CH5230: System Identification

z- Domain Descriptions 6

Finally, we have this frequency response from transfer function that is the final thing that we want to talk about with respect to transfer functions.

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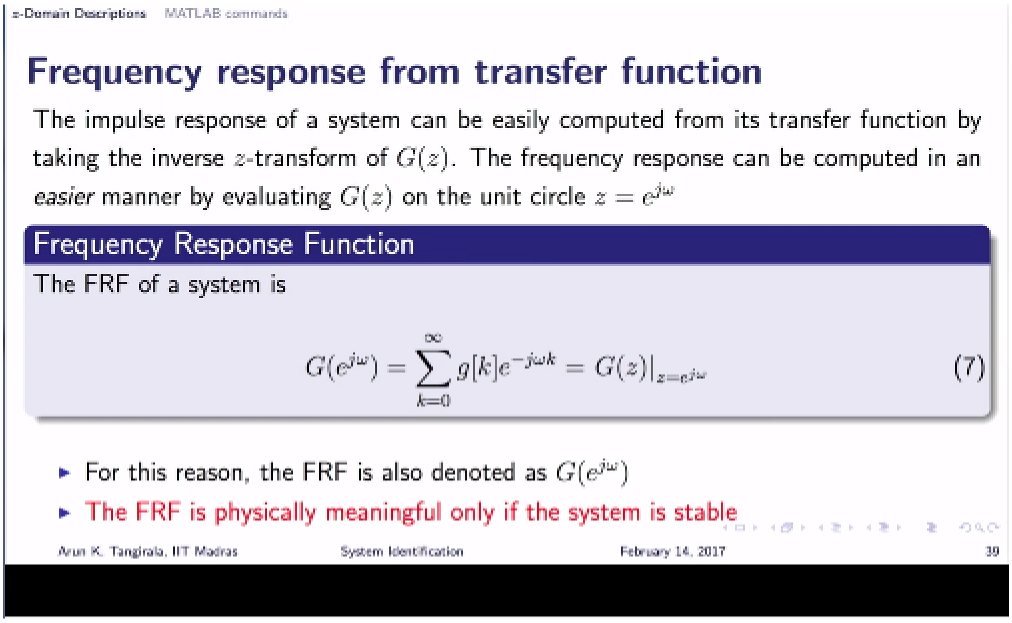

We have already talked about it. I mentioned that the frequency response function is a special case of the transfer function when it is evaluated on the unit circle that is when is evaluated on e to the j omega. I said equals e to the j omega. You know now why this is true because g of z is based on z transforms and FRF is based on the Fourier transform and Fourier transform, DTFT is a special case of z transform, when evaluated on the unit circle. Now something that you should observe. We are saying that we'll evaluate G of Z on Z equals e to the J Omega, now Z equal e to the J Omega corresponds to those signals which are oscillatory. Purely oscillatory. So that means, you are evaluating the transfer function exactly for the case of oscillatory signals and that should give you the FRF. But in doing so you have to be careful because you are assuming that this series will now converge. Remember, G of Z may have its own region of convergence. Right, because e of z is the z transform of the impulsive response. It's after all a z transform and every z transform has its own region of convergence. If the region of convergence does not include the unit circle that means this FRF won't exist. That is another way of saying, that's a mathematical way of saying that if the system is not stable then you cannot think of an FRF.

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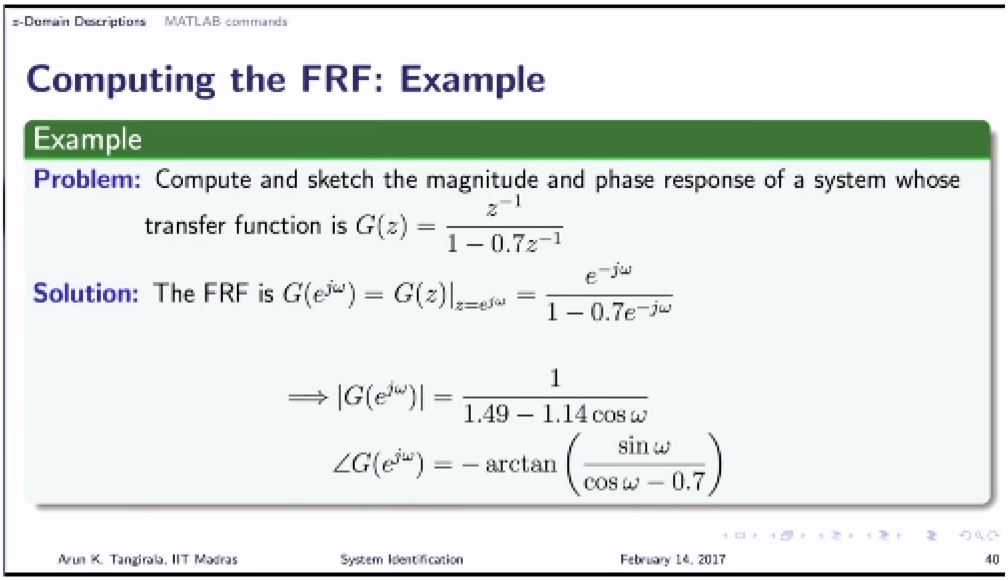

