CH5230: System Identification

Models for Identification 2

The first requirement is that the mean is allowed to change with time but should be bonded.

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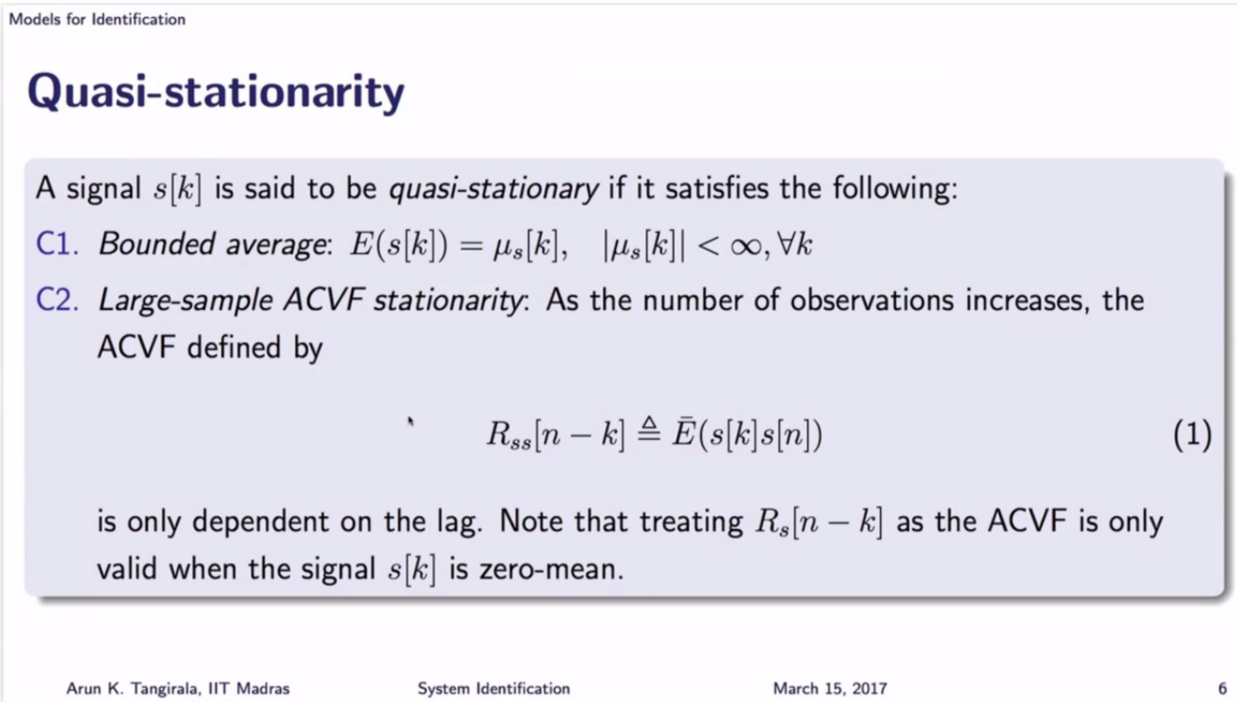

All right. Which means, what kind of inputs are ruled out in identification? We are sayingin in identification, I just can make, I can make some more corrections. One more correction, at least the tools that we are going to use to estimate those models and identification. What inputs are ruled out? Correct. A ramp input is ruled out. Because it's input, ramp continues to grow with time. The mean will go away, run away to infinity. Which means, now slowly I'm understanding in an experiment for identification what kind of inputs I should use. Or at least I should not use. The second requirement is in terms of ACVF stationarity. It's called large sample ACVF stationarity. What it says is, when you estimate ACVF you will use this formula, 1 over N sigma. Let's say for y, assume it to be zero mean you would use this formula to estimate ACVF of y or any other signal which has zero mean.

Let's say k runs from here. Zero. Our index runs from zero, in fact not zero, say l to N minus 1 because we assume y. We have observations from k equals zero to N minus 1. Correct. Now this expression is, this estimate can change with time, with the number of samples. So you may take 50 observations I may get one value for the autocovariance estimate at lag l. If I take a hundred observations it can change back but the requirement is as I include more and more observations it should settle down. That is what we mean by.

Now I have not written that expression there on the screen. Instead I have used an E bar. So we have. As you are getting comfortable with expectation I have put a bar on top of it. You can think of it is a monkey bar or whatever but E bar is called a generalized expectation. Why has this been introduced? Why can't we just live with the standard expectation which itself is giving me nightmares. Right? The reason for introducing E bar is because we are looking at a composite signal. We are looking at a deterministic plus stochastic signal. And E bar. I'll give you the definition of E bar in the next slide. It allows you to fuse both. You can apply this E bar to both deterministic and stochastic signals.

Because if I think of expectation we can only apply that to random signals. For deterministic signals there is nothing to expect because average in time. For stochastic signals what is averaging a dimension [3:38 inaudible]. This E bar allows you to walk in both dimensions depending on whether your signal is stochastic or deterministic. So what is this E bar. This is the definition of E bar.

