

NPTEL

NPTEL ONLINE COURSE

CH5230: SYSTEM IDENTIFICATION

RANDOM PROCESSES:

Review – (MATLAB) 6

ARUN K. TANGIRALA

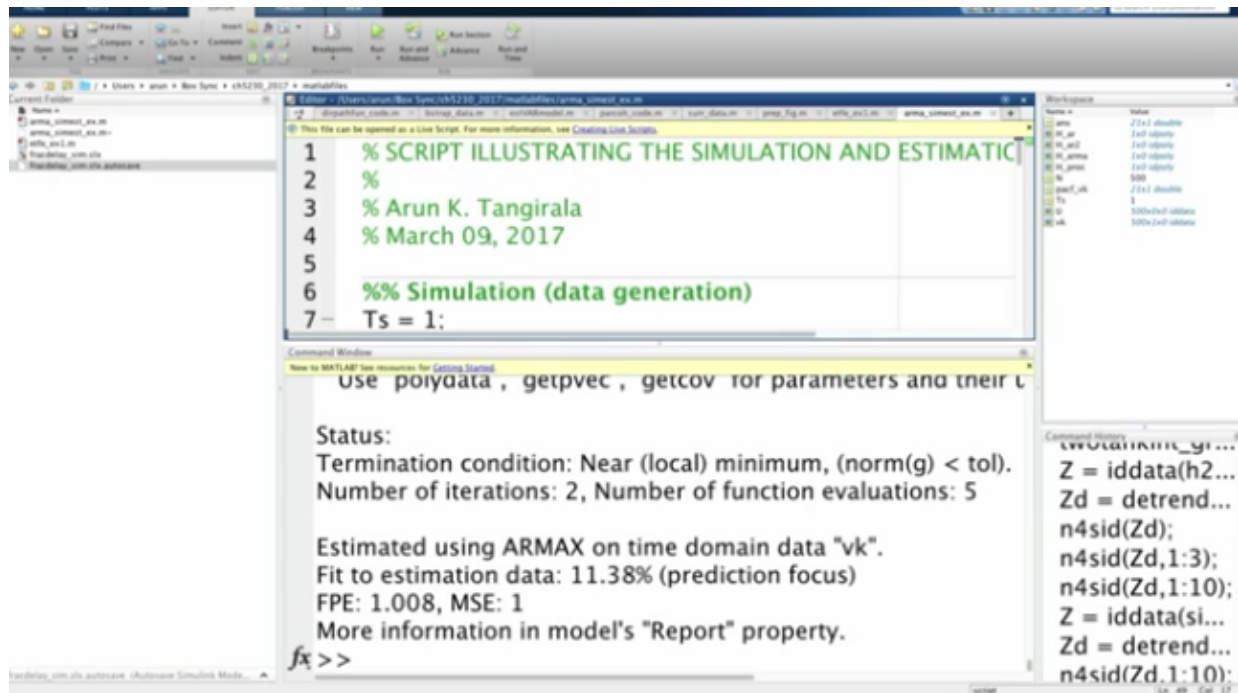
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IIT MADRAS

(Refer Slide Time: 00:19)

The image shows a green chalkboard with handwritten mathematical derivations. At the top left, the difference equation $v[k] + d_1 v[k-1] = e[k]$ is written. Below it, the solution $v[k] = e[k] + \sum_{j=1}^{\infty} (-d_1)^j e[k-j]$ is shown. A box around $|d_1| < 1$ and $\mu_v = 0$ indicates the stability condition and mean. On the right, the transfer functions are listed: $H(q^{-1}) = 1 + d_1 q^{-1}$, $H(z^{-1}) = 1 + d_1 z^{-1}$, and $H(z) = 1 + d_1 z$. The autocorrelation derivation starts with $\sigma[0]: E(v[k]v[k]) = -d_1 \sigma[1] + \sigma_e^2$, leading to $\sigma[0] + d_1 \sigma[1] = \sigma_e^2$. For $\sigma[1]: E(v[k]v[k-1]) = -d_1 \sigma[0] + E(e[k]v[k-1])$, which simplifies to $\sigma[1] + d_1 \sigma[0] = 0 \Rightarrow \rho[1] = -d_1$. An arrow points from the final result to the symbol ρ .

Alright, so let me at least quickly show you the MATLAB example, because then we can talk about cross correlations and spectral densities tomorrow.

(Refer Slide Time: 00:23)



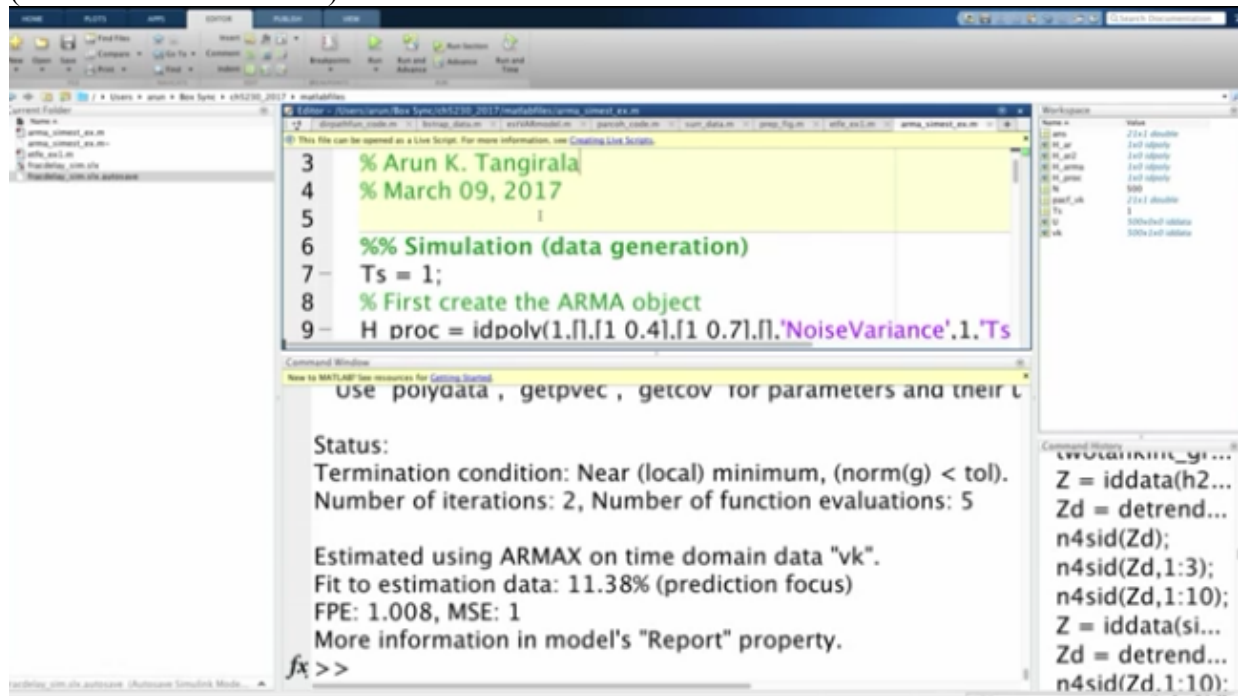
Now I have created the simple example and going to post this example, I just realize yesterday after someone told me in the class that I did not post a state space model notes before quiz one, I don't know why none of you in the class have actually told me before quiz one, none of you has actually return to me saying please post the state space model, you have the right to do that, okay, because I posted up to this sampling and discretization, I thought I have already posted the state space model part, only yesterday someone brings to my notice, to my attention that you've not posted the notes on state space models, I know text is available nevertheless, slides give you some different perspective.

