CH5230: System Identification

z-Domain Descriptions 2

Before we continue with our discussion on z-transform, just wanted to make small correction to what we learned yesterday. This is got to do with the non-parametric version of q inverse, which we derived from the convolution equation. So if you recall yesterday, we're going through this example of recovering the impulse response coefficients from the difference equation from, using the transfer function operator approach. So in that context we had define a transfer function operator. Starting with the convolution and this transfer function operated as a non-parametric form. And thanks to one of the students, who pointed out after the lecture. It's important that we maintain the sequence here, as g of n q to the minus n. When defining the transfer function operator in the non-parametric form. I had-- I mentioned that it may not matter whether you switch these two it does matter and that was just a mistake from my side. So how do you derive this transfer function operator from the convolution equation? It's pretty straightforward. You replace u k minus n as by q to the minusn uk. So that you can think of yk as being this quantity here, where n runs from zero to infinity, operating on uk. And this again by our definition has to be the transfer function operator. So any operator that operates on uk to produce the output at the kthinstant is a transfer function operator. This is a non-parametricversion. The parametric version is what we have defined starting from the difference equation, so just wanted to put things in proper place. Always you will see this non-parametric version of g of q inverse appearing in your later course of discussions. When we talk of non-parametric models for the plant and noise models, you'll see a similar kind of description for the noise model as well. What you do not want to do is to switch these two, because then that would mean the operator is operating on g, which is not correct, okay? In general although the-- if you multiply ascalar, so multiplying a scalar with the operator and taking it past or placing it before does not make a difference. Do not think of g of n as a scalar, because it is an impulse response coefficient at the nth instant, which is also tied to the exponent of q inverse. So in that sense, it is not a scalar. But otherwise alpha times q to the minus n can be thought of its q to the minus n operating an alpha, because alpha is scalar, there is no time dependence. Okay, so always remember that the non-parametric version of g or the transfer function operator is defined in this way. And whenever you are in doubt, you can go back to the convolution equation, replace u k minus n with q to the minus n operating on uk and recover your definition. Okay. You will see, of course, very soon in today's lecture. The same definition, but in terms of z inverse.So almost an identical looking expression, except that now in place of q inverse you'll see z inverse. Of course, q inverse is an operator and z inverse is a complex variable. So the one that we've written at the bottom would be called as a transfer function, there is no operative there. But we'll talk about it very soon more in detail. Alright, so let's get back to the inverse of z-transform that we were discussing today-- yesterday. We went through this example, where we obtained the inverse z-transform through partial fraction expansion.

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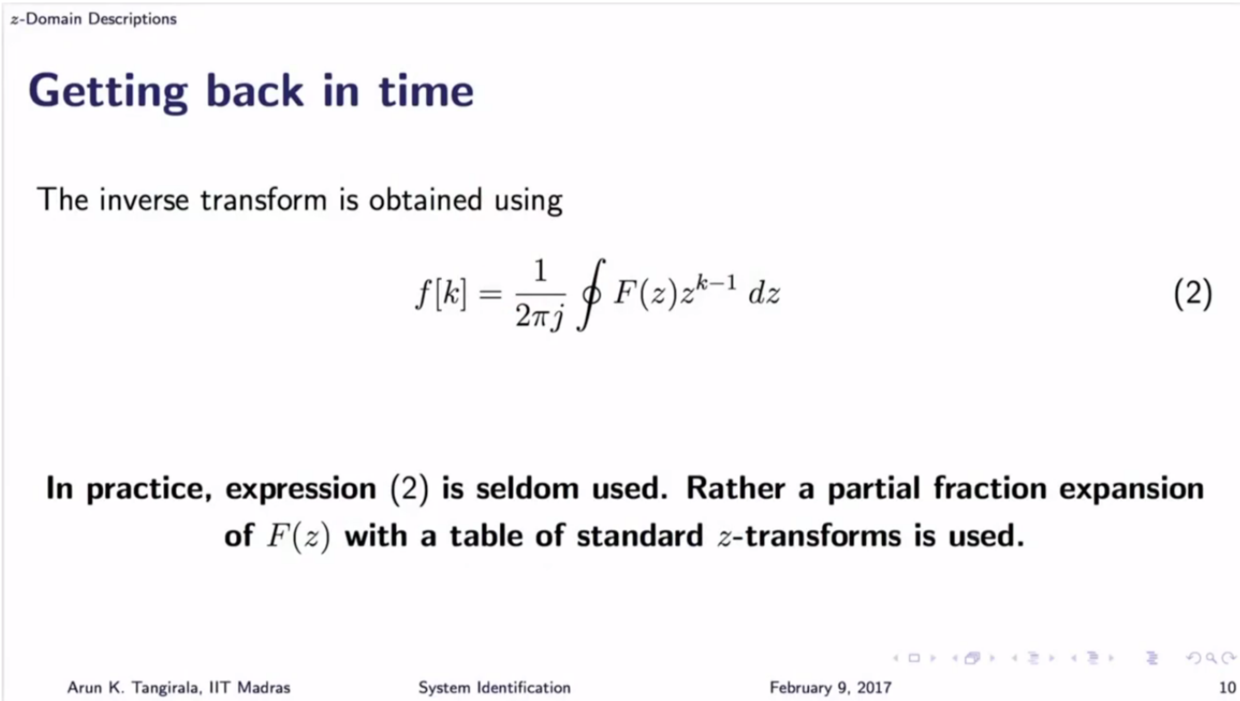

