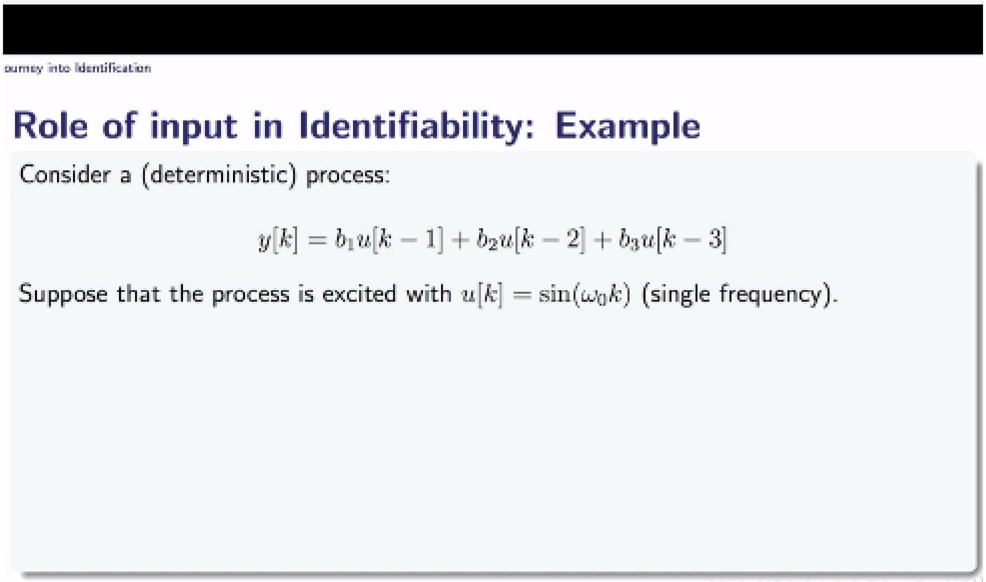
# CH5230: System identification

## Journey into Identification

## (Case Studies) 4

(Refer Slide Time: 0:15)



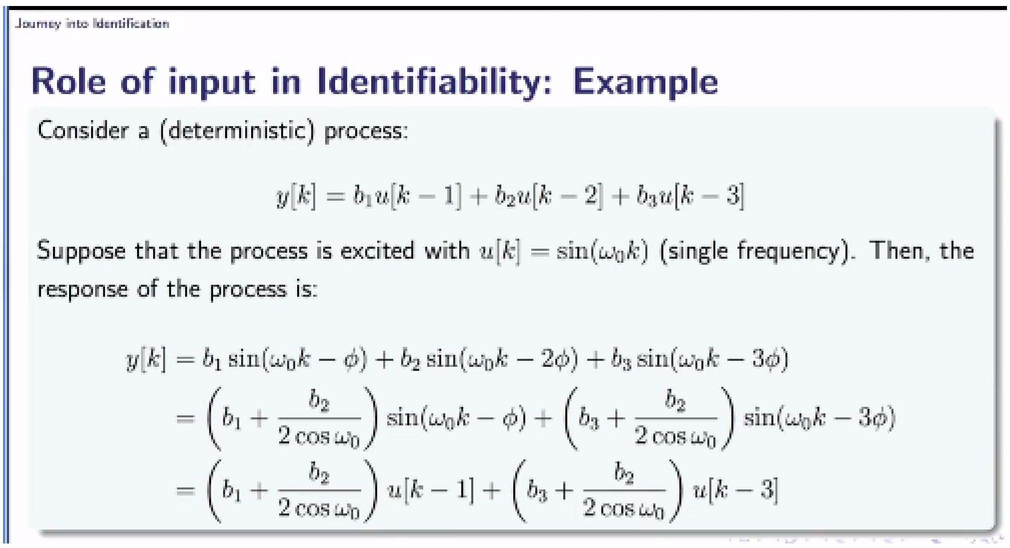

So now let's understand, now that we have got at least hopefully a flavour of what is model identifiability, I'm a bit scared to go beyond, but okay, I'll dare that. Now that we have had some feel for what is model identifiability, let's look at the second aspect, the input, role of the input in identifiability. Assume that now I have chosen this model for example, right? Now here, it's okay. In order to estimate beta, what do I need to do in my experiment? I just need to change and I just need one steady state and I can get my beta, right? That means I don't have to excite the prices too much. I just need one steady state point from the experiment and I can get beta. But suppose I had two parameters in my model. Suppose my model is of this form, right. Now if I think of the input, we'll go through the example on the screen shortly but I'm just giving you an idea of what is a concept underneath that example as well. So if I look at this model here. Obviously in order to estimate beta 1 and beta 2, I need two points, right? But if I have not done my experiment, if I've not changed the input at least twice then there is going to be a problem. I need two points on my data and to do that I have to carry out the experiment accordingly.