So please feel free to write to me, I remember Krishna Teja bugging me like anything last year, if I'm off by a day, he would write to me, sir you've forgotten to post the slides, he's not doing this time you know why? Any guess? He's auditing, okay.

But you should have been the serious auditor, fine, and the other thing that I noticed is none of you is taking any help from the TS, TS I don't know if you know the expansion, they're called teaching assistants, which means they are here to assist not only me, but also you, when you have difficulties solving assignment I know maybe you're all you know very self-reliant and determined, you want to solve on your own, but sometimes it can take time, take the help of TS and remember assignments are meant for your learning more than anything else, that 10% is simply a token, so use assignments to reinforce your learning to learn the TS are there all the time in the office, there are three TS you can bug any of them, so as an ice breaker what I had told the TS yesterday is that looks like our students are extremely shy, okay, for whatever reason, this Saturday our TS will be available for one and a half hours in MSB or 2, 4, 0 or if the room is different they'll tell you, they'll tell you the timing and email will go out, they will be available in that room for you, it's then your exclusive time, go and consult the TA in not only seeking help for solving the assignment, but for anything else related to the course, okay.

But please do make use of the TAS because they are very good, they have taken the scores, don't ask what grade I'll get, that's the only question they will not be able to answer which I also cannot answer, but please do that.

And of course I'll also distribute the quiz papers, at that time I have another announcement to make, so let's go to this example here, in this example what I'm going to show you is how to simulate an ARMA process and how to estimate in that process
(Refer Slide Time: 03:10)



The screenshot shows a MATLAB environment with a script editor and a command window. The script editor contains the following code:

```
3 % Arun K. Tangirala  
4 % March 09, 2017  
5  
6 %% Simulation (data generation)  
7 Ts = 1;  
8 % First create the ARMA object  
9 H proc = idpoly(1,[],1 0.41,1 0.71,[],'NoiseVariance',1,'Ts
```

The command window displays the following output:

```
use poiyaata , getpvec , getcov for parameters and their  
  
Status:  
Termination condition: Near (local) minimum, (norm(g) < tol).  
Number of iterations: 2, Number of function evaluations: 5  
  
Estimated using ARMAX on time domain data "vk".  
Fit to estimation data: 11.38% (prediction focus)  
FPE: 1.008, MSE: 1  
More information in model's "Report" property.  
fx >>
```

I have used the codes ACF and PCF written by me, alright, and I said as I told you ACVF, sorry ACF is obtained by using XCOV, so let's quickly look at it, it doesn't take too long, maybe about 5 minutes.

So the first part is a generation part, let me actually,
(Refer Slide Time: 03:34)

```

4  March 09, 2017
5
6  Simulation (data generation)
7  = 1;
8  First create the ARMA object
9  proc = idpoly(1,[],[1 0.4],[1 0.7],[],'NoiseVariance',1,'Ts',T
10 Generate the time-series
11 = 500;
12 = iddata([],zeros(N,0),Ts);

```

```

Termination condition: Near (local) minimum, (norm(g) < tol).
Number of iterations: 2, Number of function evaluations: 5

Estimated using ARMAX on time domain data "vk".
Fit to estimation data: 11.38% (prediction focus)
FPE: 1.008, MSE: 1
More information in model's "Report" property.
fx >>

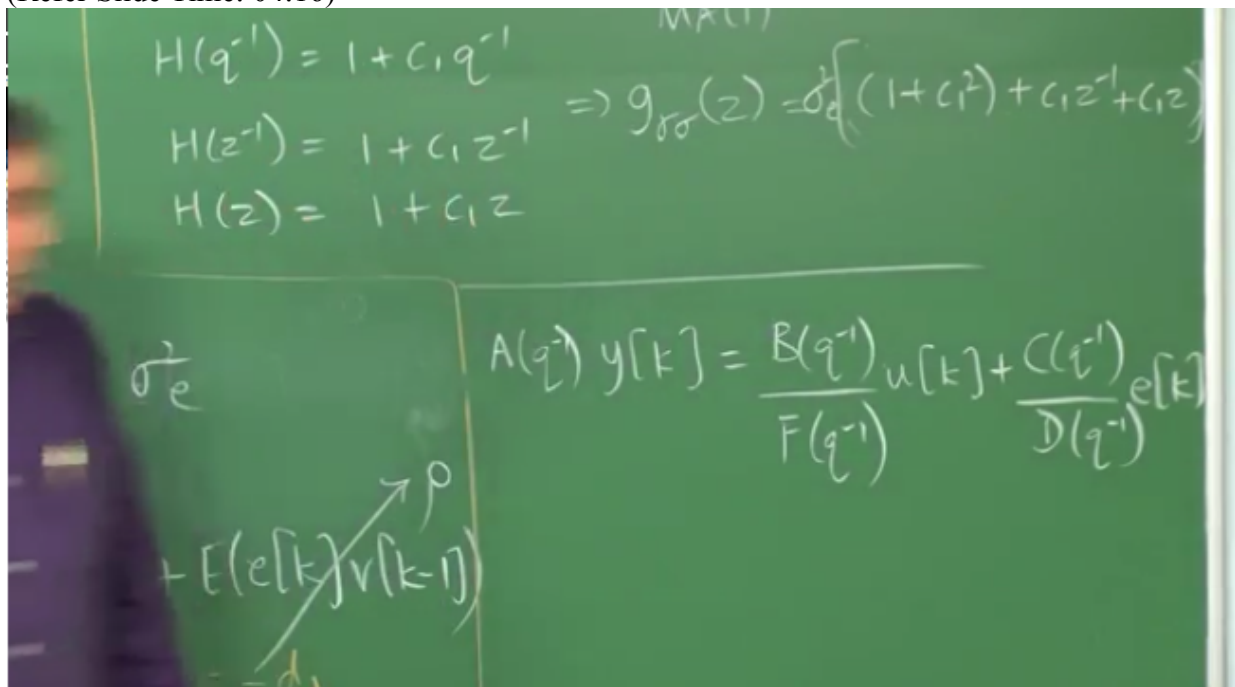
```

```

Z = iddata(h2...
Zd = detrend...
n4sid(Zd);
n4sid(Zd,1:3);
n4sid(Zd,1:10);
Z = iddata(si...
Zd = detrend...
n4sid(Zd,1:10);

