**Rheology of Complex Materials**
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**Lecture - 08**
**Stress and Strain rate**

So far when we discussed in the previous lecture what is meant by a contact force and we saw that stress tensor is a specific example of contact force. And we know that stress tensor can be specified by specifying the 9 components. And since due to balance of angular momentum stress tensor is symmetric we effectively have six components of stress sensor which are involved in any flow situation in which complex material is involved.

So, what we can do next is let us look at how are these stress components specifically, when we have a cone in plate kind of a device which we will see later on is used for measuring the viscosity of the fluid.

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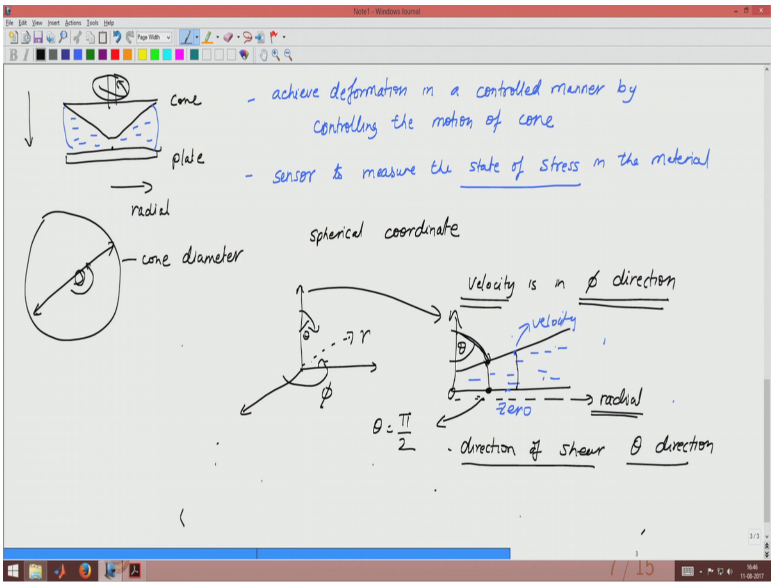


So, cone in plate as we will see later on will be a device in which case; we will have a cone which is rotated at a given controlled speed it could be also oscillated and then we have a plate. So, we have a cone and we have a plate and the fluid of interest is actually taken in the gap. So, this is where we take the fluid of interest which could be let us say ink, it could be paint, it could be polymer melt, it could be a paste, whatever is the material that is of interest and the idea of cone in plate device is to achieve deformation in a controlled manner by controlling the motion of motion of cone and then we have a sensors which can measure the forces which are generated in the material; so sensor to measure the state of stress in the material.

So, let us look at this state of overall geometry it looks like that clearly we can define this as a radial direction just to see from the top what we will have is view of this where this is the cone which is rotating and then this is the outer diameter of the cone. So, this is the cone diameter will be specified and so this cone is of course, rotating. So, the radial direction is one important direction and to describe this problem we use spherical coordinate. So, if we see this description of this problem and if we try to draw the spherical coordinate system generally what we do is we say that we measure theta from the vertical axis and then phi is measured in terms of the plane itself and then of course, we have the radial directions? So, how do we map the spherical coordinate into the cone inflate.

So, we can take this point as the center of our coordinate system. So, what do we have in this case is therefore, velocity is in phi direction because this cone is rotating we can see that the rotation direction is in the plane. So, velocity is in phi direction if you look at the cone everywhere and we have the fluid in it since this top surface is moving what we have is velocity here high while here the velocity is 0. So, that is how there is a velocity gradient set in or there is a strain rate in the material as we will define in the next lecture, but because of this the stresses general get generated in the material.

So, velocity in the phi direction; direction of shear as we will define later on also is in theta direction as you go from this point to this point. So, we have this vertical line which is similar to this from here we measure the angle theta. So, this is theta measures the top surface as well as theta measures this bottom surface. And of course, this bottom surface is theta equal to pi by 2 and so the direction of shear is theta direction of velocity is phi and of course, the radial direction itself as we go out from the center outward we have the radial direction.

So, now, the question is how in this kind of a scenario where we have the all the r theta z components how do we describe the state of the stress.