Both are equal. Obviously, the region of convergence of G of Z does not include e to the j omega which is a unit circle. No way, this is going to converge. As I said that is a mathematical statement of saying, physically that the frequency response function would not exist if the system is unstable.

So a simple example ,I'm given G of Z here. First I have to make sure, it is stable. It is stable with single pole at 0.7. Then I compute the frequency response function, the magnitude and the face, I'm just showing the plot.

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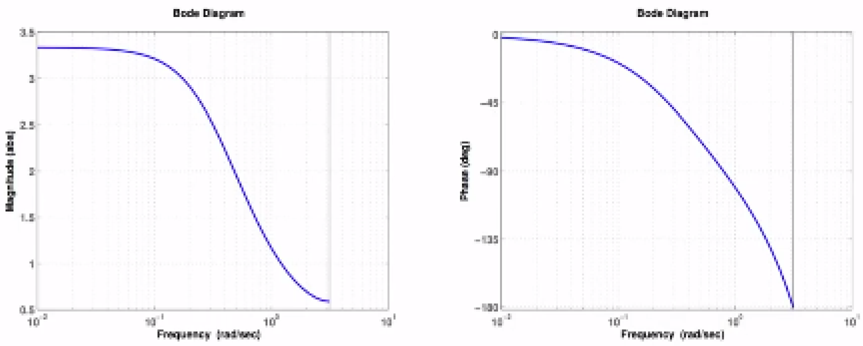
Example . . . contd.

The **Bodè** plot shows that the system has low-pass filter characteristics.



Simple thing, you can actually create this G of Z and ask for a board a plot. As a quick check in MATLAB to just see if MATLAB is alert and smart enough, you can give an unstable transfer function. This is a stable one, and ask for the board a plot and see if board a plot scolds you. If it doesn't and it just blindly like, bull computes then something is wrong. Check it out. Do it in MATLAB. Just to play around, see if board a is alert enough. Board a was enough, the board routine has to be. So it's a low pass filter. That is what we in for. Don't worry so much about the phase. Let's conclude this discussion on transfer function with the reminder for ourselves on the difference and the subtle difference between transfer function and transfer function operator.

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Transfer function operator and Transfer function

The transfer function operator $G(q^{-1})$ and $G(z^{-1})$ have identical forms. Two marked differences, however exist:

- q is a forward-shift operator, whereas z is a complex variable. Therefore, $G(q^{-1})$ is an **operator**, whereas $G(z^{-1})$ is a **multiplier**.
- Consequently, one can directly recover the difference equation from $G(q^{-1})$ whereas recovering the difference equation from $G(z^{-1})$ can have an ambiguity attached to it whenever a pole-zero cancellation occurs.

There is a subtle difference but of course, we know that G of Q investors and operator and G of Z inverse or G of Z is a multiplier. Right. Let's look at the simple example to understand the difference. So here I have a system described by difference equation.