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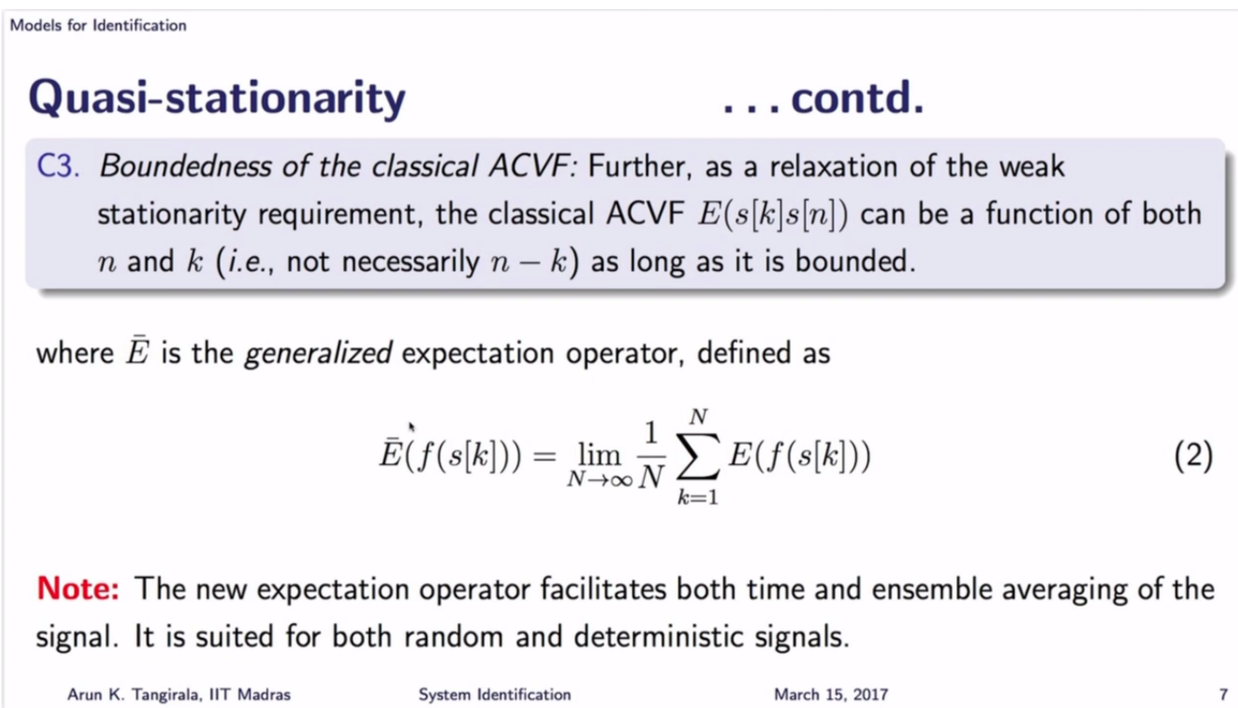

E bar of any function of any signal, any signal s. s could be stochastic or deterministic is defined in the limiting sense. So let us look at this expression here. Suppose this f of so f k. Let us say is s k itself. That means I am looking at the mean. Right? I'm looking at the mean. If s is stochastic and stationary, then what does the right hand expression simplified.

That means, so f is a general function of s. So if I'm looking at E bar of s of k as per the definition, what is the right hand side? No it's a mean. I have not said the stationary signal has a zero mean. Why will it be zero? S is purely stochastic. So the summoned here is going to be simply expectation of s k which is Mu. Because I'm assuming it is stationary. It falls out of the summation. And since I have N terms there, the 1 over N cancels out and I get expectation of s k. So E bar simplifies to the regular expectation from stationary signals.

This is good news so I know that E bar specializes itself too. That means averaging time is not done. Only the averaging in [05:31 inaudible] is done. When I look at a deterministic signal what happens? Suppose s k is some deterministic signal. And I'm looking at E bar of that deterministic signal. What does the right hand side simplified? What happens? The expectation of f of s of k or f or s is nothing but s k itself. Expectation of s k is s k itself because s is deterministic. So you're simply evaluating the time average. Correct?

And that time average depends on the deterministic signal but that is how the average is defined for a deterministic signal. Simply you look at the limiting definition 1 over N sigma k equals 1 to N s k. So that goes to show that E bar serves as an averaging operation for both stochastic and deterministic signals. It's a very beautiful definition. Since, it applies to both. Individually you should also apply to a mix. All right? So no going back to the definition here. We are seeing this so-called large sample ACVF. No the right hand side. So for all practical purposes this E bar is nothing but an averaging operator. If the signal is deterministic it averages in time. If the signal is purely stochastic it averages in the ensemble. If it is mixed it does both.

Therefore this E bar can be thought of as an expectation operator. Let us assume for all cases that s is zero mean. Then this E bar of s k times s N. What is it? It's a ACVF. Is ACVf. in general can be dependent on k and N but as I proceed to large times. The quasi-stationarity requirement is that they ACVF, the way this is being evaluated as an equation one here should only be a function of the lag. So in general, the left hand side is a function of k, N but at large samples it should be only a function of k minus n. That's all it says.
So if you don't like all of this, the simple understanding is ACVF is allowed to change with time. But at large times it should be independent of time and only be a function of the lag l. That's all. That's all it is.