```

I'd like to divide my codes into sections, so the first part as you can see is a generation part, I set the sampling interval, and in SID tool box there is a command called ID poly. What this ID poly? If you look up the help it says essentially it will create an object for you and LTI object, it's not just an LTI deterministic object, deterministic + stochastic, so the model that it assumes is this, and you know, next week we'll talk about this model in detail, B over FU + C over DE, (Refer Slide Time: 04:16)

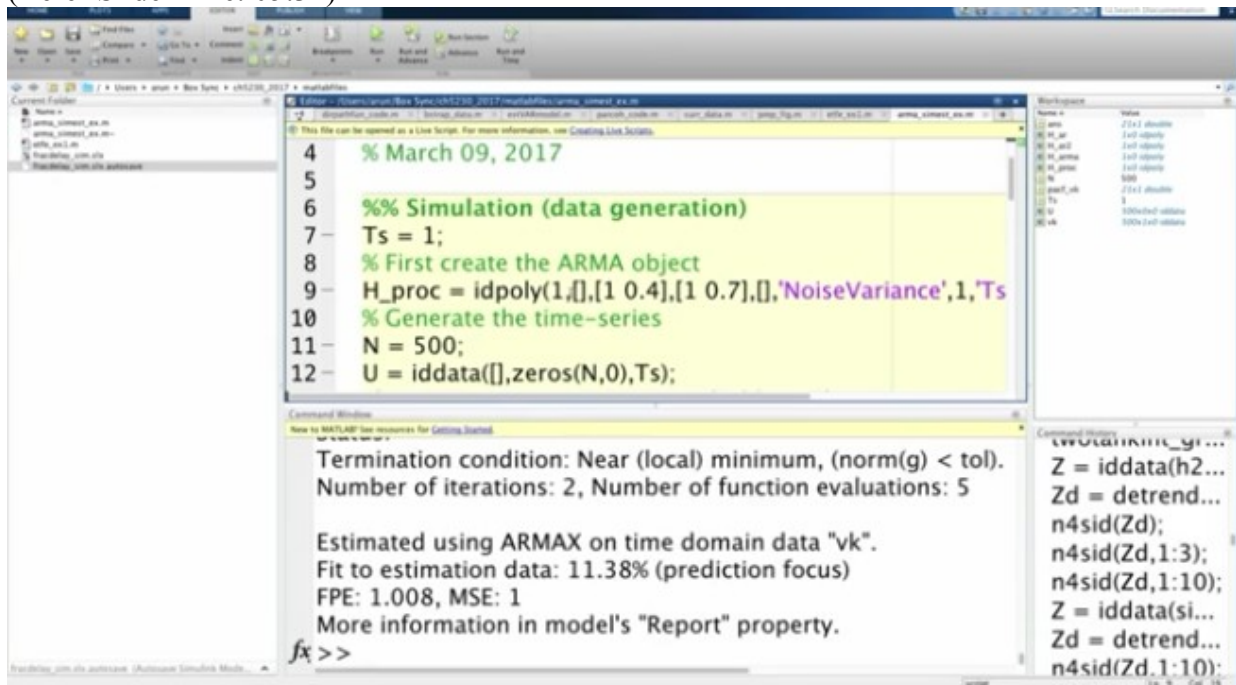


so this is the general polynomial model for deterministic + stochastic process, what you are doing is you're just fusing the deterministic and stochastic models, A, B, C, D, F are the

polynomials, if data is given they have to be estimated, if you want to simulate you'd to specify those polynomials, input and white noise you know already.

Now why can't I just write B/F and C/D, why I have taken the A out? Well A represents the common dynamics between the input that is between G and H if there is any, if there is none A is 1, okay, typically A is 1 but in some cases you can choose to have A naught = 1 which can make things easy or difficult depending on the situation, okay.

Now we want to simulate an ARMA process, that means I have to turn off this channel here, right, and ID poly it takes arguments in the alphabetical orders so A, B, C, D, F, by now you should now that all the English lower case, upper case alphabets are used up in SID, so A, B, C, D, F and if you look at the syntax here I say A is 1, because we are going to simulate an ARMA process, that means I'm only going to have C and D, (Refer Slide Time: 05:34)

