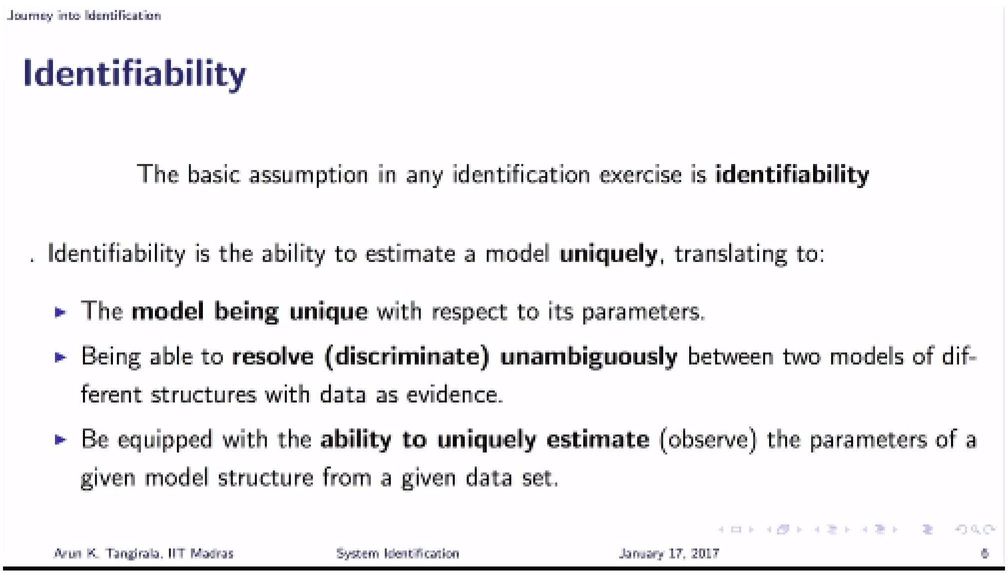
CH5230: System identification

Journey into Identification 3

Okay. Good evening. So what we'll do today is as I promised last week we'll go through a few examples to get a feel of what identification involves. It'll give you a good picture of what is expected for the rest of the course. Not only conceptually but to a certain extent technically, we will try to keep the mathematical details low but of course, you know, this is the second week. So it's time to see a few equations in action but not in full flow though. I've just constructed very simple examples to begin with. The highlight of this would be a complete case study which I would like to present upfront to give you as I said, a feel of-- an a flavor of what identification is about. And if you recall in the last class towards the end we spoke of an important concept called identify ability. Which essentially is concerned with your ability to identify and if you look at simple terms it's about our ability to identify unique model. Technically it has a lot more to it. But as I said, we will not go into the full technical details yet obtain a good picture of what it is.

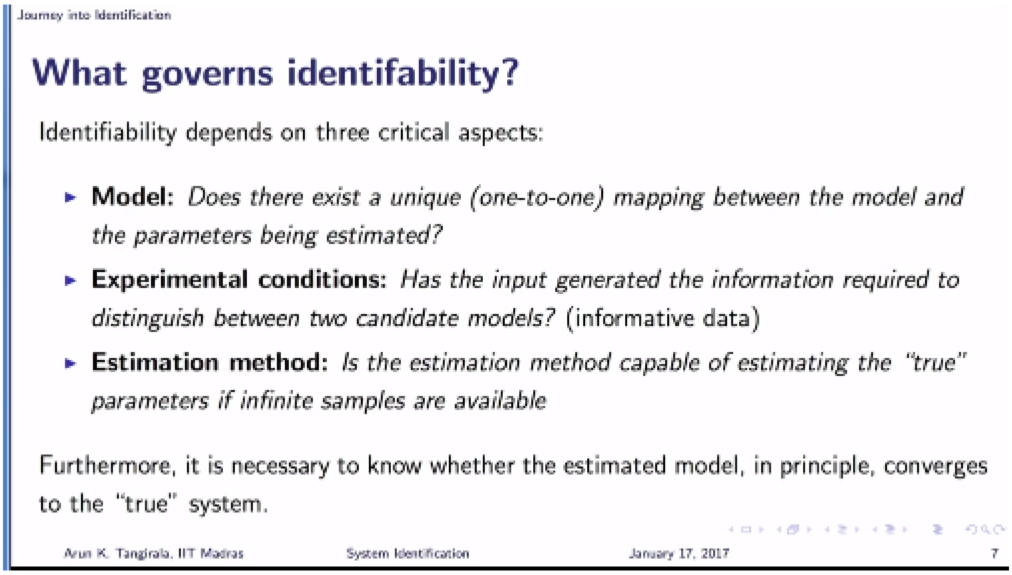
So there are three aspects of identifiability. One has got to do with the model itself that we intend to fit to the data as we discussed very quickly towards the end of the last lecture. And the second one has got to do with how we perform our experiment? Obviously, because we know data is a food [1:54] identification. So it's important to generate good quality data and we have had a couple of analogous examples particularly of the interview process and so on.