So which means, as I include more and more observations, as I observe the process for a longer time then the process in some sense achieves a stionarity [08:31 inaudible]. That's all. And the third requirement is obviously that it should be bonded. ACVF should be bounded. It cannot run away with time. Why is this being imposed? As I said one I should now know, what are the inputs that are admissible? Not all inputs are admissible in my theoretical framework that we are going to look at.

In the methods that I'm going to use. It is extremely important now to say, what class of signals are admissible, what kind of data I can handle. And this tells us that, for example, ramp inputs are a no, no. By the way, if I look at the pulse let us say any signal that is pulse. By the quasi-stationarity now you should understand has got nothing to do with deterministic or stochastic. It can be applied to both signals. Stationarity on the other hand is only talked of in the context of stochastic signals. Quasi-stationarity is applicable to all classes of signals. That's the beauty. So now the simple question to you is, if I have a pulse of finite width, is it quasi-stationarity? What do you think?

Let me ask you another question. Is a periodic signal quasi-stationarity? Yes or no? No you should not think whether it's deterministic or stochastic. You should just apply these conditions. I give you a signal and ask you for this quasi-stationary. You just have to apply these conditions. Does the first condition holds. Correct. Second condition, does it hold? We learned yesterday how to-- what are the autocovariances of periodic signals. It will hold. The second condition also hold because autocovariance is define, it will be periodic, it's only a function of the lag l. So that's fine. Is autocovariance bounded and nonzero for a periodic signal? Yes or no?

What did we learn about the autocovariance of a periodic signal yesterday? [10:57 inaudible] it is bounded. It's not zero therefore it's a quasi-stationarity. Now as a simple home you think whether a finite duration pulse. Is it quasi-stationarity. And the other question you want to ask is, if the input was a realization or not input. If I take white-noise or any stationary signal is stationary signal, quasi-stationarity. Yes or no. It has to be. Because stationary is any stricter one. The stricter one is satisfied the relaxed one is also satisfied. Correct? Not necessarily the other way round.
That means all strictly stationary signals have second order stationary. And all second order stationary signals are quasi-stationary. So which means, inputs that are generated as a realization of a white-noise. Don't confuse between now you said, inputs are deterministic Why are you talking about white-noise as an input? We are not talking of white-noise as an input. We are saying one realization of that white-noise process, I'm going to use an input. That means my input is going to be generated randomly as a random sequence.

Such a signal, that is a single realization of a stationary, sorry white-noise process. Does it qualify to be quasi-stationary? Think about it. Okay. Answer should be fairly straightforward. But think about it. The reason I'm asking you to think about is, now you should start worrying about what kind of inputs are admissible. v k we know, stationary v k, but now we are saying it can be a quasi-stationary. It's okay. But by assumption since v stationary we don't have to worry about quasi-stationarity of v. Okay. So to sum up apart from this assumptions of additive noise G being LTI, v k being stationary. We want the input to be quasi-stationary.

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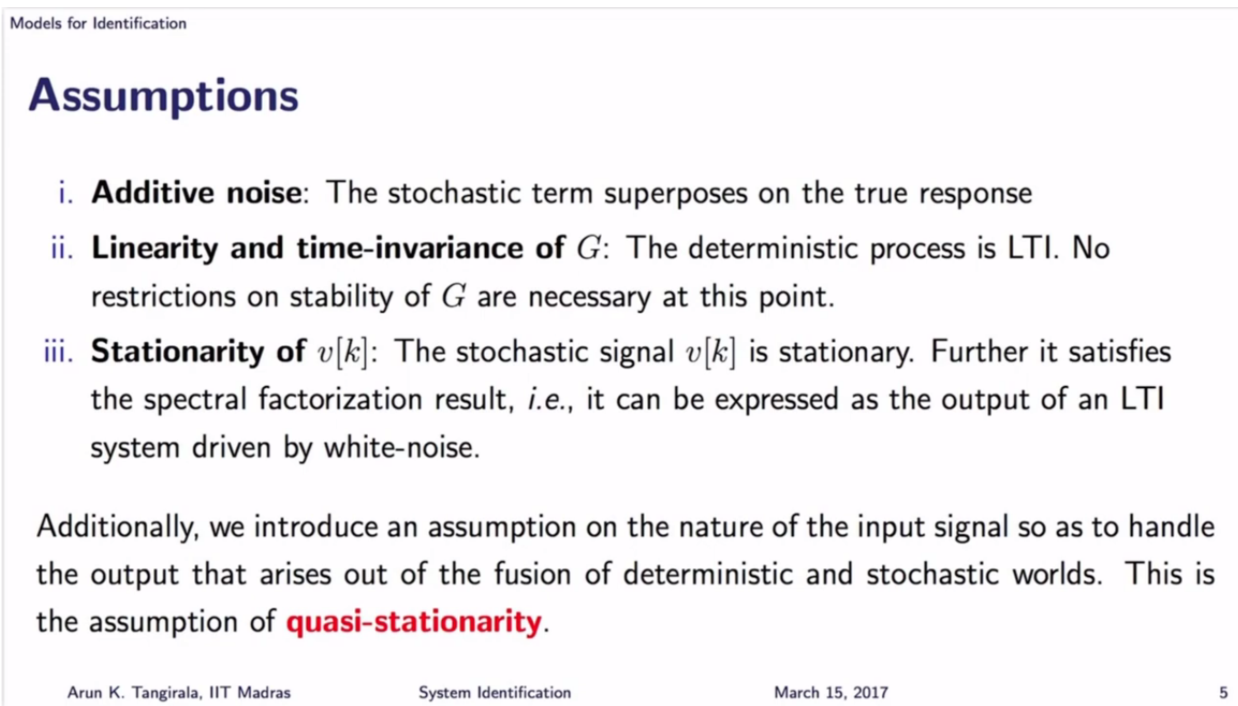

