CH5230: System Identification

z-Domain Descriptions 4

We can start. So what we'll do todayquickly is will complete our discussion onthe users of transfer function. As you recall, the transfer function can be used for in many different ways. One is in computing the response to some given input. Of course, provided the input z-transform is known. If the input is some arbitrary input for which compute the z-transform is going to be difficult,then it's not worth it.The more important use of the transfer function is in inferring the characteristics of the system, such as stability, one of the most important characteristics that we are interested in stability.

Once we confirm that the system is stable, we want to be able to say, whether the response to a certain input particularly step-input and so on, will result in some oscillatory characteristics, damped oscillatory characteristics and or whether they will be an inverse response and so on, right. Sometimes you may be interested in resonance. So what we will do is,we will quickly look at the three vital statistics of a transfer function, which will throw light on all these things including stability.And the first in order is this concept of pole.

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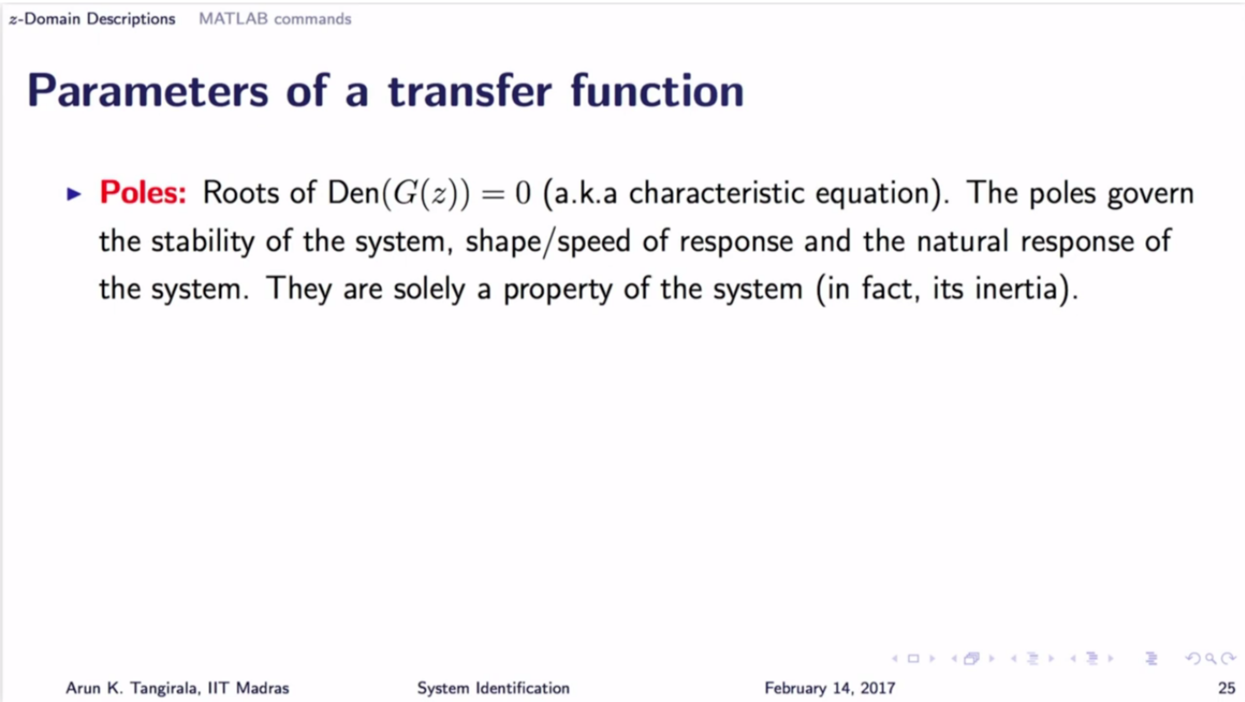

Which you must have learned in many different contexts, either in a control course or a basic linear systems theory course and soon.What are these poles? Essentially, they are the routes of the denominator of the transfer function.And for those of you who are not aware, why they are called poles, is because if you were to sketch this transfer function.Remember this transfer function is a complex valued function. It is defined in the complex variable space. When you evaluate this transfer function at different points in the z plain, you will hit uponcertain points in the z plane. Wherethe magnitude of the transfer function starts to really blow up and it blows up completely, exactly at those points have singularity, and those points have singularity are called pole. So if you were to sketch the magnitude of the transfer function in the z plane and you were to observe the shape of this magnitude, the curvature, you would see some kind of a blowing up around at the points have singularity. And from a distance they appear as if poles have been placed at those singularity points.

