CH5230: System Identification

Response-based Description 9

But let's go to this quick definition of Linear Time Invariant System from a frequency domain viewpoint. We made a remark that any Linear Time Invariant System will produce stable, stable LTI system will produce a sinusoidal waveform, after the transients have died down. Whenever the input is a sinusoidal but the most important thing is, it will produce sinusoid of the same frequency, only with amplitude and phase being modified.

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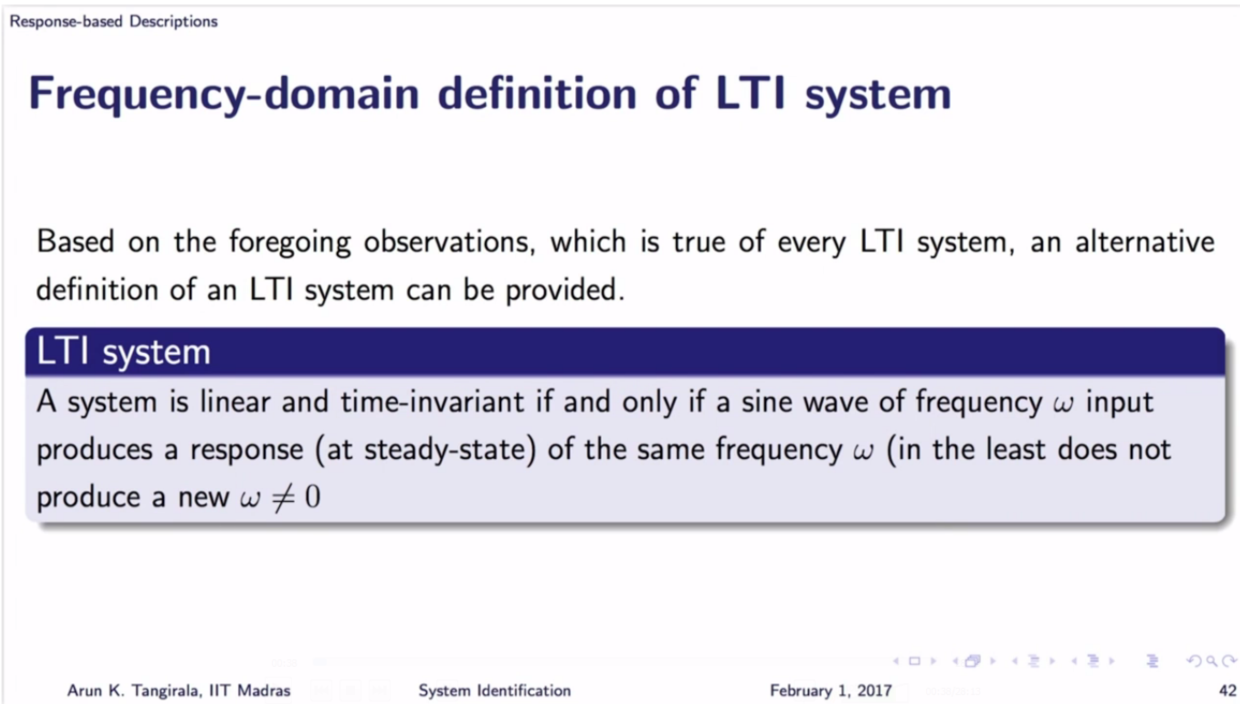

You can now state that as both necessary and sufficient conditions, a system is set to be stable LTI although I've not mentioned here stable, but it's implicitly understood it is stable an LTI, if the sine wave of frequency omega at the input produces a sinusoidal waveform of same frequency at the output after a sufficiently long time. This is one way of checking for LTI nature of the system. If I want to see how, whether the system is linear, in fact, this is used for testing nonlinearities and time varying nature of systems.

For example if I had a very simple system let's say y k equals u square k. Is it time variant or time invariant? Is it time invariant? What about linearity? Non-linear. So let's inject a sine wave at the input side. Right? What output would you obtain? We will get sine square omega. But you see always we look at frequency content from the eyes of Fourier. Right? It is like, I do not know what the frequency content is, but I'll turn to Fourier.