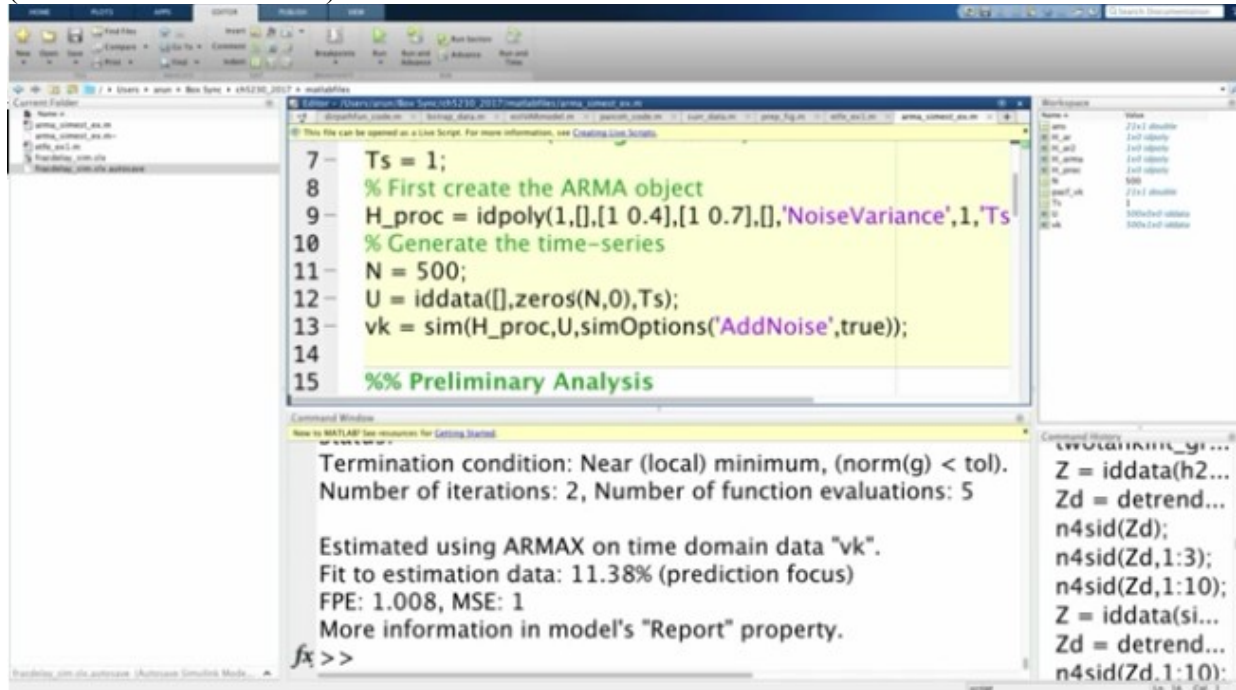


and I'm going to simulate an ARMA 1,1 process with the coefficients given here, B I have set to empty if you notice.

And if you have set B to empty so there are two ways of simulating your ARMA process, you can set B and F to some nonzero values, by the way what you supply here are the coefficients of the polynomials, you must have guessed that, so you can set B and F to some nonzero quantities and supply UK as 0, that is one option, the other option is tell the model clearly that I'm only simulating the stochastic part, where you set B and F to empty. If you set B to empty and not F to empty then it will complain, it will scold you so you have to go back and set B and F to empty which is what I've done, C and D I have chosen you can play around when I post the score, and I've set the noise variants to 1, and the way you said the noise variance is name value matching pairs, you can either do it at the time of generating the ID poly object or you can alternatively only specify A, B, C, D, F and then set the noise variance and TS later on using the dot operator, because the resulting object has a structure like shape in MATLAB, so you can say

for example if you don't set the noise variance here I can say `H_proc.noisevariance` equals 1 later on, and likewise for the sampling interval.

I've just done this in the single shot, now I want to generate the series, I only generate 500 observations of VK,
(Refer Slide Time: 07:07)



```
7- Ts = 1;
8- % First create the ARMA object
9- H_proc = idpoly(1,[],[1 0.4],[1 0.7],[],'NoiseVariance',1,'Ts
10- % Generate the time-series
11- N = 500;
12- U = iddata([],zeros(N,0),Ts);
13- vk = sim(H_proc,U,simOptions('AddNoise',true));
14-
15- %% Preliminary Analysis
```

Termination condition: Near (local) minimum, (norm(g) < tol).
Number of iterations: 2, Number of function evaluations: 5

Estimated using ARMAX on time domain data "vk".
Fit to estimation data: 11.38% (prediction focus)
FPE: 1.008, MSE: 1
More information in model's "Report" property.

```
fx >>
```

now because I have set B and F to empty, I have to pass on an empty input object to the simulation command, and the simulation command is `sim`, the `sim` is the generic command it's simulates ID poly objects, it simulates symLink objects and so on.

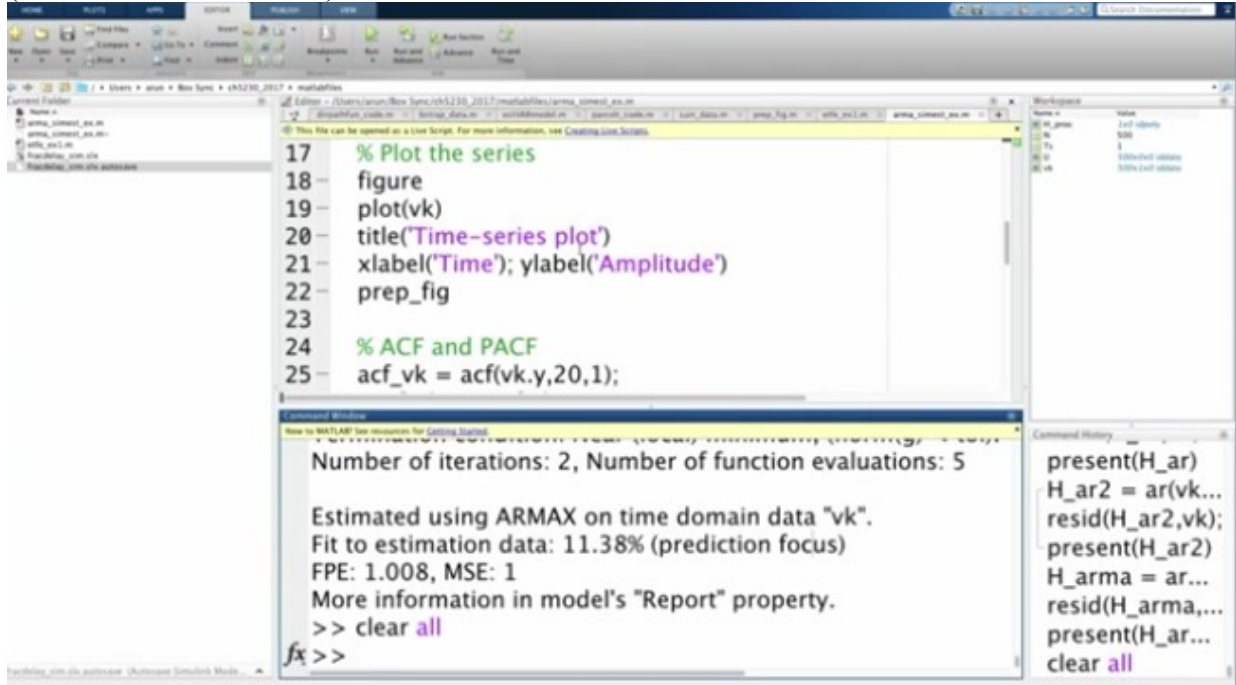
Previously there used to be an ID `sim`, but now there is absolute, at least for the last 2 years, so what you do, ID data is a routine from SID tool box, it's a very generic routine, it fuses input-output data and so on, so the syntax for generating an empty input, in fact if you look up the help on `sim`, by the way if you look up the help on `sim` it will first show you the generic `sim` that is used in symLink you have to choose to seek the help on `sim` from the identification tool box, and there I'll tell you how to simulate this, just follow the syntax that I have given here.

So I'm just creating an empty input and passing it out to `sim`, one thing you must observe is I'm saying, I'm telling the `sim` to add noise, okay, when you're simulating deterministic processes, it's extremely important to, or when you are simulating this processes it's extremely important to know whether you are performing a noise free simulation or a noisy simulation, if you do not specify this, it will do a noise free simulation in which case nothing will be, VK's will only contain zeros, because there is no input, you did not tell it to simulate noise, so therefore it will keep quiet, therefore it's important to specify this option, that's it, so it would have simulated now C/D operating on EK.

