CH5230: System Identification

Journey into Identification

(Case Studies) 12

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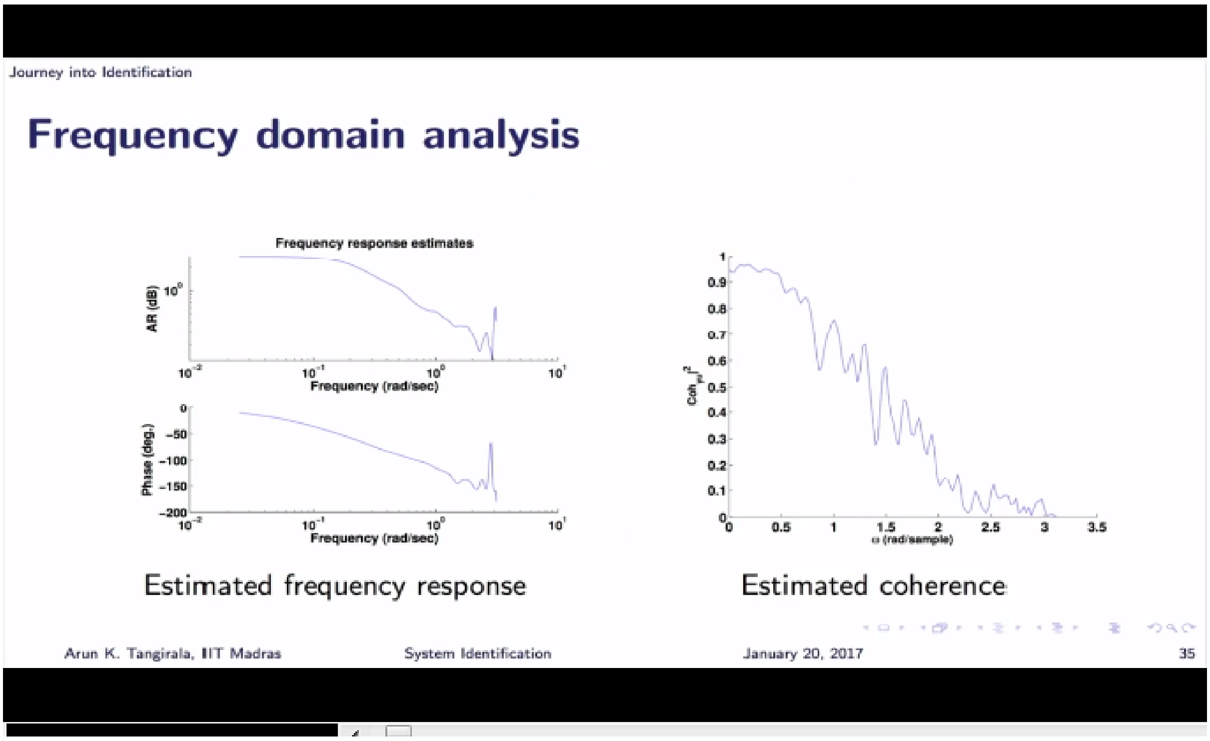

