CH5230: System Identification

Response-based Description 3

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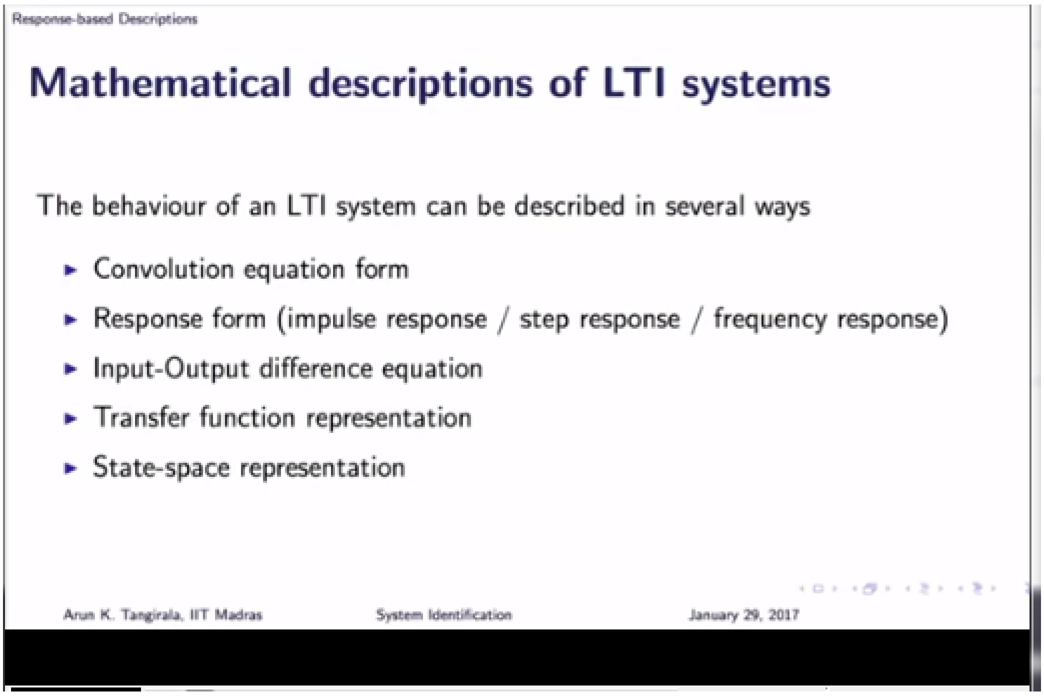

So if you recall, in the last class, we had spoken of different ways in which you can mathematically describe or represent a linear time invariant system. And the story begins to the convolution equation, from where the response based descriptions take birth and then you run into the difference equation form followed by a transfer function from. All those are an input output domain, and then finally you have the state space representation.

So let's begin with the convolution equation from. Wherein, you have the output being represented as a linear superposition you would say of or weighted superposition you can say.

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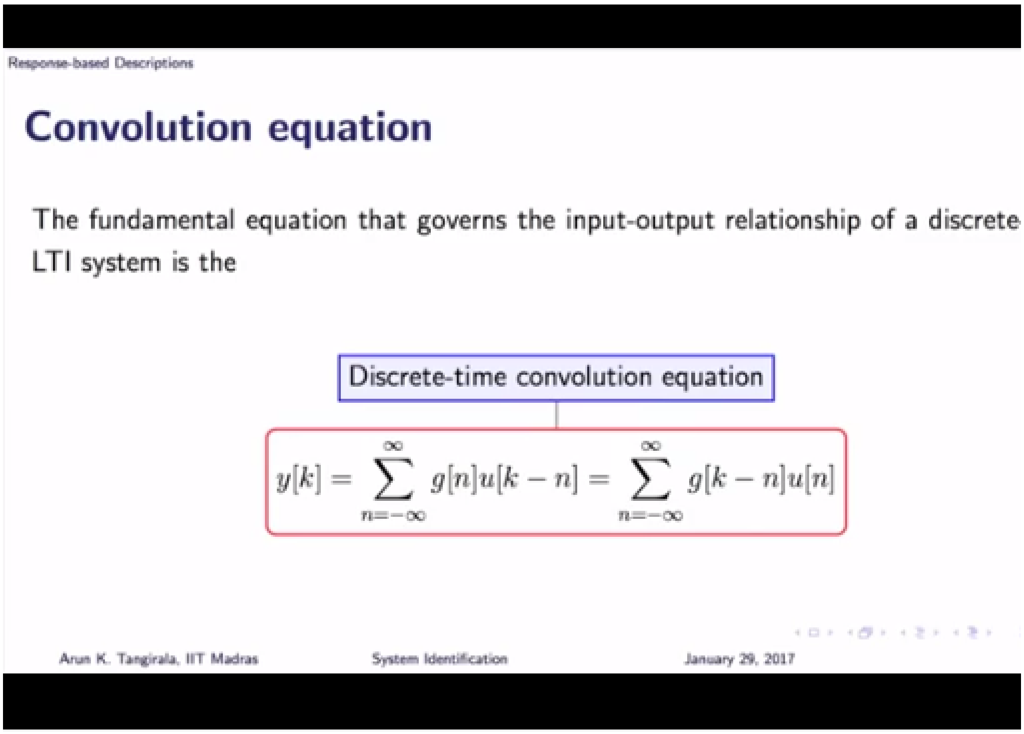