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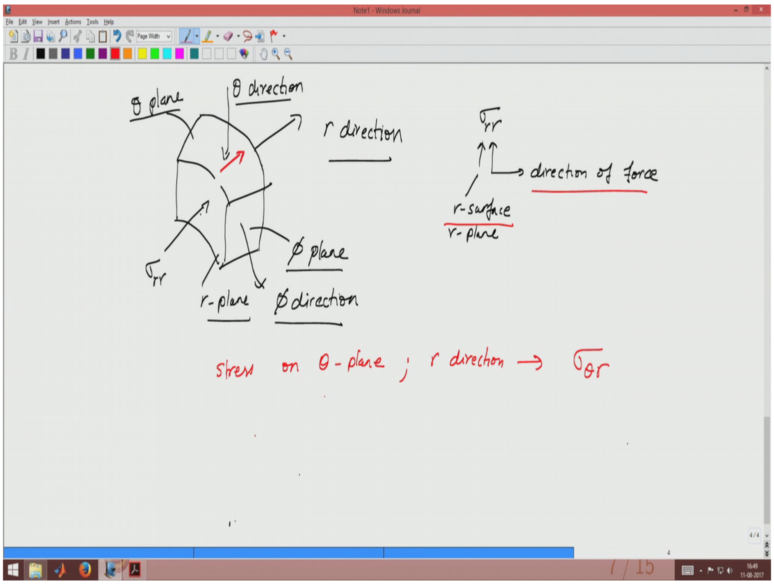


So, we can draw this in terms of a small volumetric element and because of our coordinate system now we are describing all of these. So, this is basically a radial plane this is an r plane, this will be a five plane and this will be a theta plane similarly this is the r direction. This is the phi direction and this is the theta direction.

So, now we have defined the 3 directions are theta phi and we have defined the 3 planes. So, basically we have now 9 combinations. So, the question that you can ask yourself and see is where will sigma rr be for example. So, sigma rr as we said is there is one of them indicating force one of them indicating direction.

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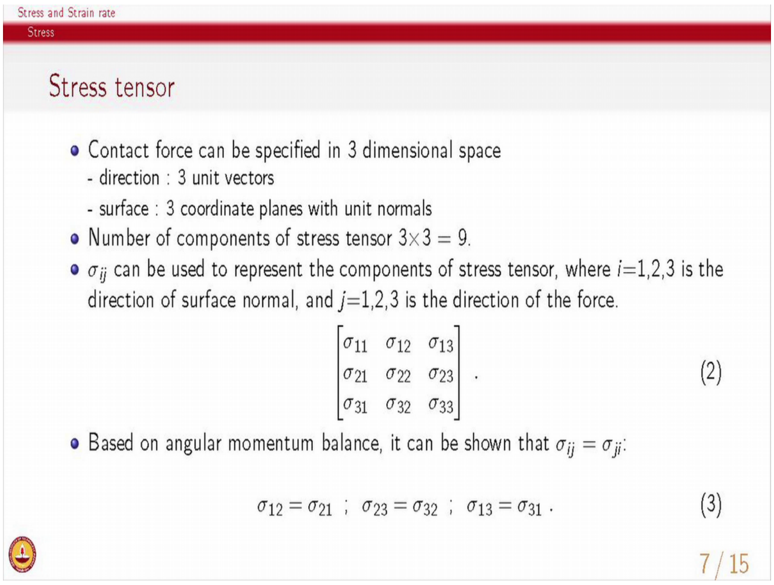


So, just to go back and see the first index indicates the direction of surface normal. So, which means this is the surface r surface or r plane and this is indicating the direction of force of the contact force and so clearly this is an example of sigma rr. Now you can also again think and try to justify as to what is being drawn here. So, what I have drawn here is what I have drawn here is basically of a stress which is on theta plane and it is in r direction.

So, clearly based on the convention that we have written now this is nothing, but sigma theta r, because the second index describes the direction the first index describes the surface. Therefore, in a cone and plate we can see that all 9 components of stresses will be active.