Arun: So we have looked at impulse response. How the system would response to an impulse input? How the system would respond to a step input? Now, I may be interested in how the system responds to a sinusoidal input? These are the three, always you have trinity, everywhere you will find this three, whether it is Carnatic music or whether it is here you- or elsewhere, you will find the trinity. So here the trinity is impulse response, step response, and frequency response. People are generally afraid of using the word frequency, okay? Quite frequently that happens, but you don't need to panic. As long as you understand that the frequency responses is yet another way of this describing the system, but now to a different input, if you're okay, with impulse and step response, why not with frequency response? Because it's just telling you, how the system would respond to a sinusoidal input. That question that you have to ask within yourself is, why I need yet another description of the system, right? Why did we move from impulse response to step response? Theoretically they contain the same information. As far as theory is concerned for LTI systems, this description is sufficient. But from a practical standpoint, each response gives me access to the characteristics of the process with a different level of convenience. So when I said, I want to read off the order, the gain, we said, step responses are more suited, right. Whereas impulse response or suited for something else. Very often, we also want to know-- remember we have spoken about this, the filtering nature of that system. In identification, that's a key piece of information. Not only an identification, in control, in all the devices that we are using phone, any communication device that you're using, be domestic, military, doesn't matter, all communication devices rest on this concept of filtering, the filtering perspective of a system. And that's best understood by turning to the frequency response. There is no other response base description that will give you a direct preview of what is the filtering nature of the system. So what you see here on the left is, what is known as a board a plot. And we have drawn this diagram earlier on the board. So let's focus on the left plot here. And let me zoom in for you. On the x-axis on both the plots you have frequency and on the top plot what we are plotting is the magnitude. I've explained to you earlier. Whenever LTI system is injected with a sinusoidal input and this is theory, will prove this later on as well.

Whenever it's injected with a sinusoidal input, the output of the system that is a response of the system at large time, so the sinusoidal waveform with the same frequency, no other frequency can manifest. The only difference is going to be in amplitude and the phase. This is an alternative way of describing an LTI system that is in fact, defining an LTI system. A system is set to be LTI if and only if a sinusoidal input produces a sinusoidal waveform at large times of the same frequency. Exactly whatever frequencies you have introduced. If you have introduced two frequencies, only two frequencies will appear. But the amplitude and phases are going to be altered. And the top plot is telling us, how the amplitudes are going to be altered. That's why it's called amplitude ratio plot. But we have a express this on a log-log scale. And when you take the logarithm of the amplitude ratio and multiplied with the factor of 20, it’s called decibel, named after Alexander Graham Bell. So, dB is 20log10 amplitude ratio. And on the x-axis you have frequency on the log scale. What do you see there? You see that as you move from low to high frequencies, the amplitude ratio falls off. What does it tell you about the system?

Student 1: Low pass filter.

Arun: It's a low pass filter, right?

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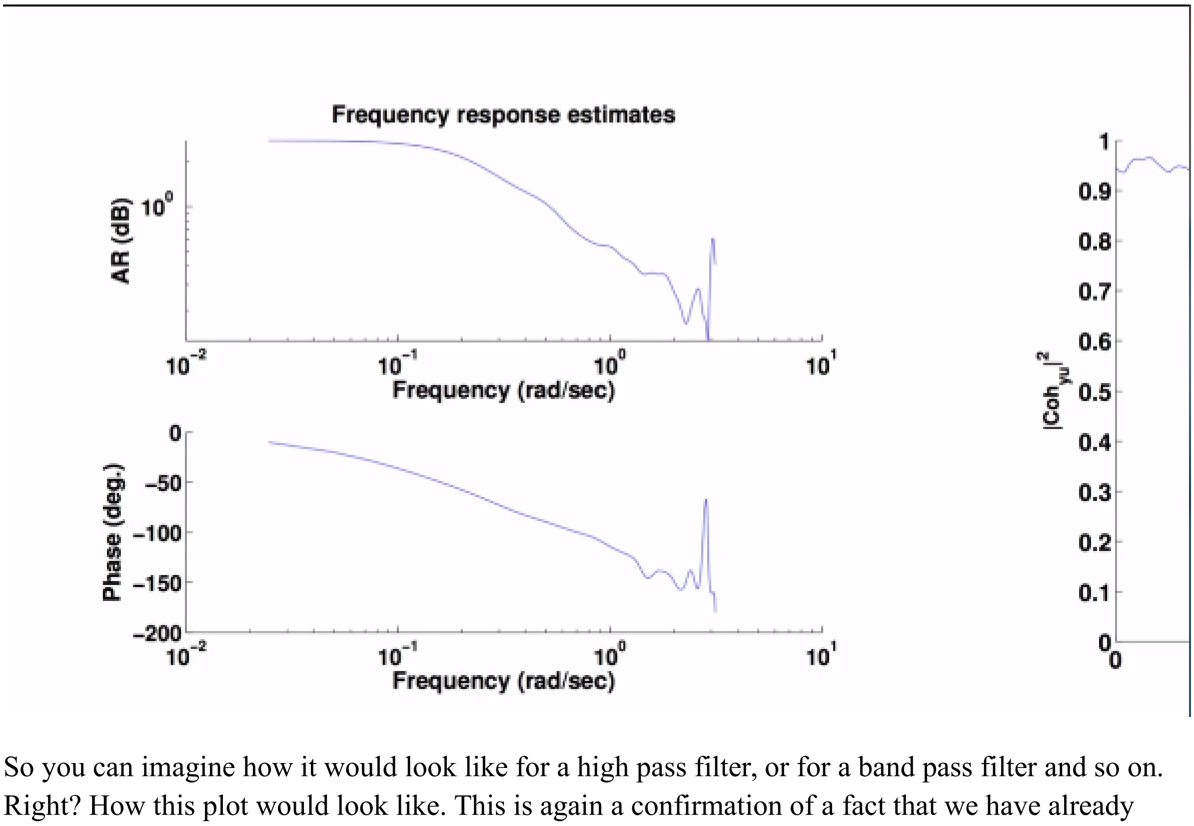