Superposition implies weighted. It's a linear combination of inputs from the past, the present, and the future. Now, if you look at this model carefully, you see that it allows you to calculate outputs, given the impulse response coefficient, so called Gs that we have been talking off. And the inputs, of course, you know, there's no point in asking the question, what will be the response without specifying the input. But that is one viewpoint that is generally used in linear systems theory, where I say, if I'm given the convolution equation, I mean, sorry, if I'm given the impulse response coefficients, then I can calculate the response of the linear time invariant system to any other arbitrary input. We have already defined in the last class, what linearity is and what is timing variants. We shall first quickly understand how this equation itself comes about, right? And then discuss the identification standpoint of this equation. What I've just said, couple of minutes ago is that, this equation tells me how to calculate the response of the system to an arbitrary input, given its response to an impulse input, that may not be the one that we are seeking. We may have another viewpoint of this equation that comes into play and identification. But before we do that, let's see how this equation comes about. Purely from the linearity and timing variants property. For this, let's actually look at the input itself.

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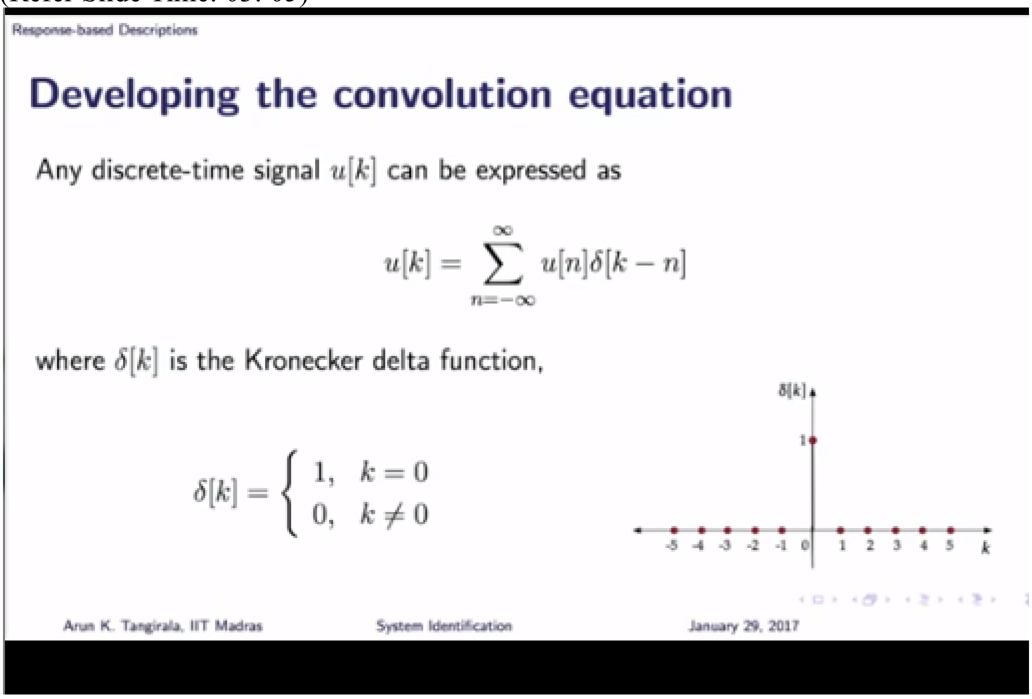

Okay. So I've already spoken about the interpretation, but we'll come back to the interpretation.