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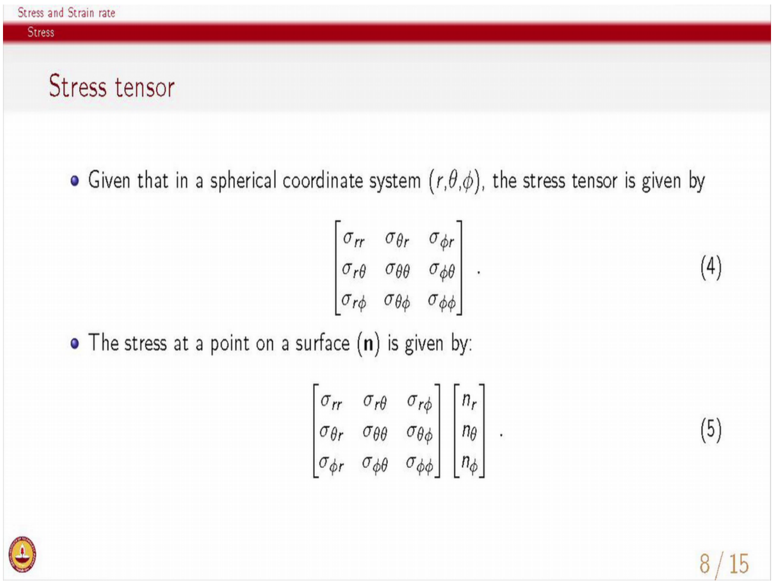


So, coming back and summarizing given that in a spherical coordinate system which will be used for cone in plate system where velocity is in the phi direction and the direction of the shear will be theta direction we will see that there are 9 components of stress tensor that can be specified. And of course, to find stress at any given point what we need to do is to multiply the stress tensor a lot with the no unit normal vector. And since now we are describing the unit normal using a spherical coordinate we have 3 components of it n r which is in the r direction n theta which is in the theta direction n phi which is in the phi direction.

So, using this operation we will be able to find the stress which is at a point on a surface given by n and as a preliminary observation we will see that cone in plate device is where shear flow is observed. So, since theta phi are the dominant components one of them being velocity direction the other one being direction of the shear we will see that sigma theta phi or sigma phi theta will be the 2 dominant components which will be discussed.

In fact, we will see that the presence or absence of normal stresses as we saw in the unusual flow phenomena lecture and where we also discussed rod climbing and other such effects which were related to the normal stresses what we will see is in fluids we will try to impose sigma theta phi or sigma phi theta using a cone in plate geometry and if we measure sigma rr and sigma theta theta, we will get a very good indication of the elasticity of the material or what are the what is the basic viscoelastic behavior of the material can be characterized; if we subject the material to sigma theta phi and then measure what are the normal stresses which are generated.

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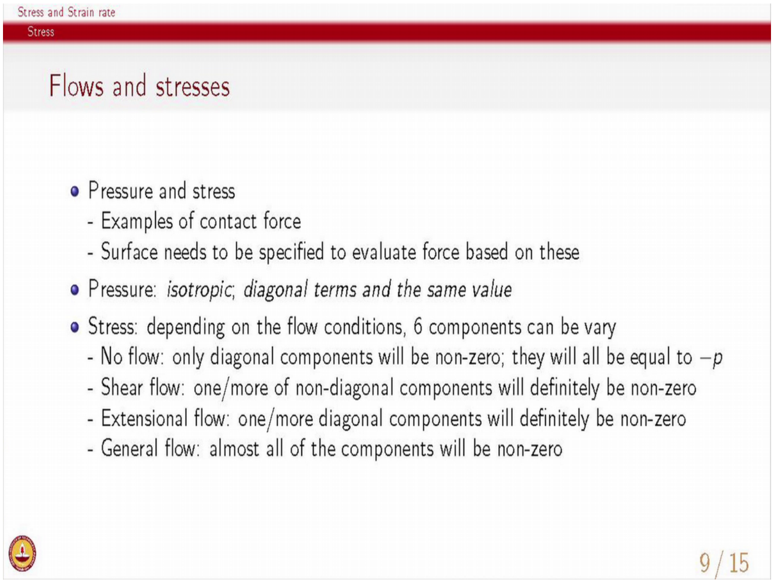


So, with this basically we can summarize now that pressure and stress are both examples of contact forces we need to specify a surface before we can specify the contact forces and we evaluate them for the balance equation once we specify the surface pressure is again isotropic they include dag they are diagonal terms and they are all equal they are all of the same value stress on the other hand is a depending on flow conditions all six components can be non-zero. So, depending on what type of condition clearly if there is no flow then only diagonal components are there which are all the same and that we call pressure. So, even in a stagnant liquid case, we will have hydro static pressure which will be the contact force when we have shear flow we will have the non diagonal elements to be non-zero.