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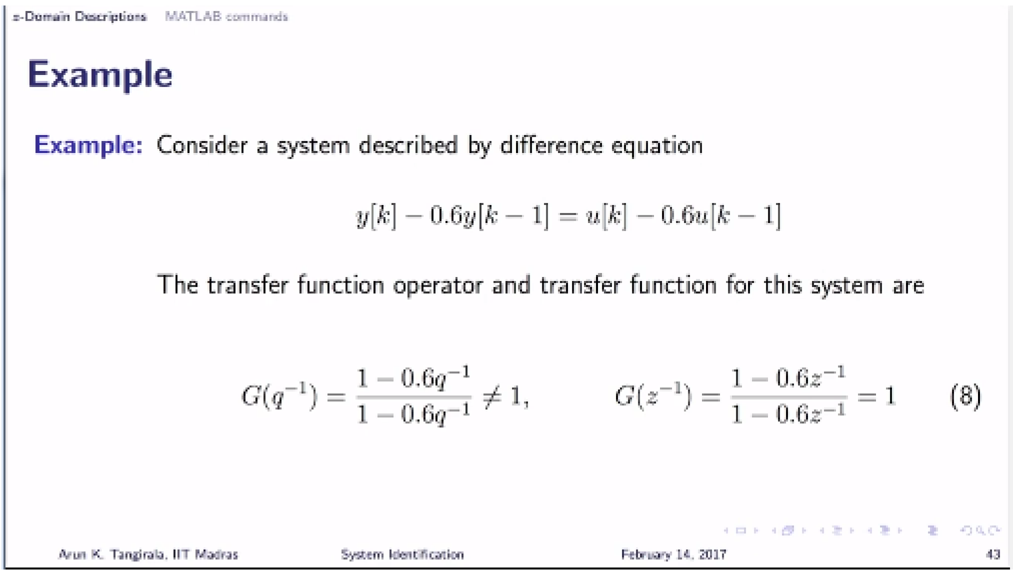

What is the nature of this system? It's stable. Fine. What else can you say? Pole and 0 are at identical locations. Good. Now, when I write the transfer function operator, it is 1 minus 0.6 q inverse over 1 minus 0.6 q inverse which you cannot cancel out and say, it is 1 because they don't cancel out. They are operators. Whereas when I write G of Z or G of Z inverse. I get 1 minus 0.6 z inverse over 1 minus 0.6 z inverse which I can mathematically cancel out because the system is stable. Generally you're not allowed to cancel zeros and polls even though they're at identical locations because you are throwing away a very important aspect of the system which is the unstable part. All right. It's okay to cancel out the zero and poll if they're at stable locations because you're not losing much information. You are. But it's okay. It's not going to cause any difference as far as stability is concerned. So you say, G of Z inverse is effectively 1. Now, if I give you G of Z inverse as 1, what would be the difference a question that you would write? yk equals UK. But that is not the true one. The true differential equation is yk minus 0.6 yk minus 1 is equal to U K minus 0.6, uk minus 1, which is preserved in the transfer function operator. But is it true that always this cancellation will mislead me? Not necessarily. In fact, G of Z inverse is equals 1 from there inferring yk equals uk is correct so long as the initial conditions are 0. When the initial conditions are 0 although this is the difference equation, effectively the system will evolve as yk equals UK. Only when initial conditions are non-zero, the system will not evolve effectively as yk equals uk. Remember, your transfer function is obtained by setting the initial conditions to 0. Whereas transfer function operator does not make any such assumptions. So that is why I always preferred to work with G of q inverse because it preserves the complete information. So remember, therefore, it's this transfer function operator and transfer function, you can say, they give you identical results only when the initial conditions are 0. This is an example that's also nicely. A similar example is nicely discussed in the book by Åström and Wittenmark on computer control systems. That's it. So that is as far as your dance functions are concerned. Hopefully, I have transferred a lot of knowledge to you on transfer functions. It's been a long discussion but it is very important to understand this because remember ultimately we are going to deal with transfer functions.

Any questions on these transfer functions and transfer function operators and so on. And we have noted that there are many ways of representing the system we have convolution equation, you have a step response, frequency response, difference equation and so on. The transfer functions are just an alternative way of representing the difference equation but they bring a lot of insights into the system characteristics. Without having to calculate I can infer a lot of things about the LTI system. So play around in MATLAB, the assignment also gives you an opportunity. There are only two topics left before we close the world of pure deterministic systems that is the comfort zone kind of ends there. And then we get into the world of random processes and then you have avial of deterministic and stochastic. The two topics that are left are, State space representations and discretization. Okay, I'll just take about five minutes to introduce notion of state and then we'll talk of state space representations in detail tomorrow.