As I said yesterday the theoretical expression involves a circular integral, but we don't use that in practice.

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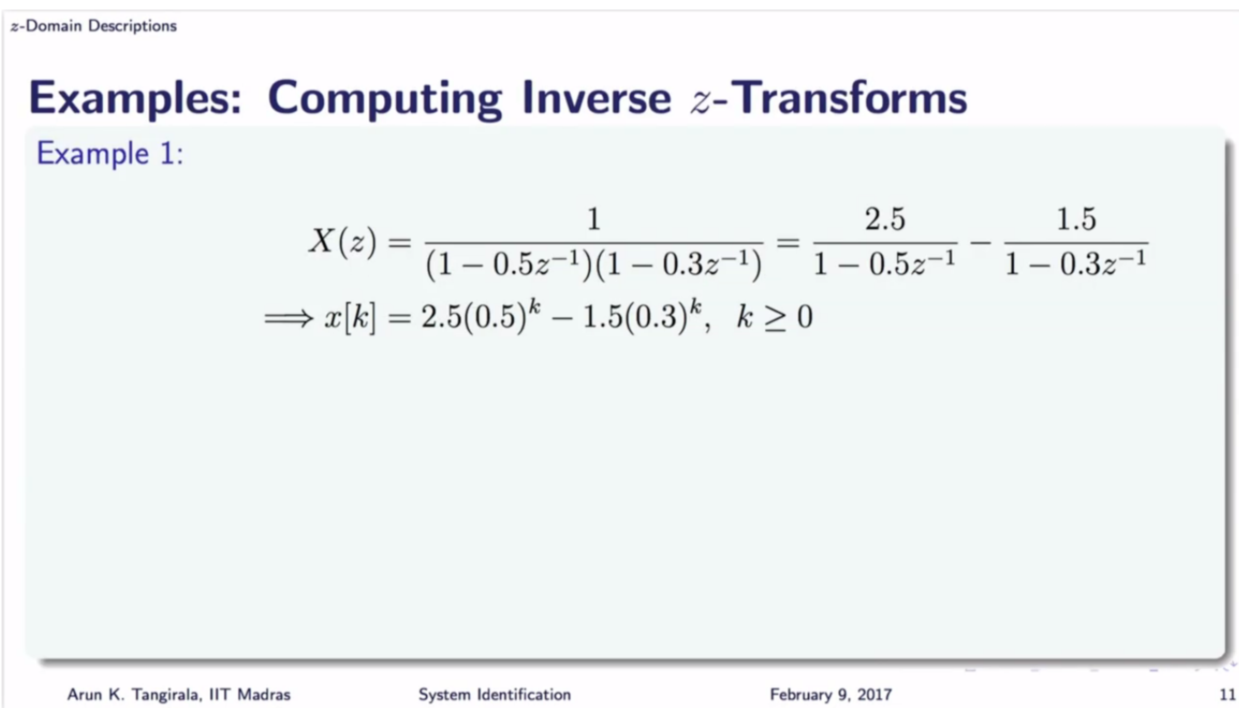