And when that is satisfied, I'll go past all of this. Then you can proceed further and we extend this notion of quasi-stationarity to joint quasi-stationarity. Why do I have to worry about joint stationarity. Because knowing when to analyze why you and you jointly, when I analyze y and u jointly I'm going to look at cross-covariance functions.

How do those cross-covariance functions behave? That has to be specified. And again the story is the same. The cross covariance function can change locally with time but at large times it should be only dependent on lag l.

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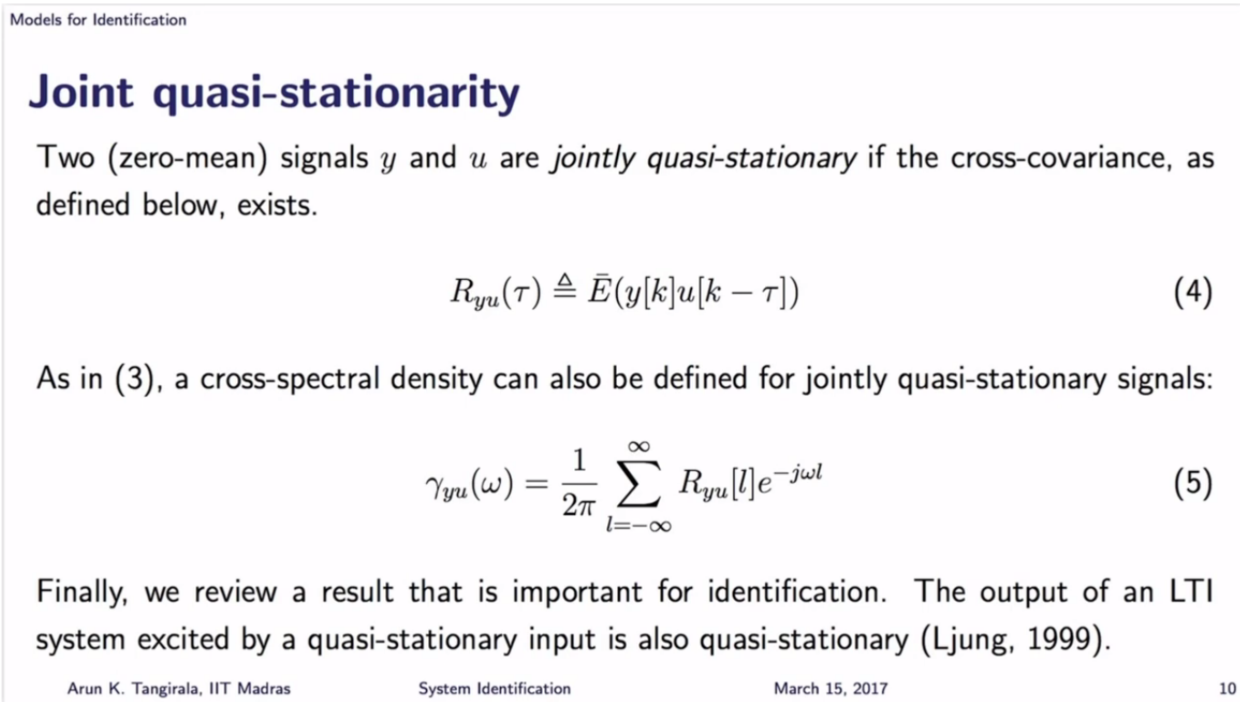

Okay. Even if you understand these assumptions in a qualitative sense you're fine. Don't worry too much about these things. Now there is an extremely important result here which says if I excite an LTI system with a quasi-stationary input u and v k is stationary, zero-mean white-noise. e k zero-mean white noise with variant sigma square e. Then you are guaranteed that the output is also a quasi-stationary. That is the key thing, right? Now I want the output to be quasi-stationary as well. In addition under open-loop conditions you can show that the spectral densities satisfy these relations that we derived yesterday.