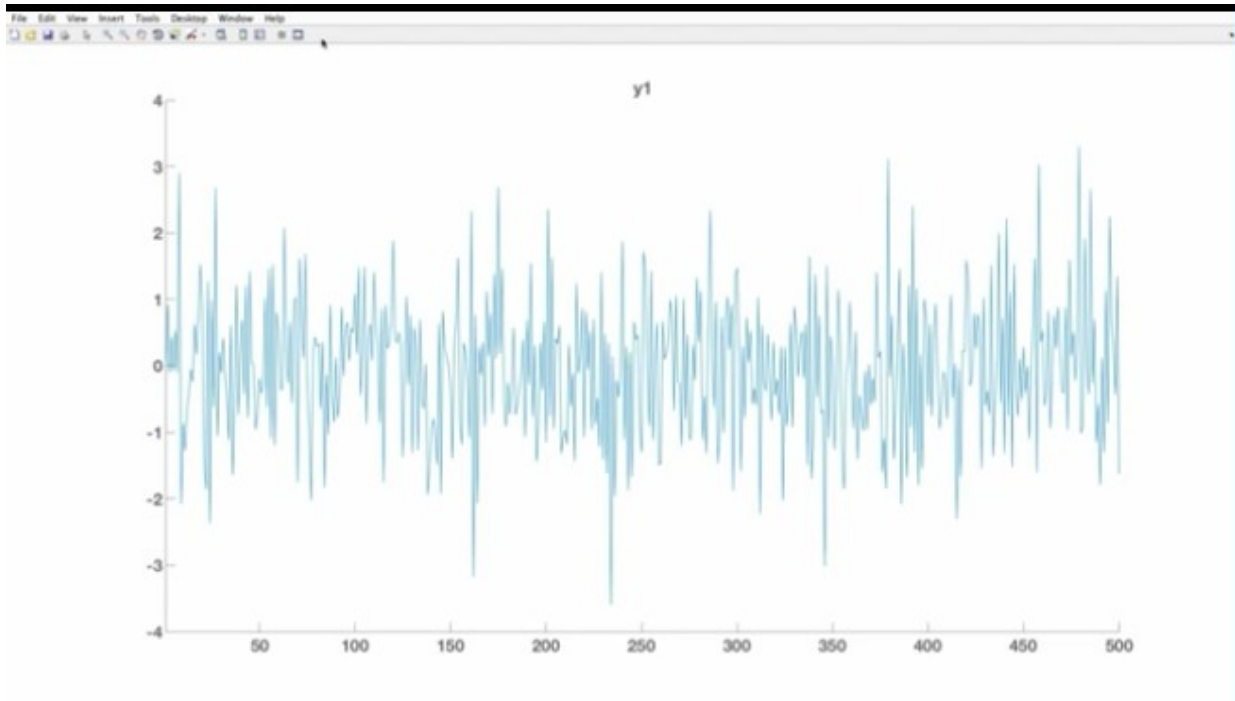
Now that the series is generated, so let me clear everything here, and I'm going to run this section here, so there is a run section on the top that's a nice thing, and it has done so you've

seen that it has generated VK, I don't know how well you can see in the environment here, but H proc, VK and U are being generated.

The next step is preliminary analysis always in time series modeling you want to first plot the series,
(Refer Slide Time: 09:04)



and then look at the ACF and PACF after at least ascertaining visually that the series is stationary, so here, I have, I'm going to just run this, alright, and this is how the series looks like, now Prep Fig, prep_fig is just a routine that I have written to decorate and make the figure look neat and this thing,
(Refer Slide Time: 09:29)



otherwise it looks very ugly, you know unkempt hair something like that I mean it just looks very ugly, so I like it this way.

By the way the one thing that you should remember is that the VK that is written by sim, where have you gone? Okay, is not your regular vector it is what is known as ID data object, so if you look at VK and if you just say get of VK, it'll show you that it has all these fields, it is not like your regular vector,

(Refer Slide Time: 10:03)

```

7 - Ts = 1;
8 - % First create the ARMA object
9 - H_proc = idpoly(1,[],[1 0.4],[1 0.7],[],'NoiseVariance',1,Ts);
10 - % Generate the time-series
11 - N = 500;
12 - U = iddata([],zeros(N,0),Ts);
13 - vk = sim(H_proc,U,simOptions('AddNoise',true));
14 -
15 - %% Preliminary Analysis

```

```

xlabel('Time'); ylabel('Amplitude')
prep_fig
>> get(vk)

ans =

    Domain: 'Time'
    Name: ''

```

```

present(H_ar...
clear all
figure
plot(vk)
title('Time-se...
xlabel('Time')...
prep_fig
get(vk)

```


remember that, if you want the vector of series accessing from VK, you have to use vk.pi, so all I am doing here is I'm plotting VK, and plot understands it's able to take an ID data object, plot is already written, so you have seen the plot fairly stationary, now it's time to look at the ACF and PACF, I'm going to just run this section, (Refer Slide Time: 10:24)

```

21- xlabel('Time'); ylabel('Amplitude')
22- prep_fig
23
24- % ACF and PACF
25- acf_vk = acf(vk.y,20,1);
26- pacf_vk = pacf(vk.y,20,1);
27
28- %% Building an AR model
29

```

Command Window

```

New to MATLAB? See resources for Getting Started.
xlabel('Time'); ylabel('Amplitude')
prep_fig
>> get(vk)

ans =

Domain: 'Time'
Name: ''