So you can imagine how it would look like for a high pass filter, or for a band pass filter and so on. Right? How this plot would look like. This is again a confirmation of a fact that we have already realized through spectral analysis, and through even physical arguments that we have made for the system, which is good. And this information is vital for a better input design, because it tells me clearly what is the bandwidth of the system. Up to what frequencies I can expect significant amplitude ratio. Of course, we will go through a detailed theoretical discussion on this later on. The bottom plot is what is known as phase plot. Now the phase plot is extremely useful in estimating what are called delays, like, we have estimated delay from impulse response estimates by reading off, unfortunately, you can't read of delay from the phase straight away. You need to do a further mathematical processing of the phase and we'll again come across method later on. As to how to estimate delays from phase and you can prove that the estimates of delays obtained from the phase plot are a lot more efficient than what you see, what you actually can estimate impulse response.

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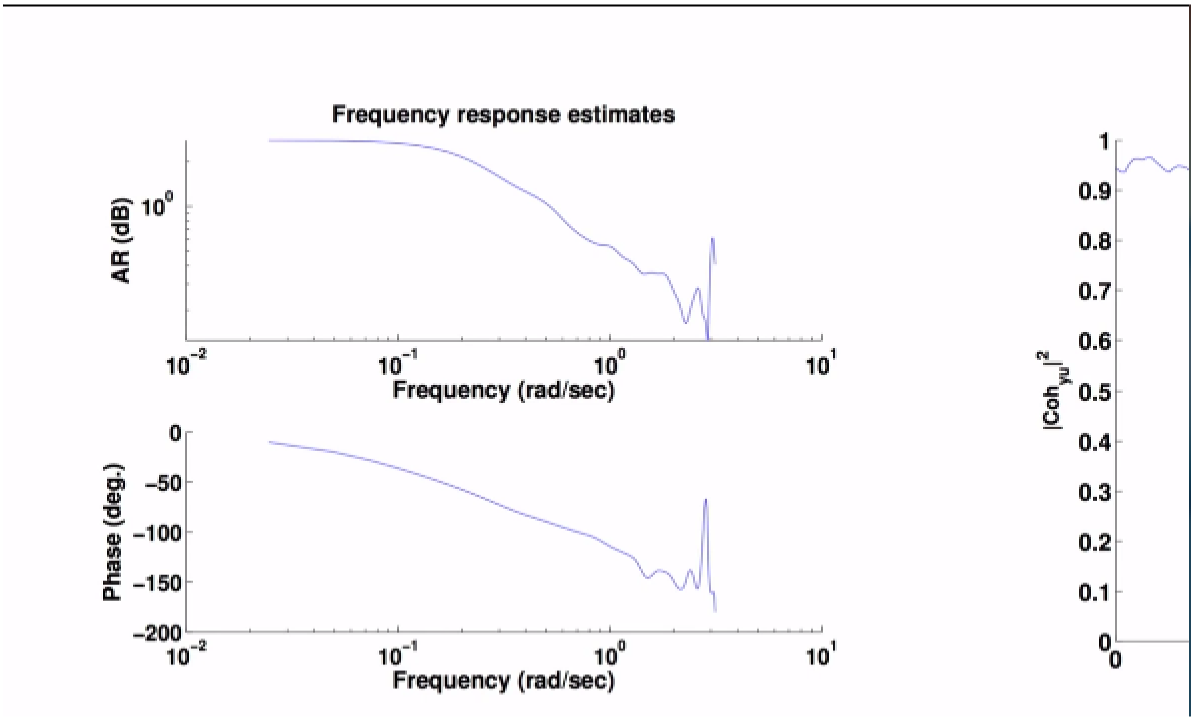