(Refer Slide Time: 2:06)



So here the input and the input design plays a central role and the third aspect of identifiability is the estimator itself. That is the algorithm. You can think of it as a formula or some technique that will help us identify the true model that is estimate the parameters. So there are three aspects to identifiability. The first one which is a model identifiability has got nothing to do with the experiment. Okay. It is just the nature of the model itself. For example, I can end up choosing a model which is not unique. And there are several examples of that. We'll go through one very simple example that we discussed last week.

(Refer Slide Time: 3:02)



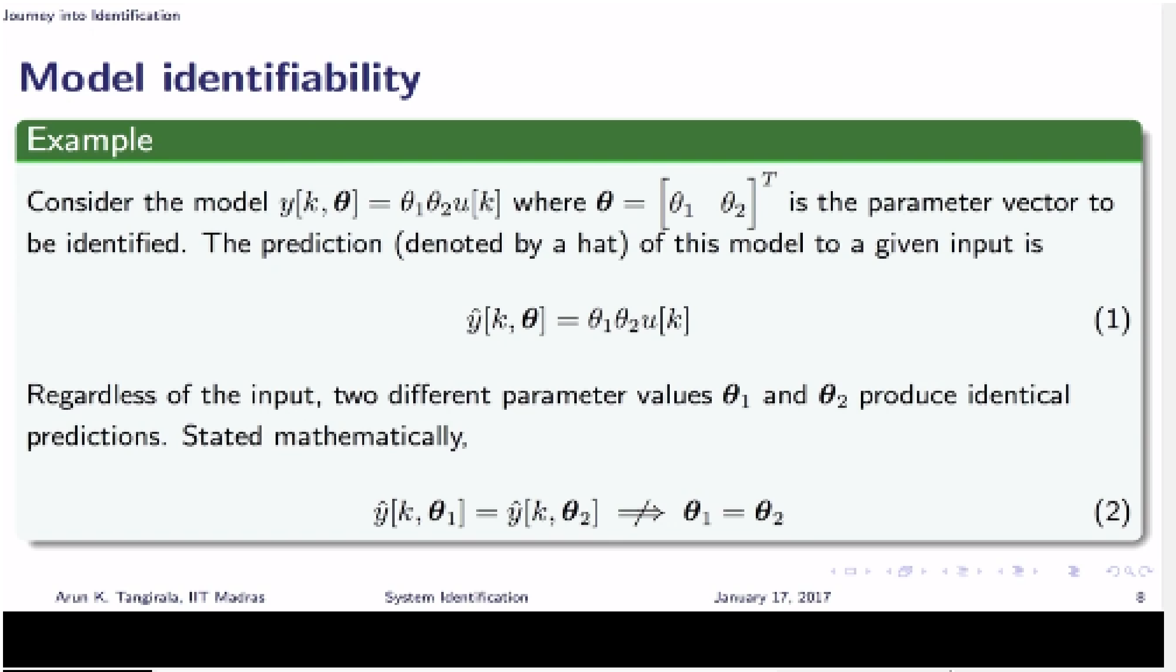
Whereas the remaining tool has got to do with data. In fact, the second one which we said--where we said, the input and input design plays a central role. There's got to do with the experiment itself. And the third one has got to do with the choice of estimator. So you would understand that there is one which has got to do with the choice of the model where we don't worry about how the data is being generated, what kind of data is being used and so on. And the second has got to do with the experiment. Assuming that the first one has been taken care of. And the third one has got to do with the estimation algorithm. Now after all of these are satisfied there is still this concern whether the final. Even if the model itself is unique and has been uniquely identified it should match with the true system. Now that's a big expectation to expect our model to match the true system, why? Because the original process for which. What even if it is the simplest of the lot usually does not fit into the framework of the model? Like the liquid level case study that we are going to take up. It is one of the most innocuous systems that you can encounter. Very simple, there's no reaction. Nothing, there's just a flow, inflow out it's a buffer system, right. It can't be simpler than that. Even if you were to look at the dynamics of such a system we know that it follows a non-linear dynamics because the [4:38] is going to be non-linear. Whereas we intend to fit linear models so it is outside the purview of the linear models. Then what do we mean by the true system belonging or matching with the identified model. So it's not a very nicely, what you say, a framed expectation so to speak. Yet, we will talk about this aspect a bit later on. And that is we'll still stick to that expectation. We'll hope that if the system is within the purview of the model have recovered the system.

