CH5230: System Identification

Response-based Description 4

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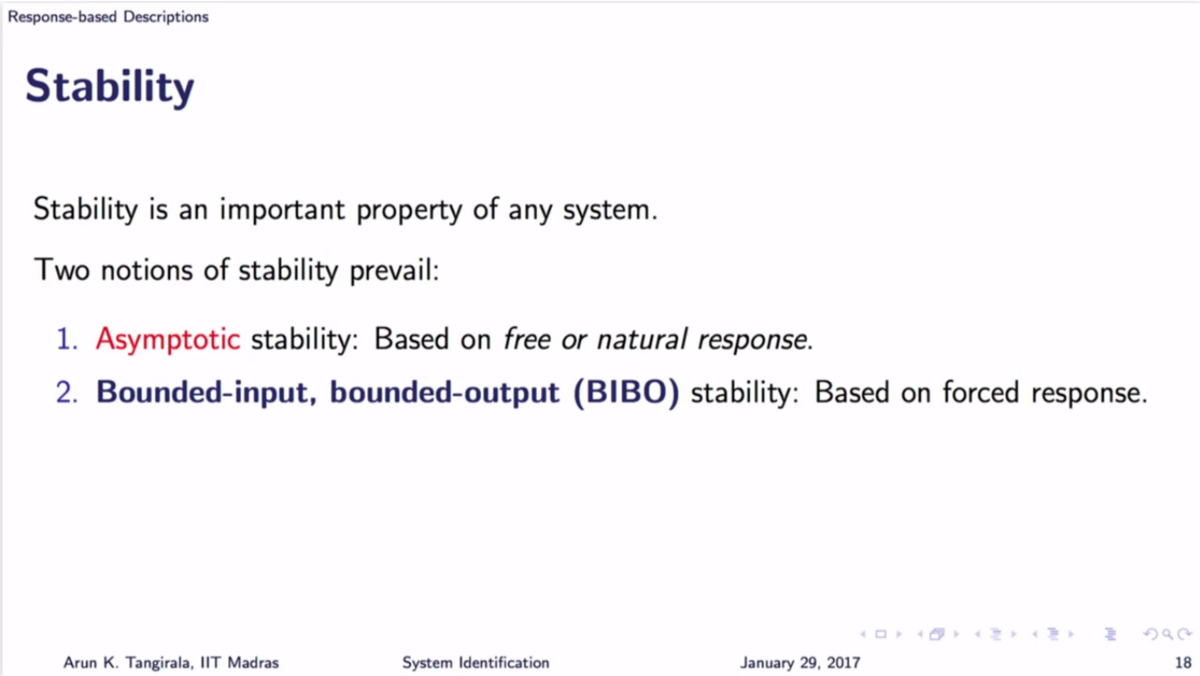

Now will talk on stability before we move on to talking of FIR's models. We have already spoken about stability but it is important now in [00:26 inaudible] To recognize that there are two notions of stability that are prevalent in the systems theory literature to be linear system or nonlinear system. Why is stability so important in identification? Why is causality important in identification? You should ask yourself. Why is causality important in identification? Why? Okay. Anything else?

Sorry.

[1:08 inaudible]

Okay. Anything else reduces the number of unknowns fix the model. Well, identifying is about knowing the system and one of the things that I need to know about the system is whether it responds to the input before it is excited, right. It's a part of your model. It's an important part of your model. So there is no question about not knowing causality. That's one of the foremost things we need to know. Then only the questions everything else comes in.

So there is not much discussion required on that. How would stability? Why is stability required? Why do we need to discuss stability in system identification? We should be only talked about them some linear system theory and so on. What is stability got to do with system identification? Any idea?

[2:14 inaudible]

What if you've told the system is unstable? Does it matter to you? So imagine, I have a pendulum. I have an inverted pendulum. Does it matter to you which system it is as far as identification is concerned or it matters. It matters, in what way?

Study state.

Okay. What happens to study state?

[2:44 inaudible] for the system it will be previous study statement and for [02:49 inaudible]

No. Stability. Why is stability important in identification?

[2:54 inaudible]

I'm sorry.

To determine what inputs we are using.

Okay.

Can you elaborate a bit more on that.

Like, we will know that for a certain input the system will become unstable. So [03:07 inaudible]

Unfortunately for linear system [03:10 inaudible] unfortunately, for linear systems stability is not a function of the input. Stability is it purely a property of the system. Only for nonlinear systems stability is not only dependent on the system or on the input that you give. For linear systems it is, if it is stable it is stable across the entire input space. He's got nothing to do with the input. Right. I mean you can easily prove that because of the linearity property.