There is always a table of standard z-transform s and when I mentioned this that first thought that comes to a student's mind is, will I be giving this table in the quiz, okay? Don't have to know the full table. The table contains a lot of properties if necessary it'll be provided. But typically you are expected to remember the fourth standard the z-transform, so the four standard signals that we discussed yesterday, impulse, step, and so on. There's no need to memorize those things. You can just derive on the spot in a few seconds, if you forget. But if you practice enough it'll just stay with you. And there are-- and along with this partial fraction expansion plus the standard z-transform s sometimes we use properties of z-transform s and we'll study those key properties of that transforms that are relevant to signal and some analysis. Before going to that one more example here.

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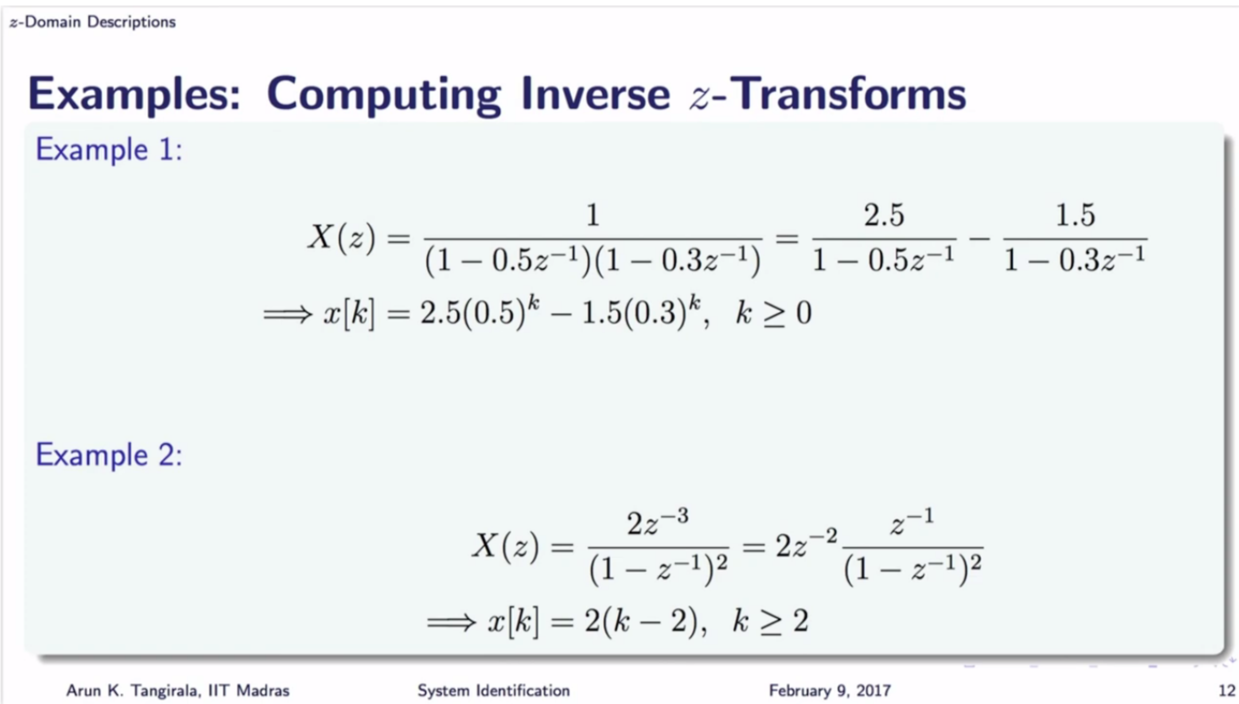