So suppose, the system was truly linear and the system was truly time invariant. Remember, we are going to fit linear time invariant models, whereas the true process even the simplest may not be linear or may not be time invariant but suppose, I imagine that there exists an approximate linear time invariant model for the process then at least I should recover that. And this was the question that was addressed in the mid 70s by [5:47] for a long time people always wanted to recover the so-called true model. But there is nothing like a true model because any process is outside the reach is beyond the reach of a mathematic, a mathematical model or even statistics. It's a lot more complicated than we can imagine ever. Therefore to expect our model to recover the so-called true system is not correct. However we would like to know for example, if we have estimated the best approximation for example that is one question we can ask. The other question--the other expectation that we can have is that if the process were linear and time invariant which is obviously only in academic settings, in a classroom setting we would expect to do that or in a theoretical setting we would like-- this question is valid even otherwise it's okay.

So if the process were truly linear and time invariant, will my identification method recover that? If it cannot do that then there is no point. There is no hope in even actually trying this and taking it to a real system. Right. Even if it cannot recover the system that is truly linear and time invariant. So that is what we mean by recovering the true model of the system. But we will not talk about it right now because first we need to understand these three concepts. Again, we are going to get only a curtain raiser on all of this. So let's talk about model identifiability. So here very simple model that we have and it's a static model. As you can see from the equation that we have and this model is characterized by two parameters. So there is a process that's at steady state. And I want to fit this kind of a model typically as you will learn in identification the moment we are given a model we would write a prediction equation before we set out to estimate parameters. As I'd explained in the last class the reason for doing that is that is parameter estimation is generally performed by minimizing prediction errors and in order to compute a prediction error, I need an expression for the prediction.

So generally what you will see is that after the model has been postulated by the user whether you do it or not the routines that are available in the tool box or any other tool box would implicitly compute the prediction from the model that you postulated and then estimate the parameters by minimizing the prediction error. So it's a good practice from now on to get used to this notion of prediction. And we shall denote predictions by a hat. And that is the convention that followed throughout in the prediction theory, in the estimation theory literature. So we'll assume that this is what is a predictor that we are looking at and what we're interested in is given input, output data now estimating this parameter theta1 and theta2. At this moment since we are talking about model identifiability, we will not worry about how the inputs were generated and so on. So now we'll not worry about how the input was generated and how these parameters are going to be estimated. But just looking at the predictor itself tells us, whether the model is unique or not because that is a question that is of interest in model identifiability whether in the parameter space.

(Refer Slide Time: 9:22)



What we mean by parameter space? In the space of theta1 and theta2. If there exists a unique point that will give rise to a given prediction. So in this space, in the parameter space theta1 and theta2 we want this parameter space to be such that one point gives rise to only one prediction. It should not be such that I have two solutions giving two sets of theta1 and theta2 giving rise to the same prediction. You can say, is there a problem? Well, there is and there is not. Depends on what you want. Well, what we want is a unique solution. When I estimate I want a unique solution so that I can then talk of the truth. Because generally when we think of truth, we expect the truth to be fixed at one point then when we estimate parameters normally what we ask is, well, there is a truth. And then I have this estimate. What is error in my estimate? But if the truth is at multiple points then the notion of an error, estimation error itself is gone. That is not valid.

So for all these reasons we want the-- anyway so formally what we want to say is that if I say that the predictions are equal at two different points then it should only occur if the parameters are equal which means a single point in the parameter space should give rise to a single prediction. Obviously this model does not satisfy that requirement. Right. And as I've stated that why hat at a set of parameters although say theta1 there that theta remember, there is a boldface theta that I've used and the boldface theta that you see, should be thought of as a vector consisting of theta1 and theta2. So it's a column vector of parameters this is a notation that we will follow for vector of parameters. So when you see a boldface theta or a boldfaced symbol then you should think of it as a vector. So formally if you look at equation 2, what it says is that prediction at two different points if they have to be equal then it should only occur when the parameters are equal but it is not occurring for this model. Predictions had two different points can be equal even though the parameters are not equal. It does not imply. So if I say y hat at theta1equals y hat at some other theta 2. It does not necessarily imply that theta 1 is equal to theta 2. When I say, theta 1 here it's not the scalar parameters it's a vector of parameters theta 1 and theta 2. So there is some confusion that the notation but hopefully you understand what we are trying to say here that there exist two different sets of parameters that give rise to the same prediction. And that means uniqueness is not guaranteed, right.