If it is stable for a class of inputs it better be stable for this class of other inputs that are scalable. So stability is important in system unification because they need to perform the identification experiment, right. If this system is unstable, how can I even think of identifying it unless I take some additional measures, right.

What is involved in identification data. Data comes from an experiment. And in this experiment I going to excite the system with a few questions. If this candidate that I am interviewing with asking just one question, the candidates started behaving violently or runs away from the interview room. That's not good know. I mean, the moment you ask a candidate a question he runs away from the interview room and he never comes back. How are you going to ask any other questions. So obviously stability is important.

So if you know this candidate is going to run away, I'm going to lock the door first with all the legal permissions. I'll lock the doors with all the legal permissions and the candidate will probably run in the room. That's okay. That's all right. I can still ask questions, right or I, in case myself into a safe bulletproof thing and so on and then ask questions. So I will take precautionary measures. I will equip this system with something more to stabilize it. In other words I have to stabilize the system before I conduct the or I'll have like they show in Bollywood movies like four people will be holding. Okay. That is called stabilization, otherwise, if we let loose he will do something.

If the person is stable I don't have to worry about stabilization. I can ask questions and the person will respond. Exactly the same story identification. If is unstable I have to first stabilize it. And only conduct experiments under stabilized conditions. Otherwise it can be dangerous. It can be detrimental, it can do a lot of damage. Therefore stability is important. Clear. Now within the stability, there are two forms of stability. One is the asymptotic stability and other is that Bounded input Bounded output stability. One is based on the notion of free or natural response. I don't know how many how many of you have taken a formal course in linear system serial systems theory.

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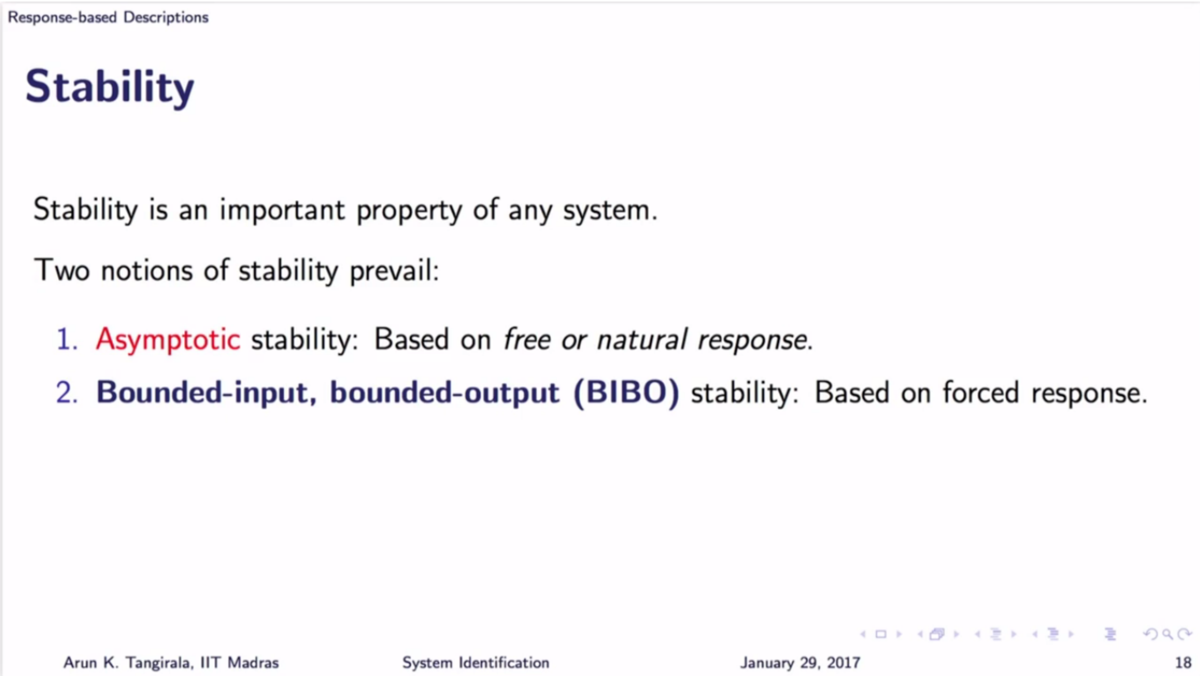

If you turn to systems theory, where the responses of the systems are talked about, you will be presented with two forms of response. One is called a free response, by itself the system is perturbed and left on its own. It'll respond. Then there is a forced response. It will only, this is force response is the response that it gives when you keep asking questions.