Now here is where I have bit kind of gone a bit ahead in terms of the discussion. In this example X of z is given as 2z to the minus 2 over 1 minus z inverse whole square, right? Now, sorry, two z to the minus three.So what I've done here is I factorize this into 2z to the minus 2 times this z inverse or 1 minus z inverse square. The reason for doing that is z inverse over 1 minus z inverse square is actually the z-transform of what is known as a ramp signal, right? So if I have a signal that looks like ramp, you know what, a ramp is it's just a straight line here, k. Straight line with the slope 1. When I have such a signal you can derive the z-transform to be z inverse over 1 minus z inverse to the whole square.Of course there is enough convergence. Suppose I didn't not-- inverse z-transform. Imagine there is no z inverse square here. 2k, right? But this z inverse square makes a difference only in one respect and of course that's an important respect, which is that as you will learn shortly with respect-- with the problem when we learn the properties of -z-transform s. That-- it appears as a result of the delay or the shift in the signal by that many units. In other words if I have a delay of D units then the z-transform is modified by a factor of z to the minus D. We will learn that very soon. Since, I havez to the minus 2 here it means that the ramp signal has been shifted by a factor-- by two units in time. And that is why the solution turns out to be 2 times k minus 2. But notice that now I have adjusted the time domain for the solution. If there was no z to the minus 2, the answer would have been 2k with the domain being k greater than equal to zero. But now that I have a z to the minus 2 there assuming that the signal is zero before the delay. So that's all assume that we don't keep saying that, you are going to shift the ramp by 2 units and write the solution as two times k minus 2. But now the solution is valid from 2 onwards k equals to 2 onwards. So that adjustment you have to make. Which means that when you're taking the inverse z-transform, some minimal level of alertness is required. There are many ways of arriving at the inverse z-transform when you go through this, but if you notice factors of z inverse and so on in the numerator and that you can factor out and attribute it to the delay, then it would be great. So you should be well-versed with some four or five standard z-transform s and some standard properties of z-transform, they are also not too many maybe two or three, that's it. So that comes as a matter of habit. It is just as a matter of practice, it's not too complicated. So let's now quickly study the properties of z-transform s.

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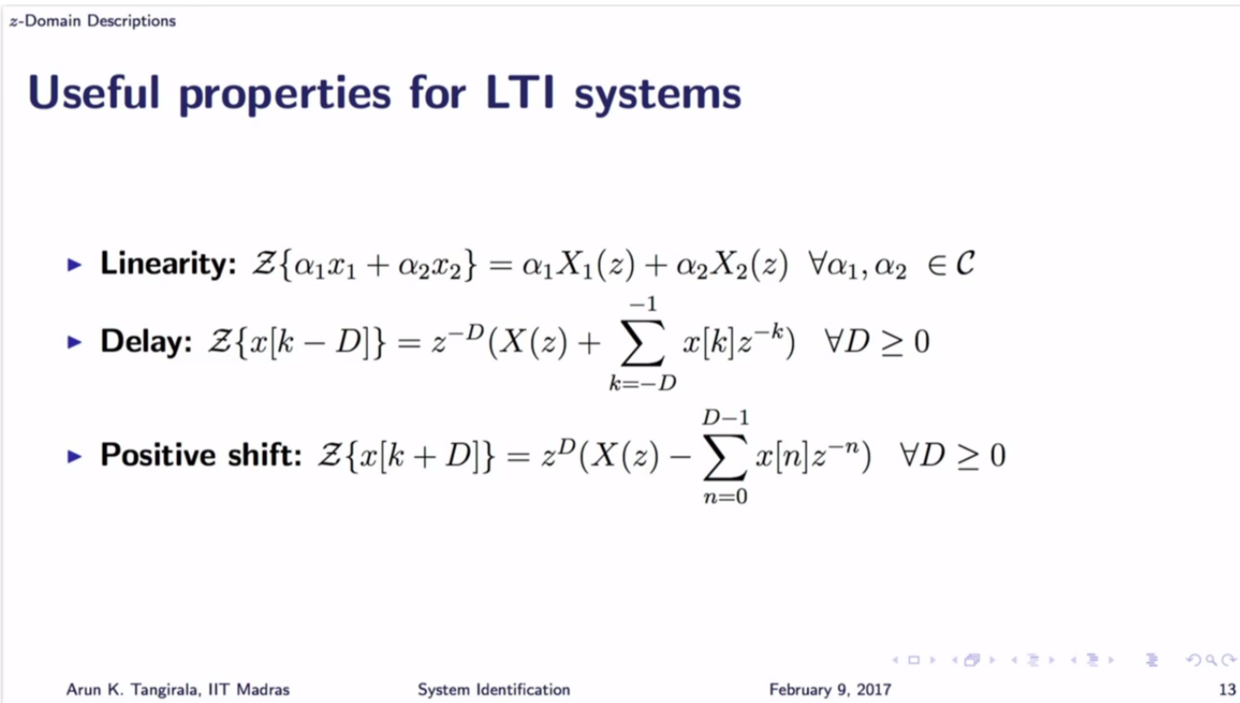