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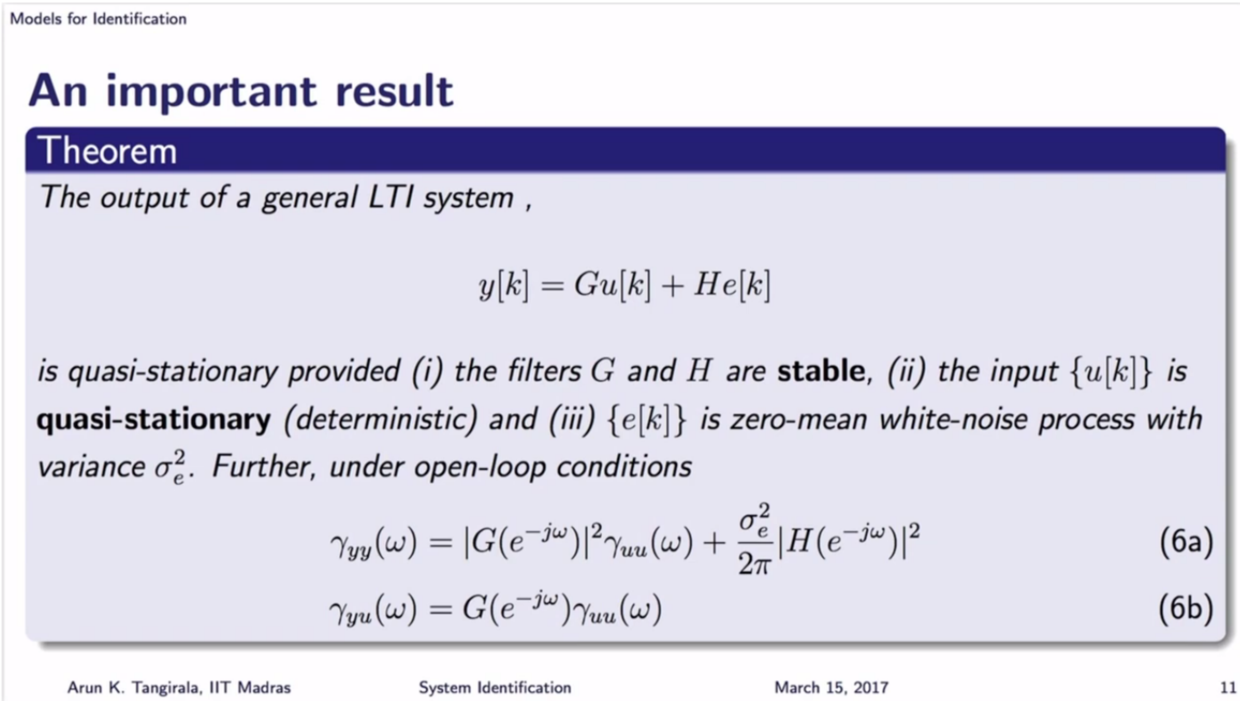

So what we are doing is, we are re-driving this results for the class of quasi-stationary segments. Yesterday's results we get for purely second order stationary signals. Because I have to be assured now when they input is quasi-stationary, output is also quasi stationary. That is important and that is guaranteed by this result. And in addition, the same results that we learned earlier also all hold. Now very quickly I just want at least windup the non-parametric part. Tomorrow we'll talk about the parametric part and the prediction.

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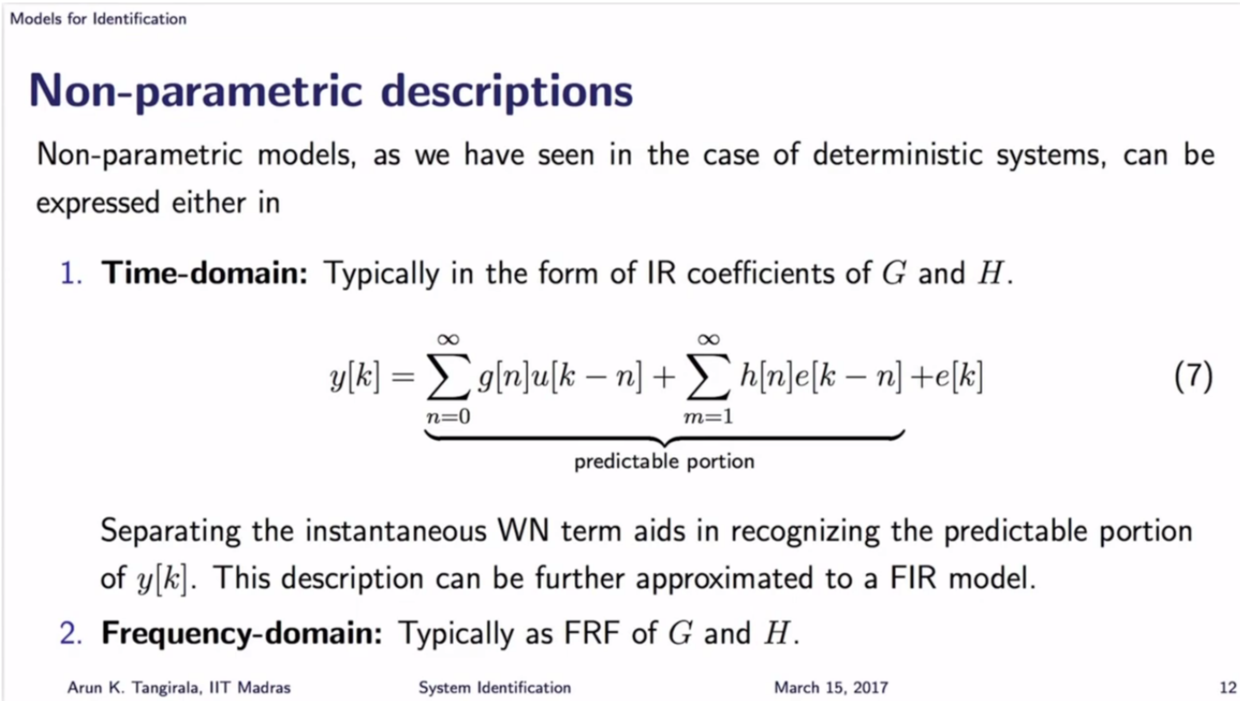