In both cases the system's characteristics are important but in the forced response case the input also plays a significant role. And the asymptotic stability is based on the notion of free or natural response and typically you'll find in the literature, asymptotic stability discussed with state based kind of representations, says state of the system is perturbed initial state and left on its own.

Now if I take a pendulum it is at some non-zero location. When I say state is at nonzero, initially the pendulum is probably to the left extreme or somewhere and left, left on its own. whatever response it gives to this nonzero initial conditions is a free response. Asymptotic stability requires that given sufficiently long time the system should return to its equilibrium position, to a perturbation in the initial condition. Does the pendulum do that? It does.

So it's asymptotically stable. BIBO stability is talking about the forced response. It says if you give a bounded input, it's very careful, like Prem said now, it depends, it's conditional whether it'll come back. If a Hulk comes and hits it, it may not come back because Hulk doesn't know how to give a even probably a simple impulse input which is gentle. So BIBO system says, if you give bounded input and the output remains bounded then the system is set to be bonded input bonded output stable.

The BIBO's that you see on water bottles and so on. So, is BIBO stability quite different from a asymptotic stability is a question that comes to our mind. What about the pendulum? If I give a bounded input will it get me a bounded output. It should. Right. Of course there are physical constraints but let's not worry about the physical constraint, length of the thread and so on. And what material it's made of. But otherwise speaking yes, it should give me bounded output, if I apply a bounded input. So in that case, the system is both asymptotically and BIBO stable.

It turns out that, asymptotic stability and BIBO stability are more or less the same for a linear system. We are only talking of linear systems. They are more or less the same except under some special cases. We'll talk about the special cases a bit later. But otherwise if the system in fact you can argue that if a system is asymptotically stable, it is always going to be BIBO stable. The general result is, if a system is asymptotically stable it's always going to be BIBO stable but not necessarily vice versa.

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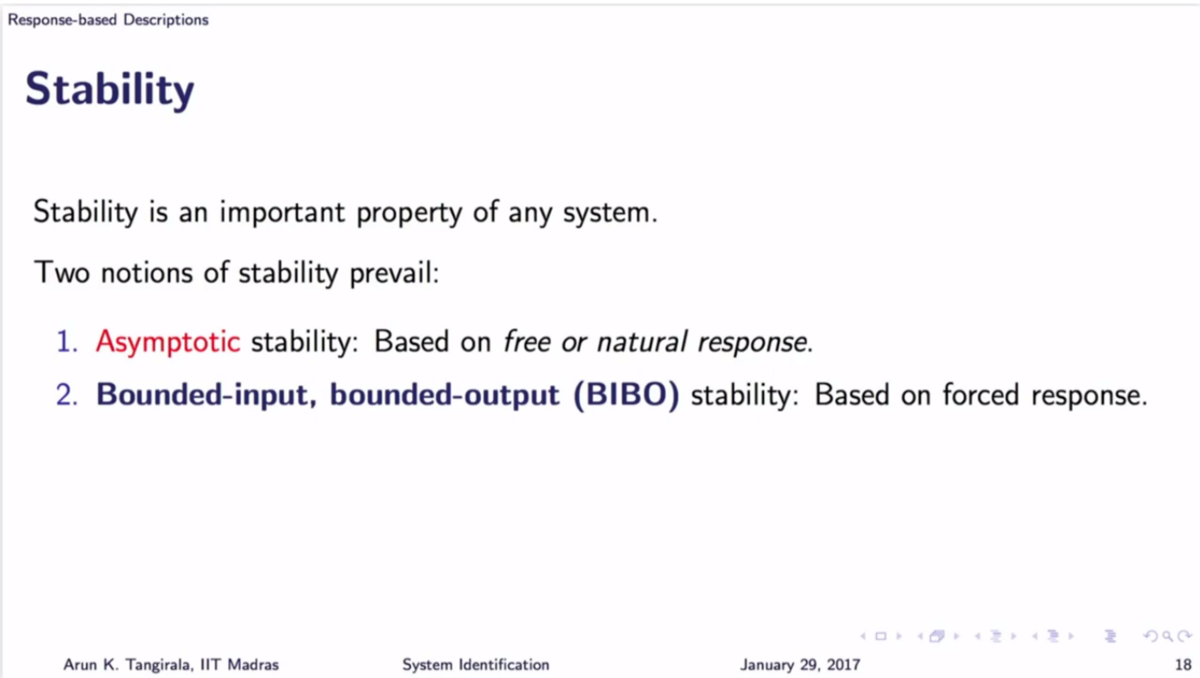

So what is the condition for asymptotic stability? Later on when we introduce poles and eigenvalues, we'll talk about that. As far as BIBO stability is concerned, a system is set to be BIBO stable, if and only if it's impulsive response is absolutely convergent.