Let's look at this example on the screen now. Assume that you have a process which is evolving according to the equation that I have shown on this screen. Y k is a linear sum of three past inputs. There are processes that have this structure. This is not a, you know, just a figment of imagination. There are many processes that have this kind of a structure known as the FIR structure, finite impulse response model kind of structure. We'll talk about FIR models later on. So assume that a process is evolving according to this. For now, to keep things simple, we'll keep noise out of the picture. Purely deterministic process. Very nice, very ideal, linear. Do you think this is a linear model? Right? When I say linear, in your minds you have to ask, what function I'm I referring to? Right? Linearity can be about anything. So here I am asking, if y is linear in u. It is but it's linear in uk minus one that is the past input uk minus 1, uk minus 2, uk minus 3.

It is also, as you shall learn later on, time invariant. Okay. Now the goal is to estimate these parameters b1, b2, b3. For this purpose, I'm going to excite the process with a sinusoidal input. Now this is quite common. That is this choice of input where we are using the forcing function -- sinusoid as a forcing function is quite common in mechanical systems, vibration systems and so on, or so called modal systems where you want to identify particular modes of the system and you do this by, let us say, you know the frequencies but you want to know these parameters b1, b2, b3 and so on. So you excite the system at those frequencies. So this is a very commonly used input, a sinusoidal input. The only point here is that we are using a single frequency sine wave. In general, you may use a multi sine wave, that is a sine wave made up of multiple frequencies. But let us say, I want to use only a single frequency and I've done so. Then what happens? That is as far as the process, as far as the mathematics is concerned, if you look at the response of this process under this forcing function. What is a forcing function? It's a sine wave of a single frequency. It's so turns out that when you view that when you're -- This is very important in identification, your input offers a window, it opens a window between you and the process, right? Imagine an interview process. Suppose the candidate comes in and I'm the interviewer. I don't ask any question, the candidate doesn't ask any question. Half an hour has elapsed, the candidate walks out.

Nothing, we are two saints who have spoken through eyes. That is not the case, right? We are ordinary people, so we need to ask questions to know. So the moment I ask a question, it opens a window of understanding or it just opens up the window for my analysis of the candidate. Likewise, here, and importantly depending on the kind of questions I ask, I'm opening up -- the windows are opening up accordingly and those characteristics of the process are being reviewed. So here I am actually exciting the system with a single frequency sine wave. It so turns out that when this process is being viewed through this window, the process appears as a two parameter model. Actual process is a three parameter model, b1, b2, b3, but when I look at it from this viewpoint, that is what am I doing?

(Refer Slide Time: 6:00)




It's so turns out that the input is such that I could have used a two-parameter model and generated the same y. You can look at it this way. This is the second viewpoint. There is a third viewpoint that I'll offer shortly. All of them lead to the same issue which is that, what is the issue that I'll run into? Why have I rewritten this way? Any idea? What would be the purpose of rewriting this way? Is there some agenda behind this?

Reducing the parameters.

Can you state it the other way? Reducing the parameters is okay, but can you state it the other way?

No, the model is identifiable on its own. There's nothing wrong here. Okay? But, okay, I'll give you 50% of the marks, but I've not done this to show that the original process is nonidentifiable and so on. Yes, it is and it is not. Regardless of the data, you should have -- what it says is, if you take the look at the original process, it is identifiable, provided you have done your experiment carefully. So now you can put together both of this and say, there is something wrong. I have not done my experiment properly because now what has happened is out of the three, I can only estimate two uniquely. That is the main problem. So with this input, output data, unfortunately, the three-parameter model is not identifiable. Now that is not the fault of the model, the fault is with the experiment. I have not asked enough questions, just let me put it that way. One frequency you can show theoretically that a single frequency sine wave allows you to estimate two parameters. I don't know how many of you are actually familiar with linear systems theory, basic linear systems theory. Even if you were to look at continuous time systems, suppose I take a transfer function, simple first order transfer function, in continuous terms, so this s is a laplace variable. All of you must be familiar with this whether you liked it or not, the first time or the last time you saw it.