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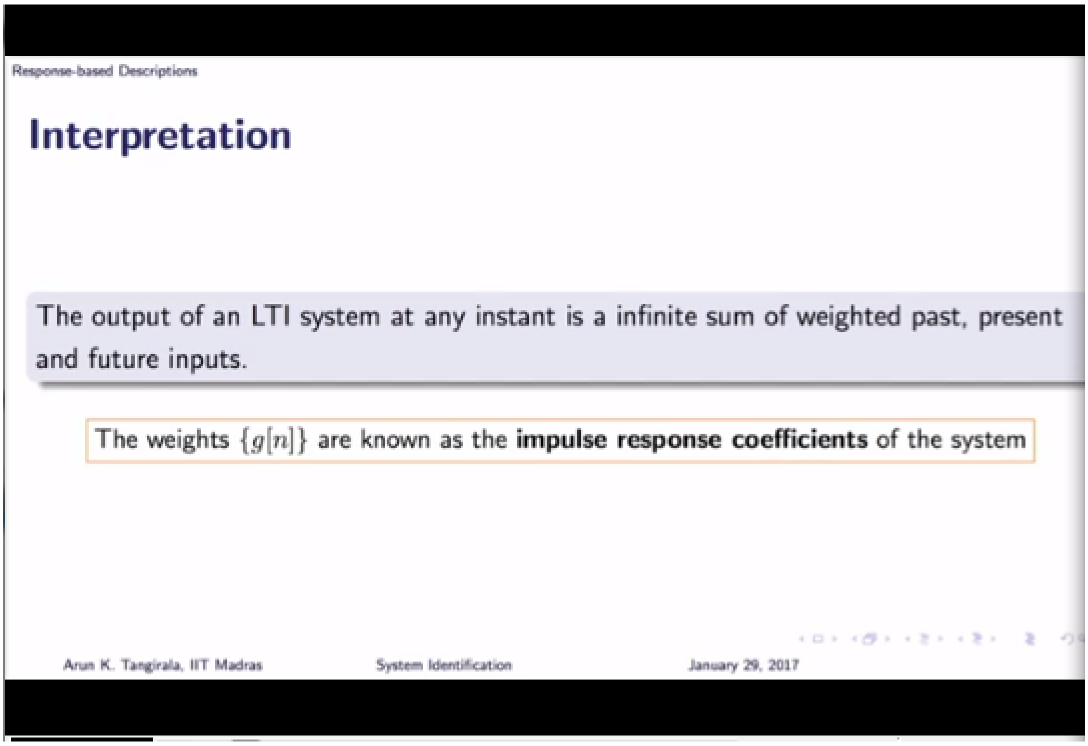

Let's look at the input sequence and see how it can be represented in terms of impulses. See, the linearity property tells me that if I know the response of the system to an input, then the responses system to escape input is simply the scale version. That is what is linearity property of the system telling me and then the time invariance tells me that if I shift the input by a certain amount in time, if I instead of giving it today, I give it tomorrow, then the response is also going to be shifted accordingly in time. These are the two properties that we are going to use along with the equation 2. So step one is to express our input itself in terms of this impulses, this as discrete time impulses, these are not your continuous time direct delta functions which are not physically realizable, they are only mathematical idealization. This Delta here is also known as a Kronecker Delta. The figure here shows you how an impulse centered are located at the origin looks like, we'll assume without loss of generality that this impulse is a unit impulse.

What this essentially means is that the signal is that there is some punch at an instant in time and after that, before and after that nothing happens. So any input that I can think of discrete time input that I can, that I have can always be represented as a linear combination of scaled and shifted impulses. That is what is the equation two means. How is that possible? Let's just take an arbitrary input. So let's take an arbitrary input here. Let's say, this is how it looks like. Maybe it exists at negative times also doesn't matter. From the x axis we have sampling instance, y axis the input itself. Noticed that, since input is discrete we draw a stem plot, we do not connect them necessarily because the by definition display time signal does not exist between two sampling instance.

Generally, in software packages, plots connect these dots to give you a good visualization field but the matter of fact is, there is no signal in between sampling instance. So if I look at this input here, arbitrary input, each of these stems here are actually impulses. Correct. Scale by a certain amount. So if I take the value of the input here, then suppose I call this as u zero, the value here as u[1] minus 1, 2, 3, 4. I have u[2], u[3] and so on. So when I look at the value of input at time zero, this is an impulse located at the origin scaled by u[0]. When I say impulse, unit impulse. Likewise, when I look at the input, let's say at [2], it is an impulse as well located at 2, but scale by u[2].

Now this located impulse, located at two is what is being represented as delta k minus n in the equation. Okay. For example, delta k minus 1, where would it be located at? So if I, if I were to expand sorry here, if I were to expand this, I would have terms all the way from negative infinity to positive infinity. Let us write a few times, let's say at n equals 0, I'm looking at the term corresponding to n equals 0. I would have use 0, Delta k. All right. Then I would have u[1] delta k minus 1.