In many movies, if the hero is blind, heroine would come and say ,"See through my eyes. See the world through my eyes." Right? You have to make this subject entertaining. So you always draw parallels with entertainment. So it's like this, I do not know what the frequency content is, Fourier is giving us the eyes to see the frequency content. So from the eyes of Fourier. So like that here, you see from the eyes of Fourier , sine square omega appears as sine 2 omega or cosine 2 omega, whatever it is. But the fact is that when I look at the power spectrum, I'll see a peak at 2 omega in sort of omega.

Why that is happening? Because from the eyes of Fourier , the world is always made up of sines and cosines. You should not blame him. That is how the Fourier transform works. If you don't like it. Leave it. But that is how it is and all power spectral analyzers are based on that concept, imagination. So a sine of frequency omega from the eyes of Fourier appears as sinusoidal waveform of double the frequency. So new frequency has appeared in the output which is not present in the input, that's a sign of non-linearity

That's a mark of non-linearity. It's not always that the old frequency will vanish. In fact, if you had a cube here what would you see? What frequencies would you see in the output? Omega and 3 omega. Right? So you will see a fundamental and then a harmonic. Okay. So, just to tell you that any violation of linearity, I've only talked about violation of linearity, what a violation of linearity leads to. But you can show even if the time invariant nature is violated, you will see new frequencies appearing. When no new frequency appears then we say the system is empty LT.

Okay. We are only talking a frequency. You're not worried about the strengths and phases and so on. Any questions? Good. So we have talked about the filtering viewpoint. As I said there are four different filters, low pass filter, high Pass filter, band pass or band reject and all pass filter.

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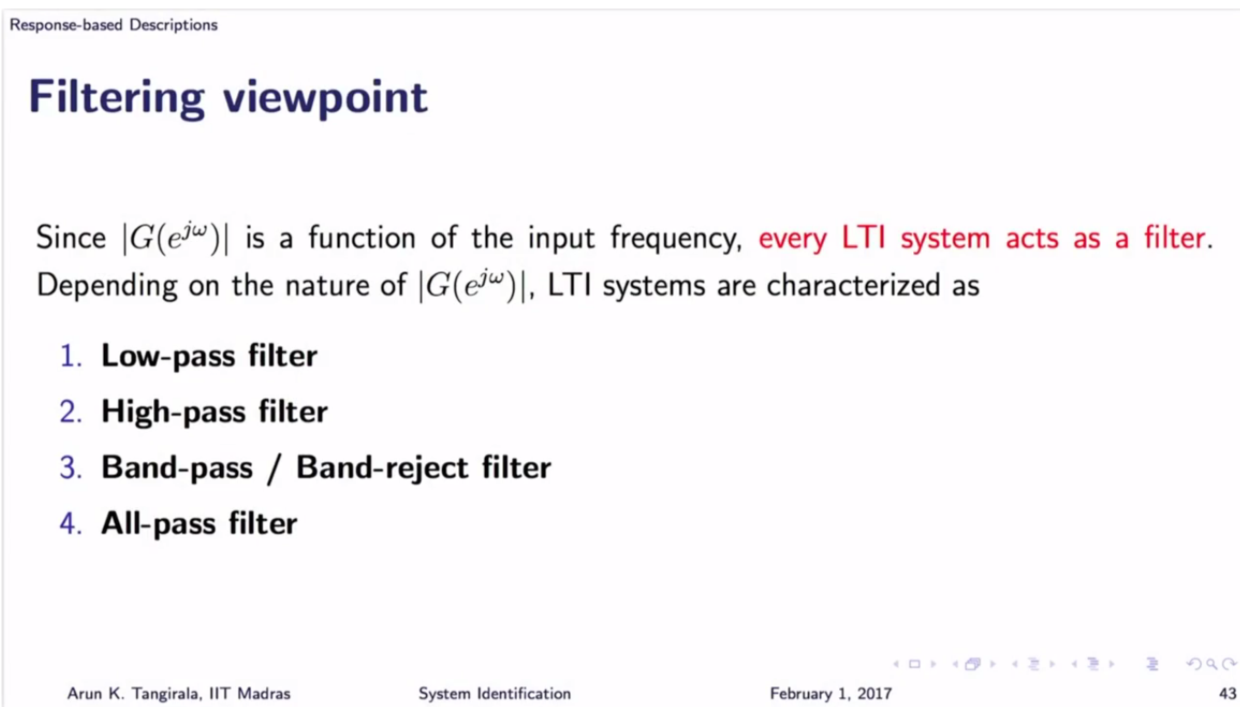

All these names are given based on how the amplitude ratio looks like as a function of omega. What kind of a filter is pure delay? All pass. It allows all frequencies to go through without any preferential treatment. Correct? So you should be familiar with all of this.