We'll again come back to the inverse z-transform s in a different context. When we are solving difference equations, we'll again solve another example, where an inverse z-transform is involved. And then that would hopefully, reinforce some of the concepts that we have been learning. The z-transform operates an operation is a linear operation, that's a very straightforward thing to show. Which means z-transform of sum of signals is a sum of their z-transform s in the single proportion. Now this is the second properties of one that I just referred too. If a signal is delayed by D units, now what would delay mean? With respect to the signal, would it be shifted to the right or shifted to the left? It will be shifted to the right. That means you have to give some time before the-- start. So, this is a ramp without delay. If this ramp is delayed by one unit, let's call this as a 0 and let's say here is 1. The ramp would-- the delayed ramp would begin this way, from here and then proceed in this way. So this would be delay draw, delayed by D equal to 1. So any signal that is delayed in time is shifted to the right. Anything signal that is advanced in time is shifted to the left.

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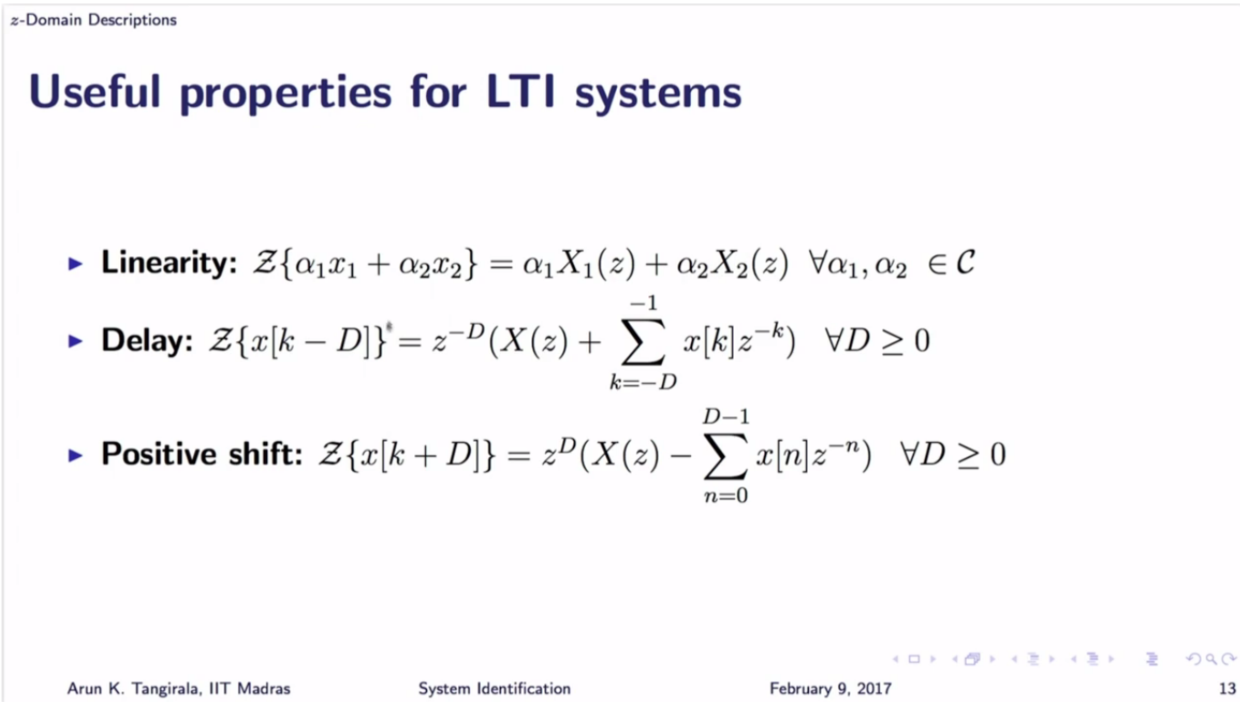