This is a fist order model with two parameters, gain k and time constantly tau. Now if I look at the Bode plot, right, where one plots the magnitude and phase. So we know very well that, I mean for this kind of is some of the genetic magnitude plot looks like this where the y axis has a magnitude and x axis has the frequency. This frequency is the frequency of the input. It tells me that as I excite the system with different frequencies, increasing frequencies, the response, magnitude of the response will go down, right? And the phase, phase also has a certain structure. How does the phase look like? So this is phase and this is magnitude plot and if this is zero and then you have 90, it saturates at minus 90. Sorry. Again here, x axis you have Omega. In fact, if you just were to write the theoretical expression for the magnitude and phase, you can show that the magnitude at any frequency which we denote by -- although I've drawn here a log scale, doesn't matter, it's k over square root of tau square, omega square plus 1. This is magnitude and the phase is minus tan inverse tau omega.

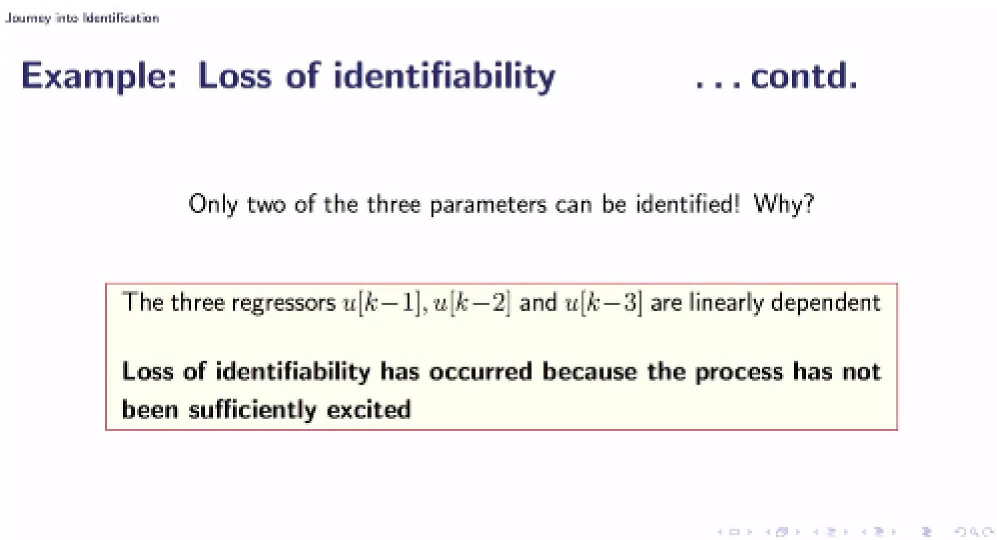
Right? Why are we going through this discussion? This is an alternative way of looking at what we have done although the system is different. If I were to excite this system with a sine wave or single frequency, I have essentially from the experiment I can figure out, what is this magnitude? It's actually the amplitude ratio, the ratio of the sinusoidal input, amplitude of the sinusoidal output that is being generated to the amplitude of the input that I've used. So that is let's say I used some frequency here. I have one point here. From the experiment I know what is amplitude ratio. And from the experiment I also know the phase. What we mean by phase is, by how much the output is lagging behind the input. So I used the sine wave, let the process come to steady state. Assuming the system is stable and then measure the amplitude of the resulting output.

The linear system theory says that if I use sinusoidal input, I will get a sinusoidal output, as you have even here. So I wait for the system to come -- for the system transcends to died on then the sinusoidal output appears at the output side. The waveform appears at the output side. Measures the amplitude, I know the input amplitude that I have used, the ratio of that is one point here on the magnitude plot.

The phase also I can calculate, that is another point. So I have two equations, right. I know these two points. What I do not know is k and tau. I know Omega that I've used in experiment. So one frequency will get me two parameters. That is a general rule and you can prove that also. This is a basic result in input design. A single frequency sine wave will allow you to estimate a two-parameter model, whether it is this continuous time system or the discrete time FIR system that you see. It's clear that a single frequency sine wave allows me to estimate two parameters.