So that sets the assumptions in place and the framework in place. You should keep this at the back of your mind. If your identification, you should see, whether your given data actually meets these assumptions to a certain extent. Right? If I give you a ramp input and then ask you to identify a model using these tools, you should straightaway say it doesn't meet the requirements. Okay. So that is something to keep in mind.

Now we start looking at the models. We know that there are two classes of models for G and H non-parametric and parametric. Under the non-parametric we have learned both for G and H there exists two descriptions, time domain and frequency domain, two classes of descriptions. Under time domain you can think of impulse response description or step response description. Frequency domain, straight away in terms of FRF. Right? So, the first class of models that we generally work with are FRF models.

This this in their knowledge of what models have available comes very handy when you are given data and when you have to choose a model. FIR model you've seen earlier. Why does it assume, it assume that the G is table. By the way in this FIR model the focus is only on G. As you can see there is no specification of H. That is one of the demerits of this FIR model. v is typically assumed to be white-noise.

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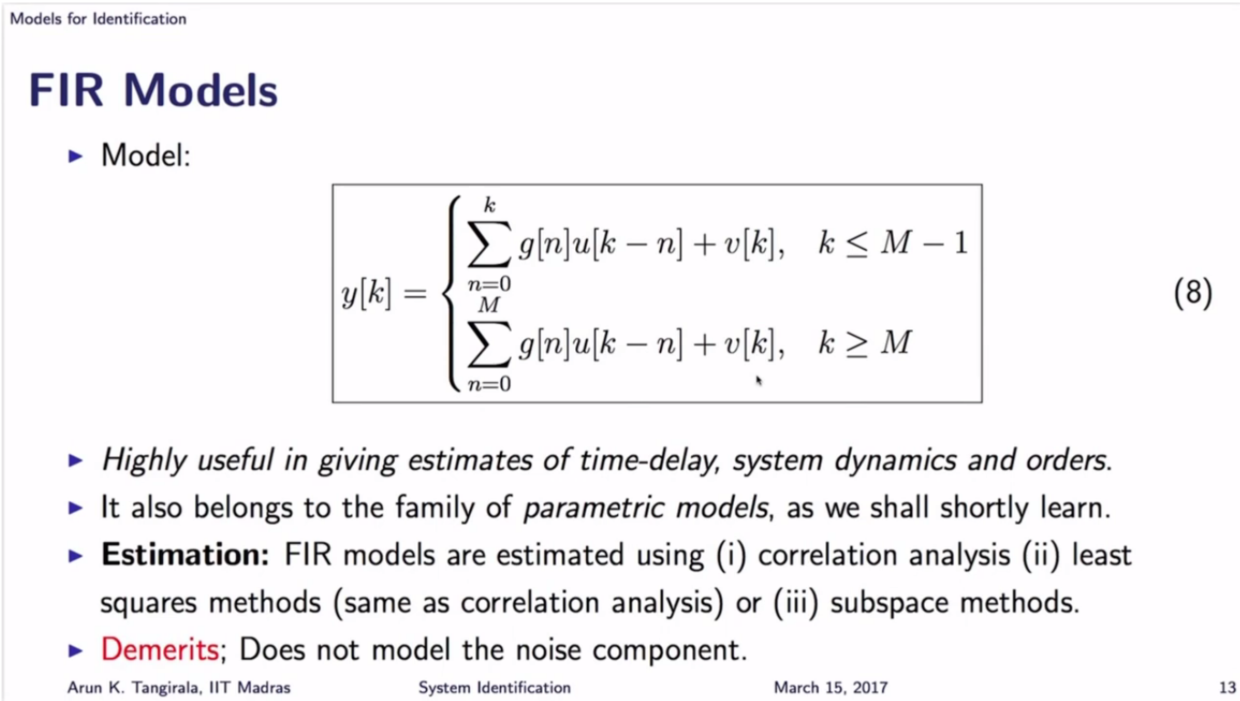

You don't worry about modeling v at all. In FIR models, if you turn to industry when they use FIR models their focus is only on G. And they use certain methods to estimate the G. And they don't really worry about estimating the noise part at all. And it's highly preferred model in the industry. It's one of the most simple models, because of its simplicity, number one. Secondly, as you see the inputs are known without error. So the regressors are known with error. What are the regressors here? The past inputs. The present and past inputs. And I do not need to specify the delay. Nothing, just M but that M is fairly easy to determine.