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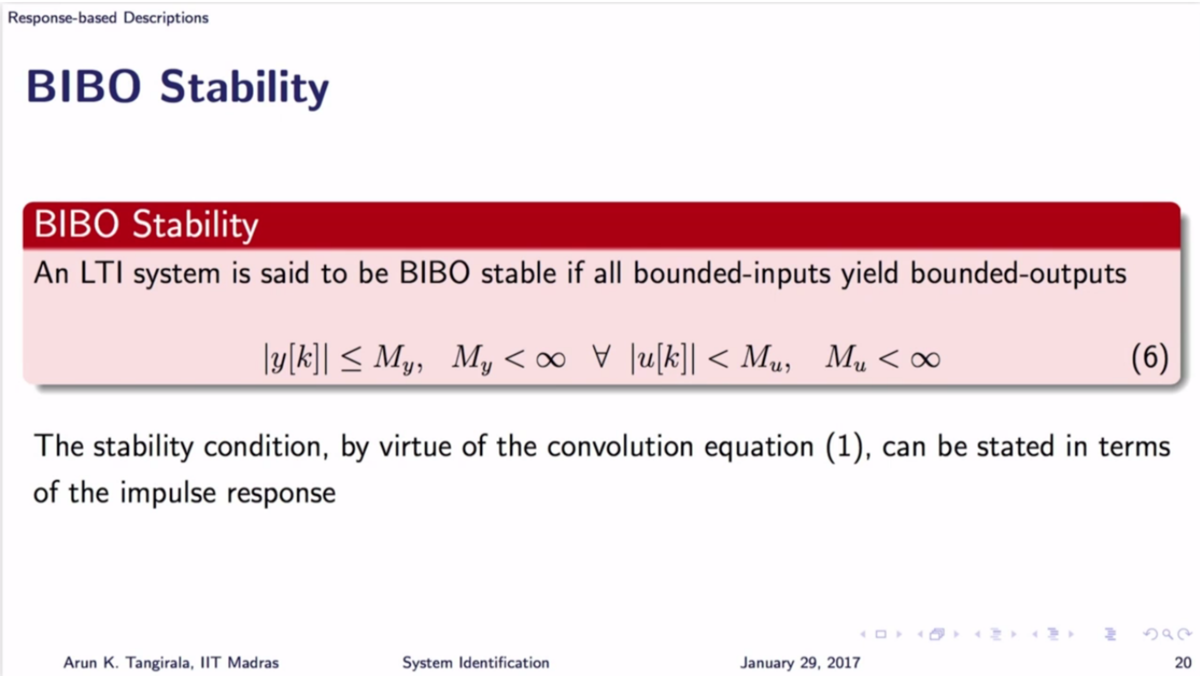

How did this result come out all of a sudden? Now we're talking of convergence of impulse response, absolute convergence of impulse response sequence. There's a sequence, right, g of k is a sequence. Why would this determine BIBO stability? What has impulse response got to do with it? Anny idea?

[10:30 inaudible]

No. No. But--

[10:32 inaudible]

No, no. Why should the impulse response play a role? Can you give me at least one hint? As to what is the connection between the impulse response and BIBO stability?

Okay. So--

The equation--

Which equation?

Convolution.

Convolution equation. When I'm giving unbounded input, then the response depends on the impulse response.

Correct. So that's a starting equation. In fact for proofs, you can actually see, in almost any proof the convolution equation is the starting point and then you require that the response be bounded and then you use the triangular inequalities and prove that this is both necessary and sufficient condition. Ultimately that the impulse response be absolutely convergent. There is a difference between regular convergence of sequences and absolute convergence, right. So your impulse response is actually going to look like this. Let's start from 0, g 0, g 1and so on.

So this the Kth coefficient, this has to be absolutely convergent. Because these impulsive responses can also be negative value. It makes a difference whether you look at absolute convergence or not. So why does this absolute convergence comes come into picture because we look at this convolution equation here and we require that, this be bounded above by M y, where M y is some not non-zero finite number for all inputs that are bounded. So if it produces a bounded response, then we say the system is bonded input bonded output stable. And it is only possible if the impulse response is absolutely convergent.