What kind of a filter the system is? The phase plot will allow me to estimate delays, which I'm not showing you here. On the right hand side, you have what is known as a coherence plot.

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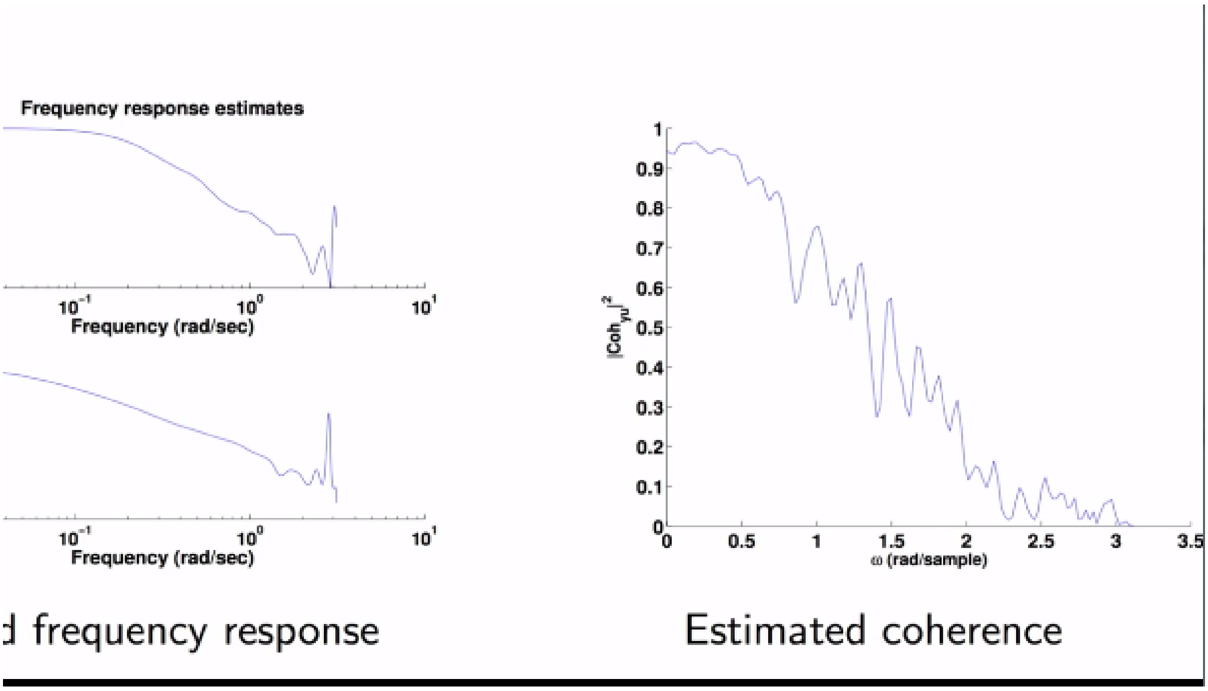