And what happens to the z-transform of delayed signals here, it is what do you do here is, strictly speaking you have z to the minus D X of z plus this term. How do you arrive at this expression? How do you derive the z- transform of a delayed signal? You go back to the definition, right? You go back to the definition. What I want is z-transform of a delayed signal, in terms of the z-transform of the original signal, which was not delayed. How do you derive this? Well you just plug in the expression in the u-- the signal into the definition of z-transform. Right? This is just the definition. Now what do we do next? Do a change of variable for the time, right? So you introduce a new variable k prime, which is k minus D, correct? So when you do that k prime is k minus D, so you assign this new variable. So that this can be written here as x of k prime time z to the, what do you have here? Minus k prime times z to the minus D, because k now is k prime plus D.Is it clear? A simple algebra, there is nothing complicated about this. Now I can factor out z to the minus D outside here of the summation because it's independent of k. However I have to adjust the limits of the summation. What would be the limits? Sorry? Correct. So it would be minus D to infinite. What is our objective? Our objective is to arrive at z-transform of the delayed signal in terms of the z-transform of the original signal. The original signal, how is this z-transform defined? X of zis sigma, so that you don't get confused. I'm just using a different dummy variable here. This is how the original signal, original I mean z-transform of the sequences defined as. But I don't-- what I can definitely see here is Z to the minus D factors out and doesn't matter whether Z to the minus D appears later or pre or post, because it's not an operator, that's okay. It is this summation that I don't find to be X of z, I would ideally like that to be. However, since k prime runs from minus D to infinity, I can separate the summation into two terms. 1 summation that runs from minus D to 0 and then the other summation that runs from 0 to infinity. The others summation that runs from 0 to infinity, will give you X of z, right? And then you have this term here. So, you have minus k-- 1 summation that's runs from minus D to minus 1 and then another summation that runs from 0 to infinity. That summation that runs from 0 to infinity is X of z and then you have this. If you are given like we did here, as like a-- even I menti1d in the previous example, if you are given that the signal has been 0 before it began, whether it's for the delayed 1 or the original 1, then what happens to the second term? 0, because before the delay the signal did not have any value. And if that is the case then only you have that z-transform of x k minus D as D X of z. It is s true that delayed-- delay in signal causes introduce as a factor of Z to the minus D. You have to be careful there can be traps in some of the applications, some of the problems. You have to be careful, you have to be sure, if you're not sure, you have to state the assumption that the signal is 0 valued before the delay. So when the signal is 0 valued before the delay, z-transform of a delayed signal is z to the minus D X of z. We will use this expression quite frequently not just in computing inverse z-transform, but also in modifying the transfer functions whenever there is a delay in and so on, clear? Thought you seem to be a bit lost, is okay, fine. Likewise we look at a z-transform of signals that are shifted to the left, which means the signals that are advanced in time. Right? Now the story is more or less the same that is now you can as a simple homework, you can derive this result, I'm not going to go through this derivation. It's a same procedure, instead of k minus D, you would have a k plus D, right? Then again you introduce a change of variable and so on. And in this case you would have to not split the summation as some of two terms, but you have to add and subtract 1 summation. And then you get this expression here. Now, what happens here? It depends on the original signal. What was the value from 0 to D minus 1 typically we don't assume that to be 0, because that may not be true, right? So we can't [17.20 inaudible] term unlike in the delayed [17:22 inaudible] where we can safely assume that the signal is 0 before the delay. Here, I cannot. The signal need not be 0 from 0 to D minus 1. The goal here, whether it is delay or positive shift or linearity or any other property, is to figure out when I perform a certain operation in time domain, what happens to the signal in the z domain? That is the X-- that is your objective in studying all of these properties. Because remember we said, we want to learn two things, how signals themselves map into z domain? That is one. How operations in time domain map to operations in z domain? That is two. So always we need to do know that, right? Even in languages that is a case, when I want to translate one sentence in one language to another language first I need a vocabulary mapping. What is one word in one-- in a language mean in another language? That is vocabulary mapping. That's what your language to language dictionary will give you. Then you need to know the grammar. When I've constructed a sentence, let's say in English we have verbs, adverbs, adjectives, prepositions, and all of the parts of the speech, I also need to know, when I am constructing a sentence, I am performing some operation, I'm taking in words arranging them in sequence and so on. I may not be doing the same thing here, right? So we know that very well. So that's exactly the case here. We has-- we have studied yesterday, how some standard signals map into z domain? And now we are studying how certain operations mapping to z domain?