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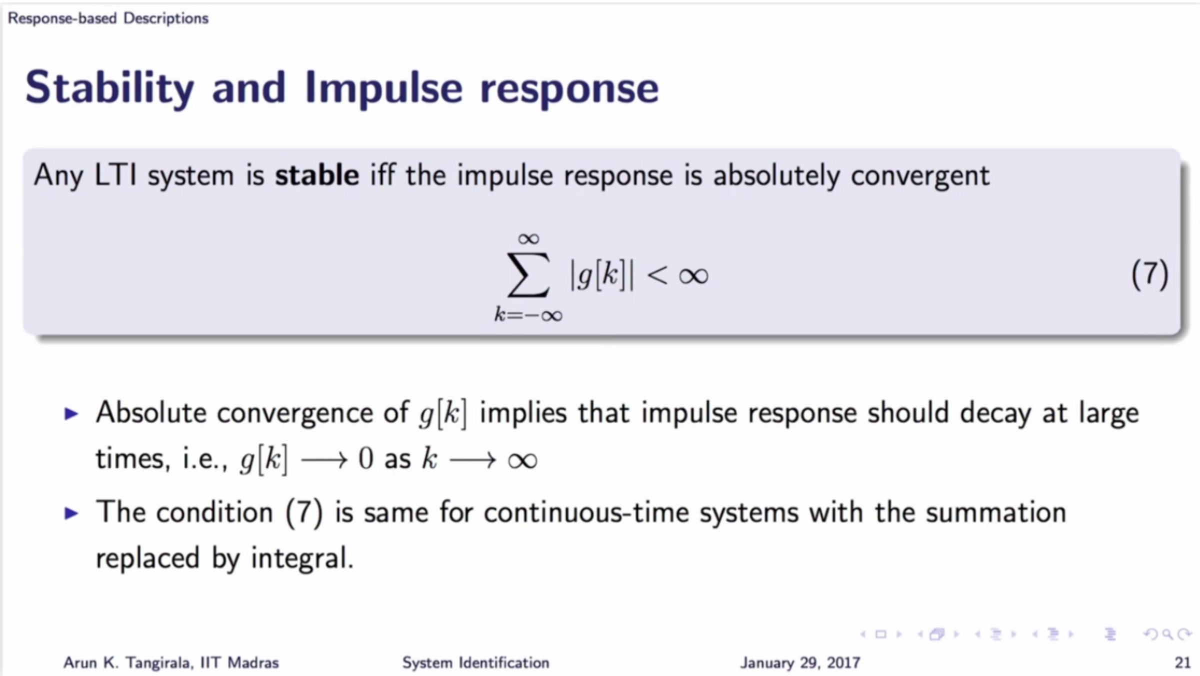

Now, I don't want to leave the result at that. I would like you to interpret this result. As in terms of how the impulse response should look like. You should get a feel of it, of the reason. For any sequence to be absolutely convergent, it has to necessarily have some property. What is that?

How should it behave at large times?

It should decay.

It should decay. Right. It cannot grow. First of all it should be bounded in itself. That's okay. We'll take that. Secondly the impulse response should decay. Not all decaying impulse responses necessarily give you that but definitely absolute convergence means it's actually decaying. And at what rate it should decay, we are not worried about it. g k goes to 0 as k goes to infinity. That is an implication. Okay. It may oscillate and go. It can do whatever it want but it should actually go to 0. Which means, eventually the system should go to its equilibrium, which is what is asymptotic stability.

So you see the connections. But there is a big difference in the asymptotic stability the setting is that there is a non-zero initial condition and the system is let on its own. Only the system characteristics will determine. Here, the system is constantly being excited by an input and the output is supposed to remain bonded. Both eventually rely on the same conditions. But what you should remember is that under some special conditions a system can be BIBO stable but not asymptotic stable. That means if you leave the system on its own, free response, it may not be stable, like your inverted pendulum.