We will see that shear a flow certainly non diagonal elements will be at least one of the non diagonal element will be non-zero, but we might also have normal stresses to be non-zero in case of extensional flow we will have one or more of the diagonal component will definitely be non-zero. So, the key word here that you should focus on is definitely. So, at this stage by just saying shear flow or extensional flow we cannot completely specify the state of stress tensor we can only say that shear flow at least one or maybe 2 components of non-diagonal elements will be non-zero.

We will see most of the time with simple shear flows that we will discuss in this course one component of non diagonal component will be non-zero, but if we have elasticity and normal stress differences then we will also have normal stresses being non-zero for a Newtonian fluid in simple shear flow we have only one component of non diagonal stress in case of extensional flow.

Similarly we will at least have off the diagonal elements non-zero, but we may have other components depending on the type of the fluid, but in general for a very normal for a complicated flow which is involved in engineering situations we will have almost all of the component non-zero. And therefore, to say that we understand the fluid flow behavior of a material we need to not only understand it under shear flow we have to also under I understand it under extensional flow. So, that the general flow where everything is non-zero can be understood based on our understanding arrived at in shear an extensional flow.

So, sometimes we will say that the general flow is a combination of shear and extensional flow because in lab situations to understand rheology we first subject the material to maybe extensional flow and then subject it to shear flow. And then try to arrive at an understanding then we say that general flow is a combination of those can we combine this understanding and then try to explain the behavior in a most general case. So, with this we come to a close in terms of what is the overall stress tensor.

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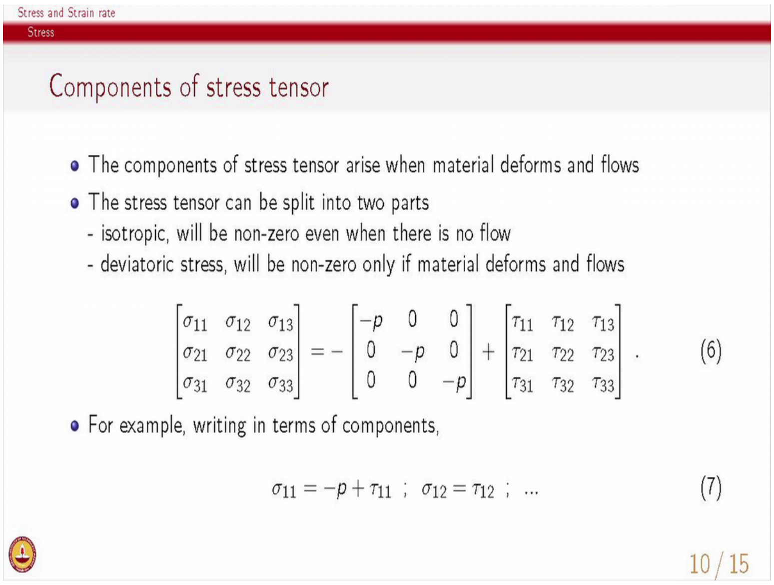


In terms of using it further for rheological analysis we will split it into 2 parts isotropic which we have already seen is pressure and this will be even non-zero even if there is no flow. And we will define another stress called deviatoric stress which will be non-zero only if material deforms and flows. So, the overall stress which is called the total stress is equal to the pressure in the material and the deviatoric stress. So, this is the deviatoric stress and this is pressure.

So, we of course, we can use the matrix notation to write it like this or vector or tensor notation and component wise we know that sigma 1 1 is minus p plus tau 1 1 and so on. So, each of the stress of the total component can be found out if we know the deviatoric stress and pressure together.

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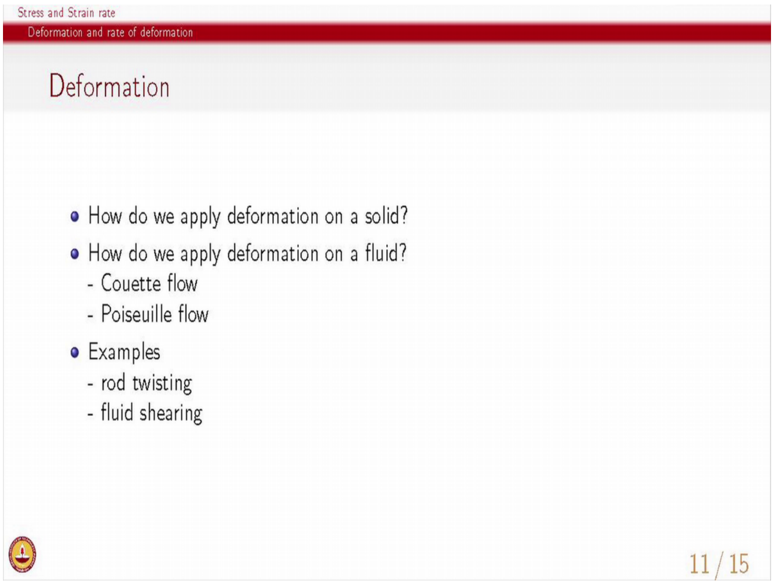