And that is why they are called poles. I don't, I mean, if you would knew, why the name-- this is not my own makeup, it is a truth,okay? That is why the name poles are given.So what you do is given a transfer function, you look at the denominatorandyou'll compute the poles. This is a mathematics part of it. Now, let's come to the physics part of it. The systems part of it. What has these-- what have these singularity points got to do with the system's characteristics? Well, first of all, we know that given a difference equation or a differential equation, there is something called a characteristic equation. And the name characteristic equation is not without a reason. It's because that the routes of the characteristic equation will govern the nature of the response of the system, whether it is the free response or the transientpart of the force response. We have seen that through an example, if you recall, right? We worked out an example in the last class.And therefore, this equation itself is called characteristic equation. That is whether you look at the difference or differential equation.

And we know now already that this characteristic equation is nothing but the denominator of the transfer function, it sits and in the denominator. And that is a connection between the poles, which are the routes of the characteristic equation and the system characteristics. And we also know at least faintly from our recollection of this theory of differential or difference equations that these routes have to satisfy certain conditions for the system to be termed as stable, correct.And those conditions change whether you're looking at a continuous time system or a discrete time system. But the fact is, these routes govern the stability of the system. The numerator of the transfer function has almost no role to play at all. I just said almost no role, but practically no role to play at all, as far as stability of the system is concerned. Why because stability at least for linear systems, we are not referring to nonlinear systems, for linear systems stability is solely a property of the system,it has got nothing to do with the input. Remember, when you write the transfer function G of z typically it's in a rational form, there is a numerator and there is a denominator.

The denominatorcomes from, where does it come from? The system-- it comes from the system's characteristics. And that is why it is called the characteristic equation. Whereasthe numerator, remember G of z isa consequence of looking at forced response. Therefore, there is an inputcomponent in your transfer function. And the numerator of the transfer functionis responsible or signifieshow this system is interacting with its surroundings. It is not a property of the system per say, it's a property of how you have wired the system with the input. So, this has got to do with the interface, withthe surroundings. Particularly-- especially the input. The numerator will tell you how the input acts on the system, whether there is a delay, whether there are input dynamics, and so on. But it hasno role to play in terms of stability. Why? Because you can or we have-- as we have already discussed, the denominator comes from the characteristic equation. In other words, if you were to write the difference equation from the transfer function, suppose I give you the transfer function, would you be able to write the corresponding difference equation? Yes or no? If I give you, let's sayin the example here, the next examplethat we are going to look at.