Now this is a very, very useful plot because it tells-- some very useful things about the process, about what's happening with the input and output and so on. Now at the moment, I do not want to go into the definition of coherence, but let me tell you something. Theoretically, this coherence is a measure of the linear relation between the frequency component in the input and the frequency component in the output. Okay, it's a normalized measure. And how should it look like for an LTI system theoretically, if there was no noise, perfect, you know, perfect deterministic LTI system? It should look flat. And flat at a value of unity. So this is how the theoretical coherence would look like for a noise free LTI system. This is coherence, in fact, this is squared coherence that I'm showing you on the plot and here is Omega. All right? This is how it should look like. What is this?

This is a theoretical coherence plot for a noise free LTI system. But you see something else here on the screen what we have estimated, how we have estimated? Don't worry about it. Why do you see a difference, any guess? So theoretically-- first of all coherence is a measure of linear relation between input and output at every frequency. And because LTI system is linear regardless of the frequencies the coherence is one, coherence is-- by the way normalized measure, it cannot go beyond one by definition. It's like your correlation. You can say, it's a frequency version of correlation. What you see on the screen is estimated coherence, correct? If the system was truly LTI and noise free and something else, fill in the blank, the estimated coherence should also-- should be identical to what I've drawn on the board. Why do you see a difference? What is the deviation? So all you have to ask is what is the deviation from the theory? This is when no noise, right? Let me write here. There's no noise here. And system is LTI.

Student 2: Nearest version of all linear system.

Arun: Okay, linear version of nonlinear system, okay, some deviations should be observed. It is not that bad. It's kind of mildly linear around operating point, right? But what is the prime difference? It's like your readers digest question, what is the prime difference between what we have drawn on the board? This is theoretical, right? What do you see on the screen? What do you see on the screen? Is it a theoretical coherence? Estimated. And estimate depends on what-- how I perform the experiment.

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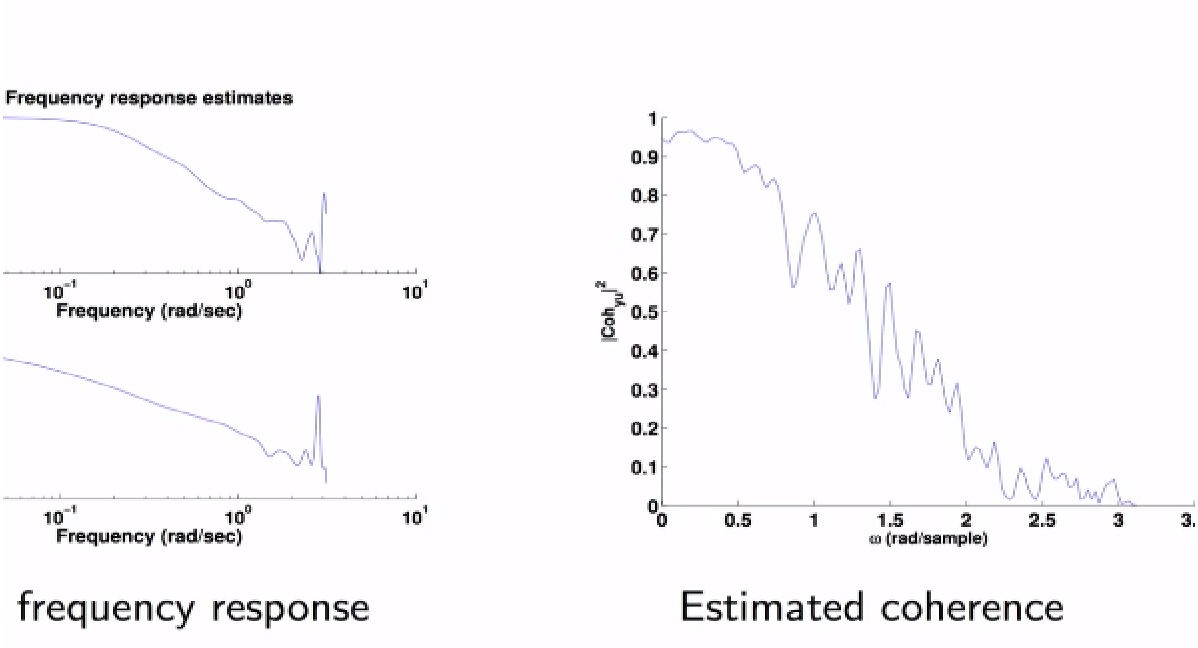