So, now in the next part of the lecture we will look at deformation. So, in general before we start and define the quantities which are which quantify deformation? Let us just ask this question is what do we mean by deformation in a solid and more importantly how do we apply deformation in a solid.

So, generally for example, when we learn strength of materials or when we look at solid mechanics what we have is a solid body which can be applied different types of deformations.

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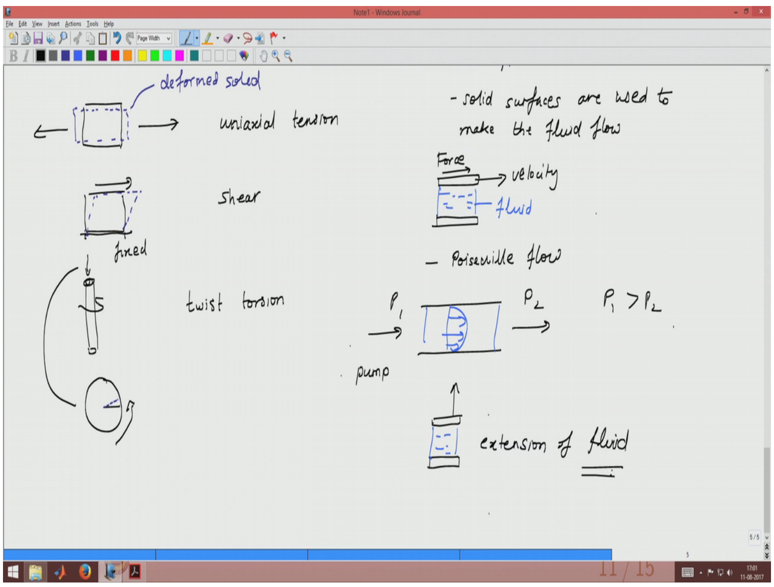

For example, we can take a block of material and we can stretch it. So, this is called a uniaxial tension or we can also take a block of solid and we can try to shear it by keeping this end fixed. So, this is called shear we can also of course, if we have and this is more understood in terms of a rod that we can twist it or torsion. So, these are all different ways in which we apply forces on the material and then material deforms.

If we look at what we would expect as a general deformation in the material we could draw that as a result of extension the material may become like this. So, this is deformed solid. Similarly in this case, the deform solid would be sheared solid and similar in this case also the material will get twisted. And if I were to look at it from this site, let us just look at it from this side and draw the cider circle and if I draw a line like this when we twist the material in this direction basically what we will have is the line would have shifted.

So, that is the twist that is been imposed on the material. So, this is generally the deformation that we apply on a solid and we see its response in terms of how much it has deform of course, sometimes we apply a fixed deformation and then measure the amount of force needed for that deformation some other time we apply a force or a stress and then we see how much deformation is there in the solid now similarly how do we apply deformation to a fluid. So, there are 2 broad classes of inducing deformation in the fluid and both of these are useful as far as rheological techniques are concerned one is called the coquette; coquette flow and coquette flow is basically similar to.

So, in this case solids surfaces are used to make the fluid flow to make the fluid flow. And so, it is very simple to see that a simple shear flow of a fluid can be done by taking the fluid between 2 parallel plates; so if I take 2 parallel plates and if I take fluid between them. So, I take fluid between 2 parallel plates and then what I do is I make the top plate move with certain velocity or I apply some force. So, that it moves; I will have the fluid also shearing and so based on this motion this is called coquette flow.