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Example: Frequency Response Function

Example: FRF

Consider the stable LTI system in (8). The FRF of the system is

$$G(e^{j\omega}) = \sum_{n=0}^{\infty} g[n]e^{-j\omega n} = \sum_{n=1}^{\infty} b(-a)^{n-1}e^{-j\omega n} = b\frac{e^{-j\omega}}{1+ae^{-j\omega}}$$

Thus an input of frequency ω will be amplified/attenuated and phase shifted according to

$$|G(e^{j\omega})| = \frac{b}{\sqrt{1+a^2+2a\cos\omega}}; \qquad \phi = -\omega - \arctan\left(\frac{-a\sin\omega}{1+a\cos\omega}\right)$$

Now let's look at this simple example, then I'll show you how to generate step impulse and bode in MATLAB. For a given LTI system. So it's a very simple example. It's the same system that we considered earlier, the first order system. Right? It says in equation eight. So let me take you back to equation 8.

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Example

The system described by

$$g[k] = \begin{cases} b(-a)^{k-1}, & k \geq 1 \\ 0, & k \leq 0 \end{cases} \tag{8}$$

where $a, b \in \mathcal{R}$, is stable for all values of $|a| <!$.

All right. So this is the system's impulse response description now. You understand now that with this definition that we have here, I definitely need to know the impulse response. Either it should be given to me directly or I should be able to infer it from the input-output equation. Remember that. So quickly, I plug in this g k into the definition of FRF. Always the starting point is that. When I do that, I end up with this result.

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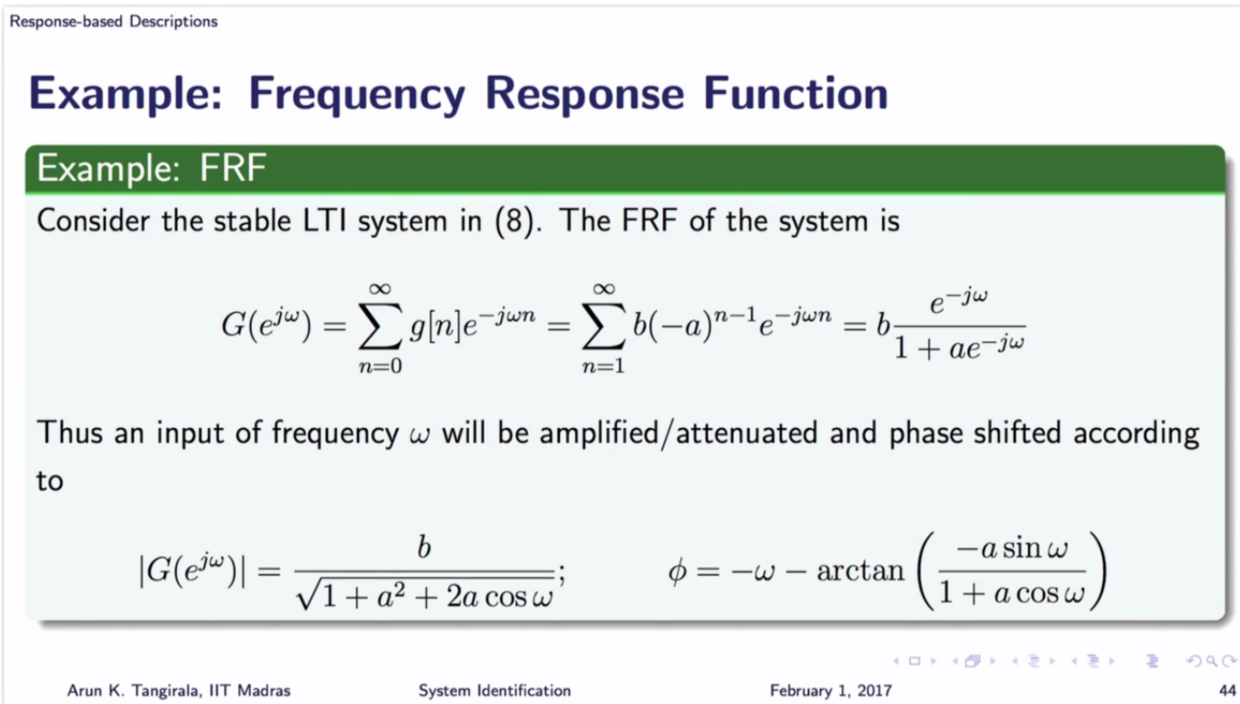