Am I right? How did I perform an experiment? BRBS, what kind of frequencies did it contain? There it contains all frequencies? So I didn't ask-- I didn't excite the system over a certain range of frequencies, right? That means, I have no information there, right? And that's why the coherence has gone down. The algorithm says, I don't know, I don't have information. I'll assume that basically from the given data the system-- there is no relation between input and output at those frequencies. Strictly speaking, I should not even report the coherence beyond the frequencies that I have not-- that I've asked, right? But I'm plotting the coherence at all frequencies.

The algorithm sees the data. And it says, I see some response at all frequencies. Why? Although, I didn't excite the system at all frequencies, the input did not contain all frequencies, the data contains response at all frequencies. Why? Very good. That is the key. You see, again noise makes-- at every stage noise makes a lot of noise. So what the algorithm says is that there is into be some response, it doesn't know that you have not excited. But in the end, the estimation algorithm is such that, it is ultimately comes to a very low coherence giving you the impression that there is no relation between input and output at those frequencies, but the fact is, you did not excite the system at those frequencies. So the coherence estimates beyond the frequencies that you have included should not be even looked at. That's point number one. Okay.

Now, let's come to the range of frequencies that we have excited, right? We do remember roughly what ranges of frequencies we have include an input. Roughly until this point. Why is not the coherence flat there? Approximately, it should be flat, right? Why is it falling off? I can understand, we have just explained, why coherence is almost nil beyond the frequencies that I've included. Because I've not excited the system, so the algorithm doesn't have information to estimate, fine. But what about the band of frequencies that I have included?

STUDENT 3: Delay induces by t.

ARUN: Sorry.

Student 3: Delay induces by t.

Arun: Delay has got nothing to do with coherence. Delay has no impact on coherence.

Student 4: Low pass filter.

Arun: Okay, expand on that. There are two aspects of the system.

Student 4: But actual coherence contains--

Arun: But it doesn't matter. Coherence is going to be one, regardless of the nature of the filter.

Student 4: Particularly, the output with that frequency doesn’t follow that so that’s why.

Arun: Even, I mean, see, just because system is low pass filter, doesn't mean that beyond a certain frequency, the system just keeps quiet. The response is going to be extremely low. That's all. And coherence will also take into account that fact. You're forgetting the fact that this is estimated, number one. And number two, you have to recall an important fact which is, let me take you back. Look at the inputs spectrum. What is happening? The power at each frequency is not the same. In whatever frequencies we have included, right? The power is diminishing, which means what? The signal to noise ratio will also fall down, right? When that is got to do the filtering nature and that point you can bring in the filtering nature as well.