So these deltas are actually positioned relative to k. So they are shifted by some amount. For simply simplicity sake, assume that I'm looking at k equals 0. So we fix k equals 0. So which means now on the right hand side, I would have u[0] I mean, on the right hand side sorry here if I were to start writing the terms let us say starting at n equals 0. At n equals 0, it have u[0] Delta 0. Then, plus [u]1, Delta at minus 1, right? And so on. And then of course, I have to the negative side as well.

Clearly, since this Delta function only exists at 0, by definition, the delta is only non 0 at the 0 index, strictly speaking, only one term prevents. But that is a point that you have to understand, although we have written this as a summation, the fact is that on the right hand side, only one term prevails. And which term prevails? Corresponding to k. Understand. So the summation writing is only for to get us to the convolution equation. But the fact is that on the right hand side, only one term will prevail and that one term will correspond to n equals k.

Because delta by definition is only prevailing at 0 at the origin. Okay. So why did we write this equation? I'm imagining now, the input to be a linear combination of scaled and shifted impulses. Correct? What will that, be get me? Well, since I have now imagined an arbitrary input to be weighted sum of scaled and shifted impulses. I will now invoke the two properties that the system has, linearity and time invariants. Time invariance tells me if I given input at some time and I get a response and I shift the put, the response also get shifted in time. If you look at this input here, one part of the input is shifted impulses. Other part is scaling. Linearity simply tells me that the response of the system to u[k] would be u m times the response to delta k minus n. Right? Linearity property tells me that the response of the system to a sum of inputs is the respective some of the responses.
What are the inputs that we are looking at, Delta k minus n. So the response to u[k] would be the weighted some of the responses to Delta k minus n. G is being exciting with u[k]. Right? Let me show you the schematic here.

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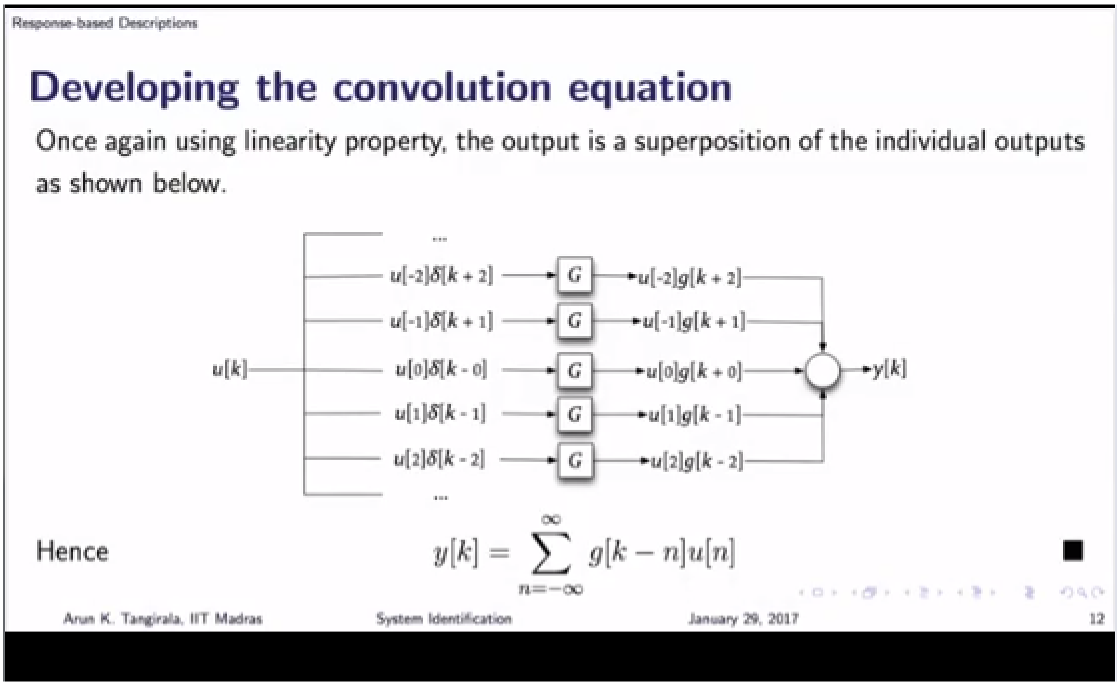

So this schematic will help you understand what's happening. I'm getting y[k] but now that I have imagined u[k] to be made up of these individual terms, instead of imagining u[k] to be exciting G, I imagine each of this is exciting G. Right? What would, for example, u at 0 zero here times Delta k minus 0 produced? It will produce u at 0 times G at k minus 0. How did I come up with this answer? Why does u 0 times delta k minus zero produce u 0 times G, k minus 0. On what basis did I write this answer here? First thing I know is Delta k producers gk, that is the definition of impulse response, remember. Delta k produces Gk, I know that. That is the definition of impulse response. If I excite the system with an impulse at time K, it will produce a response gk.