The fact that the regressors are known without errors makes a difference in estimation as we realize later on. So FIR models are simple and they are estimated typically using a correlation analysis method or a least squares method. On the other hand you have step response models which you obtain by using the relation between step response and impulse response. So all you have to do is go back to this FIR model. You remember we had a relation between step and impulsive response, right? The step response is the accumulation of impulse response. y s, if you call at the step response I have used s there in the slides, it is simply G of N with N running from 0 to k. It's simply the integration of the impulse response. So doing that will get you. I've used s as I said will get you this model. You can further rewrite this model for systems that have steady state.

Because this is a general model for y. I can, if I assume the system reaches a steady state. What happens at a steady state? The tepid response coefficients become constant. I don't have to estimate. Beyond the steady state I don't have to estimate step response coefficient so I can slightly modified this model and say if M is the number of samples or observations it takes to reach a steady state then within that period of time I have this model that means I have to estimate the response quotients. After it reaches M. Then I can take out the s M out. So there is only one coefficient to estimate. All right?

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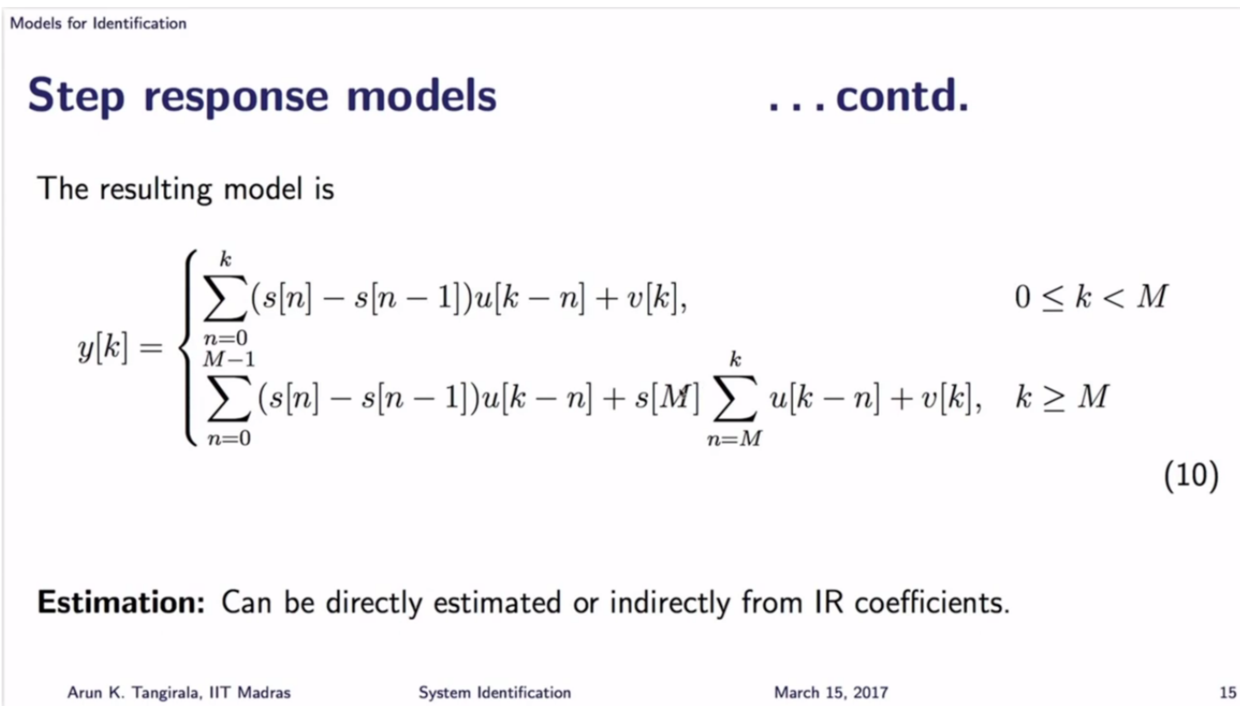

And estimation typically is done from impulse response coefficients. Actually you can estimate directly this type of response coefficients that used to be the case in [19:15 inaudible]. But now days, if you look at the [19:21 inaudible] tool box the step response is estimated from the impulse response coefficients. I just want to conclude with the frequency response descriptions.

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Frequency domain description

A frequency-domain representation corresponding to the deterministic form cannot be developed since the Fourier transforms of stochastic signals do not exist. However, a finite-sample form can be developed.

$$\boxed{Y_N(\omega_n) = G(e^{j\omega_n})U_N(\omega_n) + V_N(\omega_n) + R_N(\omega_n)} \tag{11}$$

where the Fourier transforms are the respective (unitary) DFTs of the finite-length signals, while $R_N(\omega)$ is a bounded error.