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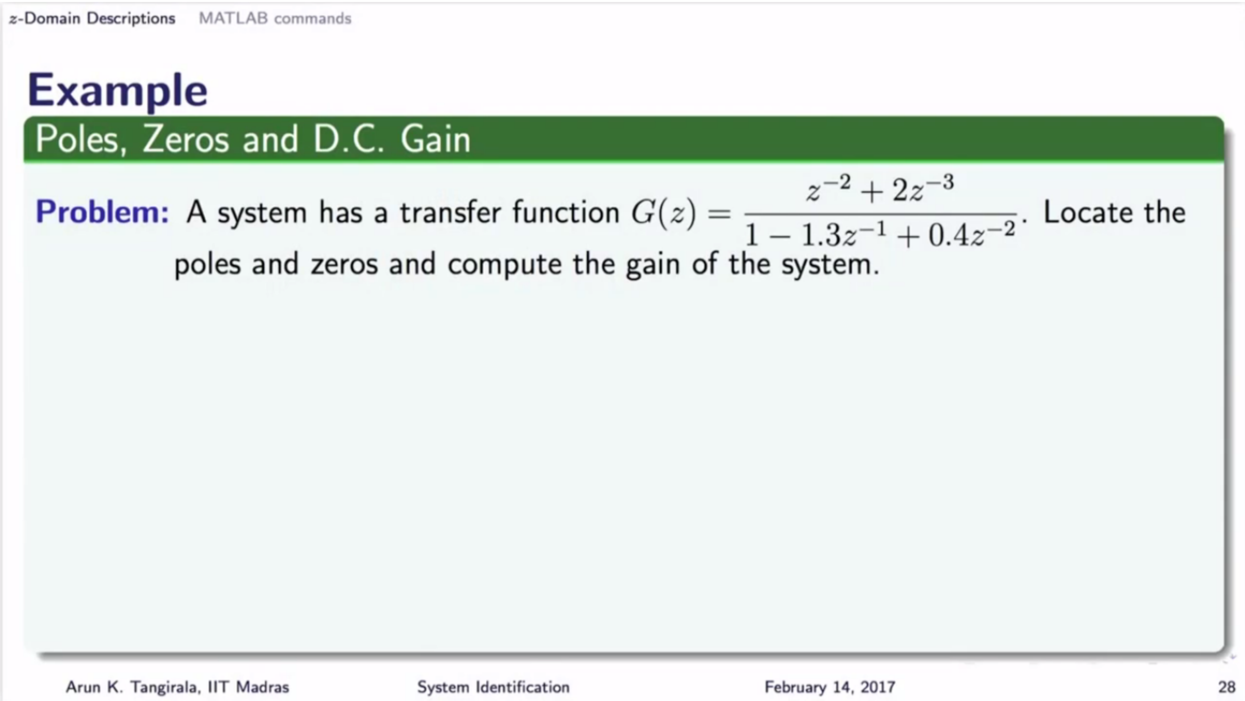

This is a transfer function. If you're given this transfer function, would you be able to write the difference equation? Right? Sowhen you write the difference equation and you want to evaluate the free response of the system, because that will tell you the stability of the system. What would you-- what would govern the free response, the numerator or the denominator? Denominator. Therefore, the denominator is solely responsible for the stability. And we know from the theory of difference equations that, if the routes of the difference equation, that is the denominator, sorry. Let us assume for the sake of convenience that these routes are distinct lambda 1, lambda 2 and so on.We know that the solution, homogeneous solution will be of the form of c1 alpha 1 times lambda 1 power k plus alpha 2 times lambda to power k and so on, right? So a general solution, assumingdistinct routes would be sigma alpha i lambda i power k. Where i is run from 1 to n, and let say it is a number of routes of the characteristic equation. This ishow the free response would look like.

And clearly from this equation, we can say that the system is stable if and only if all the routes are in the unit circle. That is a magnitude is less than 1. And that is the condition for stability. And that's why we look at poles when it comes to stability. Zeros, as I've already explained, is the second important feature of a transfer function, which tells us how the system is wired with the input. Soif you want to know, whether there is a delay, whether there are input dynamics, which means not just the present input, the past inputs are all affectingthe system and so on, you look at the zeros. There is another significance attach to thezeros. Suppose it turns out for a transfer function, we will come across such an example, sorry, such an example, where the location of the 0 exactly coincides with the location of the pole. That is mathematics. But physically, what does it mean?Every time you have to-- now train yourself, you're not just attending a pure math course, you're attending an engineering course.

So you have to have a comfort in math, and thencomfort ability to interpret that with respect to the physics. So mathematically, the pole and a 0 are at the same location. What does it mean physically? It means that, remember that poles are responsible for the shaping the response of the system. If it is free response, they are completely responsible. If it is a force response, they are responsible for the transients. Now, if there is a 0 location that coincides with the pole location, what the zero is going to do is, it's going to completely suppress the effect of the pole. Which means that, I mean math--

physically it's not gonna happen, physically in the sense that, you're one of the poles will vanish and so on. There's nothing like that. As far as your response is concerned, whatever contribution this pole which is sitting at the same zero is concerned, the effect of that pole is completely suppressed. In other words,zerosactually can play a significant role in suppressing theeffect of poles. This is one way of looking at the controller problem, controller design problem.