So, if I were to excite a system with delta k minus 1, what will it produce, by time invariance? G k minus 1. If I were to, so that is one answer. If I were to excite the system with u 0 times delta k, what would be the answer? U 0 times gk. That's all. That is why we have written those answer responses there. Simply, we have broken down the input into impulses, and used the linearity and time invariance property. Now, since the input is some of the respective terms here, okay? The terms that you see here. You can say input is a some of these terms, output is also going to be the sum of these responses. Why again, because superposition principle. That's all. So that is how now we come to the convolution equation.

This is a very simple way of developing the convolution equation. You seem to be a bit lost. That's okay. Are you? Do you understand? Oh. So if you don't understand, all you have to do is just go back to this basic definition of impulse response and use this fact equation two, look through the schematic like we just argued delta k producers Gk. Therefore, delta k minus n will produce the g [k minus n]. That is fact number one, by virtue of time invariance. And by virtue of linearity, u n times delta k minus n will produce u n times g k minus n. That's all. Okay. Very simple derivation. Okay. So now let's talk about the convolution equation itself and its role in identification. How do I deal with this and all the related matters? So the first thing that we notice is that this convolution equation is actually characterized by impulse response coefficients.

And what this tells me is, if I know the response of the system to an impulse just to 1 input, that's the beauty of an LPA system. I can predict the response of the system to all other inputs. Sounds a bit strange, but that is what the linearity and time invariance is getting you. If you, if this system does not satisfy any of this, either linearity or time invariance then you can't use this convolution equation. Okay? All right. So that's the one thing. But apart from prediction, the impulse response coefficients also get to a lot of other information about the system. For example, they tell you whether the system is causal. What do we mean by causal system?

System which doesn't need different on the nature [16:16 inaudible].

Correct. So how do, correct. That's correct. Assist the response, if the response of the system at this instant does not depend on the future input, we say that it is causal. Many manmade systems, in fact, almost all of them are causal, but many living systems, maybe non-causal. That is, I may start responding to an input that is yet to happen, right?

I imagine something happening or I know that something's going to happen in the next minute, and I'm going to react now. Or if you take the traffic light system, this guy starts moving even before the system signal changes from red to green. That's a non-causal behavior. So there are several systems like that. Now, can you say, what can you say about the impulse response of a non-causal system? How will it look like? Or a causal system does not matter. What can you say about the impulse response of a causal system? Anything? That you can say it look weird or it won't look weird. Anything.

It will not go beyond [17:47 inaudible].

What is that mean?

Impulse is responsible for [17:52 inaudible] system. No [17:52 inaudible]within the lights have more

Can you put it in a better way?

N is always [17:58 inaudible].

Can you put it in a better way? All of you're trying to say the same thing, both of you. Go by the definition of causality of a system. The system will not respond to a future input, so which means if I observed impulse responses system at negative times. If nothing is specified, always impulses introduced at times zero. And by definition, impulse does is zero value at negative times. So what should be the response of the system at negative times, if it is causal?

Zero. Because it cannot, although it may know that there is an impulse that is coming, it will not respond. 'Cause it's causal. When the impulse acts on the system, then it starts responding. Okay. What this means is that the impulse response of the system remains zero at negative times, if the system is causal. That's what they're trying to say to the left, it's going to be zero. To the left of what, it's hard to say. All you say is that the impulse response of the system is zero at negative times, because it will not respond to an impulse that's going to occur in the future. What happens after the impulse is given, we'll talk about that shortly. But what we know for sure is for a causal system, G is 0. At negative times.

Within this, there is a subclassification. If the impulse response is 0, even at the origin, and thereafter, non-zero doesn't matter. It will be non zero thereafter because there'll be some response but at 0 also, there is no response. Although you have given an impulse. We say it's a strictly causal system. That means, it will take at least one sample time to respond. Okay? So strictly causal if, the equalities also included. Otherwise we say there is an instantaneous causality. At k equal 0, there is a response. Okay, that is one feature. In other words now, impulse response not only allows me to compute the prediction, but also helps me in characterizing the system, whether it's causal or non causal.

Can you tell me anything else? Anything more. We said we'll look at the impulse response after it starts responding. Now, there are two possibilities. The impulse response can either run away with time, it can blow up or it can respond for a while and start decaying, within that there are two possibilities. But at least, broadly speaking, there are two possibilities. One that the response decays goes to zero, whether it decays in finite time or asymptotically, that's a matter of discussion for a later time. But it goes to zero or it doesn't, it blows up. What do we call such, stability. Right. So we are referring to the stability property of the system. So the impulse response determines both the stability and causality property of system.