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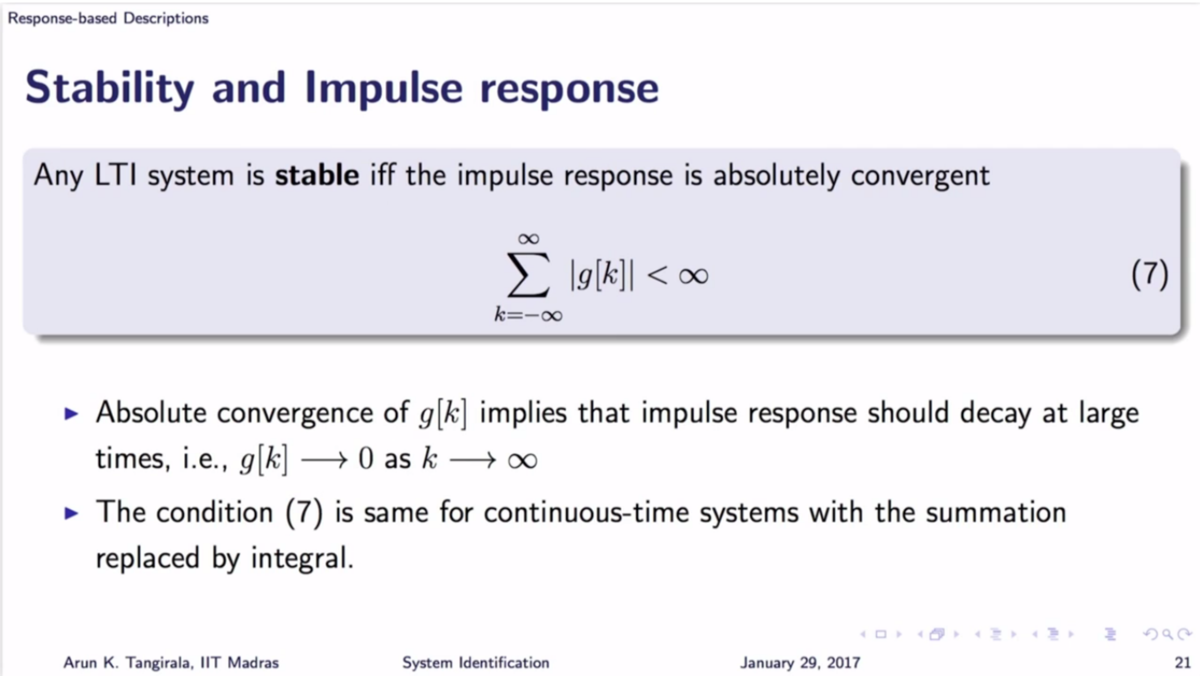

Is any inverted pendulum BIBO stable? What do you think?

Why?

It gives you bounded output. Sure. But if it is BIBO stable then the impulse response should decay to 0. Does the impulse response decay to 0? So it is not BIBO stable. But there are examples that you can give, which I'll give a bit later. Our systems that are BIBO stable but not asymptotically stable.

One thing you should understand is, from a systems theory viewpoint there is a subtle difference between free response and a forced response. Free response has got to do purely with how the system evolves on itself without any interaction with the surroundings. Whereas forced response involves an exchange of information, something, some kind of interaction of the system with the surroundings. So, this BIBO stability not being the same as asymptotic stability under some conditions means that there is somehow some wiring of the system with the input in such a way, that this wiring is not allowing the system to go unstable.

There is a system on its own and if you look at the response of the that, that's free response. Now, when you're looking at forced response, the system is externally, connected to the external surroundings to something. It could be a wire, it could be a pipe whatever but it is, there is something external to the system that's acting on it and it is responding to it. How it is wired with the input can play a significant role, particularly in the stability of the system. Okay.

And for unstable systems it is likely that you can wear it in a particular way, such that, it remains stable under forced response but it is unstable under when it's left on its own. I will only make such qualitative statements now. Later on, when we talk of poles and zeros, you will understand. You will be able to put what I say in a much better context. Poles characterize the system's own property. They're the roots of the system's characteristic equation, zeros of a system, when we talk of transfer function representation, you will understand what are zeros. They tell you how the system is interacting with the external environment.

So when a system is asymptotically unstable, we will learn that just got to do with the poles. Completely, the asymptotic stability simply rests on the pole locations, whereas BIBO stability also depends on the zero locations in addition to poles. And if there is a pole that is causing instability, I can with the proper wiring, cancel out that unstable pole with an appropriate 0. 0 occurs in the numerator, pole occurs in the denominator of a transfer function. That is why a system can be BIBO stable but not necessarily asymptotically stable. Okay.