I don't know, how deep a perspective, you have actually been given or you have gained in your control courses. Butan alternative way of design-- stating a controller design problemis that you want to place zeros, such that the dynamics of the system are suppressed. See, control is all about improving the performance. Through the feedback, you try to change the-- you can't change the poles of the system, because they're at the property of the system. If you have to change the poles of system, you have to change the system itself. Which we don't want to do, the system has-- is the one that is fixed. So if you want to suppress some dynamics of the system, you design your controller and so on in such a way, that the resulting system has zeros, which cancels out the effect of the poles. Okay. So, the bottom line is zeros not only tell us how the input system is wired with the input, but they also tell you whether-- they also aid in suppressing the effects of poles, we'll see one such example. Physicallyit is not going to really fight it out and remove one component from the system, remember that. Mathematical cancellation is different from physical cancellation.

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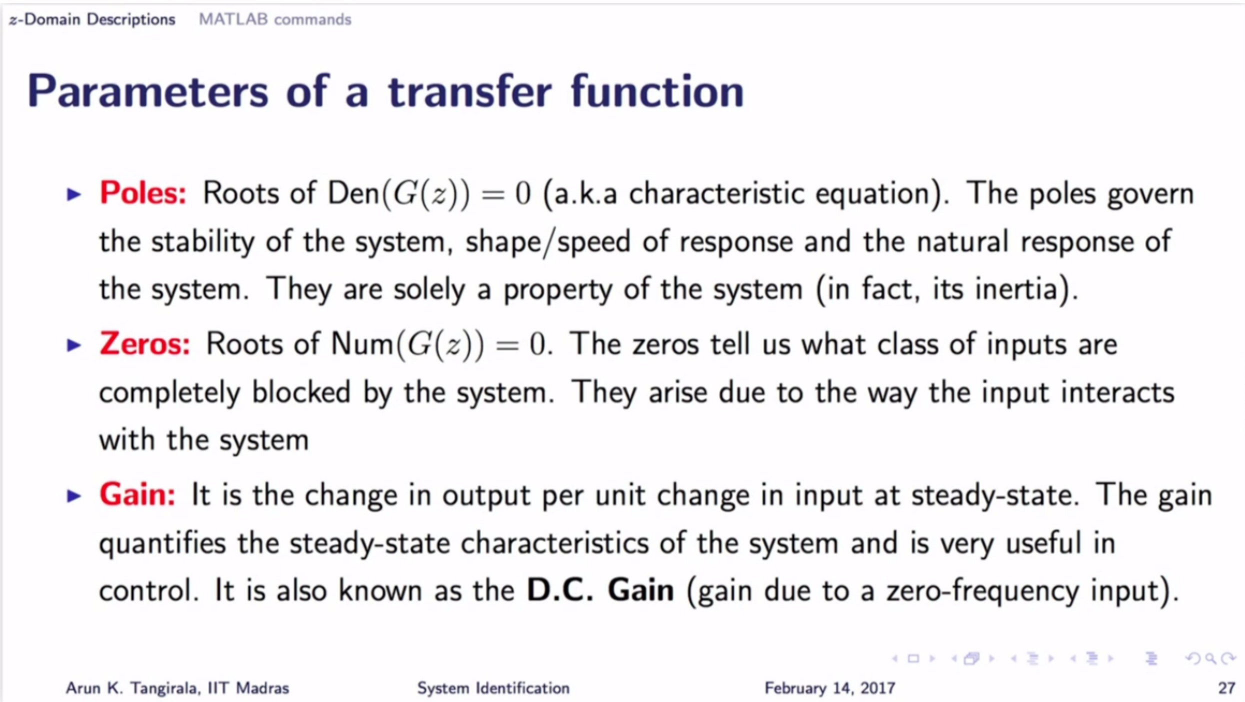