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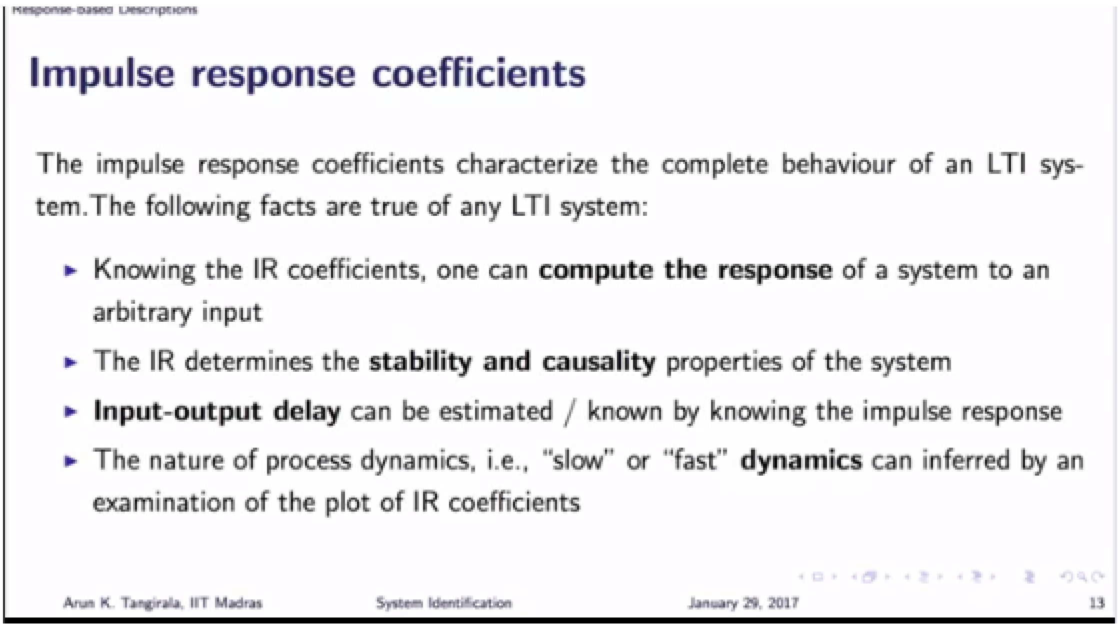

So the impulse response determines both the stability and causality property of the system. In fact, I'm just giving you as far as stability is concerned, I've only spoken about the qualitative, in a qualitative sense but I will give you a more qualitative requirement for, quantitative requirement for the stability condition. But we know definitely that the impulse response can tell me about the causality in the system, stability in the system of the system. If the impulse response starts to run away with time, then the system is unstable, obviously. Okay?

In all of this, you have to understand that we are dealing with deviation variables. So when we say impulse response goes to zero, it doesn't mean that the system becomes quiet, no response, nothing. It just comes back to its original steady state around which you've introduced a perturbation. You have to keep telling yourself that. So if I have a pendulum and I give it an impulse what happens to it? First of all is it a causal system? Is a pendulum standing there and I'm here, as I'm approaching unlike a mosquito, you know, what mosquito does? Even before you hit it, it'll fly away.

That's non causal, but it's good for survival it has it to be non causal. For our survival, it better not be. But pendulum inanimate thing until I go and hit it, it won't move. So obedient. Okay. So it's causal. Is it strictly causal? No. It just instantly response unless you know I give a punch somewhere the waves have to travel and hit them impulse [24:00 inaudible]. That's different. But if I'm directly hitting the pendulum, then it's going to be it's going to respond immediately. Okay, good. Now what about stability? Is it a stable system? I mean, does it impulse response decay to 0?

Conditionally and that is [24:24 inaudible].

Why what is conditionally What? You signed some agreement to the pendulum. What is conditional about it, man?

Impulse is closed [24:31 inaudible].

Okay. We'll talk about that. That's a good point that you made. Okay. Jokes apart, that's correct. It's a good point that you made, right? I mean, you can expect the Hulk to give an impulse and expect the pendulum to be coming back to its equilibrium. We're talking of ordinary mortals, okay. Fine, no, but that's a good point. So let us say that we have not really hit it so bad that it cannot come back. So we know it is stable. So impulse response will decay to zero. How it decays, we'll worry about a bit later on. What about an inverted pendulum? Have you seen an inverted pendulum anywhere? Have you seen an inverted pendulum? Never? You should see, you should go to our control lab or engineering design has one. Seriously.