So that is the expression that I get. You should be comfortable with dealing with complex numbers. It's not as complex as you imagine. It's fairly simple. That's it. So assuming that mod a is less than 1. I'm already given that, I can now write a simple and a single expression for this summation and from this I can derive the amplitude ratio and the phase. So, all this pure algebra there and mathematics.

So magnitude is given by this expression and phase is given by this expression. When you are computing phase, please remember that it is inverse tan imaginary by real and you have to be comfortable with that. So if you're uncomfortable, please do go through some practice of computing phases of complex numbers. Okay. So if I were to plot this now, you should also be able to draw a hand sketch. When you are drawing a hand sketch, you should be able to, you know, you should just pick some key points on the omega like omega 0, omega pi by 4, pi by 2 for which you know the cosine values. Hopefully you remember. And then simply draw the amplitude ratio.

Phase is typically lot more complicated to draw by hand. But you can still obtain some asymptotic points in the sense as omega goes to zero. What is it at omega goes to pi and so on. Now comes the most important part of the FRD. One of the most important aspects of FRF particularly for discrete time systems that you should remember, you cannot forget without fail.

So if we look at this definition here, one of the things that clearly falls out of the definition is that the FRF is periodic. So G of e to the j omega plus 2 pi. From this definition I can straightaway see that it is minus j. Is that clear, because in place of omega, if I have omega plus 2 pi, I have an additional multiplication factor here e to the minus j 2 pi by k. What is e to the minus j 2 pi k? k is an integer. Zero would be nice one.

So we say FRF is periodic for discrete-time systems only, is periodic with a period 2pi. Again for this discrete-time LTI systems. You should remember this fact. You should remember it forever. All right. What does it? Why is this occurring? It is occurring because of the nature of the complex sine waves, e to the minus j omega itself is periodic with a period 2 pi. Okay. Why this discrete-time nature is making a difference.

In fact a lot of the theory that you're learning for discrete-time systems is equally valid for continuous time systems as well. You should remember that fact as well. Which means, for continuous time systems also there exist, a notion of FRF, where [10:01 inaudible] we say simply G of omega to distinguish, I will use big omega. And here the definition would be based on the continuous time Fourier transform.

By the way, what are the units of omega in all of this in our analysis until now? You can amplify it, you can amplify your answer. What is it?

Radians per second.

Radians per second?

Radians per sample.

Radians per second. It's not Radians per second. You're dealing with samples. If it was per second then you wouldn't have a problem with cancellation of units here. Right. You can you can show that also at a later time when I talk of sampling and discretization, at that time it will becomes clear, why omega has units of radians per sample in the discrete-time world. In the continuous time world, what are the units of this big omega? Radians per time. Whatever that time is, units of time but in discrete-time it's always radians per sample. If the sampling interval is one second then the units are radians per second.

So this result here clearly says that for discrete time systems I need to only look at FRF between minus pi and pi or 0 to 2 pi. Any interval of length 2 pi is sufficient. Typically, we consider minus pi to pi and the good news is, the amplitude ratio is symmetric. In other words, if I were to draw, sketch the amplitude ratio between minus pi to pi, it would look the same with respect to the origin. So I just need to plot amplitude ratio from 0 to pi. You understand?

You check for yourself. Why the amplitude ratio is symmetric. But always remember this fact that whether it's a discrete-time system or a discrete-time signal, if the discrete-times we plot FRFs only up to 0 to pi, that is amplitude ratio. And you can say, omega in the interval minus pi to pi. If it is a signal we carry out our power spectral analysis also in the same range. We don't, it doesn't mean it doesn't exist beyond. It's just a repetition. So there is no point.

So don't be under the impression that it's not defined beyond, this periodic. It's just going to repeat what's the point in analyzing. It is, this is not true for continuous time systems. You can see, if I ask you whether G of omega plus 2 pi, is it the same as G of big omega. Is this true? What do you think? Yes or no? How do you verify? Why so much silence? How did we verify this?