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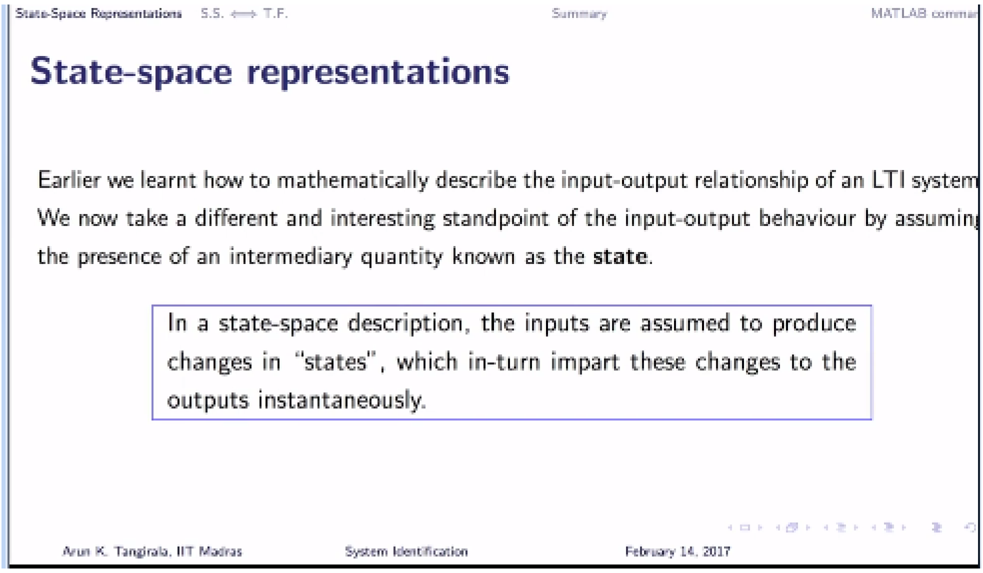

Okay. So we have until now learnt to look at input, output requests I mean these are all the equations that we have been used to where we say, there is an input that affects an output and so on. Now many a times this imagination may not help so much. There's nothing wrong with this imagination that input directly affects the output. But somebody came along and said, well, let me come up with a different way of imagining how the system is wired inside. To each his or her own imagination. Nothing wrong with it. After all these are all mathematical abstractions of how the systems work. So there's this other world of imagination where it is believed that there is a messenger in the system which is known as this state. There's a mediator. The input acts on the system. These mediators start responding and in turn they carry the message to the output. So you can think of it that way. It is not a technically perfect way of introducing a state space model but at least, it breaks the ice. That is in state space representations now we have an additional imagination at least input and output I can see visibly but the state space representations bring with them an additional imagination known as the states which are generally fictitious. In the sense, I don't necessarily see them as I said earlier also, we talk of state of mind. No he say that person's state of mind is not so good or so good and so on. I can't touch the state of mind. But I can infer about the state of mind based on what I observe and that is what is a case in state space models as well. I do not have necessarily direct access to the states. They are some kind of hidden variables but why do we use that term? It is to characterize to explain how the system is behaving. We also use the term steady state, correct. We use the term quasi steady state, transition state so as a state term we have been using. Now some mathematical imagination is a form is being attributed to it. It is still hidden by and large depending on how the state has been defined with respect to what you observe. But as far as general states based models are concerned they're hidden and sometimes they're directly observable. As I said, depends on what is the connection between what you observe and what you define your status. In the state of mind example, I cannot touch directly observe the state of mind. I can only infer through the behavior of the person. The person is very happy to see, a state of mind is very happy, is very good. Okay.

So to begin with you can think of the state of system as being some hidden variables. Why are they hidden from you? It depends on the application. We will discuss two such examples tomorrow morning. The first thing in the morning as to why the states of the systems are generally hidden from you. If they are hidden, then why should I break my head on estimating them when these models are doing the job for that? That is one thing we have to be very clear. Why should I turn to a state space model? That is point number one. Point number two is, in general given that these are hidden fictitious and so on, we'll soon showed that the state space models are not unique for a given LPI system. You may have one. Your friend may have another state space model and infinite people will have infinite different state space models. All of them describing the same input, output relation. Okay. So that is one issue with state space model. Despite these two issues one that they're hidden and that there are infinite choices state space models are extremely popular.