There is an inverted pendulum, this inverted pendulums obviously they are causal, but unstable. Unstable doesn't mean that if you hit it like it will go and hit someone already go away, fly in the air and so on. It never comes back to its equilibrium position. Assume by some magic it is actually standing like this, and you stop it or even, [25:56 inaudible]foo, that's also an impulse but of a different magnitude, right. It'll just go back, it'll go to some position and never ever come back like this, you know, this very melodramatic dialogues and soap operas and Bollywood's Phir Kabhi Main Tujhe Nahi Milungi, something like that. So it will just, that's it. It's an unstable system. How do you visualize impulse response of such a system? Will impulse response go to infinity? Interesting, right? Sorry?

[26:36 inaudible] changes.

Good. Because it's a constrained system, impulse response will not go to zero. It won't blow to infinity, but it will lead some steady state where it will not go back to its equilibrium nor it'll go unbounded. Okay. So there are systems like that as well. But we still call them as unstable. Within that there is a special class, we'll talk about and called integrating systems and so on. But look at this. With the help of impulse response, I'm able to say so many things about the system. If I give you an impulse response plot, in other words, you will be able to comment on the causality, you will be able to say the system is stable, on top of it, you can also figure out what is the delay in the system. Correct?

Whatever time it takes for the system to produce the first non-zero impulse response is the time delay, if a delay exists in the system. And on top of it, I can compute the response of the system to any arbitrary input. So, there are so many advantages of this impulse response based description. Good. So, these are just now mathematical formalization of what we have discussed, as well as causality is concerned. And we will be dealing with causal systems by and large, but you must also be aware that there are many applications in which we may deal with a non-causal version of the convolution equation. It's very important to be aware of that. What I mean to say is, many a times you may have to work with negative values of M here as well the index.

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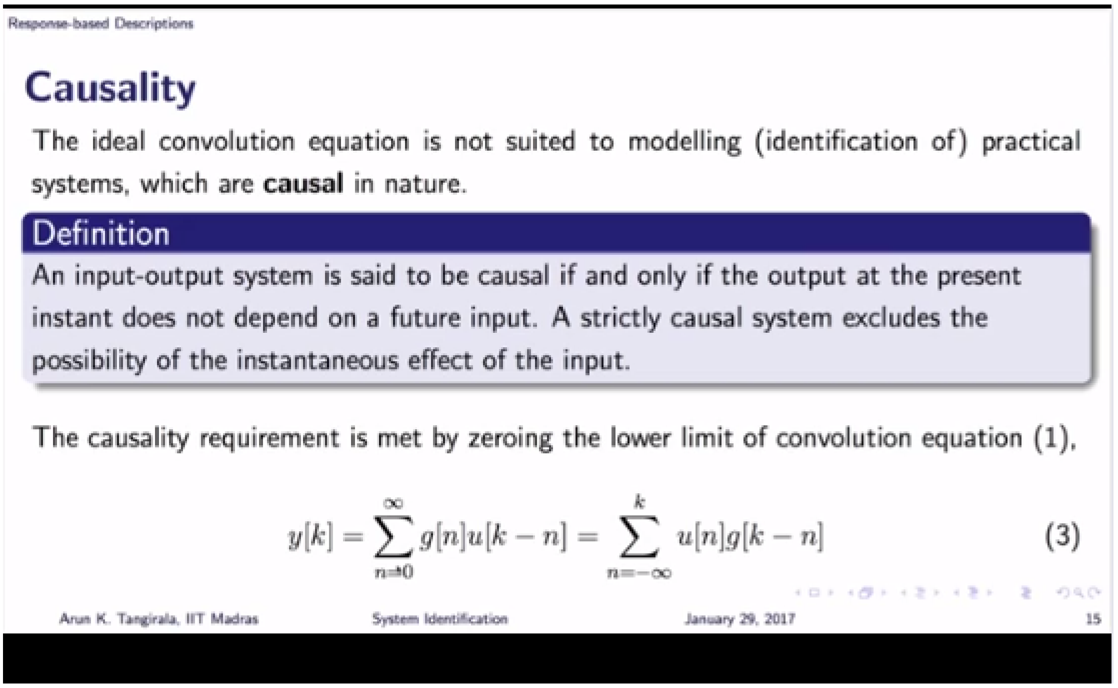

Can you think of one application where I may have to use the, sorry, non-causal version of the convolution equation? This is a causal version where we have ignored all the terms to the left of M, to the left of the origin because g at negative times zero for a causal system. But, is it possible in some application I have to use a non-causal version of the convolution equation? Can you think of one application? Let's take a very simple example, right?