Substitute there. So substitute here. What do you will get here? You'll get an additional factor of e to the minus j 2 pi t. Will that be 1? That tells you the prime difference between the behavior of discrete-time and continuous time systems. Why? Because in continuous time, t is on the real axis. Whereas in the discrete-time k is on the integer axis. Right? Why is k on the integer axis because it's a sampling instant. So there is another way of saying what we have said just now. We say that sampling introduces periodicity.

Let me write that. Sampling in time, in fact, I'm going to actually avoid the time thing, because you will see shortly that this is true even vice-versa. Sampling introduces periodicity. Very, very, important statement to remember. I didn't say sampling in time introduces periodicity in frequency. I didn't say that. I simply said sampling introduces periodicity because you will see later that, we may also have to sample in frequency which introduces periodicity in time. This is a very important statement to remember when it comes to frequency domain analysis or in general.

Yeah you can say frequency domain analysis of discrete-time systems. Okay. So how do these plots look like? The amplitude ratio looks as follows. As you see on the screen.

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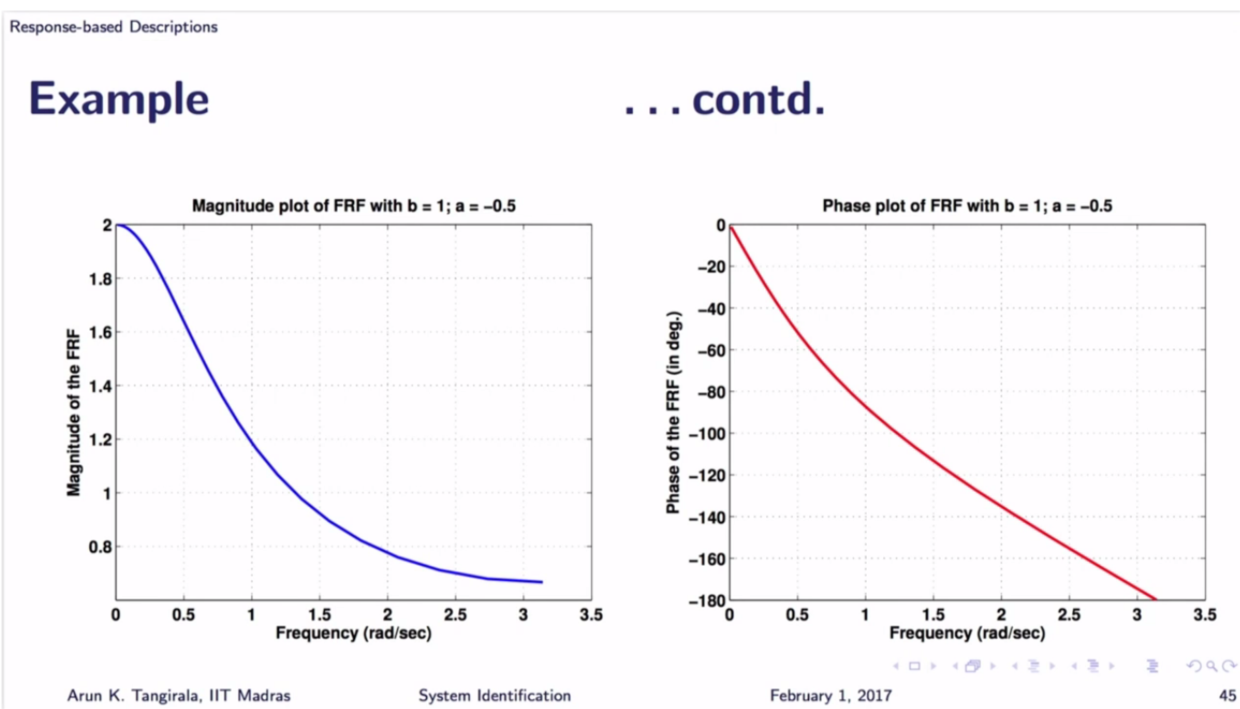

And the phase also looks like this here. What kind of a filter is this system? It's a low pass filter. Good. It's a first order system and if you change the value of a. So should play around with this example. I'll post the slides. So play around with this example and see how the amplitude ratio looks like when you change the value of a. I've given you the value of a here. Right? I don't know how well you can see. But let me zoom here minus 05. All right.