Then of course there is a third aspect, which is a gain. We know gains have a very critical role to play in many applications. We have already talked about itby default gain refers to D.C.Gainthat means we are looking at the response of the system at steady state to a step-input. Why is it called D.C.Gain? Because I'm looking at, how the system responds to zero-frequency signal.This is unlike the A.C.Gain that we have talked about, the amplitude ratio.Why is this gain important? Because of obvious reasons, the sign of the gain is extremely important. Because it'll tell me, if I increase the input, whether the output is gonna decrease or increase. If you makemistake in the sign of the gain, it can actually cause catastrophes. Imagine that you're designing a coolant flow, a coolant flow jacket, for a nuclear reactor.And if you look at thecontroller design problem, first involves modelling the flow of the coolantwith the temperature of the reactor,right?

The-- First I have to understand, how a change in coolant flow will bring about a change in the temperature of the reactor.Suppose I make a mistake, in the sign of the gain between these two, what would be the sign of the gain between the coolant flowinto this jacket and the temperature of the reactor? Negative, right? Because an increase in the coolant flow, will bring down the temperature.Suppose, I thought that negative sign was just a hyphen, okay? And I thought by mistake, somebody hit that hyphen, and I go ahead and design my controller, the chances that I will have another opportunity to correct it are going to be very small once I implement that controller, right? Unless, of course, there are very smart systems in place which will shut down the reactor, otherwise, what will happen, you're just going to help the nuclear reaction and you're going to really go along with the reactor, right? I mean, there is fission happening, there's also fission happening around.

So one has to be extremely careful with the sign of the gain, then comes the magnitude of the gain. That's why gains are extremely important. And of course, you know, students know very well, what do I gain by solving an assignment? What do I gain by sitting through these classes and so on? So, you're often puzzled with these questions of gains. But let me tell you, there are also poles and zeros that you are to worry about. Okay, so let's look at a quick example to understand these concepts, here is a transfer function. Notice that this transfer function is expressed in terms of z- inverses. Now, the simple task ahead is to compute the poles, zeros and the D.C. gain. It's a good habit; although they've written G of z, the right hand side is in terms of z-inverse. As far as pole-zero computation is concerned, it is important for you to rewrite in terms of z. What would be the poles of this transfer function? How many poles and where are they located?

Student 1: 0.8, 0. [15:59 inaudible]

So you're saying there are only two poles.

Student 2: No, three.

You're saying there are only three poles.

Student 3: 0, 0.3 [16:08 inaudible]

You always switch parties? You're sure now? Switching parties is now becoming a very rampant phenomenal. So, what is the final answer? You Know, be sure, even if you're wrong, you should be sure. How many poles?

Student 4: Three.

Three, right? So you have 0, 0.8, 0.5. As I said, first, it's important to rewrite in terms of z, I'll just quickly show you how to do this in MATLAB as well. And then you have the zero. So-- first of all, if you look at the poles, there is one pole at the origin and then you have 0.8 and 0.5.

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z-Domain Descriptions MATLAB commands

Example

Poles, Zeros and D.C. Gain

Problem: A system has a transfer function $G(z) = \frac{z^{-2} + 2z^{-3}}{1 - 1.3z^{-1} + 0.4z^{-2}}$. Locate the poles and zeros and compute the gain of the system.

Solution: (i) **Poles:** The roots of the characteristic equation, i.e., Den$(G(z)) = 0$, i.e., the poles, are located at $z = 0, 0.8, 0.5$

(ii) **Zeros:** The roots of Num$(G(z)) = 0$, i.e., the zeros are located at $z = -2$

(iii) **Gain:** Using the FVT (verify that it applies), the gain is calculated as:

$$\lim_{z\to 1} G(z) = \lim_{z\to 1} \frac{z^{-2} + 2z^{-3}}{1 - 1.3z^{-1} + 0.4z^{-2}} = 30$$

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Now this three poles is somewhat puzzling to a few of us, because when you look at the transfer function, and if you write the difference equation, what is the difference equation that would have led to this transfer function? y[k] minus 1.3 y[k] minus 1, plus point four y[k] minus 2 equals u[k] minus two plus two u[k] minus three. So when you look at this difference equation, from the general definition of order that we have been looking at it, it up is that the system is second order, right? As far as the system is, you can say, yeah, it is kind of a second order system, but there is this third pole that has come in which is located at zero. Why has that come in? Wait, what is the source of that additional pole there?