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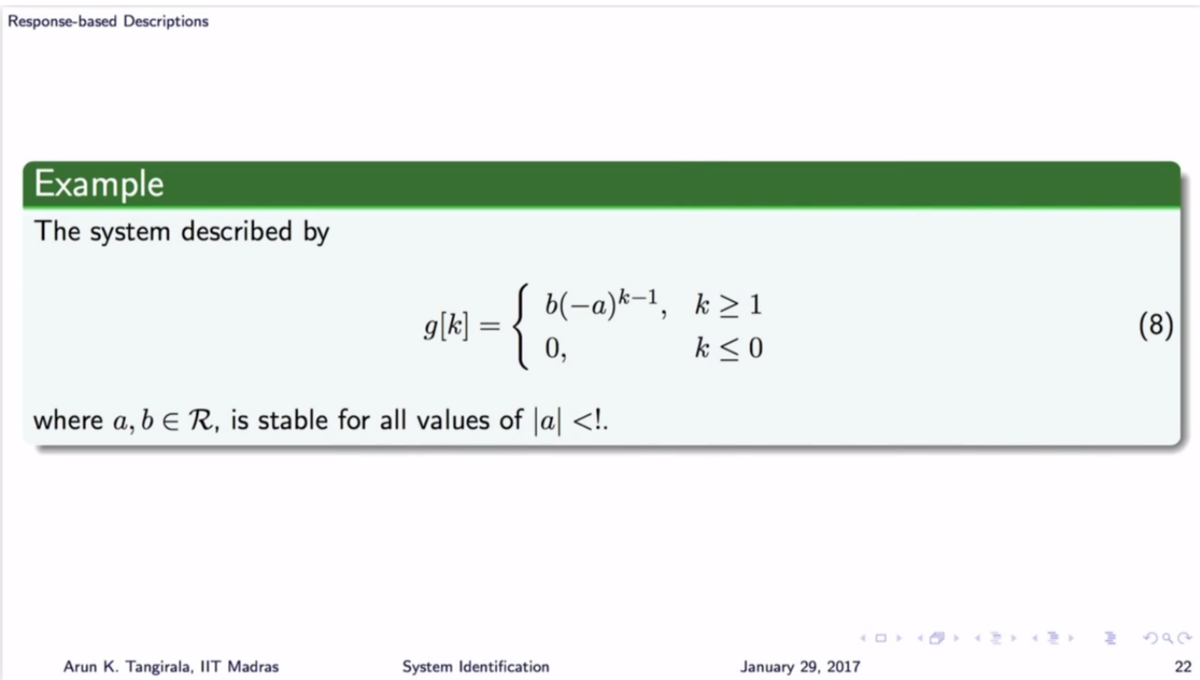

So this is an example here of a system that is BIBO stable, only when a is less than 1 in magnitude. It is the first order system we've been seeing this. It's a causal, is it's strictly causal? It's strictly causal and it is BIBO stable. Why is it BIBO stable? Because we have said magnitude of a is less than 1. Okay.

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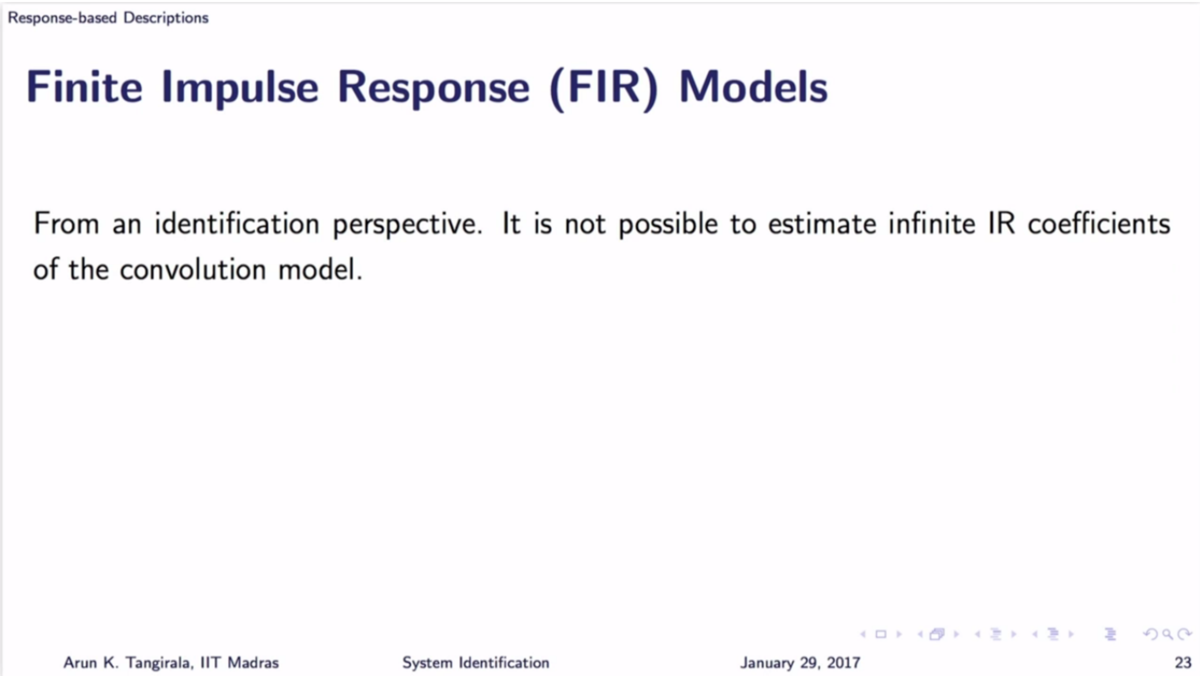