Now obviously, there are in fact infinite such possibilities here. Then we said that there is a loss of identifiability. The model is not identifiable. In fact they say, the model is not globally identifiable. In the entire theta 1 and theta 2 space it's not identifiable. Unless, I impose some kind of a constraint. Right. If I impose some kind of a constraint and suitable constraints then I can get a unique solution. That is one way of guaranteeing identifiability, I say, get me this model such that theta 1 and theta 2 satisfy some constraints. So that that constraint brings about a unique solution. That's one way of guaranteeing uniqueness. The other way of guaranteeing uniqueness is to fit a lower dimensional model not a lower order. What we mean by lower dimensional is, club these two together and call this new-- introduce a new parameter which is the product of theta 1 and theta 2. And rewrite the model now in terms of beta. So from a two dimensional parameter space we are moving to a one dimensional parameter space and in this space of parameters the model is identified. Okay. That there is no doubt about it, that means there exists one value of beta will give me only one prediction. They cannot exist two values of beta that will give rise to the same prediction. I'm I right or is there a debate or is there some question on that? Can there exist two values of beta that will give rise to the same y hat? Yes or no? What do you think? Is it possible?

Why is class so silent? It's a very simple question. There's no trick here. Do there exist two values of beta that will give rise and of course, the same input uk, that will give rise to the same y hat? No. Good. So that's a good at least a Lok Sabha

“No”, if not silent one. What is Lok Sabha “No”? Some of them agree. Majority agree. Okay.

Therefore sometimes or many times when you run in to identifiability issues, you may have to either impose constraints or what do you have to do? You'll have to go to a lower dimensional parameter space to guarantee identifiability. Now you may think this is too-- I mean probably the example that I have taken is actually quite strange and maybe exotic. You don't really encounter this but it isn't. Although the example may be quite exotic and people should be crazy to actually fit the models that I have said, theta 1 times theta 2. But it is representative of a class of models that you will run into later on known as a state space models. When you look at state space models, and even among those who are-- some of you are familiar with state space models, you should be aware that a state space model for a linear time invariant system is not unique, right. I'm not going to write the state space equation here because some of you do not know may feel left out but qualitatively if you look at a state space model then it's a fact that there exists no unique state space representation of a linear time invariant system there are infinitely different ways in which you can write a state space representation. What does it mean? If I sit to fit a state space model for a given process from input output data. Unfortunately there is no unique answer. Unless. Now what? Unless, I impose some constraints or I rewrite the state space model in terms of fewer parameters. And that is what we will realize later on which we call it structured state space models. That means, I impose a priority before even I fit the model, I will impose a priority that I want only this state space model that is in state space models, you have matrices a, b, c, d and so on. These matrices in a general case that is when you don't impose any criteria these matrices have their called fully parameterize. That means all the entries in a, b, c, d matrices have to be estimated. Okay.

But when you impose a certain structure what you are going to say is that this matrix A has only this many non-zero entries in these locations. Likewise for B and C and perhaps for D because D can we uniquely estimated. And when you do that what you would be ending up doing is guaranteeing identifiability. Again, you can't guarantee identifiability just because you are imposing a structure, you have to prove that for that structure that you specified there exists a unique state space model. So there is some work that has been done on that and so on. So the difference between a regular state space identification and a structured state space identification exists. In the regular standard state space identification, you say, I don't care really what state space model I get, I just want a state space model. Doesn't matter. But when I impose a certain structure, I am openly stating my requirement that I want-- I don't want any state space model, I want that particular state space model which has that structure for whatever reasons that we will discuss later on. And in doing so what you're doing is, you're guaranteeing identifiability. That means, you're guaranteeing a unique thing and then you can talk about errors in the entries of those matrices and so on. If you do not specify a structure on the state space model or in fact even an input output models where you look at transfer functions if you have chosen a model that has a zero pole cancellation. Then again you run into identifiability issues and so on. So it's important in summary that you should be aware that the model that you have chosen to fit is unique, is identifiable, regardless of data. Remember.