Suppose I am looking at a system like this, which is described as follows y[k] is g[n] u[k-n] and running from, let us say minus 1 to 1. Very simple. Okay, this is an impulse response. This is called a finite impulse response model because impulse response is zero, beyond minus 1 and beyond 1. The specific values of g don't matter here. Suppose there is a system which is described by this kind of an equation and I say it is possible that we may use a physical system that has this kind of a description, can you name at least one physical application, whatever it comes to your mind that has this kind of a non-causal version? What does non-causal mean here?

That means a response at this instant depends on the past, of course, present, definitely, but future also. So I'm standing at k and I'm looking to the left, which is the past, I'm looking at this instant what input is being presented to the system, also looking to the right. Any application you can think of? Here is where we need a broader perspective of what a system is. You should not think of a system all the time as an input, a physical input and a physical response, and the response of that physical system. It does not have to be necessarily that.

The notion of a system is far more broader. It is any device that takes in some signal and response. Okay, that takes in some input and processes it and gives it back to you. We have used the term processing, that processing is here, of course, through the convolution equation. And one of the other viewpoints of system is that it is a filter, right? Why do we say it's a filter? Look at the equation. Convolution equation, if you look at it slightly differently, it is some kind of averaging of the input. Am I right? It is some kind of averaging, right? I'm doing a weighted average of the… at any instant k, I'm taking the inputs to the left, the present and to the right and constructing a weighted average.

So, from that viewpoint, what happens when you average something, when you average a signal? The high frequency fluctuations can manage. Suppose there is a signal, is a lot of different components, a very fast changing competent, and then very slow one. Averaging smoothes out the fast, rapidly changing fluctuations. From that viewpoint, the system is acting like a filter. And this filtering perspective is extremely important. Within this filtering, within this broad set of filters that you have, there are a special class of filters called smoothing filters. Okay, there are two others, but we'll talk about this smoothing filter which is used extensively.

Let's take a very simple example. Suppose I go to a movie. Let's talk about Bollywood movies because there is a lot of predictability. Okay. And there is as much as we may look down upon, I'm not advertising for Bollywood movies but as much as we may look down upon Bollywood movies, you should be really thankful to them, because even if you come 15 minutes late, half-an-hour late, or you have to go out for 5, 10 minutes, get a popcorn, whatever it is, you can own ways recover that missing information. Okay. And if you are a veteran, you may even be able to reproduce a script exactly, if not the plot alone, right? How are you able to do it? So, suppose I've just gone out to get, maybe, a popcorn, or a sandwich, or whatever, whichever is you consider healthy and come back, I missed out, right? What do I do? Without asking anyone how do I recover, figure what has happened? How do I do that?

[34:12 inaudible] You refer the...

So, I know what has happened before I left. Right now, I know what is happening, right? And I know what is coming up because I'm going to sit there and watch. Whereas the instant… so that means with respect to the instant that I missed, I have information to its left, I have information to its right. I'm going to use this information and recover. I'm going to construct… I'm going to interpolate and say, "Yeah, this must have happened when I just went out." That is smoothing. When you're looking to the left and to the right of the instant, that you're interested in, and trying to recover what has happened, or you don't even have to go out, you're sitting in the movie hall and as the movie is going on, if it's a very popular hero's movie, then there are always people shouting and screaming and so on. They're so happy to see their idol on the screen and so on. They just cannot contain their excitement. So, many-a-times you don't get to hear the actual dialogues.

You must have heard something to the left before and you'll hear something to the right. And you have some information that you heard, that is your input u. Whatever you are hearing before, now and after, are all the inputs and you're processing that in your brain and you're trying to reconstruct what that dialogue must have been, what the hero must have said at that instant in time. That's a non-causal operation you must understand. That means I'm relaying on a future information to reconstruct something in the past, I mean, looking at the past future, everything I need all of that to reconstruct based on past alone, if I try to recover, then its prediction. So, when we talk of estimation theory, I'll talk of these three different problems, prediction, filtering and smoothing.

Smoothing is a non-causal operation. And in all such smoothing operations, it's an offline operation. That means you cannot do it instantly. You have to let time pass by and then only you can do it. So, in all such smoothing operations, you will have to deal with non-causal systems. Okay. So having given you some idea into where you would encounter non-causal systems, let's move on. We will again come back to the causal world. Here, you have the equation for the causal world. $y[k]$ is now a summation of $g[n]\, u[k-n]$ with n running from 0 to infinity. We'll skip this slide. We've already said what a strictly causal system is and so on. And an example of a strictly causal system is this one.

Notice that the impulse response is 0 until k equals 0. Until, it's only non-zero from K equals one onwards.