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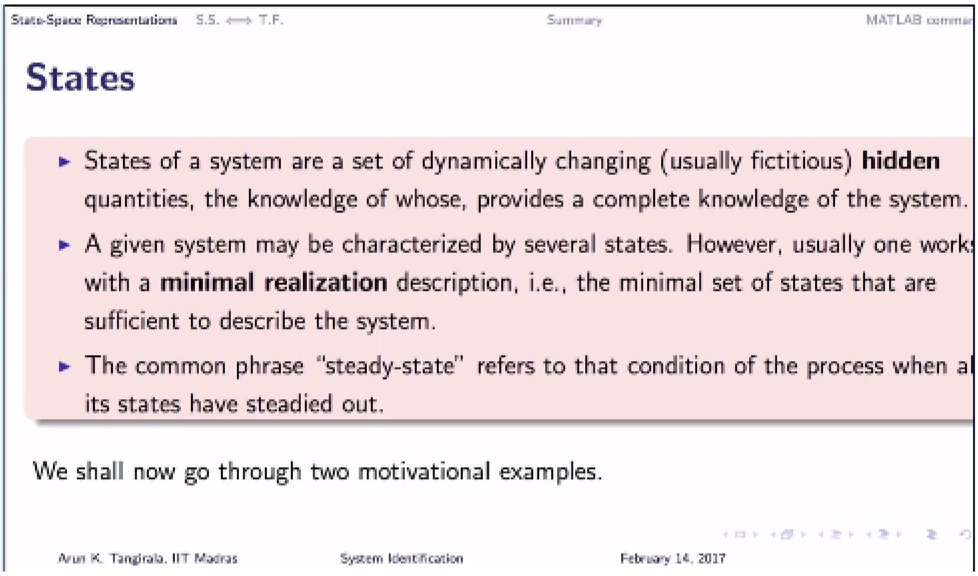

We'll figure out why tomorrow as I said, tomorrow is example but one concept that you should be familiar with in state space models is this concept of minimal realization. What is this minimal realization? When I'm given a system and I have to describe a state space model, I can actually describe in many, many different ways. That means, I can have a two state model or a three state model, a four state model and so on. Not all states may be relevant as far as what you want to describe is concern. Right. As a very simple example, suppose, I have let's say, a liquid level system again a very simple system and a flow in and flow out. And let us say additionally there is this heat being pumped in like your geyser at your home. Right? Now let us say, I'm interested in two variables liquid level and temperature. Which temperature? The temperature at the outlet stream. And here is the liquid level. Now when I am worried about so-called level dynamics that means how the level changes when there is a change in the net flow. Do I need to know the temperature? What do you think? Do I need to know the temperature to clear the level? Unless it's a flash vaporizer. Right. Unless it's a flash vaporizer for normal applications, I don't need to know the temperature to calculate the level. I just need to know the inlet floor changes and I'll be able to calculate. But on the other hand when it comes to temperature calculations, temperature dynamics I need to know the liquid level as well. In addition to the temperature inside this temperature of the hold up inside. So we say, as far as the liquid level dynamics is concerned it's a one state process.

I can give you a temperature. I can give you the color of the tank. I can give you, you know, its birth date everything. Oaky. But that has no bearing on the liquid level whereas as far as the temperature dynamics is concerned I will require-- we'll say a two state model. What are those two states? The liquid level and the temperature. Of course, here they are not hidden. In this example, the states are not hidden. But as I'm saying that sometimes they're hidden, sometimes they're not. The point here is, as far as a minimal realization description is concerned for the liquid level dynamics, one state is enough and for the temperature two states are enough. Very often we may end-up with many states that are unnecessary but what this concept says is you have to throw them out because they can mislead you in terms of inferring how high the dynamics are. In fact, we learn tomorrow that the number of states is the order of the system. Okay. Some of you must be familiar with it. So we say, liquid level follows a first order dynamics. When you write the differential equation for the liquid level dynamics, it'll be a first order. When you write the differential equation for the temperature it would be second order with the respect to change in inlet flow. So we say that the temperature dynamics is a second order system where as the liquid level dynamics is the first order that again connects the same thing. One state and the two states.

So when you have more states that are necessary you may end-up with the wrong inference on the dynamics of the system. So minimal realization is extremely important concept that will continue to appear and thirdly there is this term called steady state. What does it tell us about state? States are dynamically changing quantities in general. States are always dynamically changing variables. When this dynamically changing variables have studied out, we say, the system has reached a steady state. That's how the term steady state must have been born. But if you look at the history of state space models verses input, output model. Input, output models have been used for at least, many centuries now, two or three centuries. State space models are only popular in the last 50, 60 years. Particularly after Kalman's seminal work on how to estimate the states? See, I can come up with 100 different state space descriptions but if I cannot infer the states from the observed variables from the measurement then those models are not so useful. Only when I can start inferring the hidden variables then they become extremely useful. In fact, that is why kalman's filter is so celebrate and so popular because it came along and it said, look there are many hidden variables that you may not be measuring but I'll show you how to estimate those from what you're measuring and that's great because that means you don't need to have senses for every little variable that you want to estimate. As long as you satisfy what is known as what is an observability condition you are set to estimate those variables and Kalman's filter are used everywhere, in all aerospace applications everywhere. Right from military to space applications everywhere. So that is why this state space models became extremely popular after Kalman's work. Okay, and there are other advantages which we'll talk about tomorrow. Okay.