Change the sign and see what happens in your exploration. See whether the amplitude ratio changes whether it becomes some other kind of filter and so on. How do you generate bode plots and step response plots and impulse response plots in MATLAB. Well, there are two ways of doing it. One you can use a step impulse and bode routines respectively in MATLAB. They are a part of both control systems toolbox as well as a [16:19 inaudible] tool box. Doesn't matter where they come from, these are the three commands. Step, impulse and bode. In all of this you have to pass on the LTI system object.

You have to define the LTI system first and then only you ask for the response. The other option is to use this semi GUI I would say, is LTI view. If type LTI view it will bring up this. Let me quickly show you that in MATLAB. No I said you can bring up the LTI viewer and it's brought by typing LTI view and it launches this. This I close. It's okay.

Now at the moment it's a clean slate. I have to pass on the system that I want to analyze. Now, what I've done is, I've taken one system which has an FIR system. And this is an FIR system. That is how you define. Remember we talked about transfer functions. So one way of creating an LTI object is use the tf routine. Where you encode the impulse response coefficients. What is the transfer function? It consists of a numerator and denominator. And it is expressed in terms of q instead of q inverse or z, instead of z inverse.

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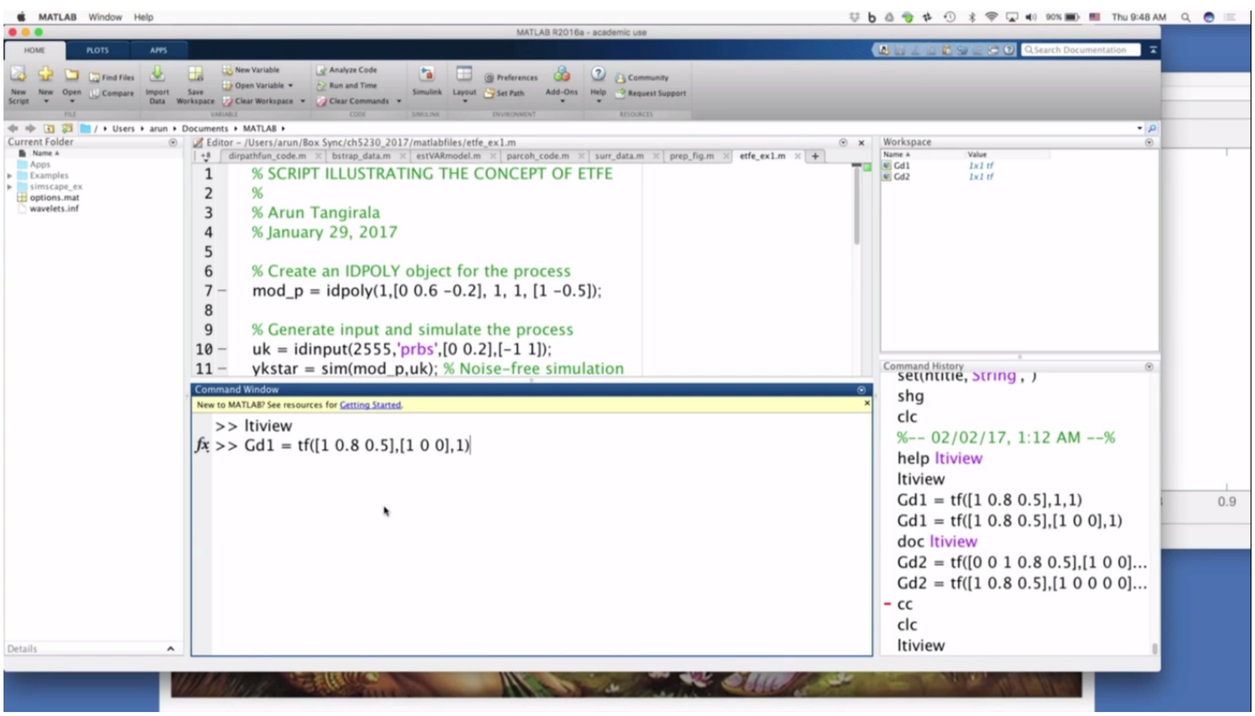

So first you write down your impulse response function. In this case it's FIR model. On many occasions you may be given difference equation form from where you have to convert it into transfer function form. How do you do that? You use your q inverse operator or q operator. Write it in numerator, denominator form. Like I showed you in the liquid level case study. Here this is an FIR model and the model that I want to look at is as follows. I hopefully I can later. So g k is 1, 0.8 and 0.5. The rest of them are zeros. It's understood, it's only the three impulse response coefficients are significant.