Student 5: To multiply numerator, denominator [18:34 inaudible]

Explain it in a better way? See, suppose the right hand side was u[k] minus 1 plus two times u[k] minus 2, what would have happened? Only two poles would have been there, right?

So, now can you explain? Where that additional pole has come through? This is very important, because later on when we talk state-space models with delays, you will get the complete picture and you'll get the complete answer. I've given you a hint now. The additional delay, so one delay, if you had only a unit delay. Right now, what is the delay? Two. If we had a unit delay, then that wouldn't have actually caused any increase in the order. An additional delay in the input side has brought about an apparent increase in the order. We talked about the other way around, right? An increase in the order causes an increase in the delay. For continuous time systems, remember, we talked about the step response of higher order systems; you're going to have sluggishness, that sluggishness manifests as apparent delay. Here the situation is reverse. The situation is that there is a delay, and it is reflecting as an apparent increase in the order, that is something to keep in mind.

The delay is still two units. Over all the system now has three poles that means it's a third order system. You will understand that in a different way, when we look at state-space models, shortly. So, this is something very important delays, additional delays: what is additional depends on the situation, but additional delays can cause an increase in the order of the discrete time system. But they do not cause infinite increase in the order. The nice thing about working with discrete time systems is delays do not bring about an infinite increase in the order. Whereas with continuous time systems, if I had a delay, what would be the transfer function? Suppose I take a continuous time system, pure delay. Suppose I had y(t) as some u of t minus D, what is the transfer function in terms of the Laplace variable?

Student 6: e to the power minus DS.

Very good. So, the transfer function is e to the minus DS, which I can rewrite as-- now you know again, what is the order of the transfer function? It is the order of the denominator polynomial. What is the order of the denominator polynomial?

Student 7: Infinite.

Infinite. So, what has happened now, a delay in a continuous time system results in infinite increase in the order. From a state-space perspective, it'll become clear also why this is so. But these are called transcendental equations, right? Transcendental things that, it just transcends. It is beyond your understandings, infinite order. In discreet time fortunately, that doesn't happen. And that's a very nice feature of discrete time models. Okay? So this is something to remember. Any questions? So, by the way, is this system stable? Yes? Good. Be sure, if you're not, then raise your hand.

Next in line is zero computation. We know that there is a single zero and the zero is located at minus two, right? Does it mean anything? Not much? Well, there is no poll located at that, so it's not going to really suppress any pole location. The fact is that there is a zero. In a pure Dynamics course, we would spend a lot more time in understanding how zeros make a difference? In fact, in a filter design course, the filters are designed, there is one design methodology, where you specify what the filter should do? You know, what the filters are supposed to be doing? They're supposed to be letting certain signals go through and reject, attenuate a certain other set of frequencies. Given these specifications, you can map it to pole zero locations.

You can understand, in fact, as you keep changing, the poles, you know, that the Bode plot will keep changing, right? In fact, as you keep changing the zeros also, the Bode plot keeps changing. So what the filter design person has, in his or her hands, is this freedom to change the poles and zeros. So there is an entire set of tools that design the filter based on zero pole locations and in such a course, we will spend more time in understanding what is the effect of poles and zeros on frequency response? At the moment, I think this suffices in this idea. The last one, in order is gain. Of course, the one important thing that you're looking at zeros is, the zero is located outside the unit circle. Sometimes that can make a difference. We'll talk about it at an appropriate time, if necessary.But that won't spoil the stability or anything. So coming back to the gain, how do you calculate the gain? Remember gain is the net change in output to a unit change in the input, which means we are looking at the steady state value of the output.