```

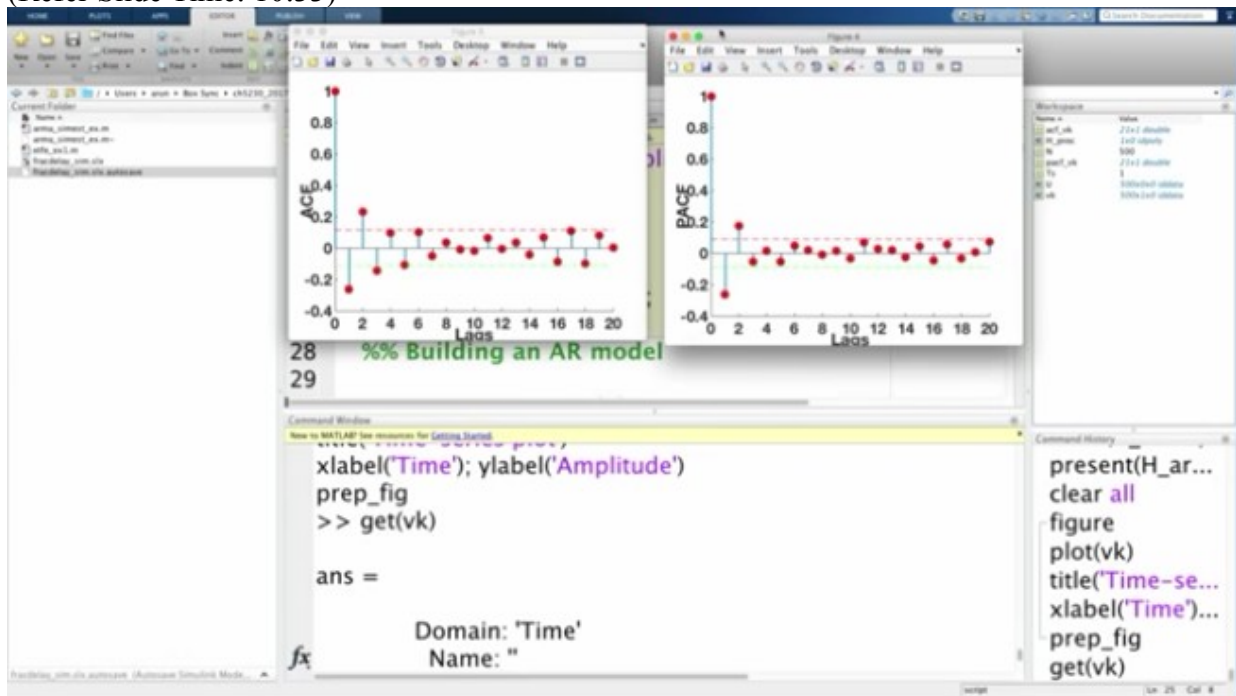
Command History

```

present(H_ar...
clear all
figure
plot(vk)
title('Time-se...
xlabel('Time')...
prep_fig
get(vk)

```

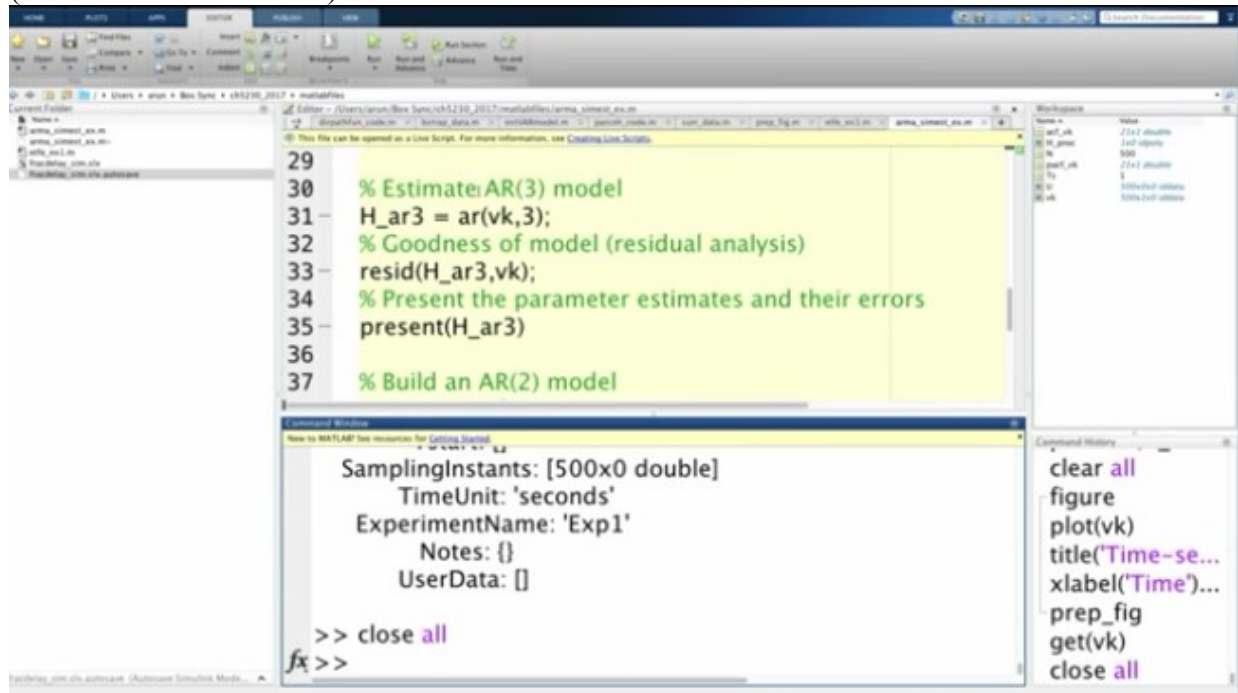
and this is how the PACF looks like, and this is how the ACF looks like, (Refer Slide Time: 10:33)



are you able to see or you want me to zoom in, so you can see ACF and PACF do not show any major signatures, right, it doesn't tell me clearly whether I should think of this as an AR or an MA, but in reality what will happen is you can say oh I look at the PACF and think of this as AR2 or an AR3.

What are those dash lines there? They're called the significance bands, I've already explained that in the liquid level case study, they help you figure out whether the estimate should be treated as insignificant or not.