Now when I write this, what this means is, in form of input-output equation, it means that y k is u k plus 0.8 u k minus 1 plus 0.5 u k minus 2. And in terms of the shift operator this is nothing but 1 plus 0.8 q inverse plus 0.5 q inverse square operating on u k. And although I say transfer function, it's okay to deal with transfer operators. So G of q inverse is in this case 1 plus 0.8 q inverse plus 0.5 q inverse square. However what MATLAB expects is, in terms of q rather than q inverse square. Actually it is expecting in terms of z, but for the moment we'll keep aside that z part.

So you rewrite this in terms of q just for the sake of keying into the MATLAB. So you have q square plus 0.8 q plus 0.5 by q square. So in other words, had use a forward shift operator. You would have obtained this. And that is. And then you enter the numerator polynomial coefficients and the denominator polynomial coefficients in that order. And tell tf that you are dealing with a discrete-time system. So the first argument here is a numerator 1, 0.8, 0.5. Is everybody able to see it see?

Okay. And denominator is 1 0 0. So you have to provide the coefficients in the decreasing order of the polynomial, here numerator and denominator. And the last one argument here that you see as 1 is a sampling interval. That doesn't matter, at the moment you have to provide some sampling interval there to tf that you're dealing with a discrete-time system. If you do not provide that it will assume it's a continuous time system. Please remember that.

So let's ask how what Gd1 looks like. Whether it is exactly what I want and it is. Right? Now I go to LTI view. And file, say import. There was another system that I've created but I will not show that at the moment. So it's a Gd1. I think it's too smaller font size. But you list the objects that you have, LTI objects that you have. And I'm picking Gd1. And you should be clear now what plot it is showing. What do you think it is showing, step, impulse frequency response? What kind of a response is this? Step. Very good.

I can change now by right clicking and choosing from the plot type, right clicking will bring you the menu. And by default it shows the step, you can ask for impulse. It's a discrete-time system therefore it will not connect by lines. It's going to at least give you a staircase plot. Okay? Now, let's move on quickly to frequency response plot where we look at bode. This is how the board block looks like. Let me maximize.

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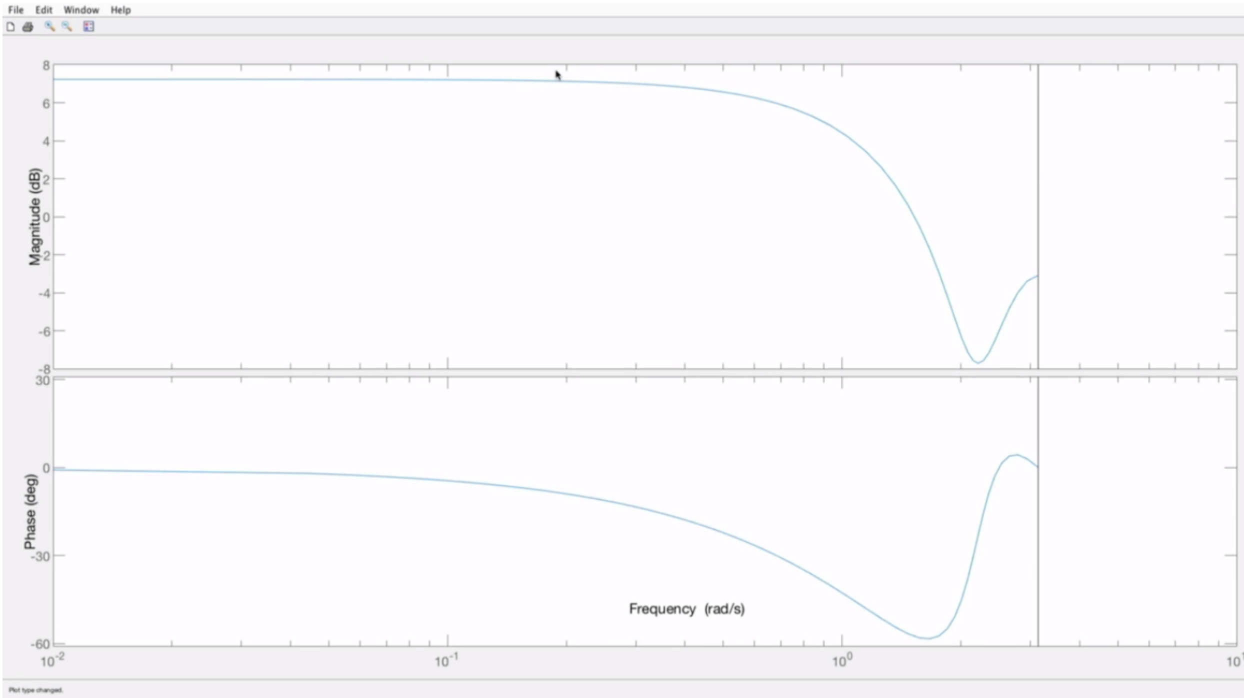