In other words, we're looking at the Final Value of the output to a step change in the input. I can talk of DC gains only when I think of step like inputs. So in other words, I am asking, what is the value of the step response in the limit as k goes to infinity? So we straight away deploy the Final Value theorem. But before we do that, we should be cautious. Are we -- is it meaningful to apply the Final Value theorem for this system? Because it's a stable system, right? So you use the Final Value theorem, what is the Final Value theorem says that this value is the same as limit z going to 1, z-1, y(z). Which y? This step response, right? And now I can replace this y(z) with G(z)U, but this is z-transform of a step. Right? And what is the z-transform of a step? Z over z-1, correct. Therefore, I end up with this expression, limit Z going to 1 G(z).

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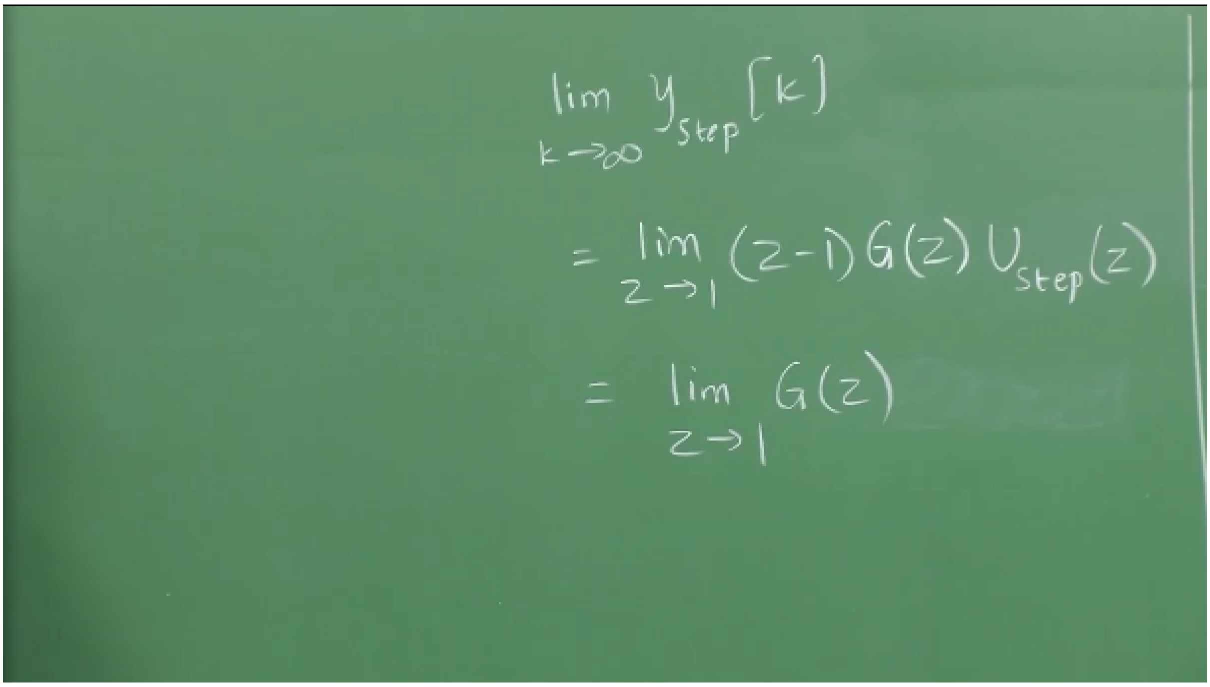

And that's what we have used. But remember, you can use this only if the system is stable. Many times this limit may exist for a given transfer function. Because remember, transfer functions can exist, even for unstable systems, we know that, we have discussed, as far as don't. So you may be given an unstable transfer function, and you will go happily because there's a limit and you really love evaluating limits. So you get a finite answer and say, here is a gain, take it. And for all you know, it may not make any sense at all because the system is unstable. So you have to check if the system is stable. There is another nice way of calculating the gain from the difference equation. How do you do that without going to the Z business? Right? So they're all equal, right? All you have to do is now set all these ys to be equal, all the us to be equal. And you're asking what is y(k) by u(k). You should check you get the same answer there. Right? That's another way. So you should always be equipped with multiple ways of arriving at the answer so that you don't wait for the instructor to evaluate and give you the surprise mark. You know for sure that yes, this is the answer. Any questions? Simple example, but quite a few things we have learned.