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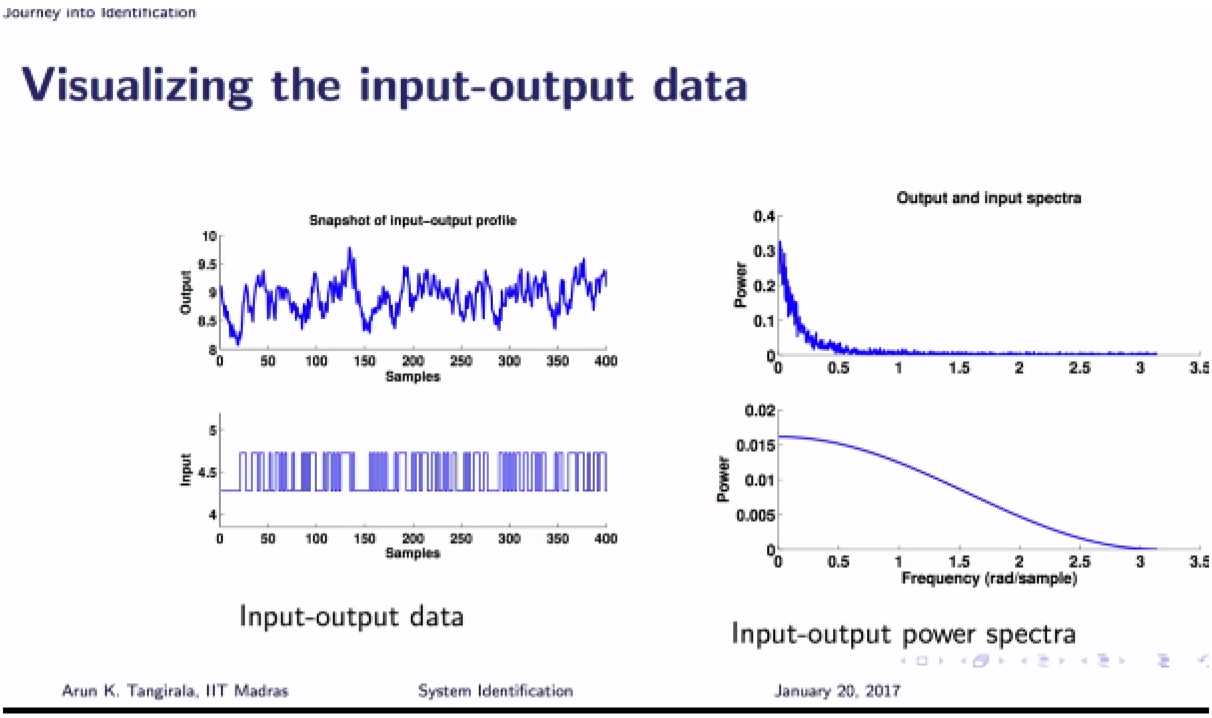

As is my input is not having uniform power at all frequencies, on top of it the system is a filter. So, look at the output. The power in the response is not the same at all frequencies, which means-- and whereas noise is going to have an impact on at all frequencies. As a result, the ability to estimate your coherence falls off, because the signal to noise ratio at each frequency will govern the goodness of the estimate at each frequency. We have earlier talked about signal to noise ratio in the time domain. Now we are referring to the signal to noise ratio in the frequency domain. That means, at each frequency how much power is present in the truth versus how much power is present in the noise. That's obviously falling off. So the coherence estimate is bound to fall off. Theoretically, the coherence is not at fall, right? So there are so many things that we learn here through coherence that first of all, what I can observe is that there is a range of frequencies only over which I can expect to build a good model, linear model. Clearly it tells me that beyond a certain frequency, do not expect your model to do well. That means, let's say, I estimate the model from this data and I want to know, how well, I mean, what kind of response this system will generate at high frequencies, your model is not going to be well suited to that. Coherence straightaway tells me, there is no relation that I have found at those frequencies. On top of it, it says, you should expect good predictions or you know, good estimates of your model in the low frequency region as compared to the mid frequency region, because the amount of linear relation is falling off. This falling off of coherence should not be construed as the system being nonlinear. Mainly, we know now the culprit is noise. But in reality, it could be a mix of both. See how complicated things can be. Together noise and nonlinearities can make life very complicated but, okay, that's good, because if there are many things that are happening and we do not know then everybody has a chance to come up with a better guess. So this is how you analyse the coherence plot. So, look at--