So, on the top you have amplitude ratio. It's not exactly amplitude ratio, you see something of that magnitude which is in decibels. Decibel is 20 times log 10 of AR. So it is not exactly plotting AR of omega. It's plotting 20 times log 10 AR. And this 20 log 10 AR has units of decibels in honor of Alexander Graham Bell. And we talk of always intensities and so on in terms of decibels. And phase is here plotted in terms of degrees not in terms of radians.

You can see that the system has a low pass kind of characterization. You see that and then phase goes from here 0. Up to what point? Are you able to see the value? Minus 60. Right? So it goes up to that point. But you can see the amplitude ratio being flat for a range of frequencies and then it starts to roll off. That range of frequencies or which you see amplitude ratio being flat and a bit of a roll off, roughly is a bandwidth of the system. After that there is significant attenuation.

So if you're going to perform an experiment to identify the system your input should have frequencies in that range. If you consider other beyond that then it's a waste of effort because the system is not going to respond. I'll just conclude by showing you again here a different system. It is nothing but delayed, the impulse response is delayed by a certain factor here .So what I'm doing here is now and delaying the impulse response by two units as you can see from the denominator. How much delay have I introduced? As you can see, delay of two units. The denominator has to change from z square to z power 4. What impact do you expect to see on the bode plot with respect to the first system. Before you plot, you should ask yourself always this question.

What impact, I mean, what kind of bode plot should I expect to see qualitatively. Which plot will change? Phase. Correct. The other plot will remain unaltered. So you choose the second one and it shows both. You can of course choose not to have one of them but you can see the phase plot has no changed but the amplitude ratio plot remains unaltered. And you can also place your cursor and you can ask for points here. For example, here I can go to a specific curve and ask for the information. In fact I can have my grind on I can ask many, many characteristics and so on.

But for a single curve. In fact, on any single curve, try this. Somehow it's not working here. You can place the cursor on the curve and ask for information there. It will tell you, what is the x axis value, what is the y axis value for with system and so on. This is how you play around with this LTI view. Get a feel of different systems. This may be useful for your assignment as well. Right? Now there's one point with which I'll close. I'll just take half a minute and then we'll close. Get back to the presentation here.

So, there is another way of defining FRF which will sow the seed for the discussion the next class, which is that it is a ration of the Fourier transforms of the output and input. Discrete-time Fourier transforms of the output and input. Where does this definition come from? Until now we have talked about FRF. Now I'm just turning to an alternative definition of FRF. The definition that we have seen is that, it is a discrete time Fourier transform of the impulse response. Right. But no I'm saying that the same FRF can be defined in a different way that it is the ratio of discrete-time Fourier transforms of output to the input. Where does this come from? It actually comes from convolution equation.

You start with the convolution equation. Take the discrete time Fourier transform on both sides of the convolution equation and use the property of Fourier transforms that the Fourier transforms of a convolution is a product. So you will end up with this kind of relation y of omega is G of e to the minus the j omega times u of omega. Even in this definition implicitly it is assumed that the system is stable, because only the system is stable, the left hand side which is y of omega will exist. Otherwise y k will run away with time. Okay. It is also implicitly assumed there.

What we will discuss next class is the practical definition of FRF that we work with. Both this and the earlier definition assume that I have in finite time with me. That means, I have infinite data with me, a both input and output which I don't. So how do I deal with finite data? What happens when I play this definition in practice? We will come across what is known as an empirical transfer function or empirical frequency response function. And I'll show you some examples there. With that we will close the discussion on FRF. Okay. Thank you.