this is an incompressible material with Poisons ratio being 0.5 what we have is in the other two direction x and y direction we have contraction and the contraction is minus half time epsilon. So, this is something which we again learn in terms of the infinitesimal strain tensor for a uniaxial extension or tensile deformation. But for a overall material deformation which is arbitrarily large in terms of stretch ratios the strain tensor E tau and E are given by the following expressions. So, with this now we have finished defining the overall strains.

Now in the next segment of the lecture we will look at the convected rates and then look finish up by looking at some non-linear models which are very useful for describing the non-linear response of materials.