So both ACF and PACF are essentially showing that there is an ARMA nature to the process, nevertheless I may want to fit an AR model, because as I have said estimating AR models is easier because you're going to solve a linearly squares problem, and therefore then last stage here I'm going to, so in a last stage here of developing an model I'm building an AR3 model, and the command for estimating AR model is AR routine here, (Refer Slide Time: 11:35)



```
29
30 % Estimate AR(3) model
31 H_ar3 = ar(vk,3);
32 % Goodness of model (residual analysis)
33 resid(H_ar3,vk);
34 % Present the parameter estimates and their errors
35 present(H_ar3)
36
37 % Build an AR(2) model
```

```
SamplingInstants: [500x0 double]
TimeUnit: 'seconds'
ExperimentName: 'Exp1'
Notes: {}
UserData: []

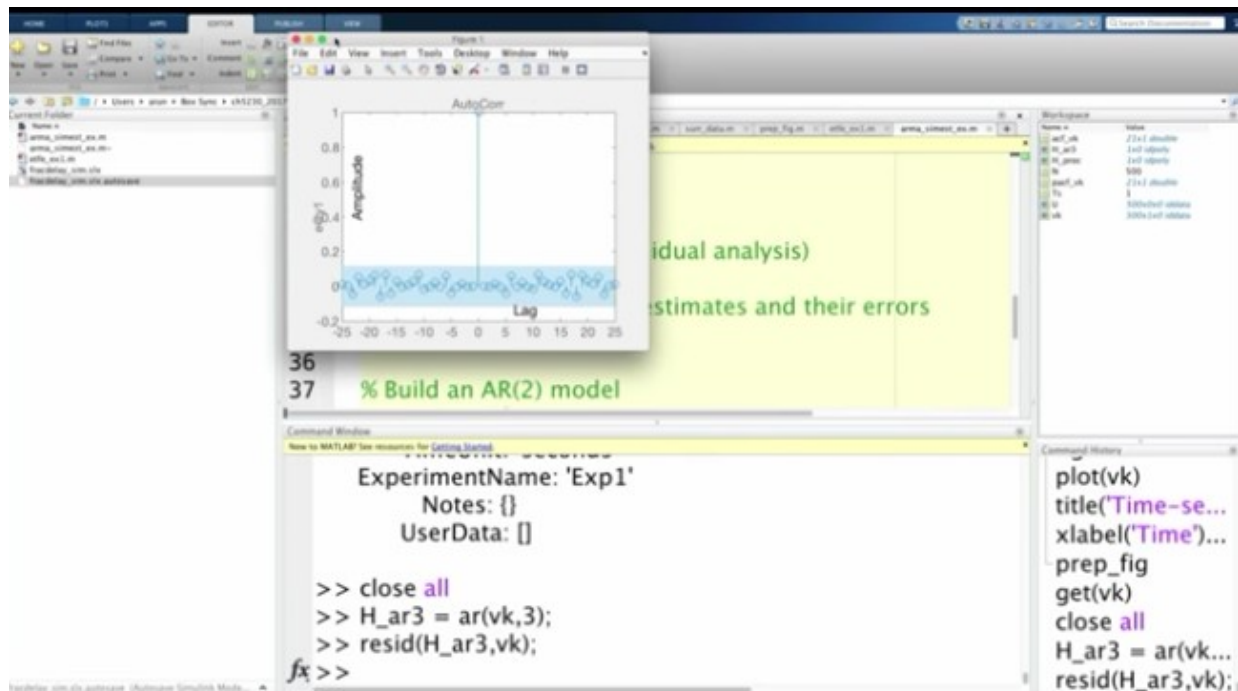
>> close all
fx >>
```

```
clear all
figure
plot(vk)
title('Time-se...')
xlabel('Time')...
prep_fig
get(vk)
close all
```

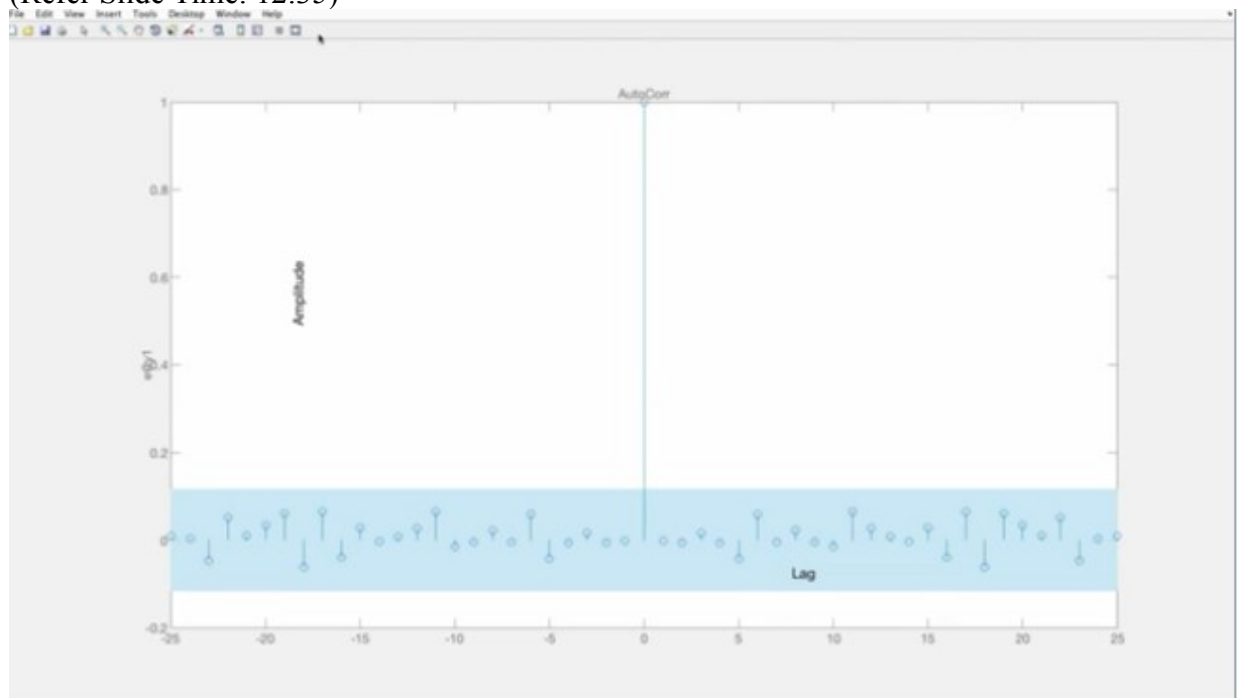
where you just have to pass on VK which is already in the ID data object format and then you specify the order.

Why did I choose 3 because I looked at the PACF alone, remember this although the underline process is ARMA to you it can manifest as an AR2, AR3 and so on, because there is an equivalence, I can always right a difference equation model as a convolution form and a convolution as difference, so the MA part can be absorbed into the AR part and therefore it appears as an AR of 2 or 3 and so on, so I'm going to start with an AR3 and see if it works, and that's what I'm doing here, the first step in any model assessment is residual analysis, alright, so I'm doing a residual analysis here, first I evaluate the selection and then do a residual analysis which brings up this plot, what is this plot showing me?

(Refer Slide Time: 12:30)



It is showing me ACF of the residuals from this model, what do you say?
 (Refer Slide Time: 12:35)



Is there anything predict, is there any predictability left in the residuals? No, so we are satisfied with this model, but there is a chance of over fitting, right.

The same story, so what do we say here? We say that we would look at the, what would we do? We looked at the errors in the parameters estimates and that's what I'm going to show you here, what do you see?