We'll just close today's discussion with just a quick preview of FIR model. So until now we have learned that the impulse response gives me quite a few valuable insights into the system properties, apart from allowing me to predict the response of the system. Now we turned to identification viewpoint. From an identification viewpoint. Let us say, I'm looking at this model, identifying this model here. Okay. So I'm just looking at this convolution model. And I'm given input data and output data. And I want to know, what are this impulse response coefficient. That's what we mean by identifying. But notice that there are infinite number of unknowns. That's the biggest hurdle that I have to cross.

I assume the system is stable for now. Don't worry. We have fixed all of that. So the first big hurdle that I have to take care off is. The fact that there are infinite number of g's. See stability means g should decay 0 at k goes to infinity. Did it ever say as in finite time it should decay to 0. No. So, stable systems can have a g, that either decay to 0, sorry, decay to 0 either in finite time or infinite time. Finite time, no problem. If g goes to 0 in finite time, what happens? This summation upper limit can I can change it to M. What about the infinite?

So, you'll find in systems theory, class of two, sorry, two classes of models. Infinite impulse response models, finite impulse response models. And they had their own advantages and disadvantages from a filtering viewpoint. From an identification viewpoint, if this system is an FIR model and has a FIR description, no worries. I just need to have sufficient input output data to estimate that many IT coefficients. But if the system is truly infinite impulse response model, what is the solution? You understand what I'm saying?

Stable system impulse response now decays to 0 only at very large times. That's what [22:11 inaudible] infinity. How do we identify such systems? What is the solution? Give me a one solution?

[22:26 inaudible]

Good. That's it. Everything is approximation only, right. Whatever I say you approximately understand. Whatever you're trying to tell me through your faces, I also approximately understand. Okay. And good. So we want to approximate. See mathematics has not only got some beautiful things to offer. I mean, It has got many beautiful things to offer. One of the most beautiful things it has to offer is the approximation symbol. Replays equally here. Seriously. Look how beautifully they're modified. It says, it's not flat. It's kind of [23:08 inaudible] which means it's approximately flight. Okay. So correct. So know we approximate but is this the only way. That's a good thing and that is what people will do, we'll end up doing. So we will force fit an FIR model to IAR system, if it is IAR. I do not know upfront if it is IAR or not. But is this the only way. Can I not exactly identify an IAR model.

Is it not possible to go about identifying an infinite impulse response model without this approximation business. Think. Think. Think. Is it possible?

[23:59 inaudible]

In what way. Give me a quick example.

[24:07 inaudible]

And then.

[24:14 inaudible]

Okay. Some where some ice has been broken. Okay.

[24:18 inaudible]

Okay. Why do I need FIR? I mean, of course, I need to know FIR. I will look at FIR. So, if you throw away the first part of your answer at least the answer to the question as to whether I can fit an exact model to an infinite impulse response system is, yes I can, through prioritization of g. The basic problem is if the system is IIR, what is the problem for me, infinite number of unknowns are there. But that is because the way I have written the model. Suppose I'm told that g has some equation, like the one that you just saw. B times minus A to the k minus 1. Suppose I'm told that.

Then I can do it, right. Because now I am mapping all the infinite unknowns to just to, how did that happen? Because I pumped in additional information. Without pumping in that additional information it is not possible to exactly identify an infinite impulse response model. That is the moral of this story.

Either I have to be told it's FIR, if I'm not told, I'm going to replace the equality with this beautiful approximation symbol or if you want exact things, identification of [25:47 inaudible] then give me some information about g. So that I can eventually whatever problem solving it should have finite number of unknowns. These are the two approaches. We will talk about the first approach that is approximating the system with an FIR description and then gradually go to the parameterization part. That's the subject of tomorrow's lecture. Okay. The topic of tomorrows lecture. Thank you.