Remember when it comes to frequency response descriptions. There are two, there is a situation that I cannot take the Fourier transform of v. So I cannot write y of omega as G of e to the j omega or minus j omega times u of omega plus v of omega. This is not possible. Right? I cannot write this. Because v of omega doesn't exist. On the other hand if I have finite number of observations I can write it and that is what I've written on the screen for you. Right. And in addition that is the remainder. Where is that reminder term coming from? Because I am using DFT. I'm not using DTFT therefore for y and u.

What I've written on the board is the DTFT when I replace y and u even in the absence of v, we have already seen there is going to be reminded term. That is your r N. Now you v of v N of omega which is the DFT of a realization of the stochastic process also comes in. That makes the ETF estimation a bit more complicated. On the other hand we know from yesterday's discussion that I can theoretically write a relation or frequency domain description in terms of spectral densities. So I don't have to write in terms of Fourier transforms of output and input. As long as the input is quasi-stationary, I know output is quasi-stationary from the previous results.

And we have already learned from the theorem, that if input and output are jointly quasi-stationary these results hold. This is the way we describe the system in frequency domain not necessarily the way we have written on the board. Okay? You can use either this expression in 11 or this this expression in 12 and 13.

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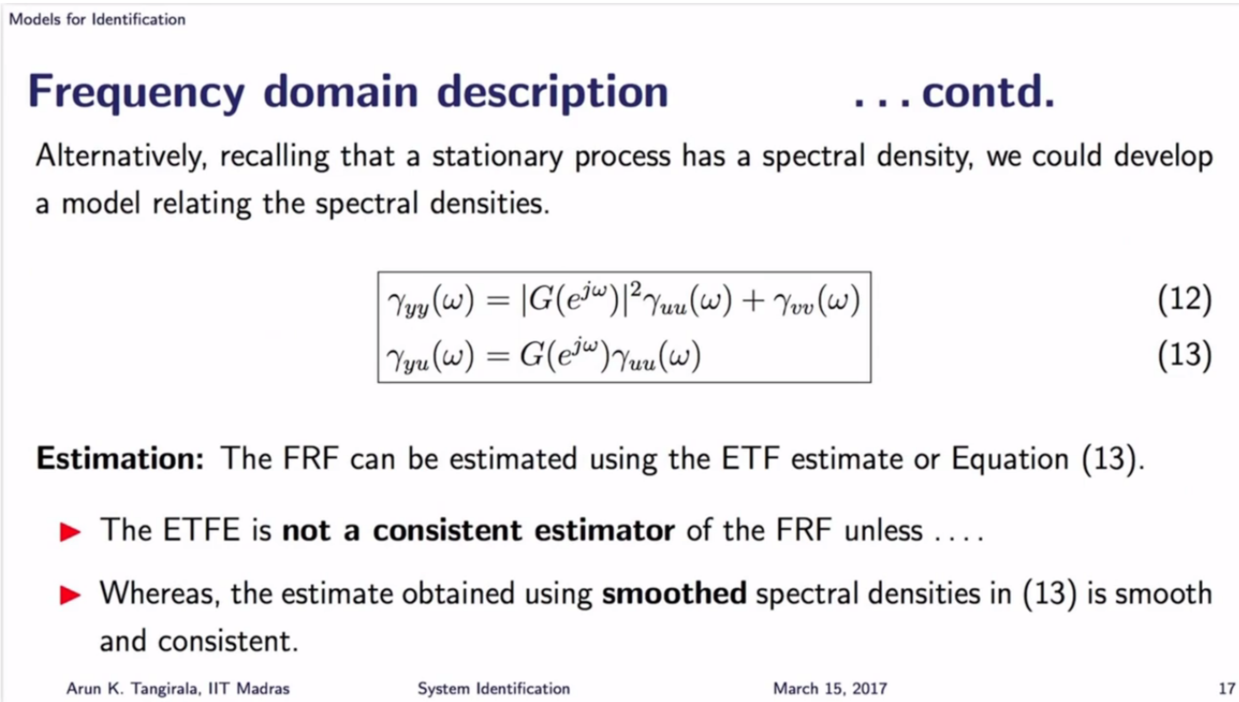

In the 13 in particular allows you to estimate FRF. Why is the v not contributing in 13/ Because we have assumed open-loop conditions and a cross spectral density between the input and the disturbance is zero. Why? Because a cross spectral density is a Fourier transform of cross covariance. Under open-loop conditions input and disturbance are uncorrelated. So the cross-correlation between v and u is zero. If cross-correlation is zero cross spectral density is also zero. Therefore the effects of noise vanish in the second relation. And that is this. Why the second relation in terms of the cross spectral density is used more frequently in estimating the FRF.

The MATLAB command SPA for example uses this idea. It uses this relation, whereas the ETFE we use as a question 11. You understand? The ETFE relies on Fourier transforms of signals and the 13 uses cross spectral density. There is a difference between the quality of estimates. We'll learn that later on. So tomorrow when we come back for the first 10-15 minutes we'll talk of parametric descriptions. And in that process we'll talk about predictions, how to construct predictors. So hopefully Friday we'll get started on estimation.