We similarly also have we also have Poiseuille flow in which case we have fluid being pushed because of a pressure gradient. So, we usually use a pump. So, that pressure at one point is higher than pressure. So, that P 1 is greater than P 2 and we take a fluid in this pipe then the pipe starts moving and so this is an example of Poiseuille flow. So, both of these cases the fluid is getting deformed and the other example that we saw when we were discussing the unusual flow phenomena was also the fact that I can take the fluid between 2 plates and I can move if they I move the top plate with a certain velocity, then again I will have extension of the fluid element. Therefore, similar to solids we can also actually achieve deformation in the fluids.

So, now let us look at 2 specific example one of them is related to rod twisting that I discussed the other one is related to fluid shearing. So, let us look at one example where we take a coordinate system.

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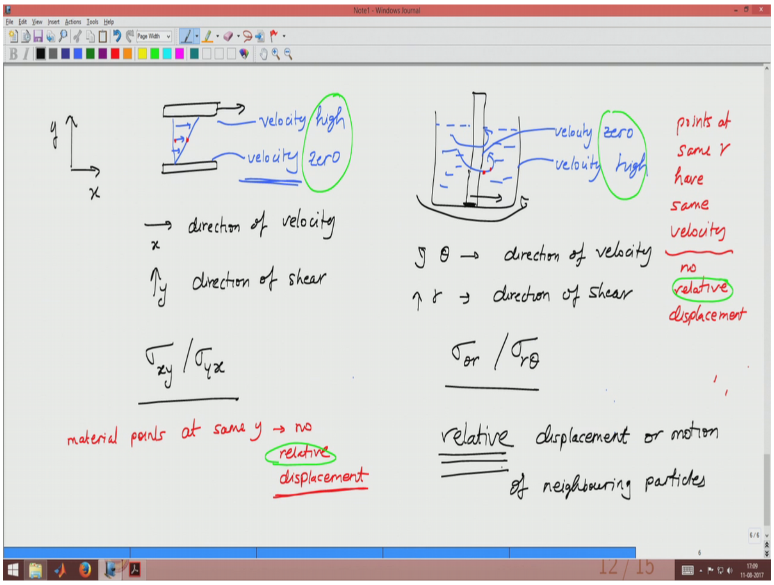


And we have 2 plates and then we take fluid in between this and then since the top plate is moving we will have the fluid also move. So, this is an example of shearing fluid we can see that how this kind of a situation would be relevant in let us say a cold cream or a shampoo in case of shampoo we will rub it. So, basically we are sharing the fluid between the surfaces in case of cold cream we are applying.

So, in general if we are designing these material systems in lab situations it is helpful to subject them to shear conditions and therefore, it is useful to do such flows and this is the most prototypical flows there that we will subject the material to and the other example that we can look at is when let us say we take a material in between let us say a rod. And so, we take the fluid and we have basically rod in the center of the beaker and now what we do is we rotate this beaker.

So, because the beaker rotates now what we will have is rotational motion of the fluid element fluids fluid itself. So, everywhere the fluid will also rotate velocity will be highest here and in this point velocity will be 0. So, this is very analogous situation because here velocity is high and here velocity is 0. So, in both of these situations what we can see is there is a direction of the velocity and then there is a direction of shear. So, in this case for example, what we have is x is the direction of velocity, but if I go in y direction, then the velocity changes.

Therefore, that will be referred to as direction of shear similarly in this case theta is the direction of velocity and r which is this direction outward is the direction of shear and. So, in terms of stresses that we had discussed earlier you would expect that in this case given that velocity and shear are involved in x and y direction. And in this case direction of velocity and shear are involved in theta and r direction we would have sigma yx or sigma yx as the stresses and in this case sigma theta r or sigma r theta as the stresses.

So, we just keep this in mind when we discuss many of these rheological techniques later on. So, having seen the different types of deformation that a fluid can be subjected to and the fact that we can look at various prototypical flows which are at either involving coquette devices or Poiseuille devices in each of them we will have certain directions of velocity, but the velocity will change in other directions which we called the direction of the shear in case of shear flows.

And so we will have to understand variations of velocity and various solutions of shear if we hope to understand the material behavior quantitatively and to that effect we need to define; what is the strain rate in the material.