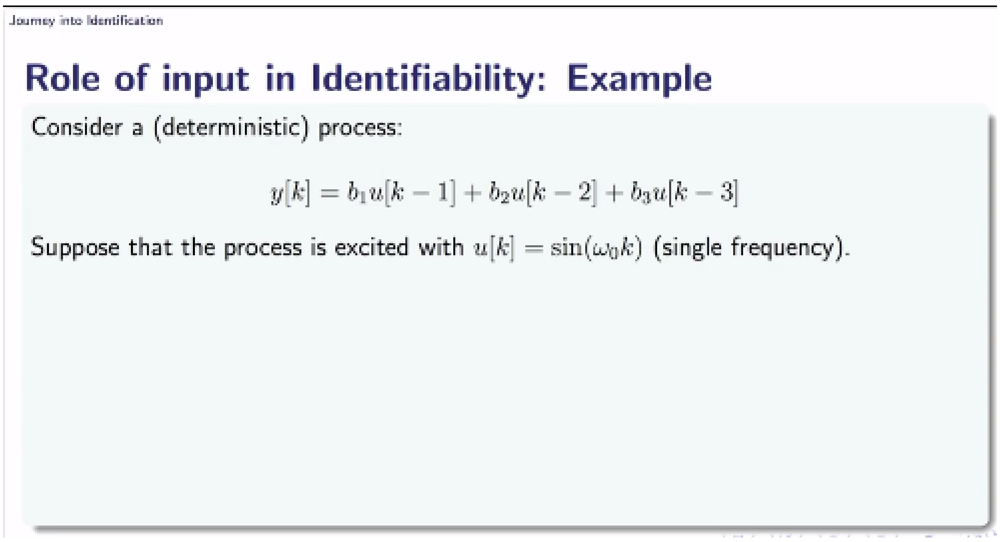
The other way of looking at it here is that we say that out of the three parameters, sorry, the three terms

(Refer Slide Time: 12:09)




uk minus 1, uk minus 2 and uk minus 3. Unfortunately, as we call them as regressors in the linear regression language. These regressors are linearly dependent. So you look at it from a linear algebra viewpoint. If you don't like any of this, go back and say, well, look at it from a linear algebra viewpoint.

(Refer Slide Time: 12:31)




Don't worry about linear systems and so on. What do I need to estimate b1, b2, b3? I need three equations, right. If I had uk minus 1, uk minus 2, uk minus three and a corresponding yk, for my linear algebra viewpoint I set up three equations, three unknowns, I should get them. Unfortunately with this input output data when you pick three different points on the time profile of the response, it turns out that when you set up those three equations, you can pick any three time points, doesn't matter. What do we mean by any three time points? On the input output profile, time profile. It turns out that when you use this input, unfortunately the matrix of regressors.

So let's say, you picked three time points k1, k2 and k3 and then you're going to set up these equations y at k1 is b1 times uk1 minus one plus b2, u at k1 minus 2 plus b3 u at k 1 minus 3. And you're going to write this for all the remaining two instants as well. Right? And then all you have to do is you say, well, in order to estimate the parameters, I'm going to write this in a matrix form, u at k1 minus 1, u at k1 minus 2, u at k1 minus 3 and likewise for k2, k3 same. Times what? What is the vector here? b1, b2, b3 equals y at k1, y at k2, and y at k3. So from my linear algebra viewpoint now this matrix unfortunately when you use a sinusoidal input is singular. That's another way of looking at it and we know when that when that matrix is singular, you do not have a unique solution. Therefore you have a loss of identifiability.

And you can show that on the other hand if you were to insert -- inject another frequency along with omega 1. Here I've used omega naught. But let's say, you used omega 1 additionally, so if you had used sine omega naught k plus sine omega one k, it doesn't matter, you can actually linearly mix them. That's not an issue but you need to have two frequencies and you can show that this matrix is going to be non-singular. Which means it will allow you to estimate all the three parameters uniquely. In fact you can go up to how many parameters based on what we discussed until now? If I were to use two frequencies, I can estimate a four-parameter model. That means if the process had a b4 uk minus 4 also I could estimate that uniquely but not b5 uk minus 5. This is the basis of input design, at least for linear time invariant systems and in general I do not know how many terms are going to be there on the right hand side, right? If you assume that there are infinite number of terms then I'll need infinite number of frequencies. That means I need all the frequencies. It's a continuum. And that is what we call as a persistently exciting signal. That is the basis behind the concept of persistent excitation. Okay? Okay, fine.