(Refer Slide Time: 13:12)

```

29
30 % Estimate AR(3) model
31 H_ar3 = ar(vk,3);
32 % Goodness of model (residual analysis)
33 resid(H_ar3,vk);
34 % Present the parameter estimates and their errors
35 present(H_ar3)
36
37 % Build an AR(2) model

```

Discrete-time AR model: $A(z)y(t) = e(t)$

$$A(z) = 1 + 0.2043 (+/- 0.04486) z^{-1} - 0.168 (+/- 0.04519) z^{-2} + 0.05384 (+/- 0.0449) z^{-3}$$

Sample time: 1 seconds

Command History:

```

title('Time-se...
xlabel('Time')...
prep_fig
get(vk)
close all
H_ar3 = ar(vk...
resid(H_ar3,vk);
present(H_ar3)

```

Do you see any over fit here? Just half a minute and it will done, do you see any over fit here? One of the estimates as high errors, the third one, so which means I have over fit, so therefore you're going to fit in AR2, I'm not going to go over that, but I have written the routine here, commands here for doing that.

(Refer Slide Time: 13:30)

```

42
43 %% Build an ARMA model
44
45 % Estimate ARMA(1,1) model
46 H_arma = armax(vk,[1 1]);
47
48 % Model assessment
49 resid(H_arma,vk)
50 present(H_arma)

```

Discrete-time AR model: $A(z)y(t) = e(t)$

$$A(z) = 1 + 0.2043 (+/- 0.04486) z^{-1} - 0.168 (+/- 0.04519) z^{-2} + 0.05384 (+/- 0.0449) z^{-3}$$

Sample time: 1 seconds

Command History:

```

title('Time-se...
xlabel('Time')...
prep_fig
get(vk)
close all
H_ar3 = ar(vk...
resid(H_ar3,vk);
present(H_ar3)

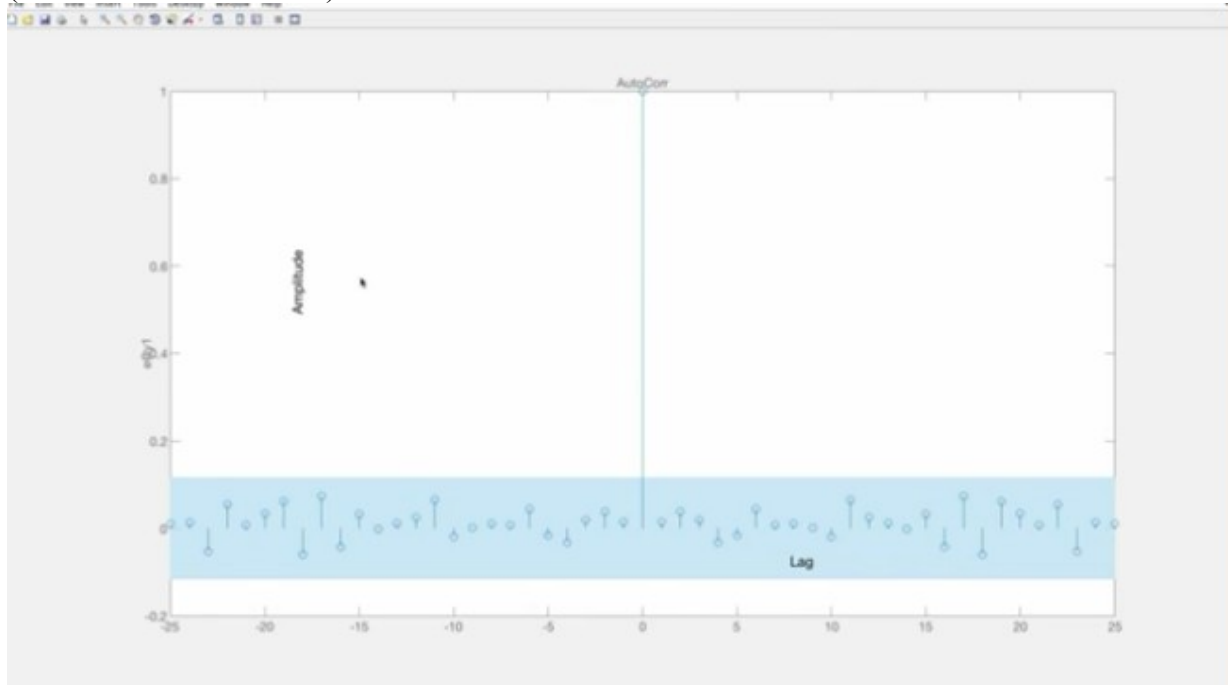
```

When it comes to estimating ARMA model, now I want to also see if an ARMA model is better suited, alright, the routine for doing that is ARMAX, that X is for taking into account the input effects, but there are no input effects here, and the syntax for ARMAX is of course pass on the

series VK and specify the orders of C and D polynomials here, I'll explain more in detail about the ARMAX routine, but as far as using ARMAX to estimate ARMA models is supply the series and specify only the numerator and denominator polynomials, be very careful because of the syntax of ARMA the first one actually corresponds to the MA part and the second one corresponds to AR part, please remember that, okay, you should be careful.

Now as usual so I estimate this and then we'll see if everything is okay, I'm going to bring up the residual analysis and see if, so this is again, now the ACF of the residuals from ARMA model,

(Refer Slide Time: 14:46)



again this shows that I've fit a good model, the only thing that I have to check is over fitting and that here, that's it, so these are the estimates of the ARMA model,

(Refer Slide Time: 15:02)


```

43 %% Build an ARMA model
44
45 % Estimate ARMA(1,1) model
46 H_arma = armax(vk,[1 1]);
47
48 % Model assessment
49 resid(H_arma,vk)
50 present(H_arma)
51

```

Command Window

```

H_arma =
Discrete-time ARMA model: A(z)y(t) = C(z)e(t)
A(z) = 1 + 0.7706 (+/- 0.07667) z^-1
      |
C(z) = 1 + 0.5551 (+/- 0.09969) z^-1
Sample time: 1 seconds
fx

```

Workspace

Name	Value
ar1_vk	21x1 double
vk	2x1 double
H_ar3	2x1 double
H_arma	2x1 double
resid	6000
vk	21x1 double
vk	2100x1 double
vk	2100x1 double

Command History

```

get(vk)
close all
H_ar3 = ar(vk,...
resid(H_ar3,vk);
present(H_ar3)
H_arma = ar...
resid(H_arma,...
present(H_ar...

```

of course pretty close to what I have, don't look at the A and C there, by the way I'm sorry earlier I said when you use ARMAX you estimate the orders because of the strange way that it is written, the first order here is for the AR and the second is for MA, earlier I said it was the other way around, so the first one is AR, the second is MA, and that's it, so you see that it has estimated with descent accuracy and none of the parameters have large errors in that, so this is how you seen there.

Now the question that you have to think about is which model I should pick, which I will talk about tomorrow, I have an ARMA model and then I have an AR2 model, right, in practice I wouldn't know, here I've simulated therefore I know, which one I'm going to pick, will answer that tomorrow. Okay.

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