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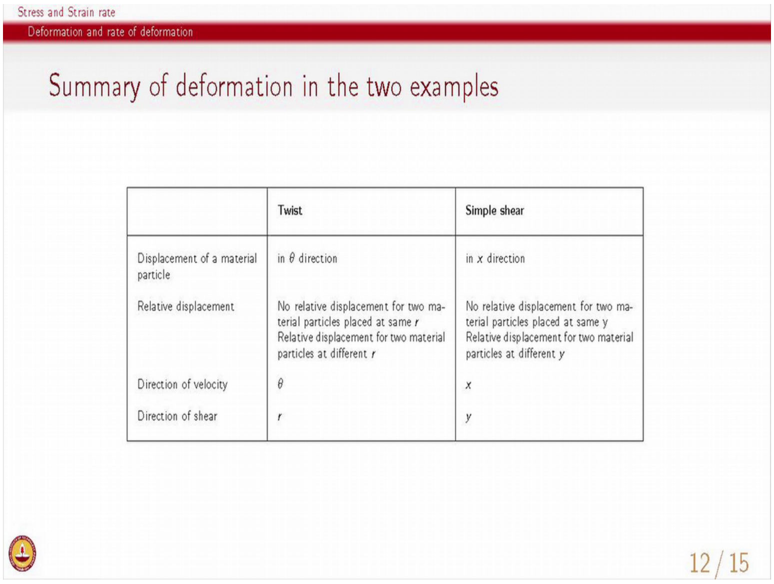


So, just to summarize what we saw was in case of twist the displacement of a material particle was in theta direction the velocity itself was in theta direction in case of a simple shear the flow was in x direction or the displacement of material particle was in x direction. So, if you look at relative displacement between material particles no relative displacement for 2 material particles plays that same r; for example, if I take 2 points which are little bit away from each other, but they are at the same r. So, points at same r have same velocity, so therefore, no relative displacement.

Similarly, here also if I take 2 points which are at the same y there is no relative displacement at same y material points at same y no relative displacement. So, you can see here we are trying to describe the kinematics of the material motion and we are using several variables i; we are talking about displacement which is how much does a material particle get displaced by we are talking about velocity which is how fast or slow the material will points are getting displaced by and clearly what is more important in all of these cases is the fact that what is the relative displacement or what is the relative velocity.

So, in both these cases; what is important is to have comparison of velocity and displacement between different material points and so this importance of relative displacement or motion of neighboring particles. So, we will see that this relative deform displacement or motion is central to defining strain and strain rates in the material. So, what we have in the next class.

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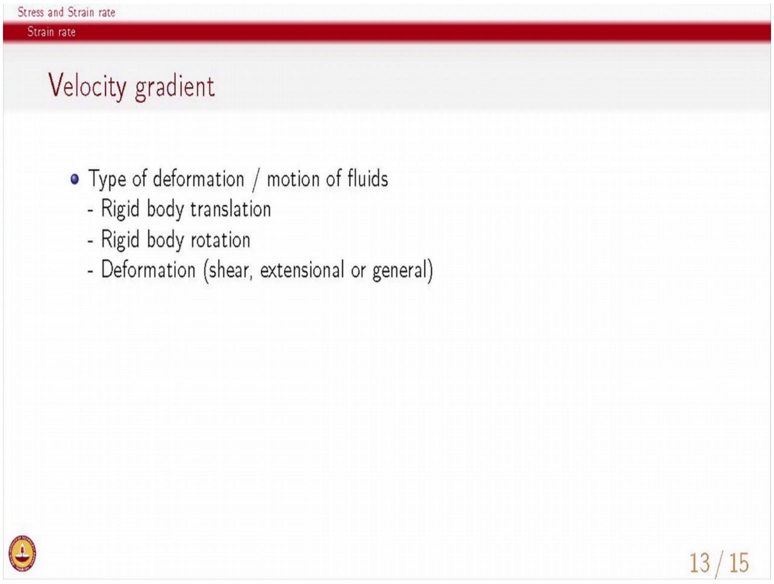


We will look at the velocity gradient and we will see that velocity gradient describes rigid body translation rigid body rotation and deformation are the 3 phenomena which have to be described for a material which is flowing. We will see that for rigid body translation there is no velocity gradient because every material point is moving at the same velocity for rigid body rotation. We will see that we will have non-zero velocity gradient. And of course, the more important case where we are deforming the material either in sheer extension or any combination of them, then we will have velocity gradient being defined for those situations.

With that we come to a close for this lecture. Next time we will define the velocity gradient and the other quantities which are based on velocity gradient.