So the loss of identifiability has occurred because the process has not been sufficiently excited. Now you had to ask the question, how do I know upfront? When a candidate walks into the interview room, how does the interviewer know that sufficient questions have been asked, right? There the number of questions is not just determined by the candidate but also by the job or the position for which the candidate is appearing. So I'm also looking at the end use. As far as end use is concerned, I mean I have asked enough questions. I don't have to ask, you know, what colour shirt he wore when he went to LKG? I don't know, maybe that question is irrelevant. So I don't have to ask all kinds of questions. The end user is telling me, if you ask these many questions it is sufficient and that is what is also the principle in identification. I don't want the full fledged model of the process. I may not worry, if I know that this model is going to be used in control. I just need a good approximate model and whatever information is sufficient is necessary for control. So when you look at control, there are some vital statistics of the process that you want to definitely gather from your experiment. Gain, time constant, time delay. If it's an underdamped system, you want to know the damping factor. These vital statistics are sufficient. So I just need to perform my experiment. I just need to ask this many questions that will get me good estimates of these vital pieces of information.

I don't have to worry about a very vigorous model that has the full information about the process and the feedback is going to take care of it. So the end user is going to really tell me what kind of experiment I have to perform but there is this theory that tells us that look, whatever be the induced for a given induce, you have to be careful, right? So if I say that I'm going to fit, let us say for control, although we're not going to talk about control in this course. But suppose I use a model in control, typically as I said, a lower order approximation is sought and I decided to fit a second order plus time delay model. How many parameters would be involved in a second order plus time delay model? Think about it. Second order plus time delay. Four, right? Gain, two time constraints, and delay. So this theory tells me, boss, you need to have at least two frequencies in your input. You can't get away with a single frequency. So that is something to keep in mind which we will visit this concept later on.

And then of course, you have the estimator. So we have discussed model identifiability. We have gone through a couple of examples that throw light on the role of input in obtaining unique models or in identification and the third we said is the role of the estimator. So I have ensured that my model structure is unique, that is it's identifiable, I've conducted the experiment nicely but now comes the role of the algorithm which will get me unique estimates. All right. Now here all was nice, there is no noise but the reality is there is going to be noise. Okay. And when there is noise, obviously, setting up just three -- picking three points in time and saying I'll estimate b1, b2, b3 will not do the job. So it's how many points should I pick? As many points as I have in my experiment. We'll show later on that more the observations, merrier is the situation. This should not be confused with fast sampling rates. The sample size is different from fast sampling rate. So more the number of observations, better the estimated it is. But that is not guaranteed all the time. It all depends on how you are going to estimate.

So there are estimators that do not necessarily get you better estimates as you pump in more and more observations. Any estimator that gets you closer to the truth. In fact, that gets you the truth as you supply theoretically infinite number of observations is said to be consistent. This consistency is a very important statistical property that -- or asymptotic property. It's a mix of both. That an estimator is supposed to possess. So when you're choosing an estimator, an estimation algorithm, you should be aware that this estimator is going to be consistent, which means, as you feed more and more data points, it should get you better and better estimator. It should take you closer to the truth. Whether it monotonically reaches and so on, that's a different thing. But ultimately as engineers we say, if I give large number of observations then it should be consistent. I should get closer to the truth.

Now having said this that is a fact, in the sense that this statement actually relies on the fact that finite number of data points will never ever get you accurate and precise estimates of the parameters. So even in the example that we just discussed, b1, b2, b3 were the parameters. Suppose I had noise in my data, which is going to be the case. I collect thousand observations, will I get precise estimates of b1, b2, and b3? No. Will I get accurate? What we mean by precise is, if I repeat the experiment, same number of points, will I get no variability at all? Unfortunately, no. When I get accurate estimates, that means for a given record and will I just get the truth? No, I won't. So the fact of estimation is from finite number of observations, we will never be able to obtain accurate and precise estimates of the parameters. There's always going to be some error and some variability, which hopefully will be small. We want the error in the estimates to be small.

In other words we are going to have estimates that are in error whenever we use finite observation, that's going to be the practical case. Now having realized, admitted that fact, the goal in any estimation exercise therefore is to reduce the errors as much as possible. When we do that, what we are seeking is known as an efficient estimator. So there are two properties, one is consistency, other is efficiency. Efficient has got to do with the ability of the estimator to procure low error estimates. Consistency has got to do with how it -- whether it gets to the truth as you supply -- when you supply infinite number of observations. Estimation theory tells us under what conditions which estimator is efficient and consistent and so on. So it has to wait. When we get into the estimation theory, we'll visit all of this.